Enhanced Dielectronic Recombination in Crossed Electric and Magnetic Fields

F. Robicheaux and M. S. Pindzola
Department of Physics, Auburn University, Auburn, Alabama 36849
(Received 5 May 1997)

The dependence of the dielectronic recombination cross section on crossed electric and magnetic fields is described. The enhancement of this cross section due to a static electric field is further increased when a magnetic field is added perpendicular to the electric field. Calculation of this field induced enhancement is presented for a realistic atomic model, and the mechanism for the enhancement is discussed. [S0031-9007(97)04035-0]

PACS numbers: 34.80.Kw, 32.60.+i, 32.80.Dz

Photorecombination is the inverse of photoionization and is the process by which a free electron simultaneously emits a photon and makes a transition to a bound state. It is the purpose of this paper to show that certain types of photorecombination processes can be greatly enhanced in static, crossed electric and magnetic fields. Besides being intrinsically interesting, this enhancement may affect the interpretation of recent experiments on photorecombination in static electric fields and affect the recombination rate in some tenuous astrophysical plasmas.

Burgess and Summers [1] suggested that under some circumstances the dielectronic recombination (DR) rate could be enhanced due to angular momentum redistribution during three-body collisions. DR is photorecombination that proceeds through the capture of an electron by an ion in an autoionizing state; the incident electron induces a transition in the ion but loses so much energy it is captured into a resonance state. Before the electron can regain the energy from the core electrons and leave the ion, a photon is emitted and the electron is captured into a bound state. Jacobs *et al.* [2,3] showed that ℓ redistribution of the autoionizing state due to plasma (electric) microfields can strongly enhance the DR rate.

The prototypical example for studying field effects is the DR of a Li-like ion. The initial state is a free electron, and the ion is in the $1s^2 2s$ configuration. The autoionizing state is $1s^2 2pn\ell$. The system is stabilized by the $2p$ electron emitting a photon, leaving the system in the $1s^2 2sn\ell$ bound state. The physical reason for enhanced DR in a static field can be understood in a time dependent picture. The electron is initially captured in an $n\ell$ state while simultaneously exciting the core. If an electric field is on, the Rydberg state will precess into states of higher angular momentum. Since the autoionization rate rapidly decreases with angular momentum, this has the effect of making photon emission more probable.

In the more traditional time independent picture, the contribution of a resonance ρ to the DR rate is proportional to $\Gamma_{a,\rho}\Gamma_{R,\rho}/(\Gamma_{a,\rho} + \Gamma_{R,\rho})$ (i.e., proportional to the probability to be captured into the state times the branching ratio for photon emission). The total photorecombination rate is proportional to the sum over all resonances ρ of $\Gamma_{a,\rho}\Gamma_{R,\rho}/(\Gamma_{a,\rho} + \Gamma_{R,\rho})$. [Each resonance is only counted once, but often the degeneracy of levels is utilized to simplify calculations. For example, in zero field, J and M are good quantum numbers so the resonance width and position are calculated independent of M and the rate is replaced by $(2J_\rho + 1)\Gamma_{a,\rho}\Gamma_{R,\rho}/(\Gamma_{a,\rho} + \Gamma_{R,\rho})$ and only one M state is counted.] The radiative decay rate of a resonance approximately equals the radiative rate of the excited core state, $\Gamma_{R,\rho} = \Gamma_R$. Thus a useful measure of the DR rate is $\Gamma_{a,\rho}/(\Gamma_{a,\rho} + \Gamma_{R,\rho})$. This ratio equals one when $\Gamma_{a\rho} \gg \Gamma_R$ and zero when $\Gamma_{a,\rho} \ll \Gamma_R$. When summed over all resonances this ratio may be thought of as the effective number of states that contribute to the total DR rate,

$$N \equiv \sum_\rho \Gamma_{a,\rho}/(\Gamma_{a,\rho} + \Gamma_{R,\rho}). \tag{1}$$

