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Possible mechanism for enhancing the trapping and cooling of antihydrogen

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We propose a usage of microwave radiation in a magnetic trap for improving the cooling and trapping of cold antihydrogen atoms which are initially produced in high magnetic moment states. Inducing transitions toward lower magnetic moments near the turning points of the atom in the trap, followed by spontaneous emission, should enhance the number of trappable atoms. We present results of simulations based on a typical experimental condition of the antihydrogen experiments at CERN. This technique should also be applicable to other trapped high magnetic moment Rydberg atoms.

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It is well known that the discrete symmetries of parity (P) and charge conjugation (C) are violated in weak interactions. However it is strongly believed that the laws of physics are invariant under the product of the parity transformation, charge conjugation, and time reversal [1] (CPT theorem). The CPT invariance predicts, among other things, that an atom and its charge conjugate, the antiatom, should have the same spectra. Atomic hydrogen, both in its hyperfine maser and in the 1S-2S transitions, allows measurements with an impressive precision, having yielded the best measured fundamental constant, the Rydberg constant [2]. The comparison of the hydrogen and antihydrogen ($\bar{\mathrm{H}}$) spectra can, thus, be a sensitive probe for CPT violation. Due to these properties and the added motivation that the abundance asymmetry between matter and antimatter in our Universe remains unexplained, three groups—ALPHA [3], ATRAP [4] and ASACUSA [5]—are setting up to study cold $\bar{\mathrm{H}}$ at CERN's antiproton decelerator (AD).

The quality factor imposed by the natural linewidth in the 1S-2S transition is 2×10^{15}, opening the possibility for precisions reaching a few parts in 10^{18} if the line could be split to parts in 10^3. For such a high-precision comparison between hydrogen and $\bar{\mathrm{H}}$ in the 1S-2S transition, we believe that ultracold trapped atoms will have to be employed [6] either in a trap, or as a source to an ultracold atomic beam, or in an atomic fountain configuration [7]. The collaborations at CERN's AD, pursuing different routes, have recently deployed multispecies traps for holding the charged particles and the neutral $\bar{\mathrm{H}}$ atoms. Cooling these atoms and producing the largest quantities of these cold and trapped atoms will be central to reaching high levels of precision.

It is known that hydrogen is not amenable to laser cooling. A one-dimensional cooling was achieved in atoms already trapped [8] requiring minutes to thermalize the other dimensions through collisions. This is why research with trapped hydrogen has so far relied on ^{3}He-^{4}He dilution refrigerators making use of the unique low binding energy of hydrogen onto superfluid helium. Unfortunately, $\bar{\mathrm{H}}$ would not be a natural candidate for this process due to annihilation. Kleppner [9] even proposed and suggested calculations for the sympathetic cooling of $\bar{\mathrm{H}}$ with trapped hydrogen but the efficiency of this process [10] turned out not very promising. Therefore a scheme that could enhance the number of initially trapped atoms would be highly desirable.

A hydrogen atom in a magnetic trap in the 1S_d low-field seeking state experiences a trapping potential equivalent to 0.67 K/T. Excited atoms, with large principal quantum numbers in the range $n=20-50$ can experience much higher potentials, proportional to their magnetic moment, which is approximately proportional to the projection of the angular momentum along the magnetic B field, m. The diamagnetic Hamiltonian term actually increases the depth of the trapping potential for the low-field seeking atoms. With the present combination schemes to produce large quantities of $\bar{\mathrm{H}}$ atoms from cold antiprotons and positrons, we expect that the majority of the atoms that remain trapped would have started with a large negative component of the magnetic moment along the magnetic field. For example, an $\bar{\mathrm{H}}$ atom at the bottom of a magnetic trap potential, with a trap depth of 1 T and with an energy corresponding to a temperature of 16 K, will be initially trapped only if $m<-24$. These highly excited atoms will slowly decay via spontaneous emission, which will be more probable near the turning points where they spend longer times. In this process the atom may change its magnetic moment which could lead either to a higher or to a lower potential energy, but the lowering of the magnetic moment is favored [11] on average. This results in an automatic cooling [12,13] by spontaneous emission. A stimulation of this cooling process was attempted with long-wavelength lasers [14], but since the typical photon energy is able to induce transitions changing the principal quantum number, n, without an upper bound, upward transitions were stimulated as much as downward transition.

