Chapter 6 - Hyperfine structure ~~and isotope shift~~

I will not go into the energy shift due to the different masses of isotopes or due to the nonzero charge radius.

The fine structure energy shifts came from the orbital angular momentum → B-field that interacts with the magnetic dipole moment of the electron. Hyperfine → B at nucleus!

I = spin of nucleus

If $I=0$, nucleus does not have magnetic dipole moment or electric moments.
$I=1/2$ has magnetic dipole moment, electric dipole moment??
$I=1$ has magnetic dipole, electric quadrupole moment; electric dipole?, magnetic quad?

Special case $L=0$ $\quad H_{HFS} = \frac{A}{\hbar^2}\vec{I}\cdot\vec{S} = \frac{A}{\hbar^2}(\vec{I}\cdot\vec{J})$

This has the same form we've done before
$\vec{F} \equiv \vec{I}+\vec{J}$
$H_{HFS} = \frac{A}{2\hbar^2}(\vec{F}^2 - \vec{I}^2 - \vec{J}^2) \longrightarrow E_F = \frac{A}{2}[F(F+1) - I(I+1) - J(J+1)]$

The allowed values of F $\quad F = |I-J|, |I-J|+1, \ldots, I+J$

Examples

72% ^{85}Rb	$I=5/2$, $S=1/2$	$F=2,3$	$\Delta E = 3.036\ldots$ GHz h
28% ^{87}Rb	$I=3/2$, $S=1/2$	$F=1,2$	$\Delta E = 6.834\ldots$ GHz h
100% ^{133}Cs	$I=7/2$, $S=1/2$	$F=3,4$	$\Delta E = 9.192\ldots$ GHz h
^{1}H	$I=1/2$, $S=1/2$	$F=0,1$	$\Delta E = 1.420\ldots$ GHz h

The hyperfine splittings are reduced because the natural unit of nuclear magnetic dipole $\mu_N = \mu_B \, m_e/m_p$. This gives
$\vec{\mu}_I = g_I \mu_N \vec{I}$ $\quad$ why opposite sign?

For alkali $l \neq 0$, there are two contributions: the orbital angular momentum gives B at the origin and the spin gives B as well

$$\vec{B} = -2 \frac{\mu_0 \mu_B}{4\pi r^3} \frac{1}{\hbar} [\vec{L} - (\vec{S} - \frac{3(\vec{S}\cdot\vec{r})\vec{r}}{r^2})]$$

This leads to a quite complicated interaction

$$H_{HFS} = -\vec{\mu}_I \cdot \vec{B} = -g_I \mu_N \vec{I} \cdot \vec{B}$$

It can be shown that H_{HFS} commutes with $\vec{F}^2$ and F_z. This means the eigenstates have F, M_F. To correctly get the full terms, you need to keep all of the terms. Only if you are interested in small B is it OK to do the following.

Effective $H_{HFS} = \frac{A}{\hbar^2} \vec{I} \cdot \vec{J} \quad \Rightarrow \quad E = \frac{A}{2} [F(F+1) - I(I+1) - J(J+1)]$

The Zeeman effect comes from

$$H_{ZE} = g_J \mu_B B J_z - g_I \mu_N B I_z$$

As in the previous chapter we can get the weak field limit.

$$E = g_F \mu_B B M_F$$

Ignoring μ_N gives

$$g_F = \{[F(F+1) + J(J+1) - I(I+1)]/[2F(F+1)]\} g_J$$

Example ^1H 1s $\quad F=1$, ($F=0$ no Zeeman)

$$g_J = \frac{3}{2} + \frac{\frac{1}{2}\frac{3}{2} - 0}{2\,\frac{1}{2}\frac{3}{2}} = 2$$

$$g_F = [(1\cdot 2 + \tfrac{1}{2}\tfrac{3}{2} - \tfrac{1}{2}\tfrac{3}{2})/(2\cdot 1\cdot 2)]\, 2 = 1$$

Example ^{85}Rb 5s $\quad F=2,3 \quad g_J = 2 \quad I = 5/2$

$$g_2 = [(2\cdot 3 + \tfrac{1}{2}\tfrac{3}{2} - \tfrac{5}{2}\tfrac{7}{2})/(2\cdot 2\cdot 3)]\, 2 = -\tfrac{1}{3}$$

$$g_3 = [(3\cdot 4 + \tfrac{1}{2}\tfrac{3}{2} - \tfrac{5}{2}\tfrac{7}{2})/(2\cdot 3\cdot 4)]\, 2 = 1/3$$

At fields $\mu_B B > A$ (but less than the spin-orbit splitting), the total electronic angular momentum decouples from the nuclear spin

$|\psi\rangle = |I M_I J M_J\rangle$

$E = A M_I M_J + g_J \mu_B B M_J - g_I \mu_N B M_I$

At intermediate field strengths, need to construct the Hamiltonian matrix and diagonalize.

See the plots for hydrogen 2P vs B

The hyperfine interaction played a key role in optical clocks that set the standard <u>seccond</u>. The microwave transition in the ground state of Cs ($6S_{1/2}$ $F=3 \leftrightarrow 4$) at ~ 9 GHz. Use Cs because heavy $\Rightarrow$ smaller Doppler width.

Optical transitions often have issues because of the line width.

In Sr the $5s^2\ ^1S_0 \rightarrow 5s5p\ ^3P_0$ is forbidden for ^{84}Sr (0.6%), ^{86}Sr (9.9%), ^{88}Sr (82.6%) because $I=0$ for all. Total ang mom $F=0 \rightarrow 0$ is forbidden exactly.

In ^{87}Sr (7.0%), $I = 9/2$. This means it's a $F=9/2 \rightarrow 9/2$ is <u>not</u> exactly forbidden, but is highly supressed!

Lifetime ~ 150 s $\Rightarrow$ $\Gamma = 2\pi\ 10^{-3}$ Hz (~ 1 mHz line width)

The frequency $\sim 4.3 \times 10^{14}$ Hz $\sim 4.3 \times 10^{17}$ 1mHz

$\lambda = 698.4$ nm