Induced decoherence and entanglement by interacting quantum spin baths

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The reduced dynamics of a single qubit or two qubits coupled to an interacting quantum spin bath modeled by an *XXZ* spin chain is investigated. By using the method of a time-dependent density matrix renormalization group (*t*-DMRG), we go beyond the uniform coupling central spin model and nonperturbatively evaluate the induced decoherence and entanglement. It is shown that both the decoherence and the entanglement strongly depend on the phase of the underlying spin bath. We show that in general, spin baths can induce entanglement for an initially disentangled pair of qubits. Furthermore, when the spin bath is in the ferromagnetic phase because the qubits directly couple to the order parameter, the reduced dynamics shows an oscillatory type behavior. On the other hand, only for the paramagnetic and the antiferromagnetic phases do the initially entangled states suffer from an entanglement sudden death. By calculating the concurrence, the finite disentanglement time is mapped out for all of the phases in the phase diagram of the spin bath.

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I. INTRODUCTION

Spin qubits are promising candidates for quantum information processing because of their long decoherence and relaxation time.[1,2] Some schemes, such as solid state spin qubits, further enjoy a potential scalability via integration with nanotechnology.[3] However, spin qubits are not totally immune from the ubiquitous decoherence. To describe the bath that causes the decoherence of spin qubits, it is known that in some cases, the bath is better modeled by spins instead of delocalized oscillators, resulting in the so-called spin baths.[4] It has been argued that the influence of spin baths may be qualitatively different from that of bosonic baths and that the non-Markovian dynamics can easily emerge.[5–7] Due to the growing interest in spin baths, the decoherence behavior and the entanglement dynamics of a few qubits coupled to spin baths have been extensively studied in recent years. Early works focus on the decoherence that is due to independent spins.[6] Here, although the proposed model formally resembles a spin boson model, the non-Markovian dynamics already emerges even when the bath modes are not interacting.[7] In real baths, however, spins are not independent. It is therefore important to include the effects due to the interactions of spins in the bath. Nonetheless, the inclusion of the intraspin interaction in the bath complicates the problem and only for some limited models with high symmetry can the exact reduced dynamics be identified.[8] Beyond models with exact solutions, the approximated dynamics was obtained by using mean-field[9] or perturbative approaches[10] to handle more generic models. For instance, within the context of electron spin decoherence by interacting nuclear spins in a quantum dot, the pair-correlation method,[11,12] as well as the linked-cluster expansion method,[13] have been developed to study the single spin free-induction decay and ensemble spin-echo behaviors in the strong magnetic field limit. The most common model employed in these works is the "central spin model," wherein the qubits are uniformly coupled to all of the spins of the bath. While analytical derivations are possible in these models, they are less realistic and are more difficult to be experimentally implemented. A nonperturbative approach that can capture the non-Markovian effects induced by an interacting spin bath with a generic coupling to qubits is, hence, highly desirable.

To overcome the difficulty associated with interacting spins, we utilize the method of time-dependent density matrix renormalization group (*t*-DMRG) [14,15] to investigate the reduced dynamics of a single qubit or two qubits coupled to an interacting spin chain. Recently, *t*-DMRG has been used to study the single qubit pure dephasing induced by an *XXZ* anisotropic spin chain.[16] The advantage of *t*-DMRG is its ability to calculate the reduced dynamics even when the spin bath is not integrable and the coupling is not uniform. Due to the accumulation of errors, *t*-DMRG will eventually run away at a large time[17] but this does not impose any serious limitation since for the study of quantum information, we are mostly interested in some smaller time scale. In this work, we apply the method of *t*-DMRG to investigate both the pure dephasing and the general decoherence model of qubits coupled to spin baths. Single qubit decoherence, as well as two qubit (dis)entanglement dynamics, is investigated. It is shown that both the decoherence and the entanglement strongly depend on the phase of the spin bath. In general, we find that spin baths can induce entanglement for an initially disentangled pair of qubits. However, when the spin bath is in the ferromagnetic phase because the qubits directly couple to the order parameter, the reduced dynamics shows an oscillatory type behavior. On the other hand, only for the paramagnetic and the antiferromagnetic phases do initially entangled states suffer from the entanglement sudden death.[18,19]

To quantify the single spin decoherence, we evaluate the evolution of the Loschmidt echo.[20] We analyze the relation between the short time Loschmidt echo decay parameter and the quantum phases of the spin bath, as it has been pointed out that these two are closely related, especially when a symmetry breaking occurs in the bath.[9] We use the temporal evolution of concurrence[21] to study the entanglement dynamics. One important issue of the entanglement dynamics is the possibility of creating entanglement through a common bath for originally disentangled qubits. It has been shown that an induced entanglement via a common bath is possible for bosonic and fermionic baths.[22,23] For spin baths, such a possibility has been explored for a noninteracting spin bath,[24] as well as for interacting ones,[8,25–27] but is restricted to uniform coupling models. It will be shown later in this paper that induced entanglement is possible for the local coupling model considered in this work. We note that the induced entanglement is also closely related to recent proposals of quantum communication and teleportation via a spin chain.[28,29]

Another important issue is the disentanglement dynamics of an initially entangled state. It has attracted much attention in recent years since Yu and Eberly[18] and Jakóbczyk and Jamróz[19] predicted that two initially entangled states without an interaction can become completely disentangled at a finite time. This feature has been termed as entanglement sudden death (ESD). ESD has been theoretically studied within various models[30–32] and has been experimentally demonstrated.[33] These models, however, are restricted to the Markovian bosonic bath or classical noise. In this work, we explore if ESD-like phenomena can occur for spin baths in the non-Markovian regime. In particular, we will show that only when the spin baths are paramagnetic or antiferromagnetic does the phenomenon of ESD occur, while when the spin bath is in the ferromagnetic phase, the concurrence shows an oscillatory behavior. As understanding the nature of the decoherence and the (dis)entanglement dynamics constitutes an important step for quantum engineering these systems, our results are of practical usage for future quantum information processing.