The reason a static electric field can increase N is that $\Gamma_{a,\rho}$ is a very rapidly decreasing function of ℓ. In Fig. 1 the ratio of the autoionization rate to the radiative rate, $\gamma = \Gamma_{a,\ell}/\Gamma_R$, is plotted for $n = 30$ states of the model discussed below. If we define $\ell_{\rm cut}$ to be the ℓ where $\Gamma_{a,\rho} \simeq \Gamma_R$ within an n-manifold, then $N \simeq \sum_{\ell=0}^{\ell_{\rm cut}}(2\ell + 1) = (\ell_{\rm cut} + 1)^2$ within the n-manifold. When an electric field is turned on, the high-ℓ states that contribute nothing to N begin to mix with the lower ℓ states and thus acquire larger autoionization rates. At high fields

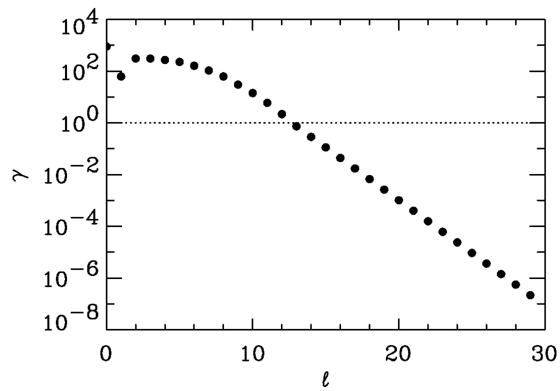

FIG. 1. The ratio of the autoionization rate to the radiative rate as a function of ℓ for an $n = 30$ manifold.

all of the ℓ's mix together, and under the best circumstances all states with $|m| \leq \ell_{\text{cut}}$ contribute to the DR rate. For a constant field in the z direction, m is a good quantum number, thus states with different m do not mix. Within an n-manifold $N \simeq \sum_{m=-\ell_{\text{cut}}}^{\ell_{\text{cut}}}(n - |m|) = n(2\ell_{\text{cut}} + 1) - \ell_{\text{cut}}(\ell_{\text{cut}} + 1)$. The increase in N over the field-free case is $(2\ell_{\text{cut}} + 1)(n - \ell_{\text{cut}} - 1)$. This increase can be substantial. For example, if $\ell_{\text{cut}} = 12$ and $n = 30$, then in zero field $N \simeq 169$ and in a strong electric field $N \simeq 594$, an increase in the recombination rate by a factor of 3.5.

Calculations of DR in constant electric fields can be performed almost without approximation [4–6]. Experiments have also been performed [7–11]. The agreement between the experiments and theory is qualitative, with the size of the enhancement being in rough agreement. This situation has led us to consider possible reasons for the differences between calculated and measured photorecombination rates in static electric fields. One possible source of this discrepancy is that the experiments are *not* performed in pure electric fields. There are also static magnetic fields present (180 G in Ref. [9], 24 G in Ref. [10], and 300 G in Ref. [11]). Since the n levels in the experiment are in the range 20–40 with a residual charge on the ion $\gtrsim 3$, the presumed wisdom was that a magnetic field of this size would have almost no effect on the total DR rate. However, it is shown below that, if the magnetic field is perpendicular to the electric field, then even very small fields may strongly enhance this rate.

The field part of the Hamiltonian for the Rydberg electron equals

$$H_{\text{field}} = bL_x + fz, \quad (2)$$

where $b = B/(4.701 \times 10^9\ \text{G})$ and $f = F/(5.141 \times 10^9\ \text{V/cm})$ are the magnetic and electric fields. The diamagnetic term [equal to $b^2(y^2 + z^2)/2$] has been dropped because it has a negligibly small effect on the dynamics. If the electric field is zero, then the coordinate system can be rotated so the magnetic field is in the z direction. In this case, the effect of H_{field} is to slightly shift the energies of the resonances by an amount bm (where m is the azimuthal quantum number); since there is no mixing between levels, the recombination rate as measured by N does not change at all. However, if neither b nor f is zero, then the fz term will mix the ℓ states and the bL_x term will mix the m states. In principle, this could lead to $N \simeq n^2$ states within an n-manifold participating in the recombination. This consideration suggests that the importance of the magnetic field should be investigated.

