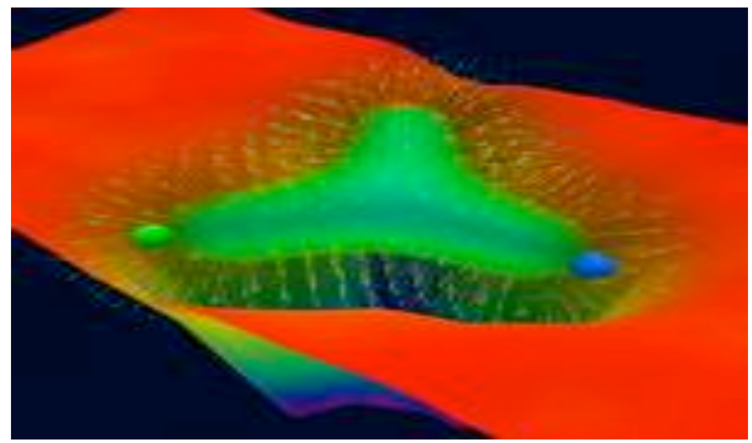
A light-cone wavefunction approach to heavy meson dynamics in the QGP

Ivan Vitev

Credit goes to my collaborators
R. Sharma and B. W. Zhang

- Phys. Rev. C 80, 054902 (2009), Rishi Sharma, IV, Ben-Wei Zhang
- Phys. Lett. B 649, 139 (2007), Azfar Adil, IV



Heavy Quark Production in Heavy Ion Collisions

Purdue, January 4-6, 2010

Outline of this Talk

- Motivation, light particle quenching and the non-photonic electron puzzle
- An alternative mechanism for open heavy flavor suppression
- Heavy mesons near the phase transition
- Distribution and fragmentation functions for open heavy flavor at zero and finite T
- Calculation of the in-medium heavy meson dissociation probability (GLV approach).
- Heavy flavor dynamics at RHIC and the LHC, examples for inclusive D/B and decay $e^{\pm}$

I. Light Particle Quenching

- So far has worked very well versus p_T, root(s), centrality, …

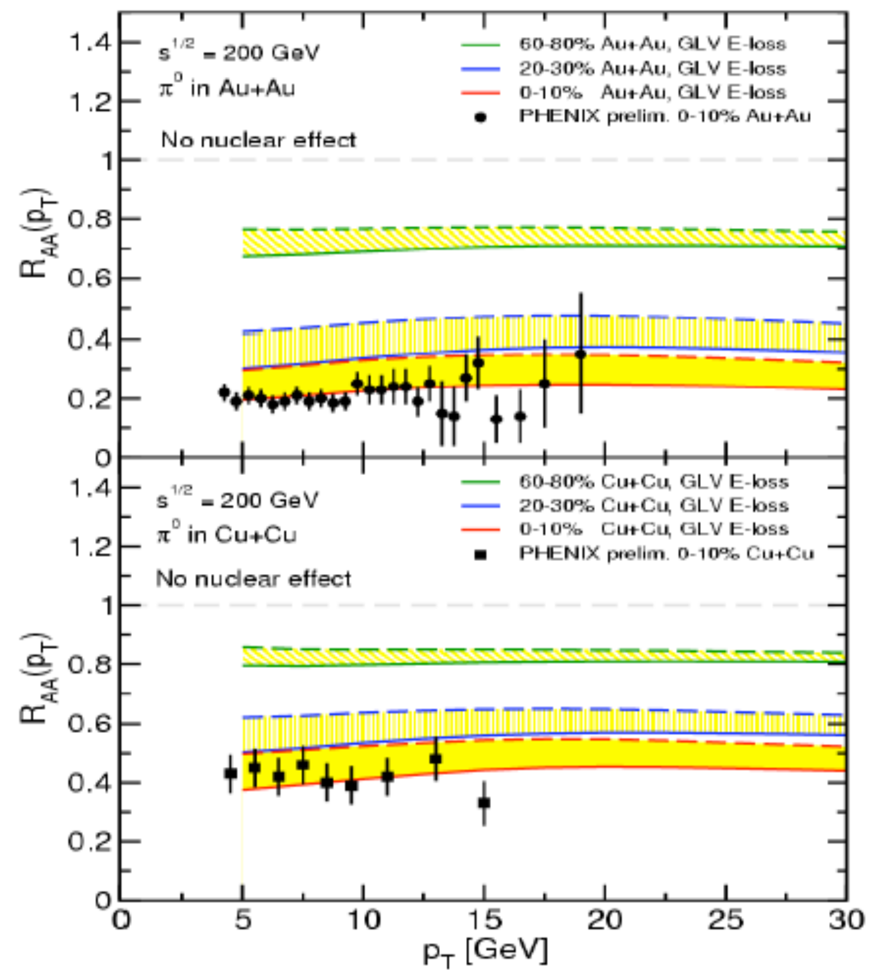
Gyulassy-Levai-Vitev (GLV) formalism

Gyulassy et al, (2001)

- R_{AA}: provides useful information for the hot / dense medium

	RHIC	LHC
T [MeV]	370	720
μ_D [GeV]	.75-1.	1.4-1.8
λ_g[fm]	.75-.42	.39-.25

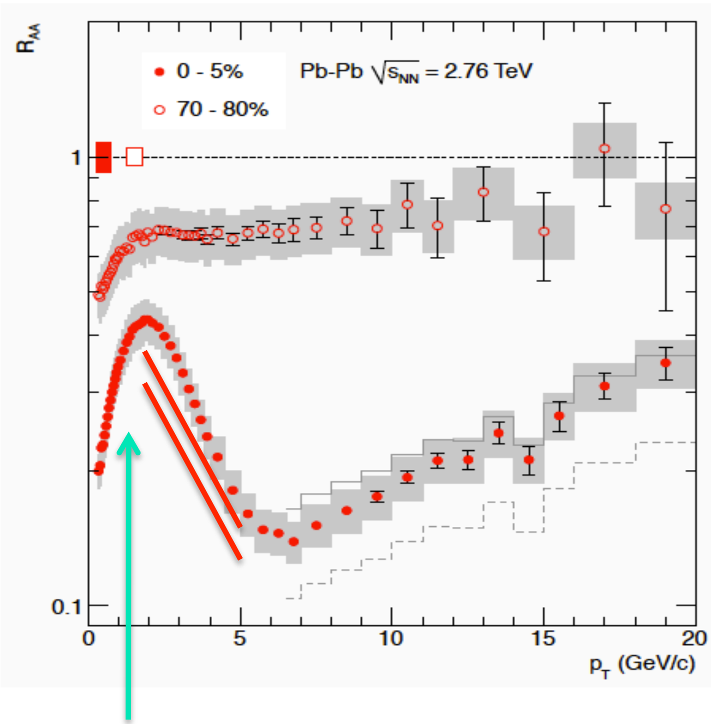
$\langle\langle \hat{q} \rangle\rangle = 0.35 - 0.85\ GeV^2.fm^{-1}$ $\langle\langle \hat{q} \rangle\rangle = 0.40 - 0.99\ GeV^2.fm^{-1}$



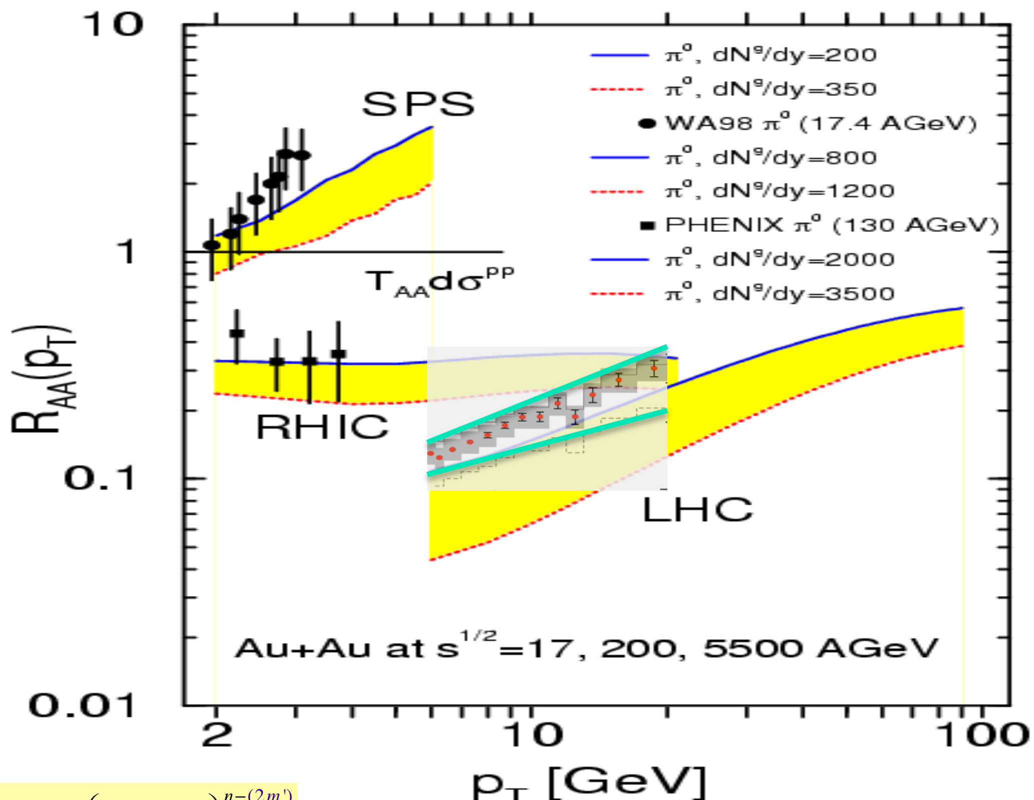
IV 2005

I. Leading Particle Suppression at the LHC

K. Amadot et al, (2011)



IV, (2005)

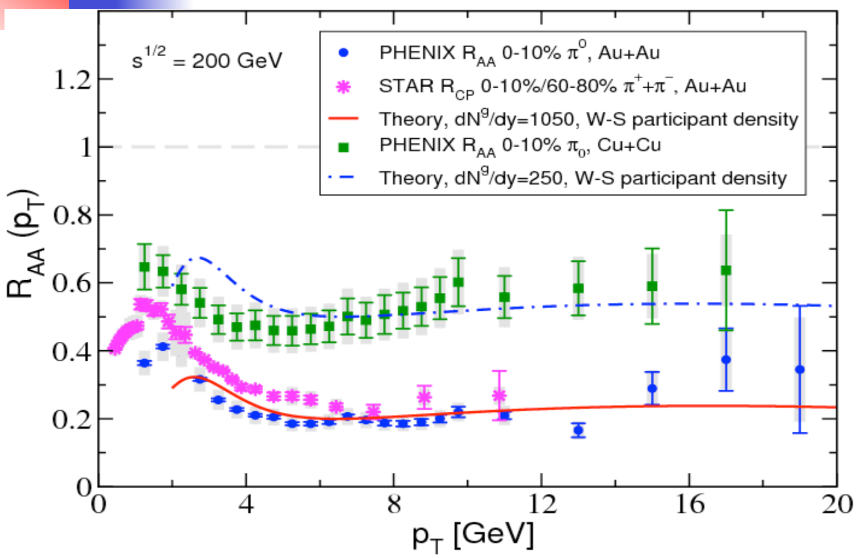


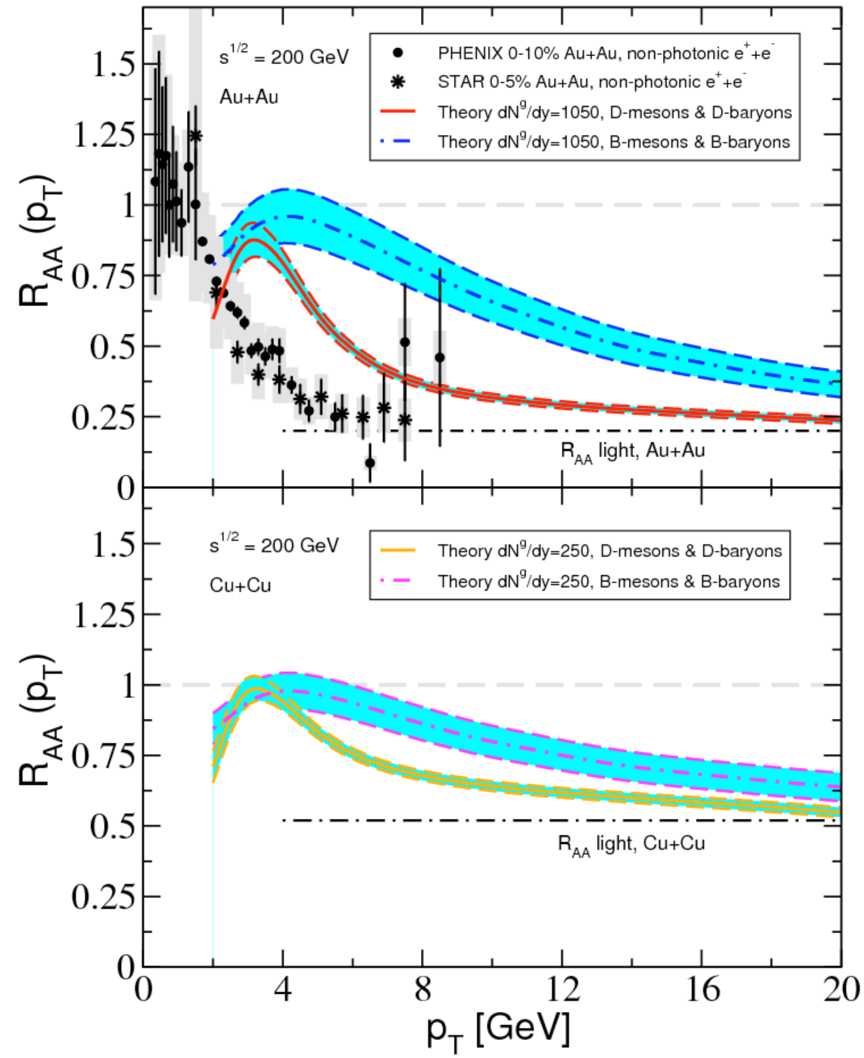
$$\text{Observable} \sim \frac{1}{E_T^n}, \; R_{AA}^{Observable} \approx \left(1 - \frac{\Delta E_T}{E_T}\right)^{n-(2m')}$$

