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ORIGINAL PAPER

Electron Spin Relaxation Due to Phonon Modulation of the Rashba Interaction in Quantum Dots

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Abstract In this work we calculate the spin-flip transition rates, considering the phonon modulation of the spin–orbit interaction. For this purpose will use the spin–phonon interaction Hamiltonian proposed by Pavlov and Firsov. We compare the contributions of the electron–phonon deformation potential (DP) and piezoelectric (PE) coupling to the spin relaxation. We reveal the importance of an appropriate description of the electron Landé g-factor in the calculation of the rates. Our results demonstrate that, for narrow-gap materials, the DP interaction becomes the dominant one. This behavior is not observed in wide or intermediate gap semiconductors, where the PE coupling, in general, governs the relaxation processes.

Keywords Quantum dots · Spin relaxation

1 Introduction

Spin dephasing is one the most critical aspects that should be considered in the elaboration of proposals of quantum computation based on single spin states as qubits in quantum dots (QD's) [1]. Due to the long electron spin dephasing times reported [2], the spin of an electron localized in a QD has been suggested as good quantum bit candidate. In the previous few years there has been an increasing effort in studying how to manipulate and control processes that involve transitions between spin states due to the recent applications in spin-polarized electronics and quantum computation [3].

In this work, we calculate the spin-flip transition rates, considering the phonon modulation of the spin–orbit interaction [4, 5]. In this approach, the Hamiltonian which describes the transitions with spin reversal due to the scattering of electrons by phonons can be written in a general form as

$$H = V_{\mathrm{ph}} + \gamma[\sigma \times \nabla V_{\mathrm{ph}}] \cdot (\mathbf{p} + e/c\mathbf{A}), \tag{1}$$

where V_{ph} is the phonon operator, γ is related with the strength of the electron–phonon interaction, $(\hbar/2)\sigma$ is the spin operator, $\mathbf{p}$ is the momentum operator and $\mathbf{A}$ is the vector potential related with the external magnetic field $\mathbf{B}$. A detailed discussion of (1) can be found in the original work of Pavlov and Firsov [4, 5] or Romano et al. [6, 7].

In QD systems where the asymmetry of the confinement potential (Rashba effect) is small or can be neglected, our main results are relevant. We consider a spin relaxation mechanism completely intrinsic to the system, since it is based on the modulation of the SO interaction by the acoustic phonon potential, which is independent of any structural properties of the confinement potential.

We evaluate and compare the contributions of the electron–phonon deformation potential (DP) and the piezoelectric (PE) couplings to the spin relaxation. We have observed the necessity to have an appropriate description of the Landé g-factor in the calculation of the rates. For instance, the variation of the InAs g-factor from negative to

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positive values (−14.4 to +0.2) modifies, in several orders the magnitude, the spin-flip scattering rates.

In the frame of the one-band effective mass approximation and considering the presence of an external magnetic field B applied normal to the plane of the QD, the electron lateral wave functions can be written in cylindrical coordinates as [8]

$$f_{n,l,\sigma} = C_{n,l}\frac{\rho^{|l|}}{a^{|l|+1}}e^{-\frac{\rho^2}{2a^2}}e^{il\varphi}L_n^{|l|}\left(\rho^2/a^2\right)\chi(\sigma), \tag{2}$$

where $C_{n,l} = \sqrt{n!/[\pi(n+|l|)!]}$ is the normalization constant, $L_n^{|l|}$ are the generalized Laguerre polynomials, n (l) is the principal (azimuthal) quantum number, and $\chi(\sigma)$ is the spin wave function of spin variable σ_z. The corresponding eigenenergies are $E_{n,l,\sigma} = (2n+|l|+1)\hbar\Omega + (l/2)\hbar\omega_c + (\sigma/2)g(E)\mu_B B$, where $\Omega = (\omega_0^2 + \omega_c^2/4)^{1/2}$, measures the competition between lateral (ω_0) and magnetic (ω_c) confinements, $\sigma = +1(-1)$ for spin up (down) orientations, μ_B is the Bohr magneton, $a = (\hbar/m(E)\Omega)^{1/2}$ is the effective length and $\omega_c = eB/m(E)$. The energy dependence of the electron g-factor and effective mass m are calculated using the Roth formulas [10],

$$g(E) = 2 - \frac{4m_0P^2}{3\hbar^2}\frac{\Delta}{(E_g+E)[(E_g+E)+\Delta]}, \tag{3}$$

$$\frac{1}{m(E)} = \frac{1}{m_0} + \frac{2P^2}{3\hbar^2}\frac{3(E_g+E)+2\Delta}{(E_g+E)[(E_g+E)+\Delta]}. \tag{4}$$