This paper is organized as follows: In Sec. II, we present our model Hamiltonian and briefly discuss how to apply t-DMRG to analyze the model Hamiltonian. In Sec. III, we present our results of single qubit decoherence, while in Sec. IV, the results of the (dis)entanglement dynamics are presented. In Sec. V, we summarize and discuss the implication of our results.

II. THEORETICAL FORMULATION

We consider a system-bath model that is described by the total Hamiltonian $H=H_{\rm sys}+H_{\rm bath}+H_{\rm int}$, where $H_{\rm sys}$ is the Hamiltonian of the single or two qubit system, $H_{\rm bath}$ is the Hamiltonian of the spin bath, and $H_{\rm int}$ represents the interaction between the qubits and the bath. We shall set $H_{\rm sys}=0$, but our method can be applied to a generic $H_{\rm sys}$. We shall assume that the spin bath is a spin chain characterized by the XXZ Heisenberg model,

$$H_{\rm bath} = J \sum (S_i^x S_{i+1}^x + S_i^y S_{i+1}^y + \Delta S_i^z S_{i+1}^z), \tag{1}$$

where $J>0$. It is known that the XXZ Heisenberg model has a very rich structure.[34] The system is ferromagnetic for $\Delta < 1$, antiferromagnetic (Ising type) for $\Delta > 1$, and critical (XY type) for $-1<\Delta<1$. It also encompasses the XY model, where $\Delta=0$. The most general linear coupling between a qubit $A(B)$ and the bath can be expressed as $H_{\rm int} = \sum_{i,\alpha} \epsilon_i^\alpha s_{A(B)}^\alpha S_i^\alpha$, where $\alpha=x,y,z$ and $i=1,\ldots,N$. Here, ϵ_i characterizes the coupling of the spin qubit to the ith spin in the spin chain. For most situations, the more interesting cases are $\epsilon<0$ (Refs. 9 and 10) and, hence, we shall concentrate on negative ϵ. Our numerical method, however, can be applied equally well to cases with positive ϵ. In our work, both the Ising and the isotropic Heisenberg couplings will be considered. The Ising coupling ($\epsilon_i^x=\epsilon_i^y=0$) gives rise to a pure dephasing model, while the isotropic Heisenberg coupling ($\epsilon_i^x=\epsilon_i^y=\epsilon_i^z\neq 0$) induces both dephasing and energy relaxation. The range of the coupling is crucial for characterizing the interaction of the qubits to the spin bath. For a uniform coupling, ϵ_i is independent of i. This is unrealistic but for a uniform coupling, the Loschmidt echo and the entanglement dynamics can be exactly calculated by using the Jordan-Wigner transformation when spin bath is of the type of the XY model ($\Delta=0$).[25,35] However, the more realistic coupling model is the local coupling model, in which only the coupling to the closest spin is nonvanishing. Nonetheless, there are no analytic solutions known for this model. The reduced dynamics is less studied but is more relevant to real experiments. In this case, if the spin bath is ferromagnetic, the qubit directly couples to the order parameter (the magnetization); while if the spin bath is antiferromagnetic or paramagnetic, the qubit does not couple to the order parameter. Hence, the reduced dynamics exhibits completely different behaviors in different phases. As the local coupling model is more relevant to real experiments, in the following, we shall concentrate on the local coupling model.

We now briefly outline the procedure to evaluate the reduced dynamics of the qubits and other derived quantities. For a given set of parameters, we first employ the static DMRG[36] to find the ground state $|G\rangle$ of the spin chain, wherein the open boundary condition is used. We assume that at $t=0$, the initial total state is a product state of the form $|\Phi(0)\rangle=|\psi_{\rm sys}(0)\rangle|G\rangle$, where $|\psi_{\rm sys}(0)\rangle$ is some particular system state that we are interested in. Formally, the evolution of the reduced density matrix can be obtained by first evolving the total state,

$$|\Phi(t)\rangle = e^{-iHt}|\psi_{\rm sys}(0)\rangle|G\rangle, \tag{2}$$

then tracing off the spin bath,

$$\rho_{\rm sys}(t) = {\rm Tr}_{\rm bath}|\Phi(t)\rangle\langle\Phi(t)|. \tag{3}$$

The Loschmidt echo and concurrence can then be evaluated from $\rho_{\rm sys}(t)$. In general, evolving such a state is a formidable task. t-DMRG, however, provides a way to efficiently evolve such a state with a high accuracy for a quasi-one-dimensional system. We note that the degrees of freedom of the qubits are exactly kept during the t-DMRG calculation by targeting an appropriate state. The dimension of the truncated Hilbert space is set to be $D=100$. For the short time decay simulation, we set $J\delta t=10^{-3}$ in the Trotter slicing; while for the entanglement dynamics, we set $J\delta t=0.1-0.5$ to balance the Trotter error and truncation error.