I.V., M. Gyulassy (2002)

- The p_T dependence and magnitude of the ~ suppression is consistent with predictions form 2002 and 2005

I. The Open Heavy Flavor Quenching Puzzle





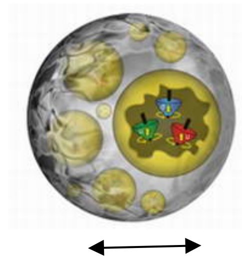
- Calculations include Cronin effect, coherent power corrections (shadowing), IS energy loss, E-loss in the QGP (FS)
- The real problem comes form B mesons that dominate at intermediate / high p_T

R. Sharma et al. (2009)

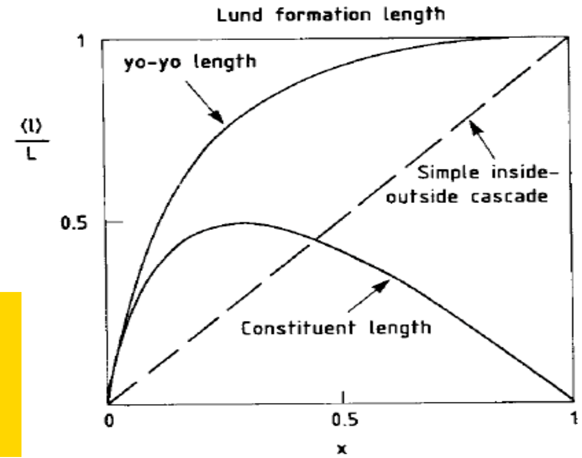
II. The Space-Time Picture of Hadronization

- Inside-outside cascade
- Outside-inside cascade

$$\tau_{\text{form}} = \tau_0 \frac{E}{m} = \tau_0 \gamma$$



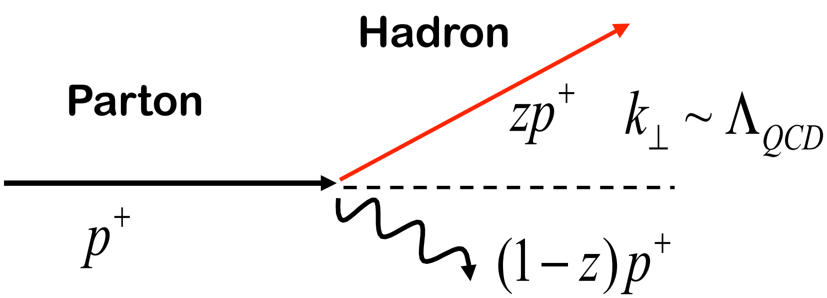
$\tau_0 \sim 1\ \text{fm}$



String models

Bjorken (1984)

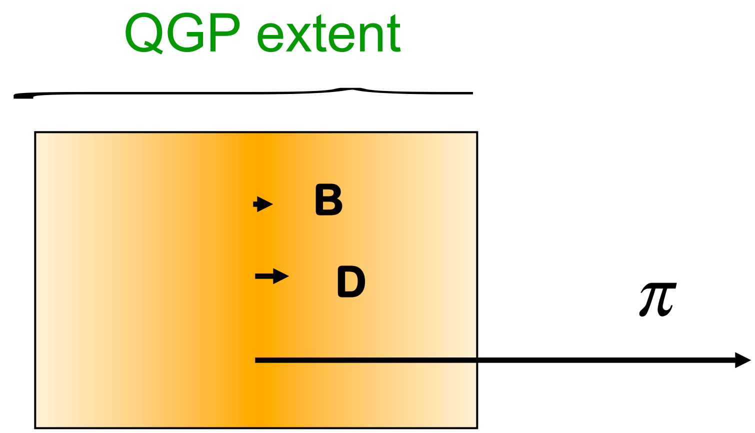
Bialas et al. (1987)



$$\Delta y^+ = \frac{1}{\Delta p^-} = \frac{(0.2\ GeV.fm)\ 2z(1-z)p^+}{k_\perp^2 + (1-z)m_h^2 - z(1-z)M_q^2}$$

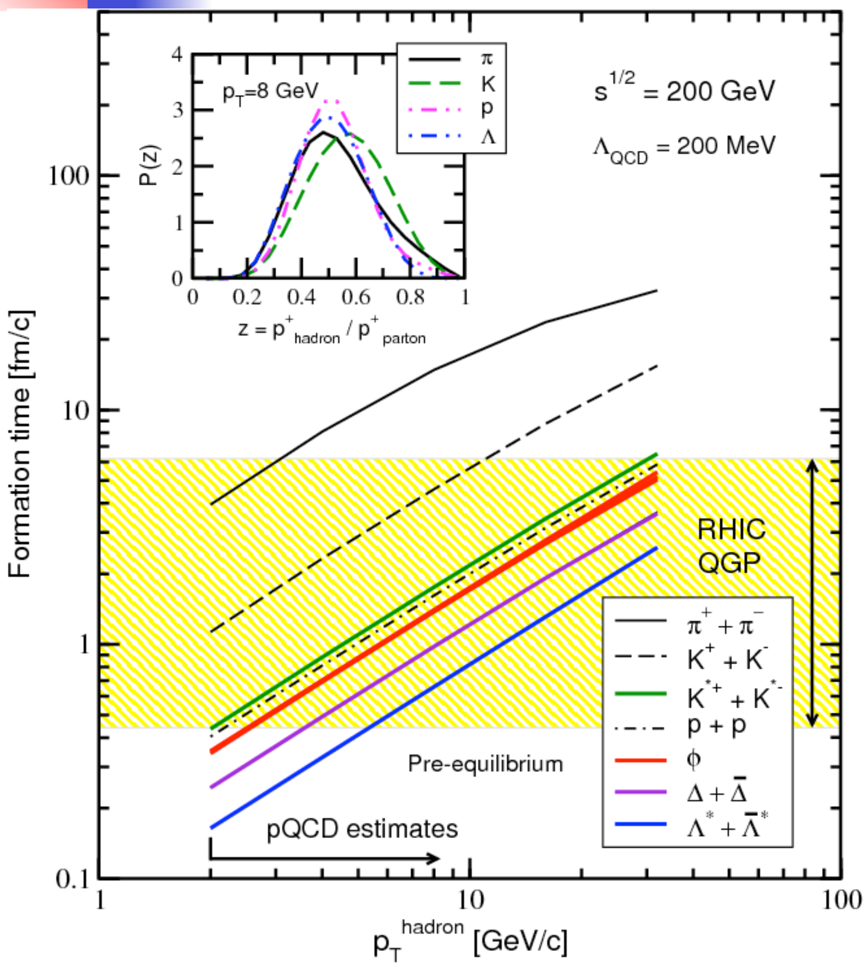
$$\tau_{\text{form}} = \Delta y^+ / (1+\beta_Q)$$

Perturbative QCD

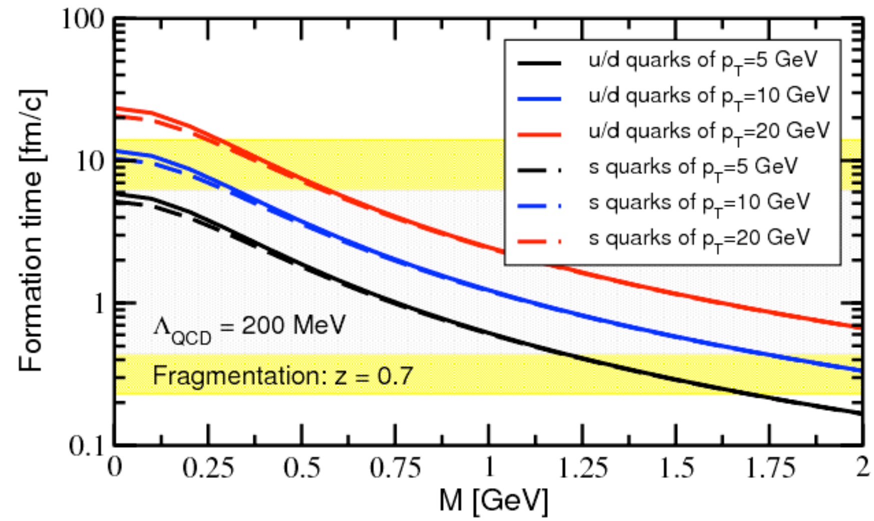


$\tau_{\text{form}}(p_T = 10\ GeV)$	π	D	B
	20 fm	**1.5 fm**	**0.4 fm**

II. The Mass and Momentum Dependence of Particle Formation



Markert et al. (2008)

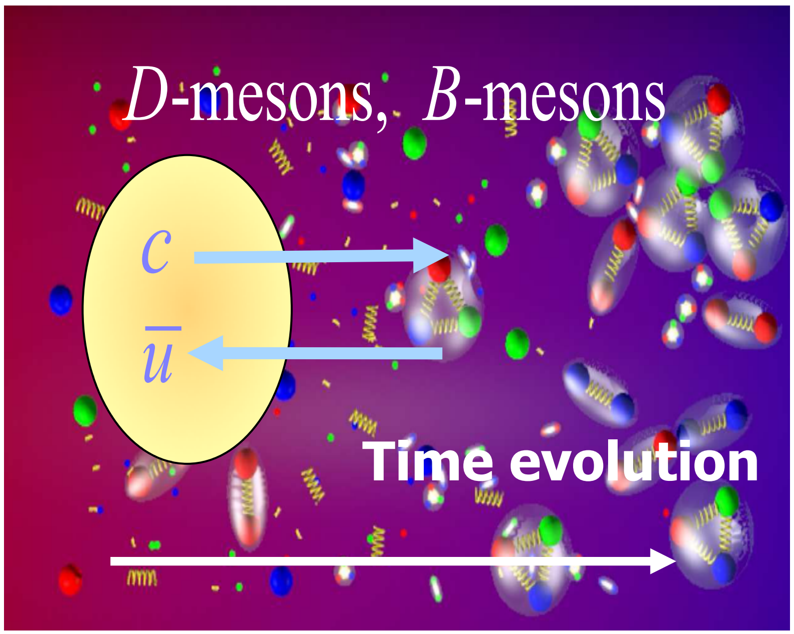


$$\Delta y^+ \simeq \frac{1}{\Delta p^-} = \frac{zp^+}{m_h} \times 2 \left[m_h + \frac{\mathbf{k}^2}{(1-z)m_h} - \frac{zm_q^2}{m_h} \right]^{-1}$$

- For particle masses ~ > 1 GeV one has to consider their formation and dissociation in the medium

II. Collisional Dissociation of D / B Mesons

- An alternative

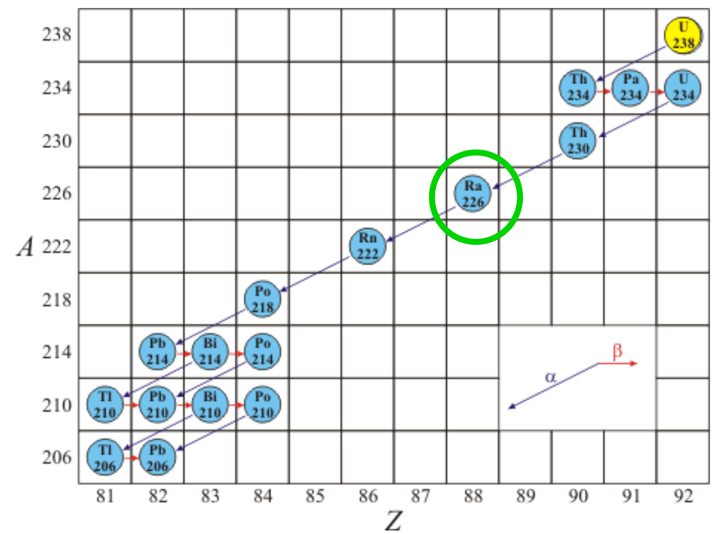


- Both emulate energy loss and lead to suppression of the final observable spectra

Adil, IV (2007)

Simultaneous fragmentation and dissociation call for solving a system of coupled equations

- Example: radioactive decay chain



$$\frac{dN_i}{dt} = \lambda_{i-1} N_{i-1} - \lambda_i N_i$$

III. Quarkonia
Near the Phase Transition

The quark-antiquark potential can be extracted form the lattice

One particular parametrization

$$V(r) = \begin{cases} -\frac{\alpha}{r} + \sigma r & r < r_{med}(T) \\ -\frac{\alpha_1(T)\exp(-\mu(T)r)}{r} + \sigma r_{med}(T) & r > r_{med}(T) \end{cases}$$

- $\sigma = 0.22\ \text{GeV}^2$, $\alpha = 0.45$, $r_{med} = 0.4 T_c/T$ fm.

