COMMENT

Comment on differential cross sections of low-energy electron–hydrogen scattering in a CO_2 laser field

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Abstract. Cionga *et al* have employed the Kramers–Henneberger gauge when performing Floquet close-coupling calculations of electron–hydrogen scattering. It is shown that with their basis the effect of the laser on the target electron is described very inaccurately: their model assumes that the electron attached to the proton oscillates freely in the laser field, whereas it is actually tightly bound for the laser intensities and frequencies considered. Such an effect could well explain the apparent breakdown they observed of the Kroll–Watson low-frequency approximation.

Cionga *et al* (1997) have recently presented calculations for electron–hydrogen scattering in a laser field showing the breakdown of the widely used Kroll–Watson (1973) (KW) soft-photon approximation, particularly at small scattering angles. These calculations were motivated by the series of measurements on electron scattering by helium and argon in the presence of a CO_2 laser field (Wallbank and Holmes 1993, 1994a, b, 1996, Wallbank 1995). These measurements investigated few-photon stimulated emission and absorption and obtained cross sections for these processes which were many orders of magnitude larger than predicted in the KW approximation. This surprising finding has stimulated numerous theoretical studies (Rabadán *et al* 1994, 1996a, b, Collins and Csanak 1995, Geltman 1995, 1996, 1997, Madsen and Taulbjerg 1995, Varró and Ehlotzky 1995, Chen and Robicheaux 1996, Robicheaux 1996, Milošević and Ehlotzky 1997).

Here we wish to point out a possible limitation in the calculations of Cionga *et al*. They made the conventional approximation of a uniform monochromatic laser field and transformed to the Kramers–Henneberger or space-translated frame (Kramers 1956, Henneberger 1968). This yielded the Hamiltonian

$$H = \frac{\boldsymbol{P}_1^2 + \boldsymbol{P}_2^2}{2m} - \frac{e^2}{4\pi\epsilon_0}\left(\frac{1}{|\boldsymbol{r}_1 - \boldsymbol{\alpha}_0(t)|} + \frac{1}{|\boldsymbol{r}_2 - \boldsymbol{\alpha}_0(t)|} - \frac{1}{|\boldsymbol{r}_1 - \boldsymbol{r}_2|}\right) \qquad (1)$$

where $\boldsymbol{r}_i$ $(i = 1, 2)$ are the coordinates of the electrons relative to the proton, $\boldsymbol{P}_i$ are the corresponding momenta and $\boldsymbol{\alpha}_0(t) = \alpha_M \hat{\boldsymbol{\epsilon}} \sin(\omega t + \phi)$ is the vector describing the instantaneous quiver motion of a free electron in the field, polarization $\hat{\boldsymbol{\epsilon}}$, angular frequency ω and phase ϕ. In their calculations Cionga *et al* employed a value of 0.1 a_0 for α_M, the amplitude of the quiver motion. The Floquet close-coupling (FCC) (Dimou and Faisal

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1987) expansion of the wavefunction was then employed. To make clear our criticism we consider an intermediate stage of this expansion of the wavefunction:

$$\Psi(\boldsymbol{r}_1, \boldsymbol{r}_2, t) = F(\boldsymbol{r}_2, t)\,\phi_{1s}(\boldsymbol{r}_1) \tag{2}$$

where, following Cionga *et al*, we have restricted ourselves to just the 1s term in the target-state expansion. For electron energies well below the first excitation threshold and for weak laser electric fields (as here) this is not a serious restriction. Exchange has been neglected. From equation (2) we have

$$\mathrm{i}\hbar\frac{\partial F}{\partial t} = \left[\frac{\boldsymbol{P}_2^2}{2m} + V(\boldsymbol{r}_2, \boldsymbol{\alpha}_0(t)) + \epsilon_{1s} + \Delta E(\boldsymbol{\alpha}_0(t))\right]F(\boldsymbol{r}_2, t) \tag{3}$$

where ϵ_{1s} is the hydrogen 1s energy,

$$V(\boldsymbol{r}_2, \boldsymbol{\alpha}_0(t)) = \frac{e^2}{4\pi\epsilon_0}\left(-\frac{1}{|\boldsymbol{r}_2 - \boldsymbol{\alpha}_0(t)|} + \left\langle 1s\middle|\frac{1}{|\boldsymbol{r}_1 - \boldsymbol{r}_2|}\middle|1s\right\rangle\right) \tag{4}$$

and

$$\Delta E(\boldsymbol{\alpha}_0(t)) = \frac{e^2}{4\pi\epsilon_0}\langle 1s| - \frac{1}{|\boldsymbol{r}_1 - \boldsymbol{\alpha}_0(t)|} + \frac{1}{\boldsymbol{r}_1}|1s\rangle. \tag{5}$$