In this Rapid Communication we propose to use low power microwaves resonant near the trap edges to enhance the transition of the atom to a lower magnetic moment state. This, together with spontaneous emission, leads to a larger number of trapped atoms and at the same time to colder samples as our simulations demonstrate. The weak microwave field ($\sim$0.1 V/cm) considered here will not cause transitions between adjacent n levels, for the range of interest ($n<50$). Very high n-state atoms, typically $n>50$, are easily ionized and do not escape the mixing positron-antiproton plasma due to the high electric fields present and, therefore, are not relevant to our discussion.

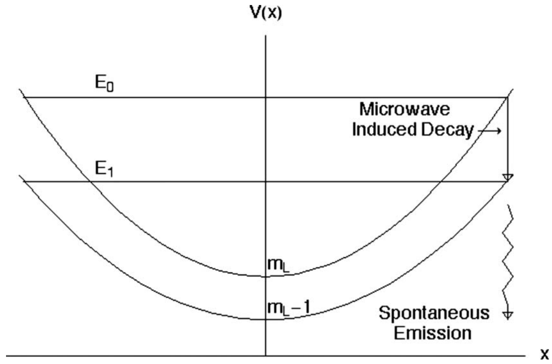

FIG. 1. Illustration of the basic cooling process. A hydrogen atom, initially at state m and energy E_0, would lose average kinetic energy in the trap if it decays to state $m-1$ at the trap edges. Other ingredients, such as spontaneous emission and reversed transitions are discussed in the text.

There are many physical processes in the $\overline{\mathrm{H}}$ traps that are not included in our simulations. The two main ones are collisions of Rydberg $\overline{\mathrm{H}}$ with the positrons and antiprotons in the trap and the absorption of blackbody radiation. The microwave transitions we propose will tend to decrease the negative effect of collisions. For example, every time the $\overline{\mathrm{H}}$ goes through the positron plasma the most likely process is ℓ, m changing collisions or charge exchange. If the $\overline{\mathrm{H}}$ is in a large $|m|$ state, these processes will tend to *decrease* the magnetic moment. Since the positron plasma will be near a magnetic minimum, this decrease in magnetic moment will lead to a decrease in trap depth and an increased likelihood of annihilation on the wall. Since the microwaves tends to decrease $|m|$, the atoms are more likely to radiate to lower n before reaching the positrons *and* are more likely to have a collision that increases $|m|$ if they reach the plasma. The blackbody radiation effect will depend on the temperature and properties of the trap. $\Delta n = \pm 1$ transitions correspond to an energy of ~ 5 K at $n=40$ and ~ 12 K at $n=30$. If the blackbody radiation is at 4 K for a liquid Helium cooled trap, then the spontaneous radiation will typically be much faster than blackbody transitions because the spontaneous decays typically involve $\Delta n > 1$ when the microwaves are on.

For convenience, the discussions that follow consider hydrogen instead of $\overline{\mathrm{H}}$ atoms. The process works as depicted in Fig. 1. An atom at rest at the turning point would lose maximum potential energy if it makes a transition to a lower magnetic moment, leading to temporary cooling, or lowering of its average kinetic energy in its new trapped state. A reversed transition would have the opposite effect. However, the higher the atom gets excited by the microwave the more it tends toward a circular state and the longer it takes to spontaneously decay. The result is that spontaneous emission complements automatically the microwave induced transitions favoring an overall cooling and it provides a permanent dissipation for the cooling process [15]. It is worth noticing that in the high magnetic fields present in these traps, the diamagnetic Hamiltonian mixes the different L states, as we discuss below. The overall process can be tuned by the proper use of the microwave frequency. While for a single atom the best situation would be to reach resonance only at its turning point, with a thermal sample one has to ensure that a large fraction of the atoms interact with the microwave. For an initially thermal sample, one can determine the optimum microwave frequency and necessary power by Monte Carlo simulation on approximate models. Our simulations clearly demonstrate a net cooling effect.