Mocsy, Petreczky (2007)

- Non-relativistic – Schroedinger

$$\frac{1}{2(m_Q/2)}\frac{1}{r}\frac{\partial^2 r\psi}{\partial r^2} + V(r)\psi = E\psi(r)$$

J/ψ can survive (color screening) to ~ 2 T_c. ϒ can survive to much higher temperatures

	T/T_c	T (GeV)	E_b (GeV)	$\langle r \rangle$ (fm)
J/ψ	1.2	0.230	0.042	0.722
	1.4	0.269	0.030	0.804
	1.6	0.307	0.024	0.869
	1.8	0.346	0.020	0.923
	2.0	0.384	0.017	0.968
ϒ	1.2	0.230	0.344	0.222
	1.4	0.269	0.301	0.232
	1.6	0.307	0.273	0.241
	1.8	0.346	0.254	0.248
	2.0	0.384	0.241	0.254

III. D and B Mesons Near the Phase Transition

- Relativistic – Dirac equation

M. Avila (1994)

Coulomb

$$V = -\xi \frac{1}{r}, \ \xi = \frac{4}{3}\alpha_s$$

Linear

$$S = br$$

$$\psi(\mathbf{r}) = \frac{1}{r}\begin{pmatrix} G(r) \\ i\sigma \cdot \hat{\mathbf{r}} F(r) \end{pmatrix} \mathcal{Y}^{j_3}_{jls}$$

$$\begin{cases} \dfrac{dG}{dr} = -(\varepsilon - V' + S' + m)F - \left(\dfrac{k+1}{r} - \dfrac{b}{2M}\right)G \\ \dfrac{dF}{dr} = \left(\dfrac{k-1}{r} - \dfrac{b}{2M}\right)F + (\varepsilon - V' - S' - m)G \end{cases}$$

B, D can survive color screening to ~ 1.5 – 2 T_c depending on the effective light quark mass

T	T(GeV)	E_b(GeV)	$\sqrt{\langle r^2 \rangle}(GeV)^{-1}$
0	0	0.730	2.374
$0.2T_c$	0.038	0.733	2.361
$0.4T_c$	0.077	0.611	2.351
$0.6T_c$	0.115	0.256	2.540
$0.8T_c$	0.154	0.098	3.202
$1.0T_c$	0.211	0.043	3.980
$1.2T_c$	0.230	0.031	4.917
$1.4T_c$	0.269	0.017	6.402

T (T_c)	T (GeV)	E_b (GeV)	$\sqrt{\langle r^2 \rangle}$ (fm)
...	...	...	...
$1.0T_c$	0.211	0.060	0.670
$1.2T_c$	0.230	0.047	0.797
$1.4T_c$	0.269	0.032	0.933
$1.6T_c$	0.307	0.024	1.044
$1.8T_c$	0.346	0.020	1.119
$2.0T_c$	0.384	0.018	1.158

IV. Light Cone Wave Functions

- Expansion in Fock components

$$\left|P^+,P_\perp,S^2,S_z\right\rangle = \sum_{n=2,3}^{\infty} \int \prod_{i=1}^{n} \frac{dx_i}{2x_i}\frac{d^2k_{\perp i}}{(2\pi)^3}\psi_n\left(\{x\}_i,\{k_{\perp i}\},\{\lambda_i\}\{a_i\}\right)\,\delta\left(\sum_{i=1}^{n}x_i-1\right)\delta\left(\sum_{i=1}^{n}k_{\perp i}\right)$$

$$\times \prod_{i,j,k\to n} ...a^{\dagger a}_{\lambda i}(x_i\vec{P}^+ + k_{\perp i})..b^{\dagger a}_{\lambda j}(x_j\vec{P}^+ + k_{\perp j})...\,\mathrm{d}^{\dagger a}_{\lambda k}(x_k\vec{P}^+ + k_{\perp k})...\left|0\right\rangle$$

Composite hadron creation operator: $a_H^{\dagger s_z}(\vec{P}^+)$

$$\left\langle P^{+'},P'_\perp,S^{2'},S'_z\right\|\left|P^+,P_\perp,S^2,S_z\right\rangle = 2P^+(2\pi)^3\,\delta^3\left(\vec{P}^+-\vec{P}^{+'}\right)\delta^{s_z s_z'}$$

The normalization then becomes

$$1=\frac{1}{2(2\pi)^3}\sum_{n=2,3}^{\infty}\int\prod_{i=1}^{n}\frac{dx_i}{2x_i}\frac{d^2k_{\perp i}}{(2\pi)^3}\left|\psi_n\left(\{x\}_i,\{k_{\perp i}\},\{\lambda_i\}\{a_i\}\right)\right|^2\delta\left(\sum_{i=1}^{n}x_i-1\right)\delta\left(\sum_{i=1}^{n}k_{\perp i}\right)$$

IV. Parton Distribution Functions at Non-zero T

- Even though the physical situation of mesons in a co-moving plasma is nor realized it is interesting to investigate theoretically the PDFs and FFs at finite temperature

Distribution function

$$\phi_{q/P}(x) = \int \frac{dy^-}{2\pi} e^{-ixP^+y^-} \left\langle P \left| \bar{\psi}^a(y^-,0) \frac{\gamma^+}{2} \psi^a(0,0) \right| P \right\rangle$$

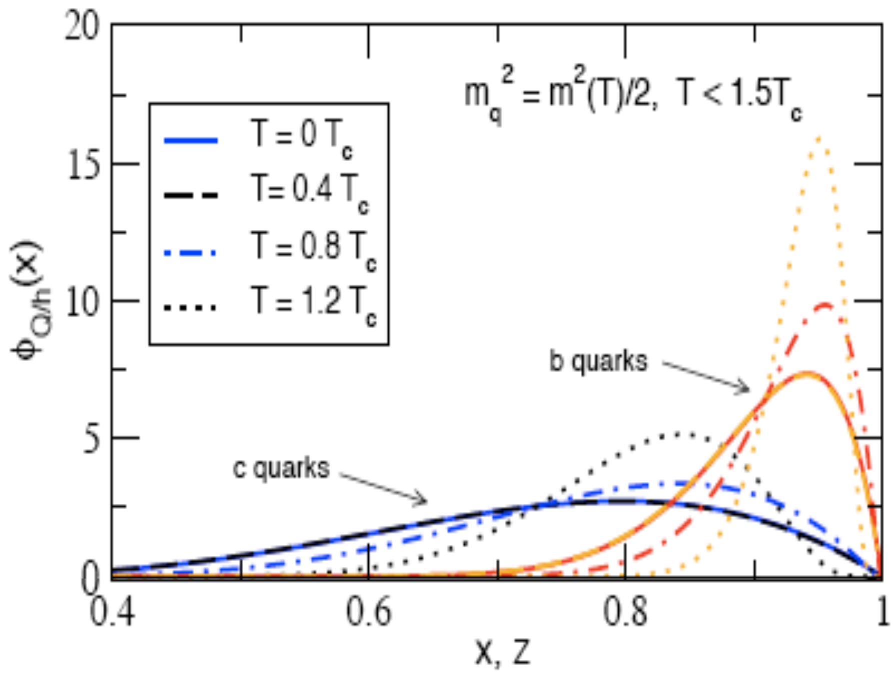
- Light cone gauge $A^+=0$, $0<x<1$

$$|\vec{P}^+;J\rangle = \int \frac{d^2\mathbf{k}}{(2\pi)^3} \frac{dx}{2\sqrt{x(1-x)}} \frac{M(j)_{s_1 s_2}}{\sqrt{2}} \frac{\delta_{c_1 c_2}}{\sqrt{3}} \psi(x,\mathbf{k}) \times a_Q^{\dagger\, s_1 c_1}(x\vec{P}^+ + \mathbf{k}) b_q^{\dagger\, s_2 c_2}((1-x)\vec{P}^+ - \mathbf{k})|0\rangle \, .$$

$$|\psi(\Delta k_\perp, x)|^2 \sim Exp\left[-\frac{\Delta k_\perp^2 + 4m_Q^2(1-x) + 4m_q^2(x)}{4\Lambda^2 x(1-x)} \right]$$

$$\phi_{Q/h}(x) = \frac{1}{2(2\pi)^3} \int dx_Q d^2\mathbf{k} \, |\psi(x_Q,\mathbf{k})|^2 \delta(x_Q - x)$$

Sharma, IV, Zhang (2009)

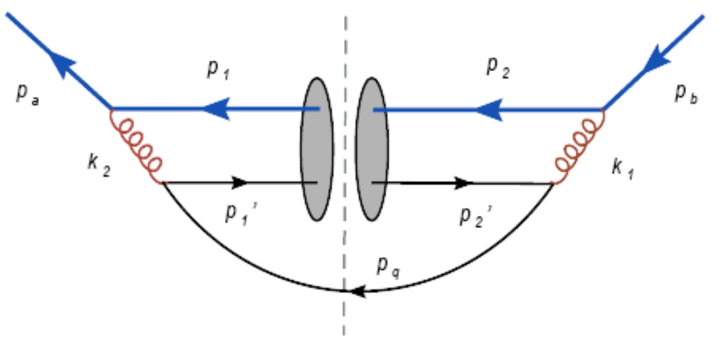


IV. Heavy Quark Fragmentation Functions at Non-zero T

Fragmentation function

At tree level: $\# \delta(x-1).\ \# \to 0$

The leading contribution:

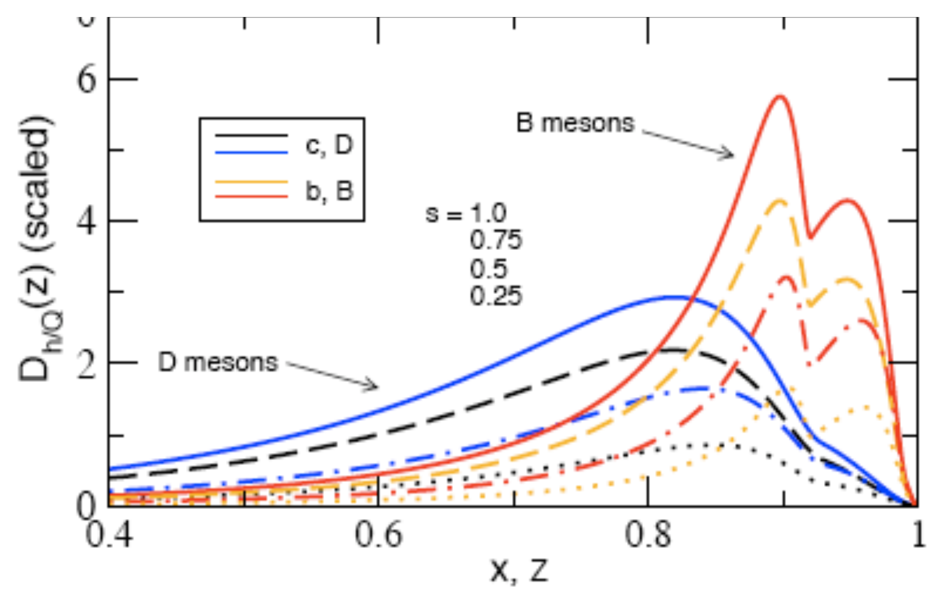


Ma (1993), Braaten et al. (1995)

- Keep the leading q_T dependence in the meson wavefunction. Complexity ~10^3 terms

$$D_{H/q}(z) = z\int \frac{dy^-}{2\pi} e^{iP^+/z\, y^-} \frac{1}{3} Tr_{color} \frac{1}{2} Tr_{Dirac}$$

$$\times \frac{\gamma^+}{2} \left\langle 0 \middle| \psi^a(y^-,0) a_H^\dagger(P^+) a_H(P^+) \bar{\psi}^a(0,0) \middle| 0 \right\rangle$$

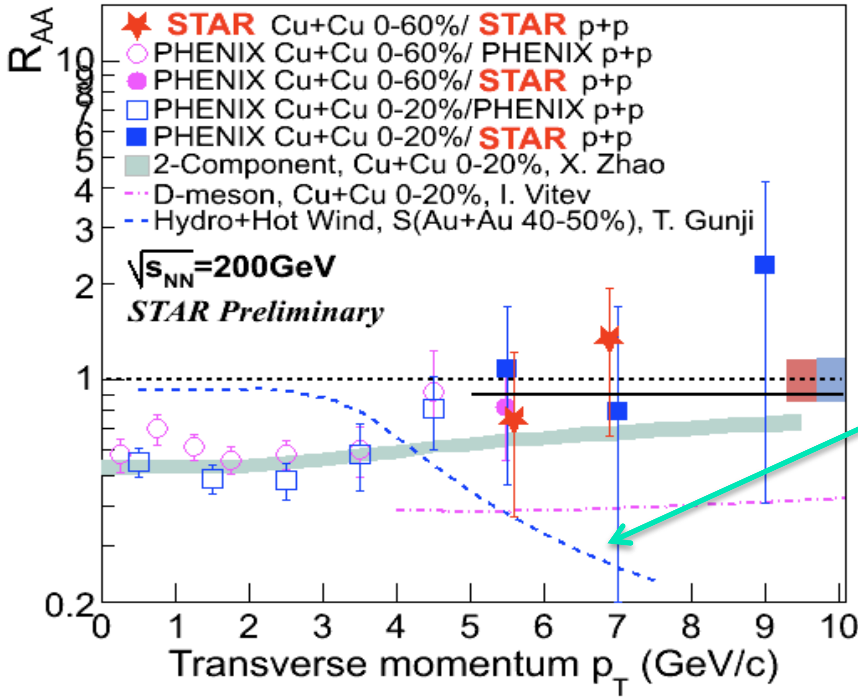


Sharma et al. (2009)

Includes: ${}^3S_0 \to {}^1S_0$

The only calculation of "modified fragmentation functions". The shape is the same as in the vacuum. Quenching comes from energy loss - not universal.