III. SINGLE QUBIT DECOHERENCE

In this section, we present our results for the single qubit decoherence, which is characterized by the Loschmidt echo.

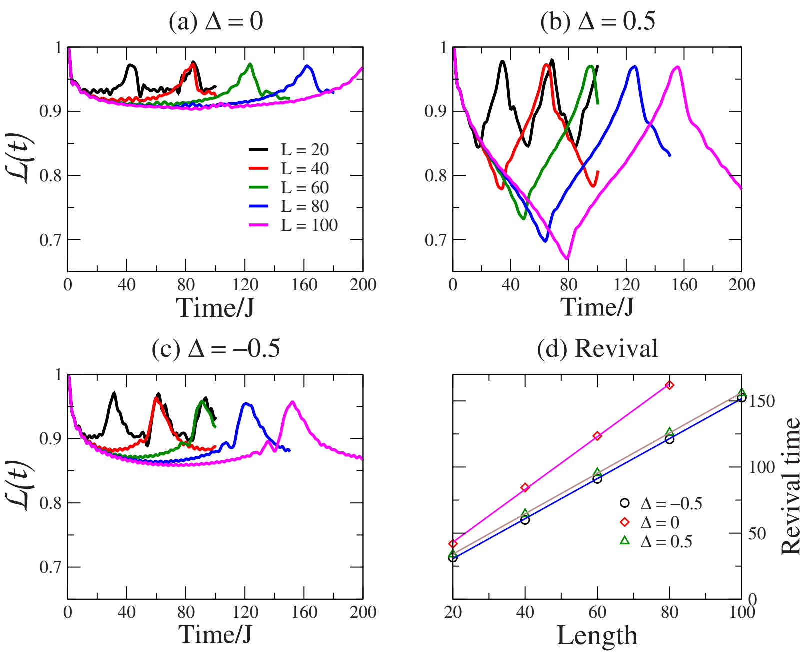

FIG. 1. (Color online) [(a)–(c)] $\mathcal{L}(t)$ as a function of time for different lengths and Δ. (d) Revival time as a function of length. The coupling is of the Ising type with $\epsilon=-0.3$.

The Loschmidt echo has been extensively used to quantify single qubit decoherence, especially its connection to the quantum criticality of the spin baths.[16,37,38] The Loschmidt echo can be intuitively understood as follows: Consider an initially disentangled total state $(C_+|+\rangle + C_-|-\rangle)\otimes|G\rangle$. At some later time t, it will evolve into an entangled state $C_+(t)|+\rangle\otimes|\Psi_+(t)\rangle + C_-(t)|-\rangle\otimes|\Psi_-(t)\rangle$. The Loschmidt echo, which is defined as $\mathcal{L}(t)\equiv|\langle\Psi_+(t)|\Psi_-(t)\rangle|^2$, clearly measures the coherence between $|+\rangle$ and $|-\rangle$. When $\mathcal{L}=1$, the qubit is disentangled from the bath; while when $\mathcal{L}=0$, the qubit is totally entangled with the bath.

We start by noting that for a numerical calculation on finite length, all dynamics will show quasiperiodic behavior. The quasiperiod is known as the revival time. Since in our numerical calculation the spin bath is a chain of finite length, it is essential to identify the revival time for each length to avoid unphysical results due to the revival. As a zeroth order approximation, the revival time is proportional to the length and inversely proportional to the maximum phase velocity of the spin chain. In Figs. 1 and 2, we plot the Loschmidt echo as a function of time for the case of the Ising and the Heisenberg couplings by using various lengths and Δ, from which the revival time of the echo can be easily identified. We also plot the revival time as a function of the length. One clearly observes the linear dependence of the revival time on the length. We find that for the Ising coupling, the minimal value the Loschmidt echo reaches is nonzero unless a very strong coupling strength is taken (not shown here), while for the Heisenberg coupling, the Loschmidt echo reaches zero if the length is longer than some Δ dependent critical length.

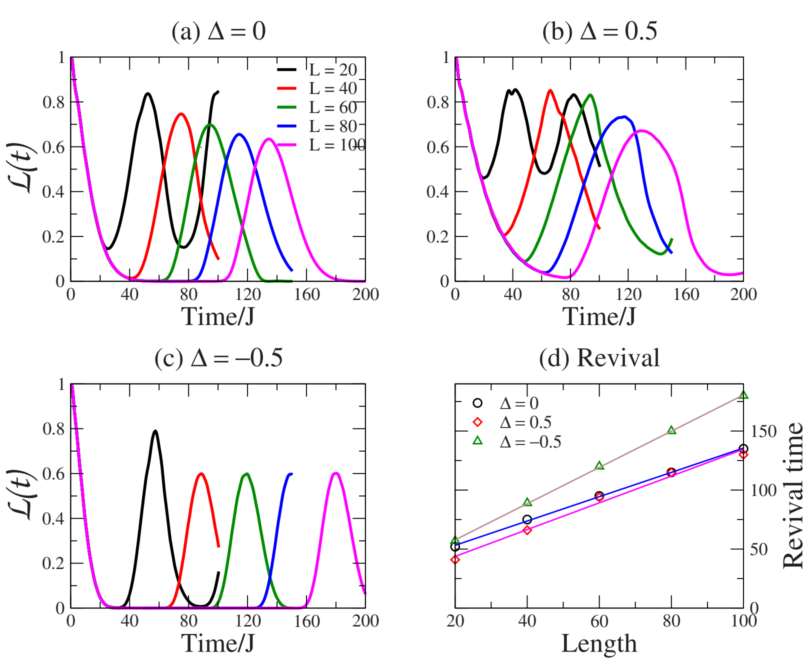

FIG. 2. (Color online) [(a)–(c)] $\mathcal{L}(t)$ as a function of time for different lengths and Δ. (d) Revival time as a function of length. The coupling is of the Heisenberg type with $\epsilon=-0.3$.