Simple scaling arguments could lead to incorrect assumptions about whether or not the magnetic field will make an important contribution. For example, within an n-manifold, the size of the matrix elements of the magnetic field scale the same as bn, whereas the electric field term scales the same as fn^2/Q, where Q is the residual charge of the ion; this implies that the effect of the magnetic field will become less important at higher n; since the high n states are the dominant contributors to recombination in static electric fields, this argument suggests that magnetic fields will not play a large role in the total recombination rate. A different argument leads to completely different results. The magnetic field acts as a mechanism to couple the Stark split states of one m to that of $m \pm 1$. This coupling scales the same as bn. The strength of the mixing between states of different m depends on the coupling strength and on the difference of the energy levels from m to $m \pm 1$. The smallest difference $\Delta E/\Delta m$ scales the same as fn/Q. Since the coupling and the energy difference scale in the same way, the amount of mixing remains roughly constant with n.

To this point, the discussion has centered on the outer electron's quantum numbers. This is somewhat misleading because the outer electron's quantum numbers couple to those of the core electrons. For example, in the Li-like case, the autoionizing states may be represented as $[(2p_j, n\ell)Ks_o]JM$ (i.e., the j of the core is coupled to the ℓ of the Rydberg electron to give K, which is coupled to the spin of the Rydberg electron to give the total angular momentum J and total azimuthal quantum number M). A proper treatment would show the effect of the fields on these full states. This has been done for DR in constant electric fields [5]. For crossed electric and magnetic fields, the resulting Hamiltonian matrix is extremely large, even when restricting the states to one n-manifold. Before performing studies on the full system (core plus Rydberg electron), we have constructed a slightly simpler atomic model which has most of the features of a real system.

In this model, all of the angular momentum, except the orbital angular momentum of the Rydberg electron, will be set to zero. Thus the autoionizing state is completely specified by $n\ell m$. For the specific calculation presented here, the charge was set to $Q = 10$. The energy of this state is given by $E_{n\ell} = -Q^2/(2n^2) + Q^2\mu_\ell/n^3$, where $\mu_\ell < 0.1$ is the quantum defect in the ℓth partial wave. To ensure realistic ℓ dependence of the quantum defect, the μ_ℓ was chosen to be the average of the quantum defect of the $(2p\epsilon\ell)^1L$ channels, where $L = \ell \pm 1$. The autoionization rate is independent of m and was chosen to be $2Q^2K_{ij}^2/(\pi n^3)$, where K is the K matrix and the ith channel is the $[2s\epsilon(\ell + 1)]^1L$ channel and the jth channel is the $(2p\epsilon\ell)^1L$ channel, where $L = \ell + 1$. The radiative decay rate was chosen to be 2×10^{-8} a.u. Finally, the eigenstates in the field were obtained by diagonalizing the Hamiltonian within each n-manifold separately because the field strengths are not large enough to cause n mixing. Because each $n\ell m$ state decays to a different continuum, the decay rate of the ρ eigenstate equals

$$\Gamma_{a,\rho} = \sum_{n\ell m} \Gamma_{a,n\ell m} U_{n\ell m,\rho}^2, \quad (3)$$

where $U_{n\ell m,\rho}$ is the eigenvector of the ρ state.