IV. The Inapplicability of Simple Mesons in Equilibrium Models



- Incompatible with equilibrium + boost assumption AdS/CFT

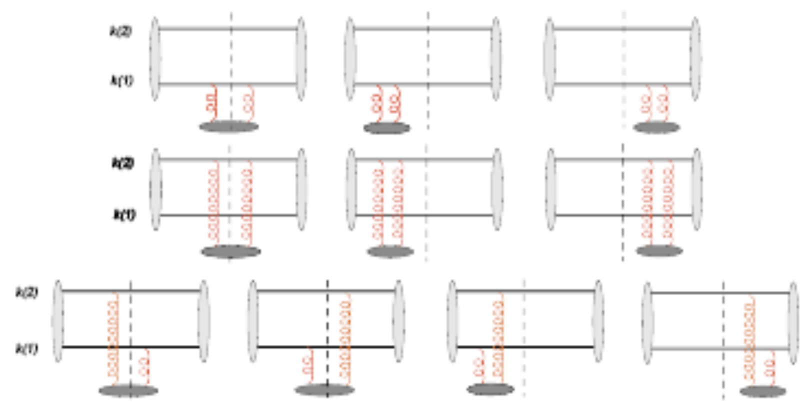
- The J/ψ yield suggests that the LQCD based results with the equilibrium dissociation
- Theoretically $\gamma r_{med} > t_{form}$. Leads to “instant” wavefunction approximation

- It is not the screening, but the subsequent collisional dissociation that reduce the rates of open heavy flavor and quarkonia

V. Calculating the Heavy Meson Dissociation Probability

- Calculate the kernel for the system evolution in the GLV approach

$$|\psi_f(\mathbf{K},\Delta\mathbf{k})|^2 = \sum_{n=0}^{\infty}\frac{2^n\chi^n}{n!}\int\prod_{i=1}^{n} d^2\mathbf{q}_i\frac{1}{\sigma_{el}}\frac{d\sigma_{el}}{d^2\mathbf{q}_i} \times\left[\left(e^{-\mathbf{q}_n\cdot\vec{\nabla}_{\mathbf{K}}}-\hat{1}\right)\cosh\left(-\mathbf{q}_n\cdot\vec{\nabla}_{\Delta\mathbf{k}}\right) + \left(e^{-\mathbf{q}_n\cdot\vec{\nabla}_{\Delta\mathbf{k}}}-\hat{1}\right)\right]|\psi_0(\mathbf{K},\Delta\mathbf{k})|^2 \, .$$



Opacity (mean # scatterings): $\chi = \frac{L}{\lambda}$

- One can resum the interactions in impact parameter space

$$\int d^2\mathbf{q}\frac{\mu^2}{\pi(\mu^2+q^2)^2}e^{iqb\cos(\phi)} = b\mu\, K_1(b\mu) = 1 - \frac{b^2\mu^2}{2}\left[\ln\left(\frac{2e^{-\gamma_E}}{b\mu}\right)+\frac{1}{2}\right] + \mathcal{O}(b^4\mu^4)\, .$$

Rutherford tail enhancement $\xi = \ln(2e^{-\gamma_E}/(b\mu)) + 1/2 \qquad (\xi = 3-5)$

Master equation $|\widetilde{\psi_f}(\mathbf{B},\mathbf{b})|^2 = |\widetilde{\psi_0}(\mathbf{B},\mathbf{b})|^2 e^{-b^2(\chi\mu^2\xi)}e^{-B^2(\chi\mu^2\xi)}$

V. The Physics of In-medium Heavy Meson Scattering

Initial condition:

$$|\psi_i(\Delta k_\perp,x)|^2 = \left[\delta^2(K_\perp)\right]\times\left[\text{Norm}^2 e^{-\frac{\Delta k_\perp^2}{4x(1-x)\Lambda^2}} e^{-\frac{m_1^2(1-x)+m_2^2 x}{x(1-x)\Lambda^2}}\right]$$

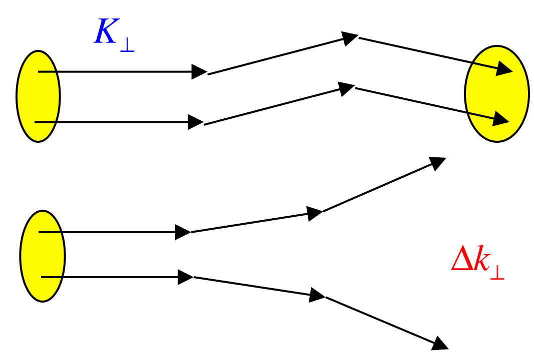
Result:

$$|\psi_f(\Delta k_\perp,x)|^2 = \left[\frac{e^{-\frac{K_\perp^2}{4\chi\mu^2\xi}}}{4\chi\mu^2\xi}\right]\times\left[\text{Norm}^2 \frac{x(1-x)\Lambda^2}{\chi\mu^2\xi + x(1-x)\Lambda^2} e^{-\frac{\Delta k_\perp^2}{4(\chi\mu^2\xi + x(1-x)\Lambda^2)}} e^{-\frac{m_1^2(1-x)+m_2^2 x}{x(1-x)\Lambda^2}}\right]$$

- Two physics effects arise form meson broadening

- Heavy meson acoplanarity: $\langle K_\perp^2\rangle = 2\left(2\mu^2 \frac{L}{\lambda_q}\xi\right)$

- Broadening (separation) the q-qbar pair:



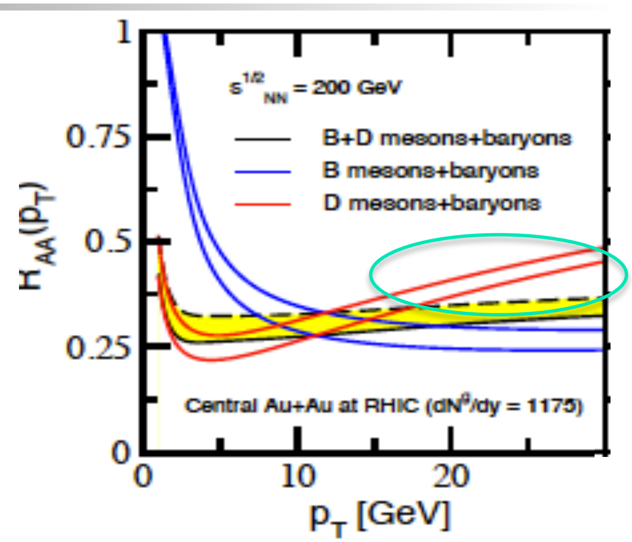
- Dissociation probability $P_{\text{diss}}(t) = 1 - |\langle\psi_t^*(\Delta k, x)|\psi_i(\Delta k, x)\rangle|^2$
- $\tau_{\text{diss}} = \frac{1}{P_{\text{diss}}(t)}\frac{dP_{\text{diss}}(t)}{dt}.$

$$\psi_f(\Delta k_\perp,x) = a\psi_M(\Delta k_\perp,x) + (1-a)\psi_{q\bar{q}\ \text{dissociated}}(\Delta k_\perp,x)$$

VI. Coupled Rate Equations and Initial Conditions

$$\partial_t f^Q(p_T,t) = -\frac{1}{\langle \tau_{form}(p_T,t)\rangle} f^Q(p_T,t) + \frac{1}{\langle \tau_{diss}(p_T/\bar{x},t)\rangle}\int_0^1 dx\ \frac{1}{x^2}\phi_{Q/H}(x) f^H(p_T/x,t)$$

$$\partial_t f^H(p_T,t) = -\frac{1}{\langle \tau_{diss}(p_T,t)\rangle} f^H(p_T,t) + \frac{1}{\langle \tau_{form}(p_T/\bar{z},t)\rangle}\int_0^1 dz\ \frac{1}{z^2} D_{H/Q}(z) f^Q(p_T/z,t)$$



Adil, IV (2007)

- The subtlety is how to include partonic energy loss
- It cannot be incorporated as drag/diffusion

- We include it approximately as MODIFIED INITIAL CONDITION

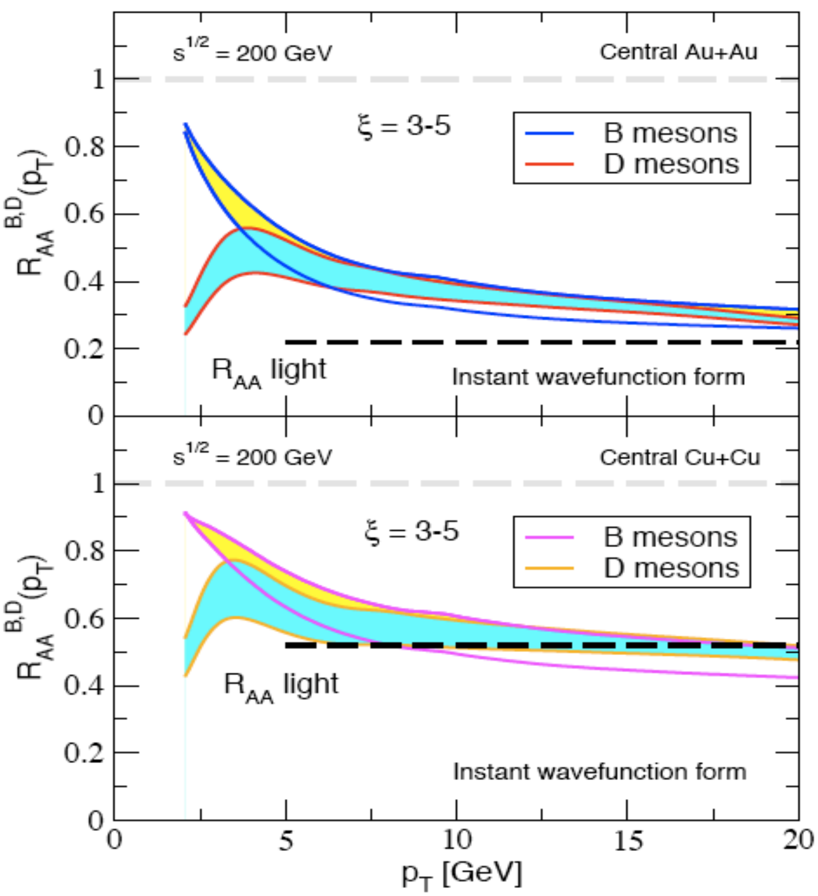
$$f^Q(p_T, t=0) = \frac{d\sigma}{dyd^2p_T} f^Q(p_T, QUENCHED)$$

$$f^H(p_T, t=0) = 0$$

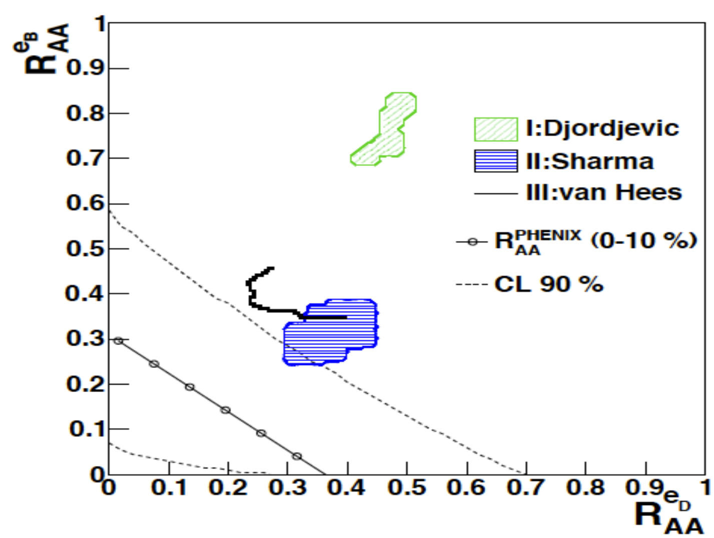
Sharma et al. (2009)