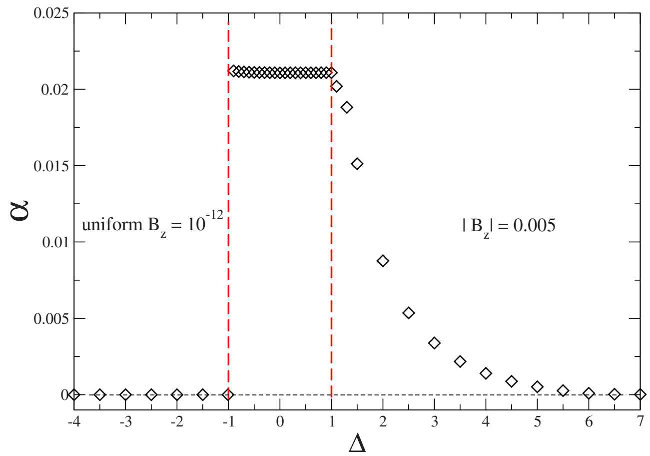

FIG. 3. (Color online) Decay parameter as a function of Δ for the case of the Ising coupling. Here, $\epsilon=-0.3$, and length $N=80$.

To compute the single qubit decoherence, we couple the qubit to a single site of the spin chain, which is taken to be the middle site of the chain to suppress boundary effects. We tune the spin bath to different quantum phases by changing the parameter Δ. In the ferromagnetic regime, a small uniform external field is introduced during the static-DMRG calculation but is turned off during the time evolution. It is numerically checked that the numerical results reported below are insensitive to the magnitude of the applied external field. Similarly, when the bath is in the Ising antiferromagnetic ground state of the *XXZ* model, a small staggered external field is applied to lift the twofold degeneracy in the ground state.[39]

When the spin is in the Ising antiferromagnetic regime or the *XY* critical regime, we find that in a short time, the behavior of the Loschmidt echo decay is Gaussian, $\mathcal{L}(t) \sim e^{-\alpha t^2}$, where α is the decay parameter.[40] In Fig. 3, we plot the decay parameter as a function of Δ for the case of the Ising coupling. In the ferromagnetic regime ($\Delta<-1$), the qubit decay is completely suppressed ($\alpha=0$). This is a consequence of the Ising coupling in which both $|+\rangle\otimes|G\rangle$ and $|-\rangle\otimes|G\rangle$ are eigenstates to the system and, hence, $\mathcal{L}(t)=1$.

Clearly, the decay parameter is largest in the critical regime ($-1<\Delta<1$) and it gradually decreases to zero as one moves into the antiferromagnetic regime ($\Delta>1$). For the single link scenario, the decay parameter is almost featureless within the critical regime. Our numerical results also show that if the qubit is coupled to multiple sites, the decay parameter acquires a weak dependence on Δ and the transition near $\Delta=1$ becomes less sharp (not plotted). Note that the decay parameter becomes sensitive to the magnitude of the small staggered field applied when the spin bath is close to the phase boundary ($\Delta\sim 1$). This is due to the fact that for finite N, the barrier between two degenerate ground states is finite and approaches zero as Δ approaches 1. The ground state obtained by static DMRG includes a small mixture of the degenerate state, which is sensitive to the strength of the staggered field. For larger Δ, the barrier between two degenerate ground states increases as one moves deeper into the antiferromagnetic regime. As a result, the decay parameter becomes less sensitive to the strength of the stagger field for $\Delta \gg 1$.

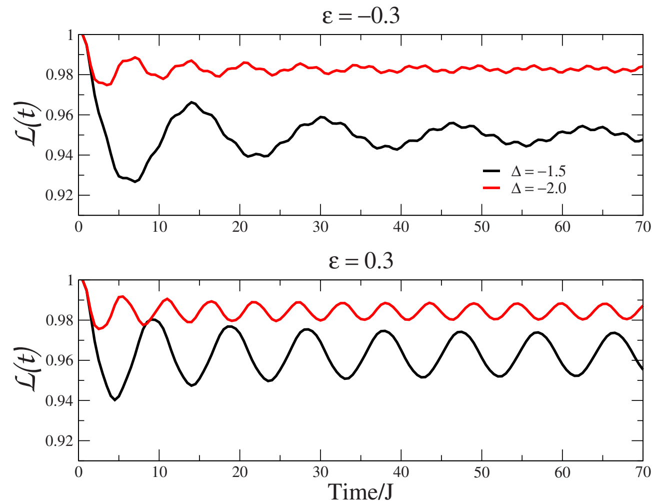

FIG. 4. (Color online) $\mathcal{L}(t)$ as a function of time when the spin bath is in the ferromagnetic phase for the case of the Heisenberg coupling. Here, the length $N=80$ and the coupling strength $\epsilon=-0.3$ (upper) or $\epsilon=0.3$ (lower). Clear oscillatory behaviors are seen for both positive and negative ϵ.