Here $P = (\hbar/m_0)\langle iS|p_z|Z\rangle$ represent the inter-band matrix element, and $E = E_{n,l,\sigma}$ is the electron energy measured from the bottom of the conduction band. For InAs the value of Δ is comparable to the fundamental gap, thus we can expect significant variations of the electron g-factor with the size parameters. The available experimental results have revealed the strong influence that small dot sizes exert on the values of g-factor. In our model, we also consider the effects of the Dresselhaus contribution that provides additional admixture between spin states. For 2D systems, the linear Dresselhaus Hamiltonian can be written as

$$H_D = \frac{\beta}{\hbar}(\sigma_x p_x - \sigma_y p_y), \tag{5}$$

where $p_i = -i\hbar\nabla_i + (e/c)A_i$ with $i = x, y$ and β is the Dresselhaus coupling parameter for this confinement. If the confinement potential in the z-direction is considered highly symmetrical, then $\nabla V_z \sim 0$ and the Rashba contribution can be safely ignored. We calculated the spin relaxation rates, W, via the Fermi Golden Rule at $T \sim 0$-K considering transitions between ground-state Zeeman levels. The temperature dependence for one-phonon emission rate is determined from $W = W_0(n_B + 1)$, where n_B is the Bose distribution function and W_0 is the rate at $T = 0$ K. As well known, when the temperature $T < 5$ K, one can safely approximate the Bose function as $n_B + 1 \approx 1$ and thus, $W \approx W_0$ [9].

2 Results and Discussion

The calculations were performed for a parabolic InSb QD at $T \sim 0$ K. We only have considered electron transitions between ground-state electron Zeeman levels $(0,0,\uparrow) \rightarrow (0,0,\downarrow)$ and $(0,1,\downarrow) \rightarrow (0,1,\uparrow)$. In Fig. 1(a) we have plotted the spin relaxation rates for the ground-state Zeeman transition as a function of the effective lateral QD size $r_0 = \sqrt{\hbar/m(E)\omega_0}$ and B. We clearly identify a region of strong spin coherence, defined by $B > 1$ T and $r_0 > 100$ Å. In this regime, the relaxation times τ are in the ns order and this is an important feature for spin qubit engineering. In the $B < 0.1$ T regime, the relaxation times are approximately of few μs. This spin frozen region is not robust against the temperature and will disappear whenever the thermal energy is larger than the spin transition energy. The plot in Fig. 1(b) shows the spin rates for $(0,1,\downarrow) \rightarrow (0,1,\uparrow)$ transition. As in the previous case, the strong coherence regime is defined approximately by $B > 1$ T and $r_0 > 100$ Å. However, the relaxation times, in this region, are two orders of magnitude faster than for $(0,0,\uparrow) \rightarrow (0,0,\downarrow)$ transition.