VI. Numerical Results for D/B mesons at RHIC

- D/B mesons (and non-photonic electrons): show similar suppression Au+Au, Cu+Cu at RHIC



- Cronin effect is very important.
- Reduces the suppression at intermediate p_T



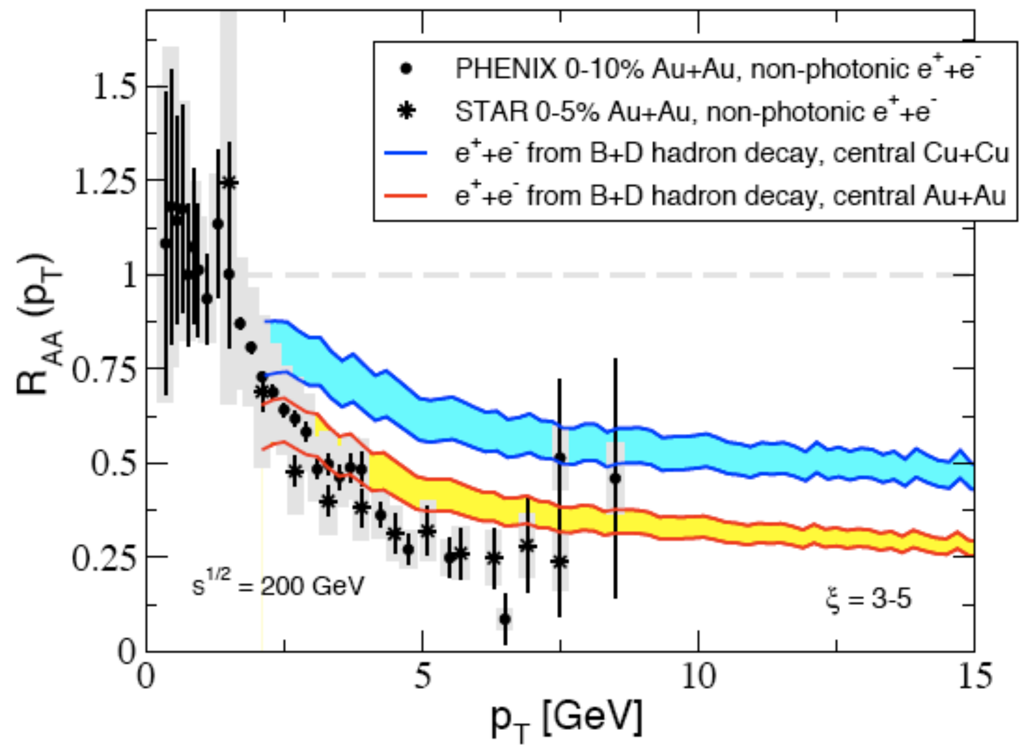
M. Aggarwal et al. (2010)

VI. RHIC Results on Non-photonic Electrons

- Employ a full simulation of the D and B meson semi-leptonic decay,. PYTHIA subroutin.e

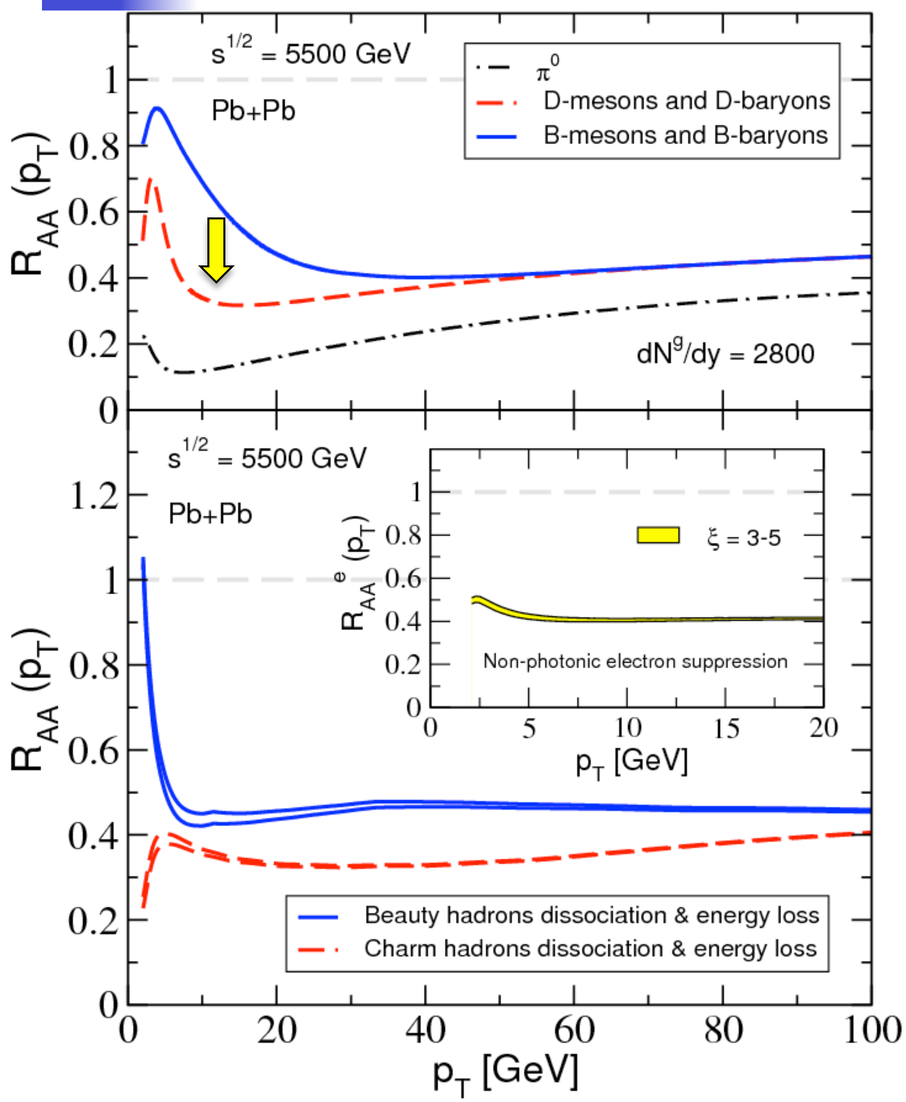
T. Sjostrand et al (2006)

- The predicted suppression is still slightly smaller than the quenching of inclusive particles
- It is compatible with the experimental data within the error bars
- Improved direct measurements are needed to pinpoint the magnitude and relative contribution of B/D



R. Sharma et al. (2010)

VI. Open Heavy Flavor Suppression at the LHC



- The main effect is the increased suppression in the intermediate p_T range 5 GeV to 30 GeV
- At low p_T the predicted D/B (electron) suppression is smaller than the one for light particles
- At p_T > 40 GeV they become comparable. (Residual matching model dependence)
- LHC is critical to cover the p_T range needed for b-quark, B-meson dynamics

Conclusions

- Theoretical predictions of leading light particle suppression in the GLV approach have worked very well from SPS to the LHC
- In contrast, the suppression of non-photonic electrons is significantly underpredicted. The problem is in the B mesons. Common to all e-loss approaches
- Color screening by itself does not imply disappearance of quarkonia to ~ $2T_c$ and heavy mesons to 1.5- 2 T_c Presented first calculations of heavy meson distribution and fragmentation functions at zero and finite temperature (beyond the point like approximation)
- There is no universal (T) medium modification of fragmentation functions. Suppression comes from processed that are/emulate energy loss

Conclusions (continued)

- Theoretical and phenomenological considerations suggest that for energetic mesons fragmentation may not be affected by the thermal medium.
- Calculated the dissociation probability of mesons moving through the medium associated with its collisional interactions
- Partonic energy loss and meson dissociation are now combined. This is not straightforward – as quenched quark initial conditions
- The main prediction remains equal and large B and D meson suppression. Matches smoothly onto the quenching region. Compatible with RHIC results.
- Predictions for LHC presented

IV. QCD on the Light Front

The free theory

- Quarks
- Anti-quarks
- Gluons

$$\psi^a(\vec{x}^-) = \int \frac{dp^+}{2p^+}\frac{d^2p_\perp}{(2\pi)^3}\sum_\lambda \left(a^a_\lambda(\vec{p}^+)u_\lambda(p)e^{-ip\cdot x} + b^{\dagger a}_\lambda(\vec{p}^+)v_\lambda(p)e^{+ip\cdot x}\right)_{|x^+=0}$$

$$\bar{\psi}^a(\vec{x}^-) = \int \frac{dp^+}{2p^+}\frac{d^2p_\perp}{(2\pi)^3}\sum_\lambda \left(b^a_\lambda(\vec{p}^+)\bar{v}_\lambda(p)e^{-ip\cdot x} + a^{\dagger a}_\lambda(\vec{p}^+)\bar{u}_\lambda(p)e^{+ip\cdot x}\right)_{|x^+=0}$$

$$A^a(\vec{x}^-) = \int \frac{dp^+}{2p^+}\frac{d^2p_\perp}{(2\pi)^3}\sum_\lambda \left(d^a_\lambda(\vec{p}^+)\varepsilon_\lambda(p)e^{-ip\cdot x} + d^{\dagger a}_\lambda(\vec{p}^+)\varepsilon^*_\lambda(p)e^{+ip\cdot x}\right)_{|x^+=0}$$

Commutation relations and normalization of states

$$\left\{a^{a'}_{\lambda'}(\vec{p}^{+'}), a^{\dagger a}_\lambda(\vec{p}^+)\right\} = 2p^+(2\pi)^3\delta^3\left(\vec{p}^+ - \vec{p}^{+'}\right)\delta^{aa'}\delta^{\lambda\lambda'} \qquad \left\{b^{a'}_{\lambda'}(\vec{p}^{+'}), b^{\dagger a}_\lambda(\vec{p}^+)\right\} = 2p^+(2\pi)^3\delta^3\left(\vec{p}^+ - \vec{p}^{+'}\right)\delta^{aa'}\delta^{\lambda\lambda'}$$

$$\left[b^{a'}_{\lambda'}(\vec{p}^{+'}), b^{\dagger a}_\lambda(\vec{p}^+)\right] = 2p^+(2\pi)^3\delta^3\left(\vec{p}^+ - \vec{p}^{+'}\right)\delta^{aa'}\delta^{\lambda\lambda'}$$

- States: $\left|n, \{\vec{p}^+_n\}, \{\lambda_n\}\{a_n\}\right\rangle = \prod_{i,j,k\to n} \ldots\, a^{\dagger a}_{\lambda,i}(\vec{p}^+_i)\ldots b^{\dagger a}_{\lambda,j}(\vec{p}^+_j)\ldots d^{\dagger a}_{\lambda,k}(\vec{p}^+_k)|0\rangle$
    - Implicit: quark flavor, (anti)symmetrization
    - Normalization trivially obtained from above

I. High p_T (E_T) Observables

- Understanding high p_T particle suppression (quenching)

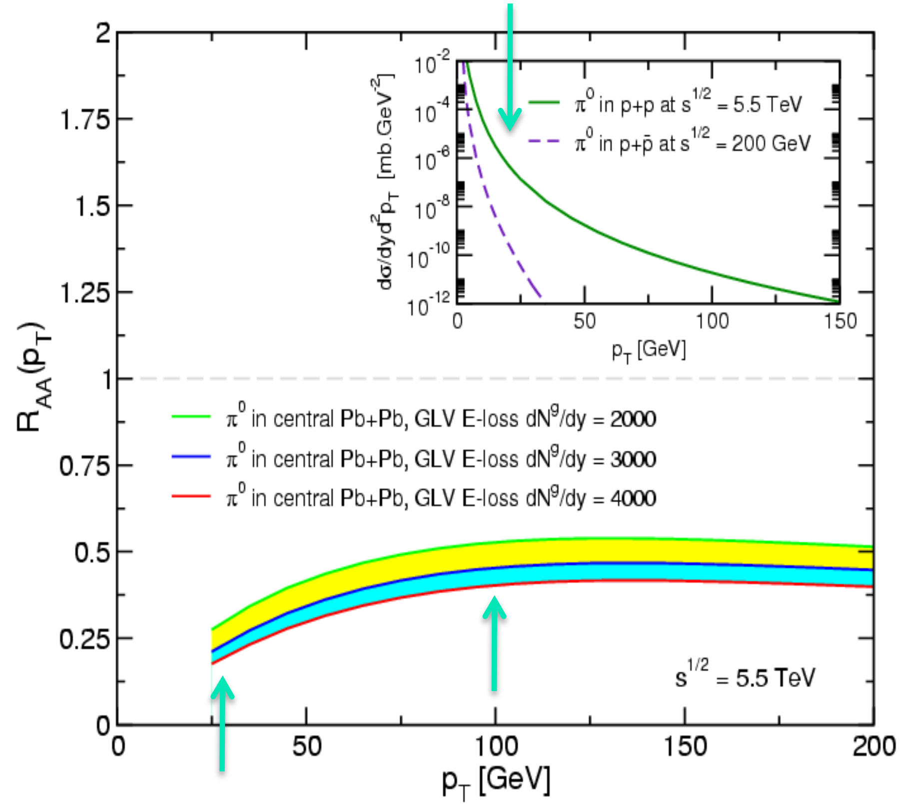
Power laws: $n = n(\sqrt{s}, p_T, system)$ $\frac{d\sigma}{d^2 p_T} = \frac{A}{(p_T + p_0)^n} \approx \frac{A}{(p_T)^n}$