Let us define M, $\vec{R}$, $\vec{P}$ as the total mass, the center of mass position and the center of mass momentum of the atom, respectively and μ, $\vec{r}$, $\vec{p}$, $\vec{L}=\vec{r}\times\vec{p}$ as the reduced mass, the relative position, the relative momentum and the relative angular momentum of the electron, respectively. The total Hamiltonian of the hydrogen atom inside a magnetic trapping field $\vec{B}(R)$ in the z direction and subjected to the action of a microwave field $\vec{E}(t)$ is, in SI units,

$$H = H_0 + \frac{P^2}{2M} + \gamma(\vec{R})(L_z + 2S_z) + H_1(\vec{R}) + H_{MW}, \qquad (1)$$

where $\gamma(\vec{R})=eB(\vec{R})/(2\mu)$, S_z is the z component of the electron spin, the Coulomb internal Hamiltonian $H_0=p^2/(2\mu) - Ke^2/r$, with $K=1/(4\pi\epsilon_0)$, the diamagnetic term $H_1(R) = (\mu/2)\gamma(\vec{R})^2(x^2+y^2)$, and the microwave interaction $H_{MW} = e\vec{r}\cdot\vec{E}(t)$.

In the simulated trap the magnetic field energy will be low enough so that the internal energies will be approximately given by $E_n=-e^2/(2n^2a_0)+H_d(R)$, with [16]

$$H_d = \frac{\mu n^2 a_0^2}{4}\gamma(\vec{R})^2[n^2 + 3 + (L_z^2 + 4\vec{A}^2 - 5A_z^2)/\hbar^2], \qquad (2)$$

where a_0 is the Bohr radius and $\vec{A}$ is the Runge-Lenz vector, given by

$$\vec{A} = \frac{1}{(-2\mu E_n)^{1/2}}\left(\frac{(\vec{p}\times\vec{L} - \vec{L}\times\vec{p})}{2} - \frac{\mu K e^2 \vec{r}}{r}\right). \qquad (3)$$

To investigate the efficiency of this microwave induced cooling process, we performed simulations that used a mixture of classical and quantum mechanical ideas. In this model, the center of mass motion of the atom is computed using classical equations of motion with the force arising from the changing magnetic field. As in Ref. [12], we used realistic three-dimensional magnetic fields matching the trapping magnet used in the ALPHA collaboration [3] which is an octupolar variation of a Ioffe-Pritchard trap. This choice for an octupolar trap is dictated by requirements for the stability of the charged particles Penning trap, which cannot afford much field asymmetry in the region where the particles are trapped (Ref. [3], and references therein). The force acting in the center of mass of the atom is, neglecting the contribution from $H_1(R)$ to the potential, given by $-e/(2\mu)\vec{\nabla}[\vec{L}(t)\cdot\vec{B}(\vec{R})]$ and it was computed numerically. The average value of $\vec{L}(t)$ in a magnetic field and microwave field can be calculated using the Heisenberg equations of the motion for the operators $\vec{L}$ and $\vec{A}$. The microwave is treated throughout the trajectory. Within a given n-manifold we find

$$\frac{d\vec{L}}{dt} = \frac{e}{2\mu}\vec{B}\times\vec{L} + \frac{3ea_0n}{2\hbar}\vec{E}\times\vec{A} + \frac{e^2a_0^2n^2}{8\mu\hbar^2} \times [(\vec{B}\cdot\vec{L})(\vec{B}\times\vec{L}) - 5(\vec{B}\cdot\vec{A})(\vec{B}\times\vec{A})], \quad (4)$$

$$\frac{d\vec{A}}{dt} = \frac{e}{2\mu}\vec{B}\times\vec{A} + \frac{3ea_0n}{2\hbar}\vec{E}\times\vec{L} + \frac{e^2a_0^2n^2}{8\mu\hbar^2} \times [(\vec{B}\cdot\vec{L})(\vec{B}\times\vec{A}) - 5(\vec{B}\cdot\vec{A})(\vec{B}\times\vec{L}) + 4B^2(\vec{A}\times\vec{L})]. \quad (5)$$