We now turn to the case of the Heisenberg coupling. We first note that in the ferromagnetic regime, the qubit couples to the order parameter. Therefore, the qubit is effectively in an average magnetic field $\langle \vec{S}_i \rangle$. As a consequence of the Heisenberg coupling, the qubit will precess about $\langle \vec{S}_i \rangle$. Since magnons are generated at the same time when the qubit evolves, $\langle \vec{S}_i \rangle$ starts to deviate from 1/2 and results in oscillations in the reduced dynamics. Figure 4 shows some typical oscillating behaviors of $\mathcal{L}(t)$ in this scenario. Clearly, the reduced dynamics is no longer Gaussian. Therefore, we shall not mark the ferromagnetic regime in the following.

In Fig. 5, we plot the decay parameter as a function of Δ for the case of the Heisenberg coupling. The overall behavior is very similar to the case of the Ising coupling except that the decay parameter weakly depends on Δ in the critical regime. This is different from the Ising coupling case shown above but is similar to the multiple sites in the Ising coupling

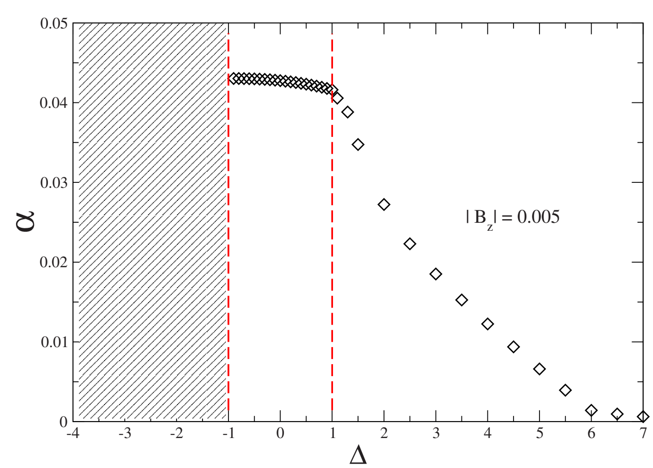

FIG. 5. (Color online) Decay parameter as a function of Δ for the case of the Heisenberg coupling; $\epsilon=-0.3$, and length $N=80$.

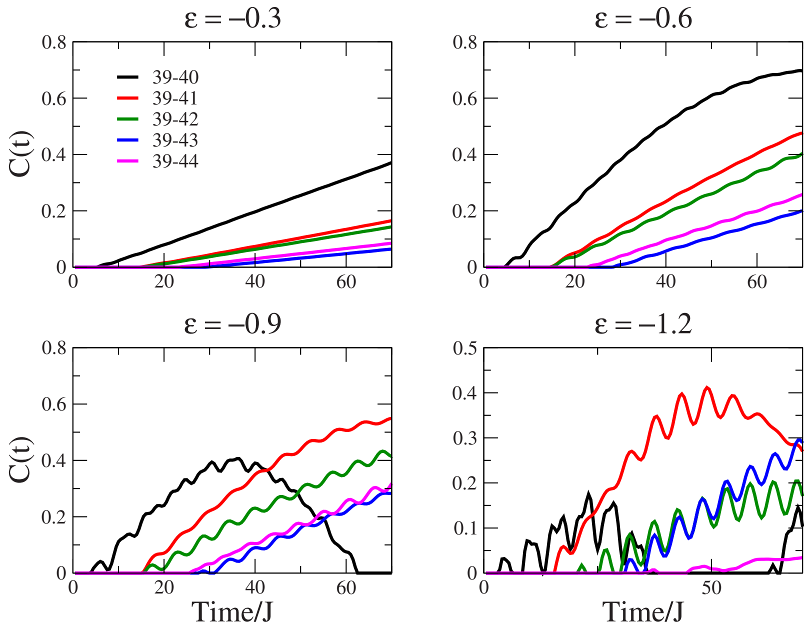

FIG. 6. (Color online) Entanglement dynamics for an initially disentangled pair of qubits for the case of the Ising coupling. Here, $\Delta=0$, $N=80$, and number pairs (such as 39-44) indicate positions of two qubits.

case. For both the Ising and the Heisenberg couplings, we find a discontinuity in the behavior of $\mathcal{L}(t)$ at $\Delta=-1$ and a first derivative discontinuity at $\Delta=1$. These discontinuities coincide with the phase boundary of the underlying spin chain. The different behavior at $\Delta=\pm 1$ can be traced back to the different natures of the ferromagnetic and antiferromagnetic transitions, and the close relation between the decoherence and the quantum criticality of the bath is clearly demonstrated.