Quenching factor correlated to spectra

$$\text{m-Particle Observable} \sim \frac{1}{E_T^n}$$

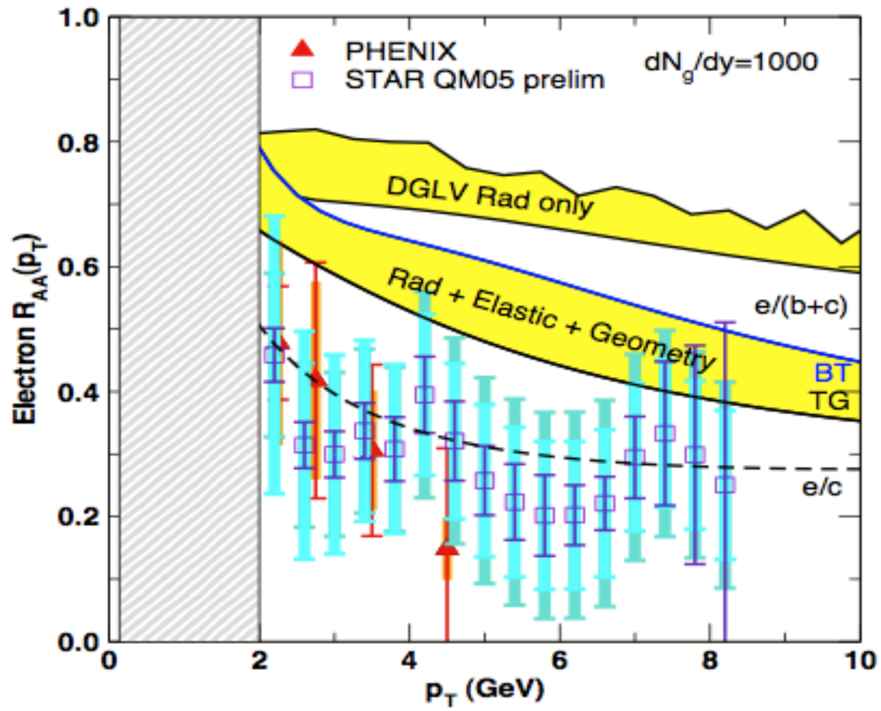
$$R_{AA}^{Observable} \approx \left(1 - \frac{\Delta E_T}{E_T}\right)^{n-(2m')}$$

- Some models' R_{AA} varies with the underlying power law spectrum
- High p_T suppression at the LHC can be comparable and smaller than at RHIC
- Complete absorption models produce a constant R_{AA}

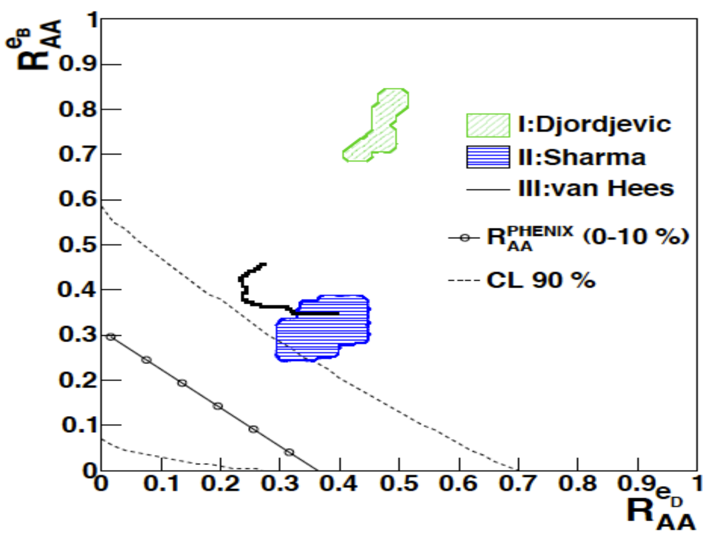


I. The Open Heavy Flavor Quenching Puzzle

- Large suppression of non-photonic electrons – incompatible with radiative/ collisional e-loss
- … one can fit heavy flavor only (no doubt)



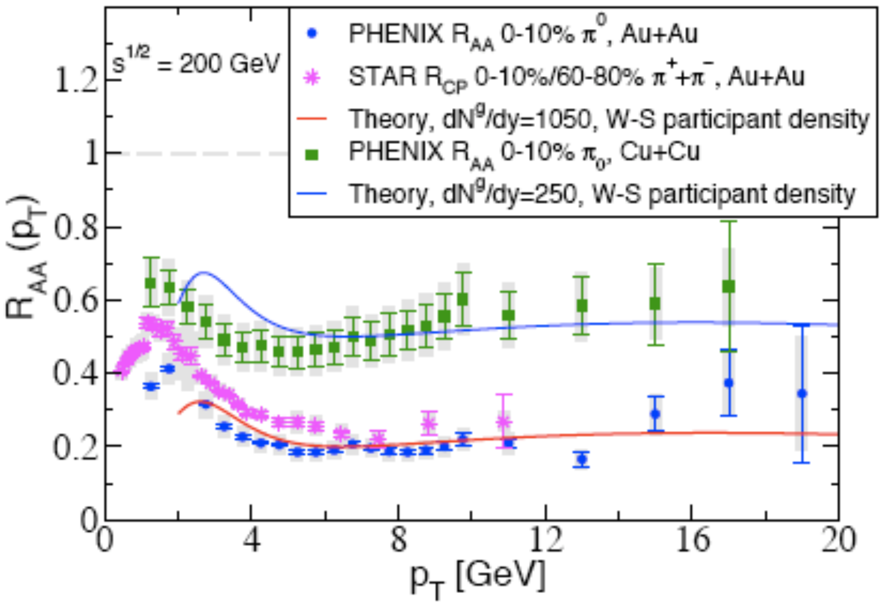
M. Djordjevic et al. (2006)



M. Aggarwal et al. (2010)

But it doesn’t work. Data favors B meson suppression comparable to that of D mesons

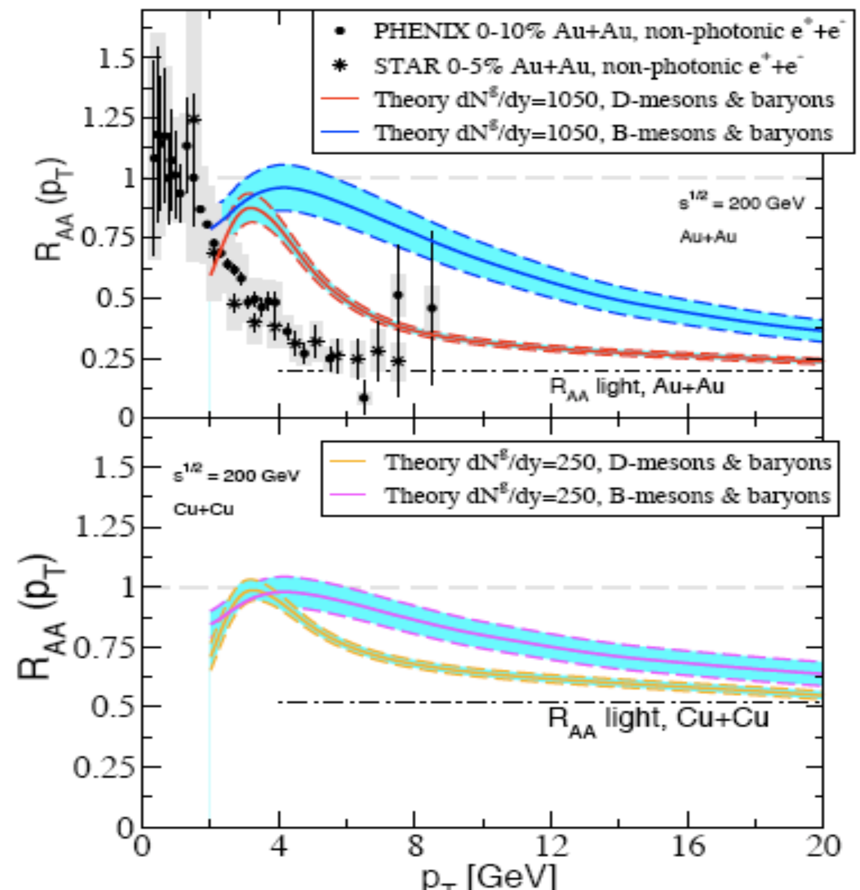
"Instant" Approximation

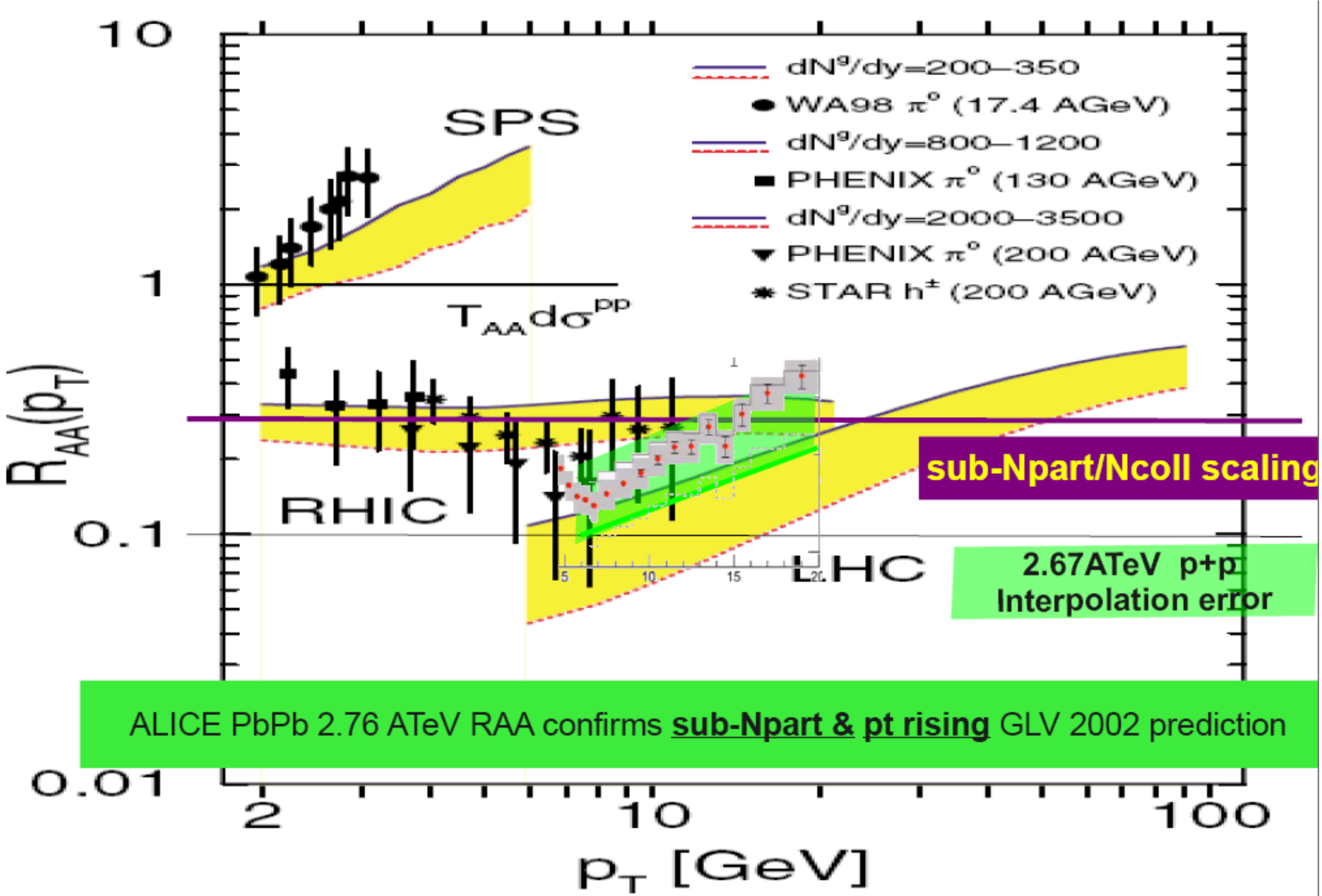


- $\frac{dN^g}{dy} = 1050$ for Au and $\frac{dN^g}{dy} = 250$ for Cu.