In our simulation the magnetic trap had a minimum field of ~1.1 T and the magnetic field at the wall was ~2.28 T (an ideal case in the ALPHA experiment [3]); thus, the ground-state well depth was ~0.8 K. In the experiments realized so far, the m distribution of the $\bar{\mathrm{H}}$s is not known. Atoms with low values of n would decay very rapidly by spontaneous emission to the ground state. They would not have time to be cooled down in the trap either by microwaves or by spontaneous emission and only those with negative magnetic moments and initial kinetic energy lower than ~0.8 K would remain in the trap. For these, the microwaves would have no or little effect. From the fraction of the atoms that are highly excited, those with $n>50$ are easily ionized and those initially with low magnetic moment have a small chance to lose kinetic energy and be trapped. Here, we only considered atoms produced with specific large values of n, l and m, with typical values ranging from $n=17-40$, $m=16-39$, and assumed a thermal kinetic energy distribution at temperatures of 4, 8, and 16 K.

Each trajectory was run both without microwave radiation and with the fixed microwave field of 10 V/m and resonant at a magnetic field of 1.4 T (≈19.6 GHz). These parameters worked well for the $n=17$ circular state and were used for all the other states without further optimization. The spontaneous decay to lower n was handled using a Monte Carlo method as in Ref. [12]. We used both a classical decay formula as well as a quantum mechanical table of decays, for which we approximate the average value of L_z by the closest integer. The two approaches gave very similar results.

We let the differential equations evolve either until the total center of mass energy is greater then the potential at the wall and the atom is lost or until the quantum number n decreases to $n=2$ when we assume that the atom decays very quickly to the 1S low-field seeking state. Notice that in our treatment we have not considered any spin dynamics or interactions up to this last stage where the atoms are taken with $m_S=+1/2$ so that they could be trapped once they reach the $n=1$ state.

In Fig. 2 we show results of a simulation. Our interest is to compare the effect of the microwave radiation on the trapped number rather than to quote absolute numbers. We plot the histogram of the number of particles as function of the final energy (in Kelvin) with respect to the trap bottom (for $m=1$) for two values of the microwave electric field: 0 and 10 V/m. The curves clearly show cooling of low energy atoms and an enhancement of the number of trapped atoms (whose energy is below 0.8 K) when the microwave field is

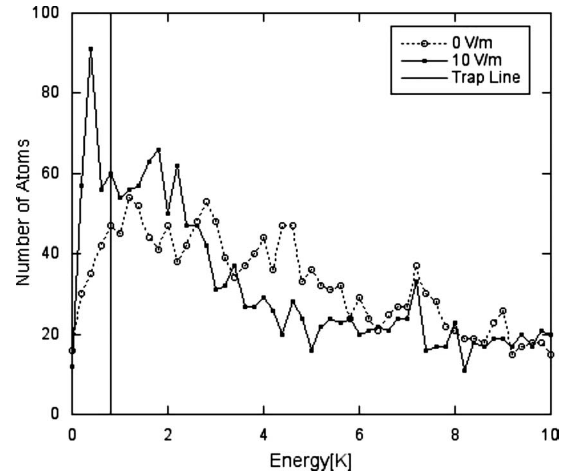

FIG. 2. Final energy distribution, with and without microwave power for a thermal sample of 4096 hydrogen atoms initially in the state $n=25$, $l=24$, $m=24$ at 16 K. The trap holds atoms with energy up to 0.8 K. At the end, 213 and 139 atoms are trapped in the 1S state with and without microwave power, respectively.

turned on. These data were obtained, with the initial values of $n=25$, $l=24$, $m=24$. The microwave intensity corresponding to this 10 V/m field is 0.14 W/m^2, which is very reasonable, compatible with the cryogenic environments used for the traps, and is easily generated using off-the-shelf commercial solid-state devices.

