

Reply to “Comment on ‘Recurrences without closed orbits’”

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We discuss some of the difficulties in using the orbits proposed by Main [preceding paper, Phys. Rev. A **60**, 1726 (1999)] to calculate the recurrence spectra in Robicheaux and Shaw [Phys. Rev. A **58**, 1043 (1998)]. [S1050-2947(99)06407-0]

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Recently, we presented quantum spectra of alkali-atoms in a static electric field [1]. We restricted the investigation to cases where the electron has at least one unit of angular momentum in the field direction. We found strong features in the recurrence spectra at short actions which do not correspond to real closed orbits in this system. We dubbed these features “recurrences without closed orbits.”

In Main’s Comment [2] on this paper, he demonstrates that orbits exist at the correct action to account for the recurrences in our spectra. These orbits come from the bifurcation of the closed uphill and downhill orbits observed in the $m=0$ photoabsorption spectra of hydrogen in an electric field into real and complex periodic orbits in the $m=1$ and $m=2$ spectra. The uphill and downhill orbits no longer exist for $m \neq 0$, but each is split into a real periodic orbit and a complex orbit in a type of tangent bifurcation. The real periodic orbits formed in this bifurcation, unlike the original orbits, no longer reach the nucleus. Main proposes that these periodic orbits are the origin of the “recurrences without closed orbits” described in Ref. [1]. This is a reasonable inference and we think that a theory of the short action recurrences must account for these orbits.

However, there are conceptual questions that need to be addressed before it can be shown whether or not these orbits actually generate the short action recurrences. The role of the complex, “ghost,” periodic orbits found by Main is interesting and may play a role in the explanation of the recurrences in H. These complex orbits are connected to the tunneling of the wave function into the forbidden region and thus to the overlap of the wave function with the initial state. This would be in line with the approximation made in [1] of treating the on-axis orbits as if they did exist even though there were classically forbidden. A theory based on Main’s orbits should also explain the success of the approximation in reproducing the recurrence strengths and the scaling law followed by the “recurrences without closed orbits” we deduced from the approximation. A possible problem with

these orbits is that the electron never gets closer than 12.8 a.u. to the nucleus for $m=1$; thus they do not overlap with the $1s$ initial state which has a size of 1 a.u.

There are also difficulties in ascribing features to the real periodic orbits. It is misleading to describe the real periodic orbits in Ref. [2] as *closed*. Closed orbit theory, as developed by Du and Delos [3] and Gao and Delos [4] requires the radially outgoing wave excited by the laser to go out from and return to the vicinity of the nucleus (or more precisely the vicinity of the compact initial state). The orbits found by Main are *periodic*, but they are not *closed* in the sense required to apply standard closed orbit theory. They neither begin nor end near the nucleus. Main himself states this when he discusses the real orbit: “in Fig. 1 the nearest distance from the origin . . . is about 13 Bohr radii . . .” At this point of closest approach, the trajectory is almost perpendicular to the radial direction as well, so it is difficult to connect a radially outgoing wave function to a wave traveling along this orbit. Since these orbits are not closed, it would be better to call them periodic or self-retracing. A theory for the recurrences from periodic orbits that do not close but only pass by the nucleus requires an extension of closed orbit theory.

The pattern of recurrences in the alkali-metal atoms showed a strong increase in the role of short action orbits. This was ascribed [2] to scattering by the non-Coulombic core into the real periodic orbit. This seems reasonable to us but the difficulty with this interpretation is that the electron on this periodic orbit does not travel through the core region (closest approach 13 a.u. compared to core size of 3–5 a.u.) and does not move radially near the nucleus. Thus, it is not clear that waves will be scattered onto this trajectory.

In conclusion, we think that Main has made an important step towards understanding the origin of the short action recurrences. Although the action is correct, other properties of the proposed orbits suggest there may be difficulties in this interpretation. Advances in semiclassical theory will be necessary to test these ideas.

[1] F. Robicheaux and J. Shaw, Phys. Rev. A **58**, 1043 (1998).

[2] J. Main, preceding paper Phys. Rev. A.

[3] M. L. Du and J. B. Delos, Phys. Rev. A **38**, 1896 (1988); **38**, 1913 (1988).[4] J. Gao, J. B. Delos, and M. Baruch, Phys. Rev. A **46**, 1449 (1992); J. Gao and J. B. Delos, *ibid.* **46**, 1455 (1992); **49**, 869 (1994).