IV. ENTANGLEMENT DYNAMICS

In this section, we investigate the entanglement dynamics of two qubits that couple to the spin bath. There are two central issues to be addressed. The first issue is the possibility of entanglement creation via the common spin bath for a pair of initially disentangled qubits without a direct interaction. The second issue is the disentanglement dynamics of an initially entangled state. In particular, we would like to address the issue if qubits influenced by spin baths also suffer from entanglement sudden death and if the entanglement sudden death depends on the quantum phase that the qubits couple to.[18] To characterize the entanglement, we shall use concurrence as a measurement of the entanglement.[21] For a given reduced density matrix $\rho(t)$, the concurrence is defined as $C=\max\{\lambda_1-\lambda_2-\lambda_3-\lambda_4,0\}$, where $\lambda_1\geq\lambda_2\geq\lambda_3\geq\lambda_4$ are the square roots of the eigenvalues of the operator $\rho(\sigma^y\otimes\sigma^y)\rho^*(\sigma^y\otimes\sigma^y)$ and ρ^* is the complex conjugation of ρ.

A. Entanglement creation

It has been shown that entanglement can be created without a direct interaction if two qubits interact with a common bosonic bath[22] or a fermionic bath.[23] Of particular interest to us is the onset time of the entanglement, the strength of the induced entanglement, and the time scale wherein the induced decoherence eventually takes over. These considerations are important in determining if such an induced entanglement is useful in a real quantum computation. The issue is also closely related to the proposals of induced interaction via a common bath,[41,42] wherein the effect of the induced decoherence from the same bath is usually neglected during the derivation.

In Fig. 6, we plot the concurrence as a function of time by using various coupling strengths and interqubit distances. The coupling between qubits and the spin bath is of the Ising type, which gives rise to a pure dephasing model. We assume that the coupling strength is the same for two qubits ($\epsilon_1=\epsilon_2$) and the initial state is taken to be $\frac{1}{\sqrt{4}}(|00\rangle+|01\rangle+|10\rangle+|11\rangle)$. We shall set $\Delta=0$ but similar results can be obtained for $\Delta\neq 0$. Before we discuss our findings in more details, we would like to comment that if the Markovian approximation or the uniform coupling assumption are taken, then one can no longer discuss the interqubit distance dependence. The relation between the entanglement dynamics and the interqubit distance, however, is gaining interest since people began to explore the non-Markovian effects of a bath.[23,43,44] We terminate the simulation at half of the revival time, where usually the Loschmidt echo reaches its minimum, to avoid the unphysical dynamics due to the revival. We also numerically check the finite size effect by comparing the entanglement dynamics from different chain lengths. We find that the results from different chain lengths agree with each other reasonably well. The length of the chain mainly sets an upper bound for the simulation time.

We find that for this configuration, it is possible to create entanglement via the spin bath. In particular, for a weaker coupling strength, the induced entanglement more slowly rises but can reach a higher value; while for stronger coupling, the induced entanglement more rapidly rises. The maximal concurrence reached, however, is lower. This is because a larger coupling strength also leads to a stronger decoherence. It is also evident from Fig. 6 that the entanglement creating rate decreases as the interqubit distance increases, which is typical for this kind of induced interaction. We find that for a large enough ϵ and a small enough

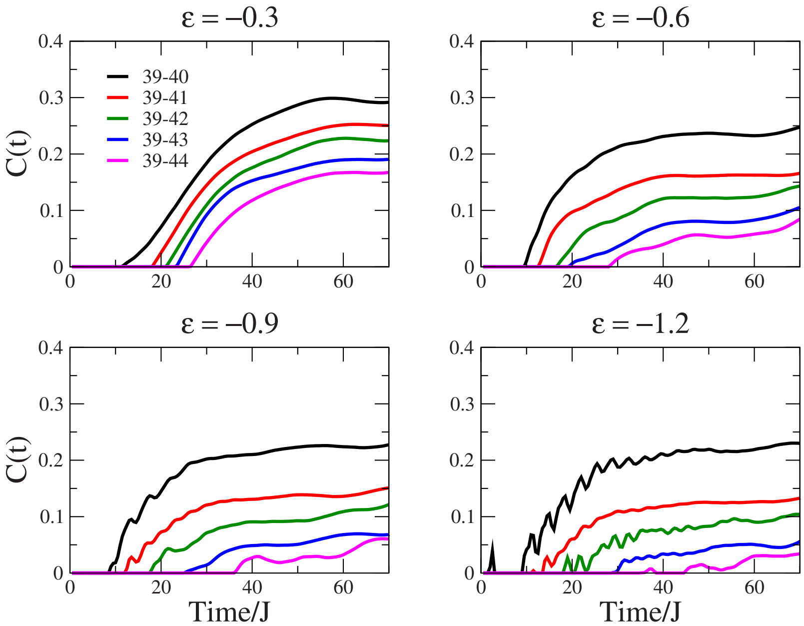

FIG. 7. (Color online) Entanglement dynamics for an initially disentangled pair of qubits for the case of the Heisenberg coupling. Here, $\Delta=0$, $N=80$, and number pairs (such as 39-44) indicate positions of two qubits.

interqubit distance, the concurrence shows an oscillatory behavior. In these cases, the coupling is strong enough to create concurrence oscillation but also weak enough to prevent the bath from totally disentangling the qubits. The delicate interplay between the induced decoherence and the induced entanglement indicates that using such an induced entanglement for a quantum computation is a tricky task. One has to tune the coupling to be within the right window to balance the effect from each side.