In order to see that there was no special effect of starting with a circular state, we also ran simulations with noncircular states. In these calculations, the atoms initially have a flat distribution in ℓ from m to $n-1$.

The results of all the different simulations are shown in Fig. 3 where we plot the ratio, R_T, of the number of trapped atoms with and without microwave power. The microwave induced cooling effect decreases, as expected, away from the circular state, but it is still present. In these simulations we used the same values of microwave field and frequency men-

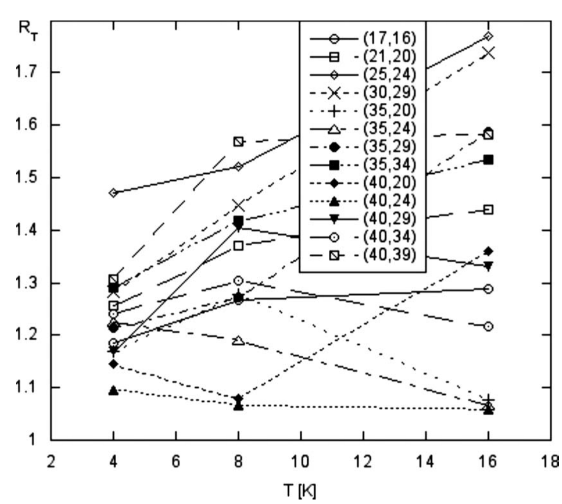

FIG. 3. Ratio, R_T, of trapped atoms with and without microwave power as a function of initial temperature, for different initial (n,m)s. The connecting lines are just to guide the eyes. Typical statistical errors are ~5% at 4 K and ~11% at 16 K.

tioned before, with no attempt for optimization, which makes a trend analysis difficult. We restricted the simulation to $n \leq 40$ so that we could trust the approximations made. In all of the cases we checked, the microwaves enhanced the fraction that would finish the radiative cascade as trapped atoms. The largest enhancement is for atoms that start as nearly circular states while the enhancement is least for the low m distributions. Since the atoms will be formed with a unknown distribution of m, the expected enhancement for a given experiment cannot be known.

Since these simulations are quite computer intensive, we did not try different microwave power schemes or fully optimized parameters. In a real experiment, the use chirp or of a comb of frequencies in the microwave field, with a tailored power distribution, should increase the number of trapped atoms since atoms with different velocities will have a large chance to make microwave transitions closer to the turning point. Thus, the results of Fig. 3 likely underestimate the amount of enhancement that can be achieved. The actual optimization of this cooling scheme can be done experimentally once trapped $\bar{\mathrm{H}}$ atoms are available. And, in fact, it should be done experimentally, since the exact experimental conditions in these traps cannot be fully captured by a theoretical model as, for example: (i) the distribution of the kinetic energy and of n, l, and m are not known; (ii) some atoms are field ionized and recycled; (iii) collisions with the positron plasma will be a heating mechanism (this is expected to be less severe with the microwave as it accelerates the radiative decay between passages through the plasma).

In conclusion, we have proposed using microwave radiation resonant near the trap edges to enhance the number of magnetically trapped $\bar{\mathrm{H}}$ atoms formed at high quantum numbers and magnetic moment states. A Monte Carlo simulation model using the complete diamagnetic Hamiltonian and realistic trapping fields, like the one employed by ALPHA [3] at CERN, confirms these predictions. Since all the developments and techniques to further cool and trap $\bar{\mathrm{H}}$ atoms will be important for the achievement of a high-precision comparison of $\bar{\mathrm{H}}$ and hydrogen, we believe this simple-to-implement scheme could be of benefit to this difficult quest. The necessary microwave power is small, easily provided by commercial components and compatible with the cryogenic environments of the traps. Finally, it is worth mentioning that, while we focused the discussion on $\bar{\mathrm{H}}$, the technique could, in principle, also work for high magnetic moment Rydberg atoms in an inhomogeneous trapping field.

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