In Fig. 2 we have plotted the effective number of states that participate in the photorecombination rate, Eq. (1),

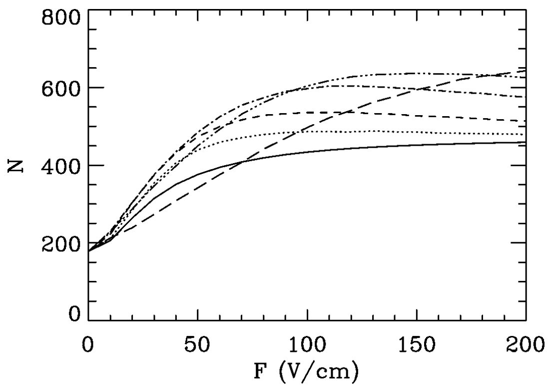

FIG. 2. The effective number of states in the $n = 30$ manifold participating in recombination as a function of static electric field strength for $B = 0$ (solid line), $B = 50$ G (dotted line), $B = 100$ G (dashed line), $B = 200$ G (dot-dashed line), $B = 300$ G (dot-dot-dot-dashed line), and $B = 600$ G (long dashed line).

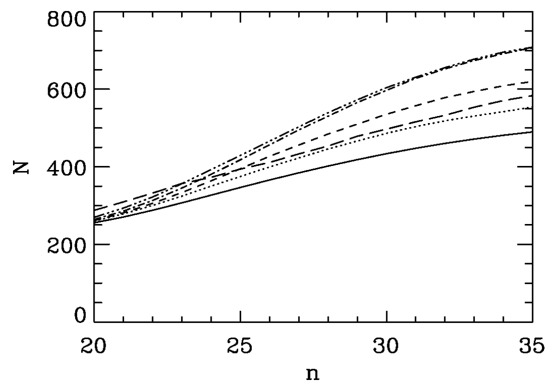

FIG. 3. Same as Fig. 2 but as a function of n for $F = 100$ V/cm.

for the $n = 30$ manifold. The largest that N can be is $n^2 = 900$. N is plotted versus the constant electric field strength for $B = 0$, 50, 100, 200, 300, and 600 G. It is clear that the enhancement over the zero field value, $N \simeq 179$, is strongly dependent on the magnetic field strength over almost the whole range of F. The effective number of states that participate in DR is $N \simeq 376$ for $B = 0$, $F = 50$ V/cm and $N \simeq 434$ for $B = 0$, $F = 100$ V/cm. When the magnetic field is increased to 200 G, the number of states increases to $N \simeq 485$ for $F = 50$ V/cm and $N \simeq 597$ for $F = 100$ V/cm.

Several features of this figure can be understood in a qualitative way. The first aspect is that the zero field number of states that participate in DR implies $\ell_{\rm cut} = 12$. Another feature is that all curves have the same $F = 0$ value; this is because the field Hamiltonian cannot mix levels if $F = 0$, thus the autoionization rates are unchanged. Another feature is that the high-F limit for $B = 0$ is not quite as high as might be expected; the simple counting argument suggests that the high-F limit should be $N = 594$. The reason this value is not reached is that the $10 \leq |m|$ states have such small autoionization rates that mixing with all $\ell \geq \ell_{\rm cut}$ gives states with autoionization rates less than Γ_R: The autoionization rates become too diluted. Another interesting feature is that the $B \neq 0$ curves have a maximum value. This feature probably arises because as F increases the smallest energy difference between different manifolds increases, which tends to reduce the mixing between the different m states.

The total N summed over all manifolds $20 \leq n \leq 35$ is very similar to Fig. 2. At zero field, $N \simeq 2998$. At $F = 100$ V/cm, the effective number of states increases to $N \simeq 6149$ for $B = 0$ and $N \simeq 8116$ for $B = 300$ G. In Fig. 3, the effective number of states participating in photorecombination for $F = 100$ V/cm is plotted versus the n-manifold. This figure shows the relative importance of the perpendicular magnetic field. As n increases, $N^{B=300}/N^{B=0}$ rapidly increases from $n = 20$ to 30 then becomes relatively constant. But since the high n states dominate DR in fields for $\Delta n = 0$ excitations, this figure argues that a perpendicular magnetic field may be important.