In Fig. 7, we plot the concurrence as a function of time for the case of the Heisenberg coupling, starting from the same initial condition. Qualitatively, the behavior is similar to the case of the Ising coupling. We find that the maximal entanglement that can be reached is smaller. This is because, for the Heisenberg coupling, the Loschmidt echo always decays to zero regardless of the coupling strength; while for the Ising coupling, the minimal Loschmidt echo value is a decreasing function of the coupling strength and is not zero. We also find that the onset time is roughly proportional to the interqubit distance. This is expected as the excitation of the spin chain, which mediates the entanglement generation, travels with a finite phase velocity. The time the excitation reaches the other qubit would be proportional to the interqubit distance. However, the concurrence oscillation is absent, indicating that the induced interaction is weaker for the Heisenberg coupling. We note that it is difficult to write down an exact form of the induced interaction unless a Markovian approximation is taken. In general, the induced interaction is time dependent and is accompanied by a complicated decoherence effect. It is, however, possible to experimentally or numerically perform a quantum state tomography to extract the Kraus operators. The Kraus operators can then be used to design quantum operations without directly using the form of the induced interaction.

B. Entanglement decay

Here, we present our results for the disentanglement dynamics of an initially entangled state. To investigate the possibility of an ESD in the spin bath, we start from an initial state of the form $|\psi_{\mathrm{sys}}(0)\rangle=\alpha|00\rangle+\beta|11\rangle$, with an initial concurrence $C(0)=2|\alpha\beta^*|$. Two qubits are set be 20 sites apart so that the decoherences of the individual qubits are nearly independent of each other and the coupling is of the Heisenberg type. We first show a typical behavior of the concurrence in the ferromagnetic regime in Fig. 8. Clearly, as for the reduced dynamics for a single qubit, the concurrence shows oscillatory behaviors. Hence, the qubits in the ferromagnetic regime do not suffer from ESD and the envelope of the entanglement exponentially decays.

In Fig. 9, we plot the disentanglement dynamics of two states in the *XY* critical and antiferromagnetic regimes. All of the computations start from the same initial concurrence with two sets of coefficients, $\alpha/\beta=1/\sqrt{3}$ or $\beta/\alpha=1/\sqrt{3}$, corresponding to two different initial states. We find that in the critical regime $(-1<\Delta<+1)$, both states suffer from ESD.

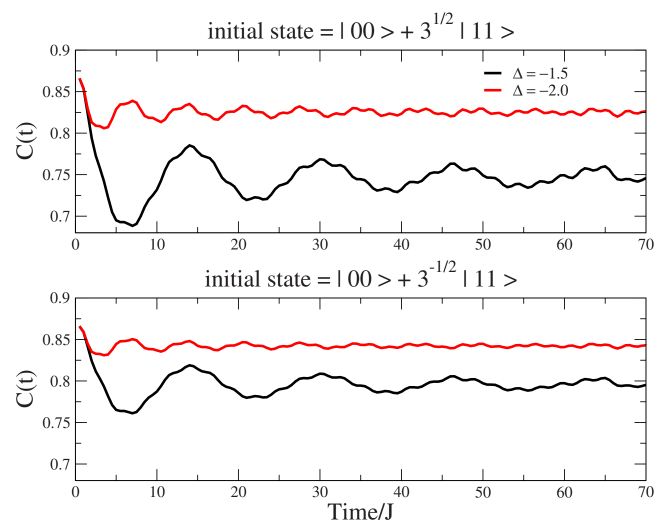

FIG. 8. (Color online) Disentanglement dynamics for an initially entangled pair of qubits when the bath is in the ferromagnetic phase. The coupling is of the Heisenberg type; $\epsilon=-0.3$ and $N=80$.

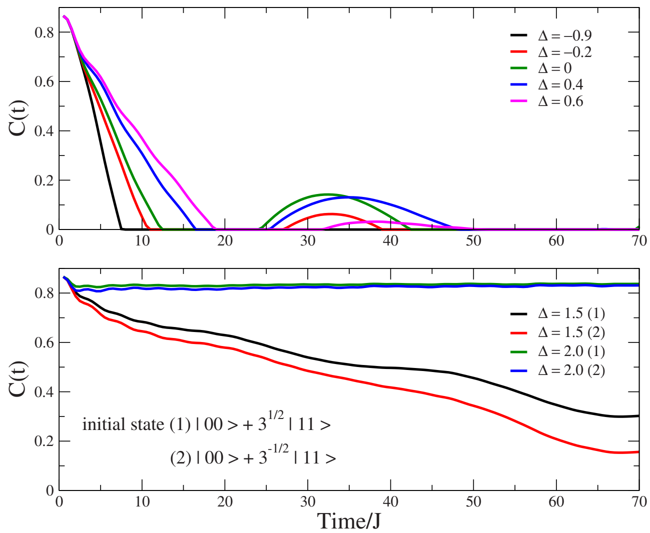

FIG. 9. (Color online) Disentanglement dynamics for an initially entangled pair of qubits when the bath is in the antiferromagnetic or the XY critical phase. The coupling is of the Heisenberg type; $\epsilon = -0.3$ and $N=80$.