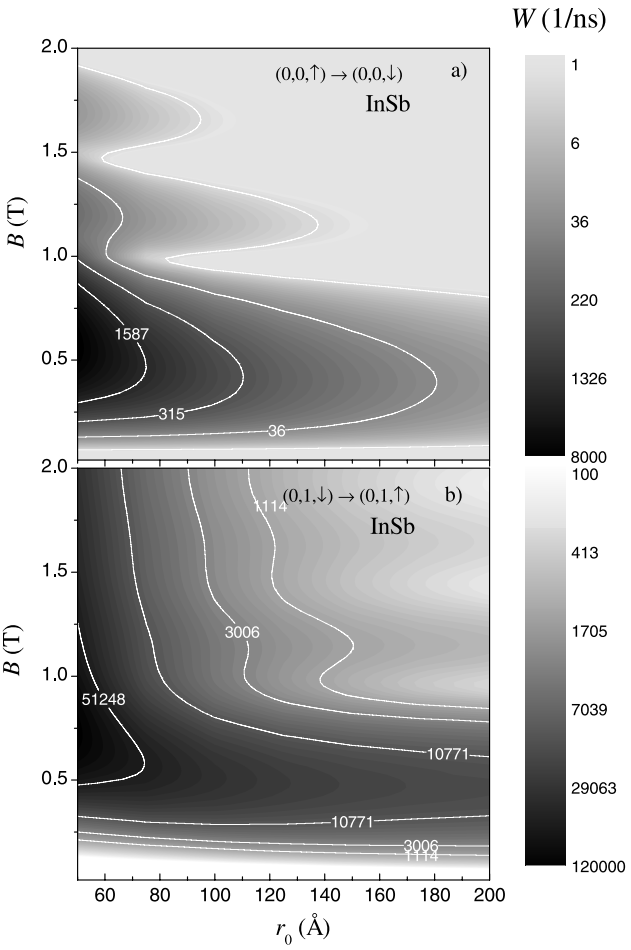

Fig. 1 Contour plot of the spin relaxation rates as a function of magnetic field B and lateral size r_0 for transitions: (**a**) $(0,0,\uparrow) \rightarrow (0,0,\downarrow)$ and (**b**) $(0,1,\downarrow) \rightarrow (0,1,\uparrow)$

The rates show a strong dependence on the magnetic field. This fact can be explained from the dependence of the rates with the transition energy ΔE. In general, we obtain that $W \sim [g^* \mu_B B]^n = (\Delta E)^n$, n being an integer number that depends on the electron–phonon coupling process and g^* the effective g-factor. As can be seen in Fig. 1, when the magnetic field increases, the rates also increase until reaching a maximum near $B \sim 0.5$ T. The position of this main maximum is defined by transition energy conservation: $E_{nl\sigma'} - E_{n'l'\sigma'} = \hbar v q$, where q is the phonon wave vector and v is the average sound velocity. The rate dependence with the lateral QD size r_0, are related to the interplay effects between the spatial and magnetic confinements. These competing effects are contained in the electron–phonon overlap integral, $I \propto \int f^*_{n',l',\sigma'}(\rho) \exp(i\mathbf{q} \cdot \mathbf{r}) f_{n,l,\sigma}(\rho)\, d\mathbf{r}$. For large fields, the magnetic confinement causes a gradual decrease in the overlap integral as the r_0 increases. For small magnetic fields, the spatial confinement is dominant. Thus, when r_0 diminishes the wave functions become more localized and the overlap integral should increase.

We also calculate the spin-flip scattering rates for InAs and GaAs QDs (not shown here), considering only transitions between the ground-state Zeeman levels $(n = 0, l = 0, \sigma = -1) \rightarrow (n = 0, l = 0, \sigma = +1)$. The material parameters for the InAs and GaAs systems are listed in Ref. [11]. In this case, due to the large values of the InAs g-factor, and their connection with the phonon vector $q_0 = \mu g(E) B/(\hbar v)$, we verify that the spin relaxation due to the DP coupling becomes larger than the PE scattering rates. The rates due to PE processes are approximately two orders of magnitude larger than for the DP ones. For GaAs quantum dots, the electron effective g-factor is weakly dependent on the dot lateral size. For $R > 150$ Å, the spatial confinement effects on the g-factor are negligible. Also, the g-factor deviates from its bulk value ($g_{\text{bulk}} = -0.44$) for magnetic fields $B > 5$ T [12]. Thus, the approximation $g = g_{\text{bulk}}$ appears to be valid for the regime of sizes and fields considered in our calculation. The rates for GaAs are, in general, several order of magnitude smaller than for InAs rates.

3 Conclusions

In conclusion, we present the spin relaxation rates calculated via the phonon modulation of the SO interaction. This mechanism has proved to be an efficient and intrinsic channel of spin-flip relaxation in quantum dots. The rates exhibit a strong dependence on the magnetic field and with the lateral dot dimension. The spatial dependence of the rates is a consequence of confinement effects on the electron g-factor and effective mass. It is important to mention that the spin relaxation channels considered in our work can provide important contributions to the overall spin-flip process in systems where the SO does not produce significant admixture between spin states. This situation can be reached in symmetrical samples in the absence of electric fields and in materials with a negligible Dresselhaus contribution, such as elements of group IV.

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