In order to show the effect of the fields on the autoionization rates, the ratio $\gamma_{a,\rho} = \Gamma_{a,\rho}/\Gamma_R$ is plotted as a function of $\Delta E_\rho = E_\rho + Q^2/(2n^2)$ for the $n = 30$ states; E_ρ is the eigenenergy of the ρth state obtained by diagonalizing the atomic plus field Hamiltonian. In Fig. 1, this ratio is plotted versus ℓ. In Fig. 4, $F = 100$ V/cm, $B = 0$; in Fig. 5, $F = 100$ V/cm, $B = 300$ G. The horizontal line is where the autoionization rate equals the radiative rate; states well below this line contribute nothing to the recombination cross section. In Fig. 1, the energy and rate only depend on ℓ so at each point there is a $2\ell + 1$ degeneracy. In Fig. 4, the states with m are degenerate with states $-m$. The very low-ℓ states, $\ell \leq 4$, are not shown in the figure since they fall outside the energy range shown because of their relatively large quantum defects; these states don't play a large role in the dynamics since they only weakly mix with the higher-ℓ states because of the large energy difference. In Fig. 5, there is no degeneracy. It is clear after comparing Figs. 4 and 5 that the magnetic field has moved a large number of states above

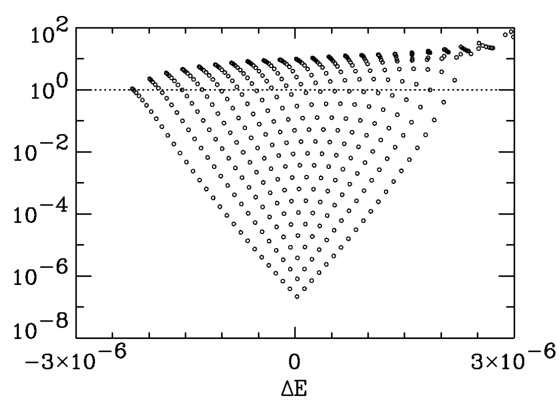

FIG. 4. The ratio of the autoionization rate to the radiative rate versus the difference in energy of the resonance from the $E = -Q^2/(2n^2)$ value. $F = 100$ V/cm, $B = 0$, and $n = 30$.

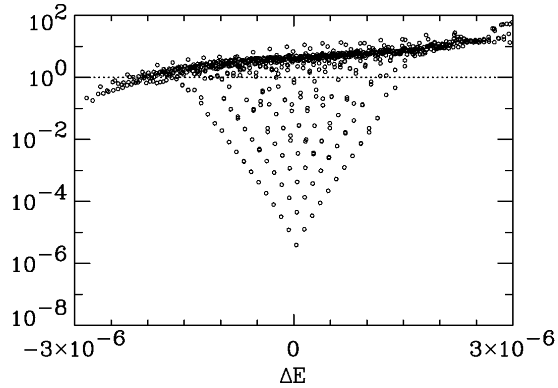

FIG. 5. Same as Fig. 4 but for $F = 100$ V/cm, $B = 300$ G, and $n = 30$.

the $\gamma = 1$ line which will increase the total recombination rate. In zero fields, $N = 179$; with $F = 100$ V/cm and $B = 0$, $N = 434$; with $F = 100$ V/cm and $B = 300$ G, $N = 603$.

We have performed similar calculations for recombination of C^{3+} (only including the angular momentum of the Rydberg electron) summing from $n = 10$ to 35. The results are $N = 2060$ for $F = 0$, $B = 0$; $N = 6190$ for $F = 12$ V/cm, $B = 0$; $N = 8050$ for $F = 12$ V/cm, $B = 24$ G; $N = 6210$ for $F = 30$ V/cm, $B = 0$; and $N = 9430$ for $F = 30$ V/cm, $B = 180$ G. These results suggest that the $B = 0$ theoretical results should be multiplied by $8050/6190 = 1.30$ when comparing to Ref. [10] and by $9430/6210 = 1.52$ when comparing to Ref. [9]. This brings experiment and calculations into much better agreement.