- Compatible with measured multiplicities

- Full treatment of cold nuclear matter effects





VOLUME 89, NUMBER 25 PHYSICAL REVIEW LETTERS 16 DECEMBER 2002

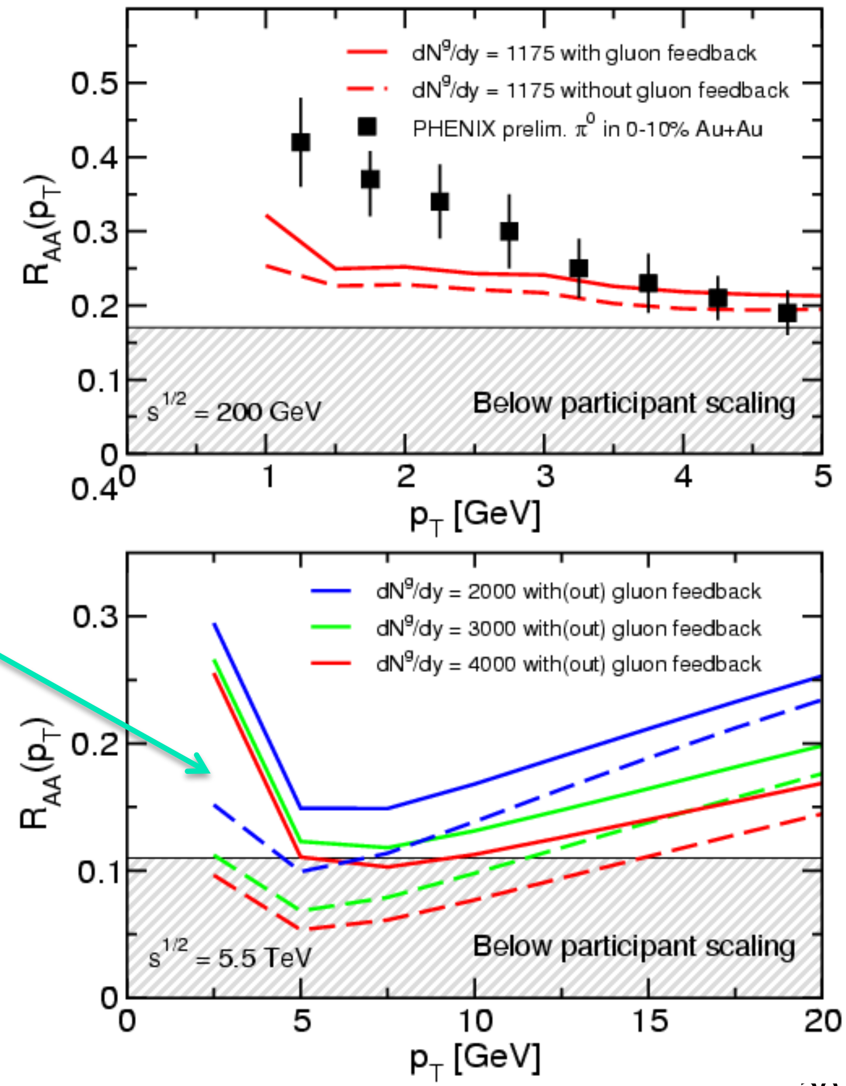
High-p_T Tomography of d + Au and Au + Au at SPS, RHIC, and LHC

Ivan Vitev[1,2] and Miklos Gyulassy[1]

Gluon Feedback to Single Inclusives

- Note that $N_{bin} / (N_{part}/2) = 0.11$

- The redistribution of the lost energy is very important at the LHC. 100% correction and p_T<15 GeV affected



- Can quickly eliminate some models or indicate very interesting new possibilities for modeling

Calculating the Meson Wave Function

- Relativistic Dirac equation

$$S' = S - \frac{3}{2}\frac{VS}{M_Q} - \frac{m}{M_Q}V \qquad \kappa = \begin{cases} -l+1, & j = l+1/2 \\ l, & j = l-1/2 \end{cases}$$

$$V' = S - \frac{1}{2}\frac{S^2}{M_Q} - \frac{m}{M_Q}S$$

Coulomb

$$V = -\xi\frac{1}{r},\ \xi = \frac{4}{3}\alpha_s$$

Linear

$$S = br$$

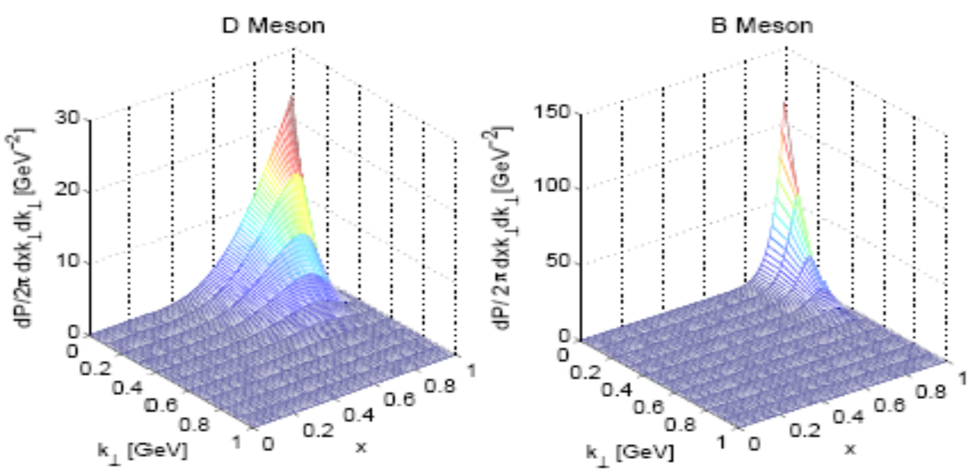
Reduces to:

$$\begin{bmatrix} \dfrac{dG}{dr} = -(\varepsilon - V' + S' + m)F - \left(\dfrac{k+1}{r} - \dfrac{b}{2M}\right)G \\ \dfrac{dF}{dr} = \left(\dfrac{k-1}{r} - \dfrac{b}{2M}\right)F + (\varepsilon - V' - S' - m)G \end{bmatrix}$$

Reduces to:

- Radial density: $\rho(r) \sim (F^2 + G^2)$

$^1S_0 \quad ^3S_1$ $\qquad D^0, \bar{D}^0, D^- D^+, D_s \ldots The^*, \ \ldots.\ Same\ \ for\ \ B$

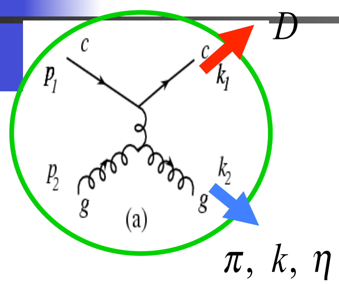


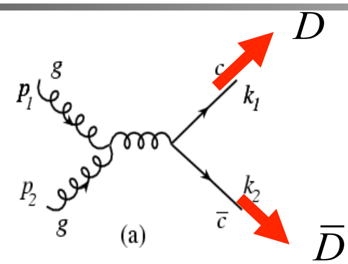
Boost with large $P^{+\,-}$ end up at the same longitudinal rapidity

$$|\ \psi\ (\Delta k_\perp, x)\ |^2 \sim Exp\left[-\frac{\Delta k_\perp^2 + 4m_Q^2(1-x) + 4m_q^2(x)}{4\Lambda^2 x(1-x)}\right]$$

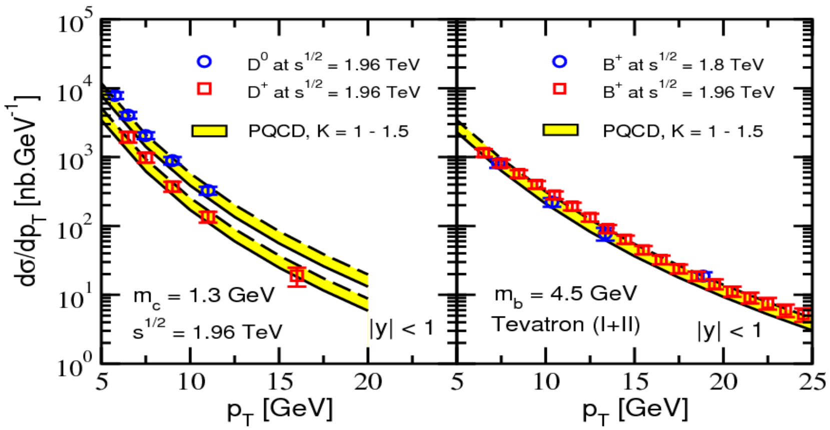
M. Avila, Phys. Rev. D49 (1994)

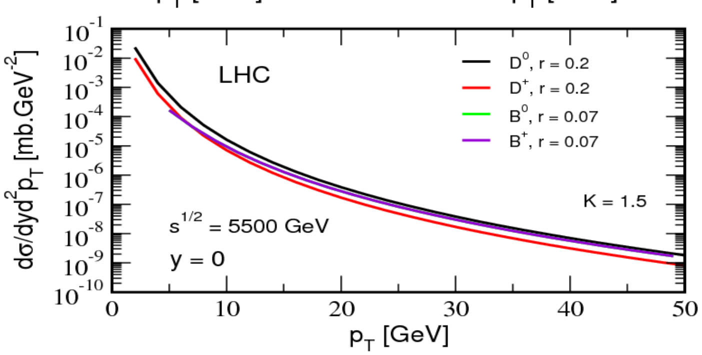
Heavy Quark Production and Correlations

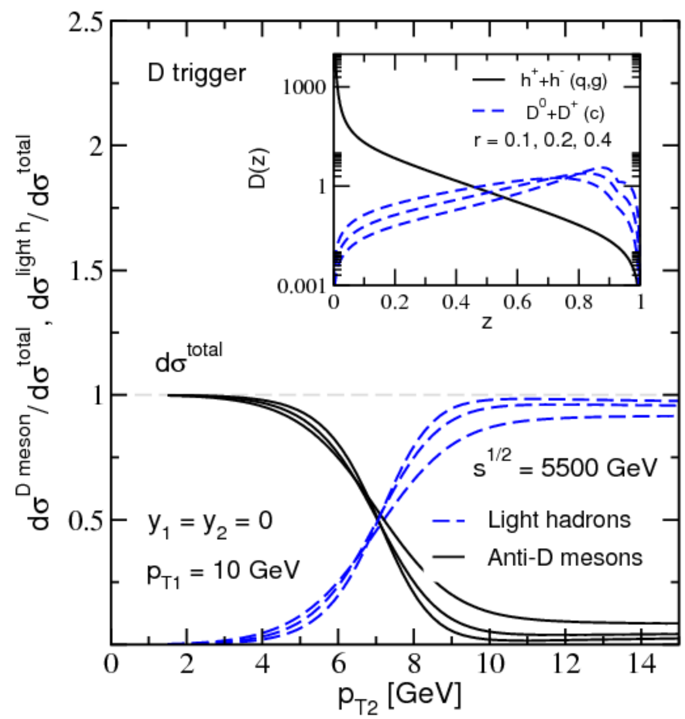




- Fast convergence of the perturbative series
- Possibility for novel studies of heavy quark-triggered (D and B) jets: hadron composition of associated yields







Quenching of Non-Photonic Electrons

- Full semi-leptonic decays of C- and B-mesons and baryons included. PDG branching fractions and kinematics. PYTHIA event generator

$$R_{AA}^{e^{\pm}}(p_T) = \frac{d\sigma_{AA}^{e^{\pm}} / dyd^2p_T}{\langle N_{\rm coll}\rangle d\sigma_{pp}^{e^{\pm}} / dyd^2p_T}$$

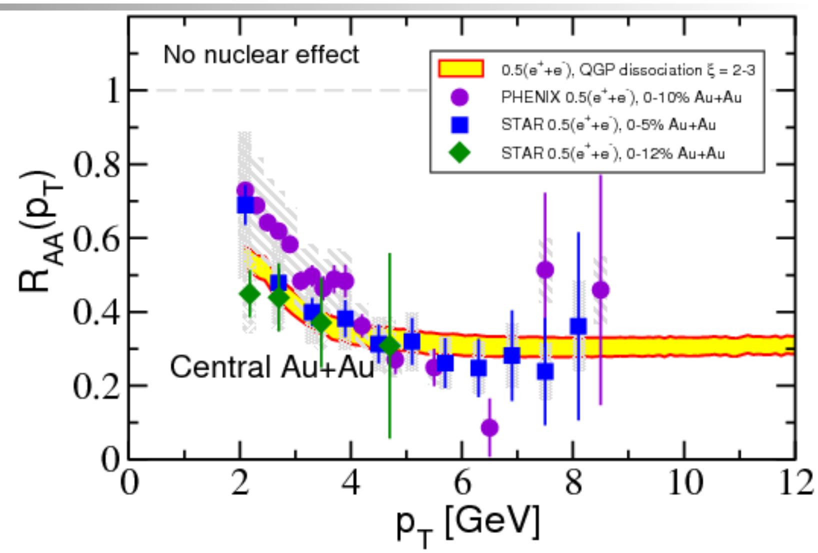
- Similar to light π^0, however, different physics mechanism

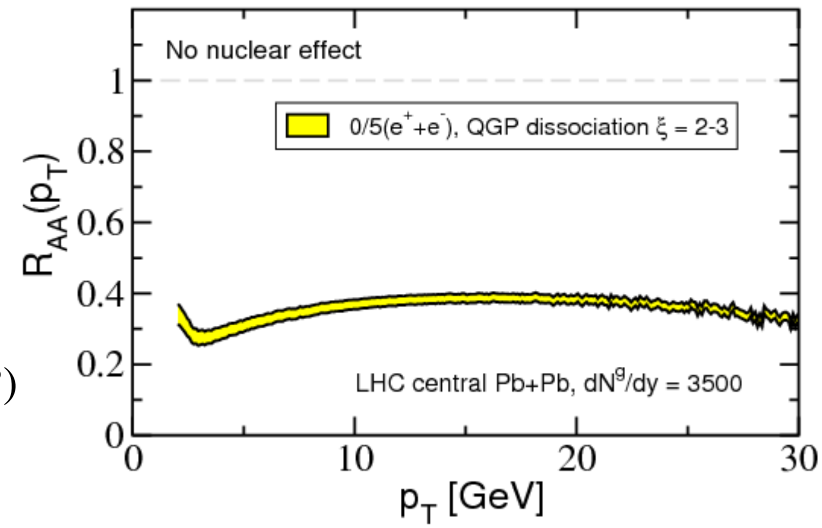
- B-mesons are included. They give a major contribution to $(e^+ + e^-)$