Here ΔE represents the laser–target interaction, independent of the projectile. This appears to give a non-vanishing coupling asymptotically, although for small values of α_M this coupling will be small. The principal problem lies with V, where the space-translated form of the projectile–nucleus interaction is combined with the field-free electron–electron interaction. One consequence of this is the appearance of the long-range interaction, $(\boldsymbol{\alpha}_0 \cdot \boldsymbol{r}_2/r_2^3)$, discussed by Cionga *et al* (1998). This interaction is independent of the state of the target. It arises because with the approximation employed in equation (3) it is assumed that in the electron–electron interaction both electrons are free to move in phase in the laser field so their separation is unaffected. However, the target electron is *not* free to move, being firmly bound to the proton and only very slightly affected by the weak, low-frequency, laser electric field. This was shown by direct numerical calculation of the H ground state in a low-frequency field (Robicheaux 1996); the incident electron scatters from an oscillating H atom, with the H atom being only slightly polarized. This polarization, while again yielding an asymptotic off-diagonal $1/r^2$ potential, is much too weak for the laser fields of interest to produce significant scattering (Rabadán *et al* 1994, Geltman 1995).

A similar use of the acceleration gauge led to the appearance in the work of Varró and Ehlotzky (1995) of a spuriously large polarizability, as noted by Dickinson (1996).

A more physically appropriate expansion would be

$$\Psi(\boldsymbol{r}_1, \boldsymbol{r}_2, t) = \mathcal{F}(\boldsymbol{r}_2, t)\,\bar{\phi}_{1s}(\boldsymbol{r}_1, \boldsymbol{\alpha}_0(t)) \tag{6}$$

where $\bar{\phi}_{1s}(\boldsymbol{r}_1, \boldsymbol{\alpha}_0(t)) = \phi_{1s}(|\boldsymbol{r}_1 - \boldsymbol{\alpha}_0(t)|)$. Then

$$\mathrm{i}\hbar\frac{\partial \mathcal{F}}{\partial t} = \left[\frac{\boldsymbol{P}_2^2}{2m} + \bar{V}(\boldsymbol{r}_2, \boldsymbol{\alpha}_0(t)) + \epsilon_{1s}\right]\mathcal{F}(\boldsymbol{r}_2, t) \tag{7}$$

where

$$\bar{V}(\boldsymbol{r}_2, \boldsymbol{\alpha}_0(t)) = \frac{e^2}{4\pi\epsilon_0}\left(-\frac{1}{|\boldsymbol{r}_2 - \boldsymbol{\alpha}_0(t)|} + \left\langle 1s\middle|\frac{1}{|\boldsymbol{r}_1 - \boldsymbol{r}_2 + \boldsymbol{\alpha}_0(t)|}\middle|1s\right\rangle\right) \tag{8}$$

$$= V_{st}(|\boldsymbol{r}_2 - \boldsymbol{\alpha}_0(t)|) \tag{9}$$

where V_{st} is the static electron–hydrogen potential (Mott and Massey 1965). The appearance of V_{st} is physically intuitive as giving the effect of scattering in a close-coupling expansion without exchange when only the target ground state is retained. By expanding $\mathcal{F}$ in the usual FCC fashion modified close-coupling equations can be obtained. These are comparable in number to those employed by Cionga *et al* (1997), but should give a better description of the laser-assisted scattering. With the expansion from equation (6) there are no finite couplings asymptotically. In the unsymmetrized wavefunction, it is much more accurate to use the length gauge for the electron bound to the proton and the acceleration gauge for the scattering electron.

Hence, until the calculations of Cionga *et al* (1997) are repeated using the more appropriate potential $\bar{V}$ it appears premature to assume that the application of the FCC method will necessarily explain the breakdown of the Kroll–Watson approximation. We note that Collins and Csanak (1995) reported at the 19th ICPEAC that FCC results using the dressed Yukawa potential for broadly the parameters of interest for the Wallbank and Holmes' experiments were consistent with the KW approximation.

Since Rabadán *et al* (1996b) have shown that double scattering could explain the experimental results and Milošević and Ehlotzky (1997) have confirmed this using a different electron–atom field-free scattering potential, we feel that further theoretical work may be premature until the possible importance of double scattering has been confirmed or refuted experimentally.

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