The only previous study of the effect of magnetic fields on the DR rate is Ref. [12]. In this paper, they considered how the diamagnetic term would affect the rate in strong magnetic fields, 5–100 T; these fields are over 100 times stronger than those considered here. Since the diamagnetic term mixes ℓ (like the static electric field), the expectation was that this would increase N. However, even fields up to 10 T had relatively little effect on the rate. This shows that the breaking of the cylindrical symmetry by having crossed electric and magnetic fields is necessary for the enhancement; magnetic fields parallel to electric fields have little effect on the recombination rate unless the magnetic fields are very strong.

In this paper, we have shown that the DR rate in crossed electric and magnetic fields can be substantially enhanced (30–50)% in a realistic atomic model over the rate with only a static electric field. We have thought of two possible mechanisms which may invalidate this conclusion for a real atom. The first possibility is that the angular momenta ignored here will conspire to suppress mixing. The Zeeman interaction with the spin of the Rydberg electron and the total angular momentum of the core electrons has been ignored; however, it seems most likely that these interactions will provide shifts of n-manifolds and not affect the mixing very strongly. The second mechanism is the approximation inherent in the assumption that the DR rate is proportional to $\Gamma_{a,\rho}\Gamma_{R,\rho}/(\Gamma_{a,\rho} + \Gamma_{R,\rho})$ for an individual resonance ρ. This assumption will fail if two states mostly radiate to the same final state *and* their energy difference is less than their total width. In Fig. 5, it is clear that there are many states whose separation is less than their width. However, in the photon emission process, the core goes from an excited state to a final state, leaving the Rydberg electron's wave function almost unchanged. This means that all of the autoionizing states photodecay to different final states when states of different n-manifolds do not mix; the pattern of decay becomes more complicated when the fields are strong enough to mix states from different n-manifolds. Our calculations strongly suggest that weak magnetic fields cannot be ignored in DR if there is a perpendicular electric field; the more difficult task of showing this effect in a real atomic system remains to be addressed.

We acknowledge an insightful discussion with V. L. Jacobs about DR in static fields. This work was supported by DOE EPSCoR Grant No. DE-FC02-91ER75678.

[1] A. Burgess and H. P. Summers, Astrophys. J. **157**, 1007 (1969).
[2] V. L. Jacobs, J. Davis, and P. C. Kepple, Phys. Rev. Lett. **37**, 1390 (1976).
[3] V. L. Jacobs and J. Davis, Phys. Rev. A **19**, 776 (1979).
[4] K. LaGattuta and Y. Hahn, Phys. Rev. Lett. **51**, 558 (1983).
[5] C. Bottcher, D. C. Griffin, and M. S. Pindzola, Phys. Rev. A **3**, 860 (1986).
[6] K. LaGattuta, I. Nasser, and Y. Hahn, Phys. Rev. A **33**, 2782 (1986).
[7] D. S. Belic, G. H. Dunn, T. J. Morgan, D. W. Mueller, and C. Timmer, Phys. Rev. Lett. **50**, 339 (1983).
[8] A. Müller, D. S. Belic, B. D. DePaola, N. Djurić, G. H. Dunn, D. W. Mueller, and C. Timmer, Phys. Rev. Lett. **56**, 127 (1986).
[9] P. F. Dittner, S. Datz, P. D. Miller, P. L. Pepmiller, and C. M. Fou, Phys. Rev. A **35**, 3668 (1987).
[10] D. W. Savin, L. D. Gardner, D. B. Reisenfeld, A. R. Young, and J. L. Kohl, Phys. Rev. A **53**, 280 (1996).
[11] T. Bartsch, A. Müller, W. Spies, J. Linkemann, H. Danared, D. R. DeWitt, H. Gao, W. Zong, A. Schuch, A. Wolf, G. H. Dunn, M. S. Pindzola, and D. C. Griffin, preceding Letter, Phys. Rev. Lett. **79**, 2233 (1997).
[12] W. A. Huber and C. Bottcher, J. Phys. B **13**, L399 (1980).