Note on applicability

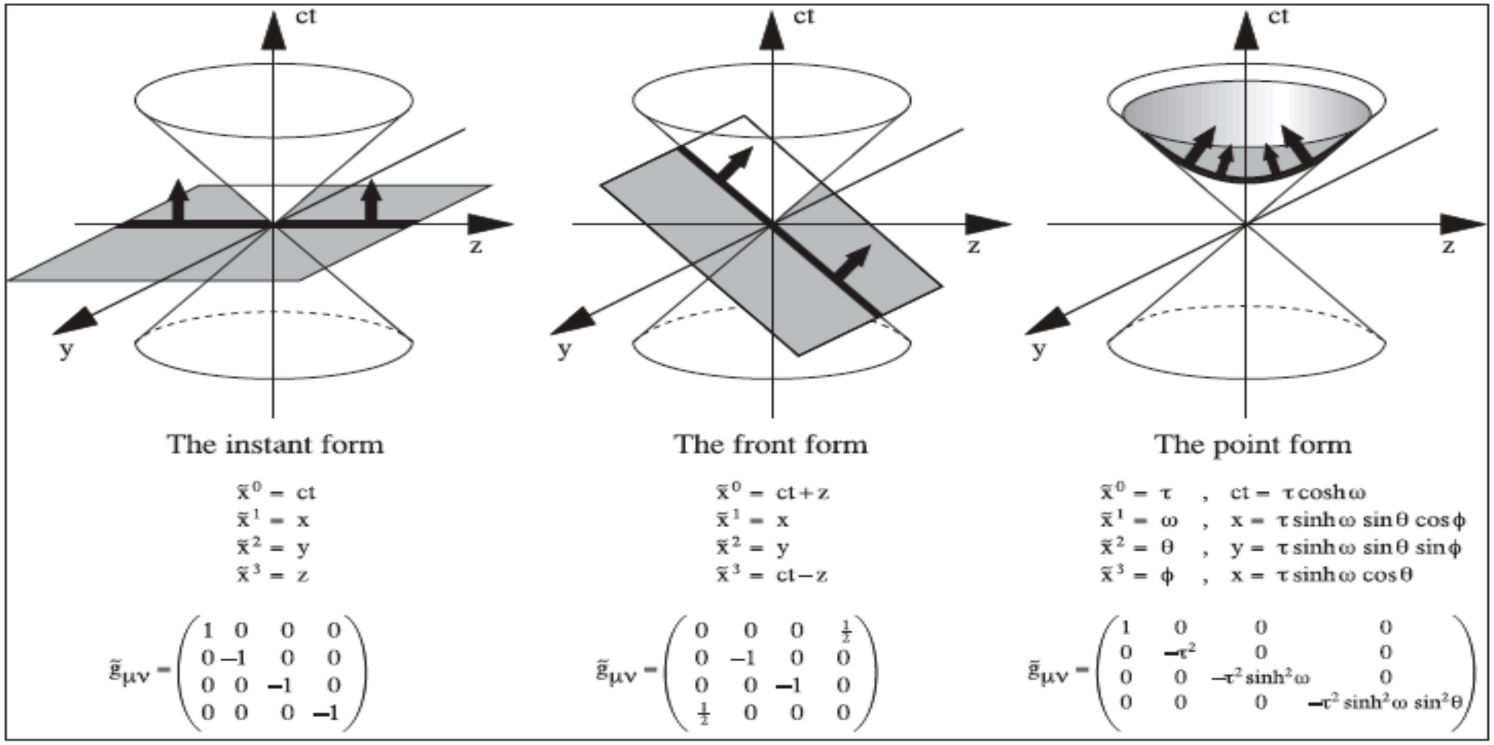
D-, B-mesons to $R_{AA}(D) = R_{AA}(B)$

$(e^+ + e^-)$ to 25 GeV





Light Front Quantization



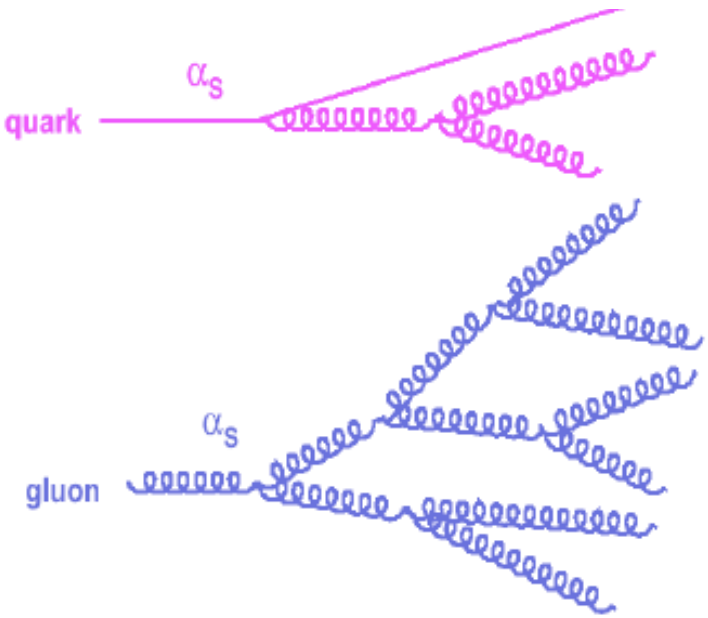
- Advantages of light front quantization: simple vacuum, the only state with $p^+=0$
- Full set of operators, commuting: $M^2 = 2p^+p^- - p_\perp^2,\ p^+,\ p_\perp$
  $S^2,\ S_z$

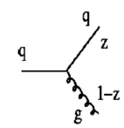
S.Brodsky, H.C.Pauli, S.Pinsky, Phys. Rep. (1998)

From Low to High Fock Components

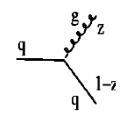
- Perturbative generation of the higher Fock states

$$dP_a = \frac{\alpha_s}{2\pi}\frac{d\rho^2}{\rho^2}\frac{d\phi}{2\pi}dzP_{a\to bc}(z)$$

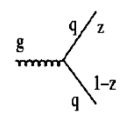




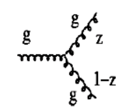
$$P_{qq}^{(1)}(x) = C_2(F)\left[(1+x^2)\left(\frac{1}{1-x}\right)_+ + \frac{3}{2}\delta(1-x)\right]$$



$$P_{gq}^{(1)}(x) = C_2(F)\,\frac{(1-x)^2+1}{x}$$



$$P_{qg}^{(1)}(x) = T(F)\left[(1-x)^2 + x^2\right]$$



$$P_{gg}^{(1)}(x) = 2C_2(A)\left[\frac{x}{(1-x)_+} + \frac{1-x}{x} + x(1-x)\right] + \left(\frac{11}{6}C_2(A) - \frac{2}{3}T(F)n_f\right)\delta(1-x),$$

At the QCD vertexes: conserve color, momentum, flavor, …

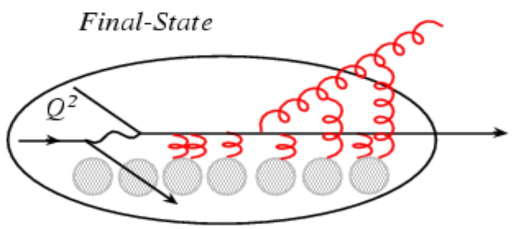
- The lowest lying Fock state (non-perturbative) – the most important

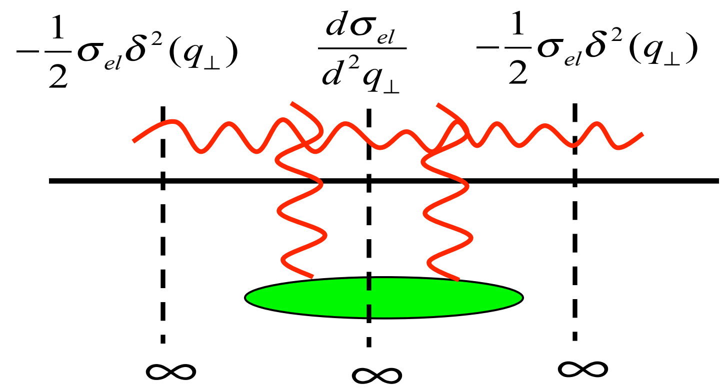
Correct quantum #s carry over to higher states

Medium-Induced Radiation: Theory

- Includes interference with the radiation from hard scattering

$$\hat{R}_n = \hat{D}_n^{\dagger}\hat{D}_n + \hat{V}_n + \hat{V}_n^{\dagger}$$





Number of scatterings **Momentum transfers**

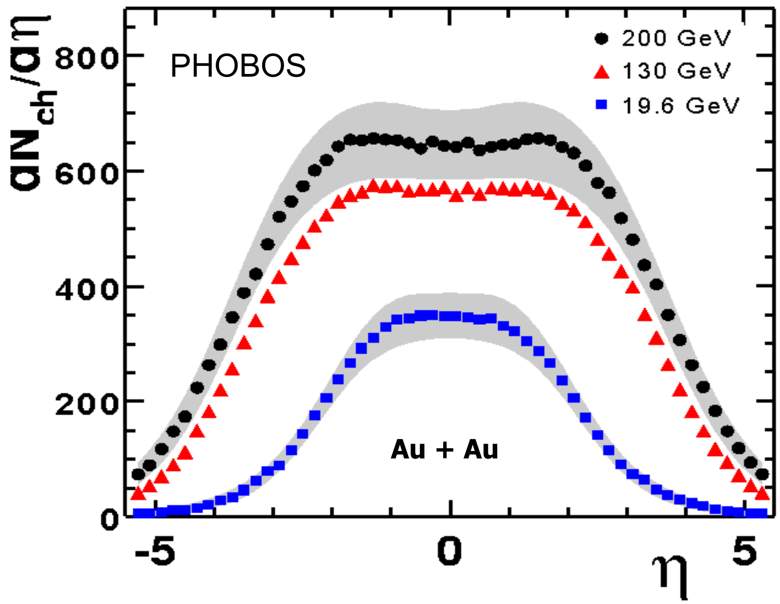
$$k^+ \frac{dN_g}{dk^+ d^2 k_\perp} = \sum_{n=1}^{\infty} k^+ \frac{dN^n{}_g}{dk^+ d^2 k_\perp} = \sum_{n=1}^{\infty} \frac{C_R \alpha_s}{\pi^2} \left[\prod_{i=1}^{n} \int_0^{L-\sum_{j=i+1}^{n}\Delta z_j} \frac{d\Delta z_i}{\lambda_g(z_i)} \int d^2 q_i \left(\frac{1}{\sigma_{el}} \frac{d\sigma_{el}}{d^2 q_i} - \delta^2(q_i) \right) \right]$$

$$\times \left[-2 C_{(1\ldots n)} \cdot \sum_{m=1}^{n} B_{(m+1\ldots n)(m\ldots n)} \left(\cos\left(\sum_{k=2}^{m} \omega_{(k\ldots n)} \Delta z_k \right) - \cos\left(\sum_{k=1}^{m} \omega_{(k\ldots n)} \Delta z_k \right) \right) \right]$$

Color current propagators

Coherence phases (LPM effect)

The Soft Medium



- **Local thermal equilibrium**
- **Local parton-hadron duality**
- **Gluon-dominated soft sector**
- **Bjorken expansion / approximate boost invariance**

$$\rho_{\exp}(\tau) = \frac{1}{A_\perp \tau}\frac{dN^g}{dy}, \qquad \frac{dN^g}{dy} = 1200, \quad A_\perp = 120 \;\; fm^2, \quad \tau_0 = 0.6 \;\; fm$$

$$\mu_D \approx gT, \;\; g = 2 - 2.5 \;\; (\alpha_s = \frac{g^2}{4\pi} = 0.3 - 0.5)$$

$$\sigma^{gg} = \frac{9\pi\alpha_s^2}{2\mu_D^2}, \quad \lambda_g = \frac{1}{\sigma^{gg}\rho}$$

$$\Rightarrow \;\; \rho_{\exp}(\tau_0) = 17 \;\; fm^{-3}$$

$$\rho_{theory}(T) = \# DoF \int_0^\infty \frac{1}{e^{p/T} - 1}\frac{4\pi p^2 dp}{(2\pi)^3} = \frac{\# DoF}{\pi^2}\varsigma[3] \times T^3$$

where $\# DoF = 2(polarization) \times 8(color), \;\; \varsigma[3] = 1.2$

	RHIC	LHC
T [MeV]	370	720
μ_D [GeV]	.75-1.	1.4-1.8
λ_g [fm]	.75-.42	.39-.25