Furthermore, the entanglement dynamics of the two states are identical in the critical regime, which is due to the reason that the rotational symmetry is not broken in the critical regime and $|00\rangle$ is equivalent to $|11\rangle$ by the Z_2 symmetry along the quantization axis. In both the ferromagnetic and antiferromagnetic phases, where the rotational symmetry is broken, we find that the entanglement dynamics for these two states start to deviate from each other. In most of the antiferromagnetic regime, both states do not suffer from ESD. When Δ is close to the phase boundary, however, both of the two states that correspond to $\alpha/\beta = 1/\sqrt{3}$ and $\beta/\alpha = 1/\sqrt{3}$ suffer from ESD and have slightly different disentanglement times. In Fig. 10, we plot the inverse of the disentanglement time, which is defined as the time when the concurrence becomes zero, as a function of Δ. Starting from $\Delta = -1$, it shows a monotonic decrease. Across the phase boundary, $\Delta = 1$, the inverse of the disentanglement time develops a small bump

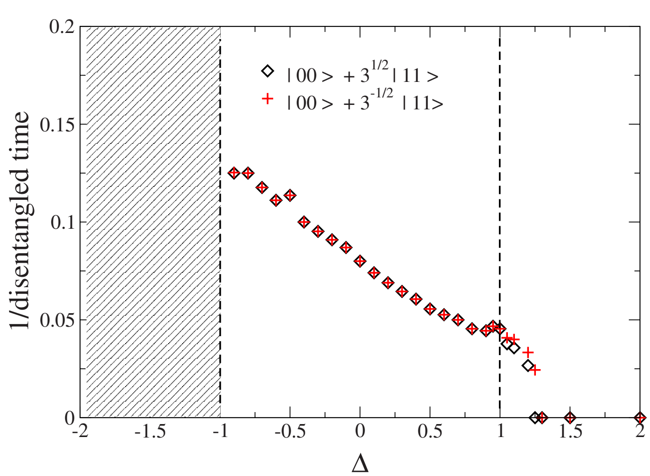

FIG. 10. (Color online) Inverse finite disentanglement time as a function of Δ. Here, the coupling is of the Heisenberg type, $\epsilon = -0.3$ and $N=80$.

and persists into the antiferromagnetic phase, and finally decreases to zero around $\Delta = 1.2$. The existence of a finite region with a finite disentanglement time in the antiferromagnetic regime is due to the fact that when Δ approaches 1, the barrier between the two degenerate ground states approaches zero. For a finite N and a finite value of $\Delta - 1$, the ground state obtained by the static DMRG includes a small mixture of the degenerate state so that it resembles the XY critical state and results in a finite disentanglement time. In the thermodynamic limit ($N \rightarrow \infty$), the region with a finite disentanglement time in the antiferromagnetic regime shrinks down, resulting in the discontinuity at the phase boundary $\Delta = 1$. The overall behavior found in the above is different from those reported in an early work,[30] in which two states investigated in the above possess different entanglement dynamics and only one of them suffers from ESD. The difference is due to the reason that the model adopted in Ref. 30 includes the effect of spontaneous decay, breaking the symmetry between $|0\rangle$ and $|1\rangle$; while for the spin bath in the XY critical regime, such symmetry breaking is absent. It is important to note that there exists a subregime, which is roughly around $-0.3 < \Delta < 0.6$, in which an entanglement shows revival after some dark period. Note that the entanglement revival after some dark period was also reported in Ref. 45, wherein a photonic multimode vacuum bath is assumed and the revival is attributed to the two photon decay. In our work, the origin of the revival is less clear. We believe that the existence of such a subregime is due to the competition between the entanglement decay and the Loschmidt echo decay. Within this subregime, the Loschmidt echo more slowly decay giving the system a chance to revive after the first ESD.

V. CONCLUSION

In summary, the decoherence and (dis)entanglement dynamics induced by spin baths are nonperturbatively investigated by using t-DMRG. For both the pure dephasing model (Ising coupling) and the general decoherence model (Heisenberg coupling), we calculate the short time decay parameter of the Loschmidt echo. We find that in both cases the decay parameter is closely related to the phase of the underlying spin chain. In the ferromagnetic regime, the reduced dynamics shows an oscillatory behavior; while in the XY critical and antiferromagnetic regimes, the decay parameter shows a first derivative discontinuity at $\Delta = +1$. We evaluate the entanglement dynamics of a pair of initially disentangled qubits that are close to each other. We demonstrate that it is possible to induce entanglement via their common interaction with the spin bath. The competition between the induced decoherence and the entanglement can be easily seen in the coupling strength dependent behavior of the entanglement onset time, the growth rate, and the maximal entanglement reached. Finally, we investigate the disentanglement dynamics of a pair of initially entangled qubits, which are far from each other. For the two initial states we studied, we find that their disentanglement dynamics are identical and suffer from ESD in the critical regime. Their disentanglement dynamics begin to deviate from each other in both the ferromagnetic and the antiferromagnetic regimes. They no longer suffer from ESD in the ferromagnetic regime but still suffer from ESD if the chain is near the antiferromagnetic transition. It is shown that the inverse of the finite disentanglement time has a close relation to the phase of the spin bath and shows the monotonic decrease behavior as one that moves into the antiferromagnetic regime. It should be noted that this work represents a successful application of *t*-DMRG to study the decoherence and the entanglement dynamics induced by interacting one-dimensional models. We would also like to comment that it is straightforward to include the self-Hamiltonian of the qubit into the simulation. Consequently, one can easily generalize the method used here to study the efficiency of various pulse sequences[46,47] proposed to eliminate the decoherence induced by an interacting spin bath.

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