PHYS 416 (Fall 2026) Test 2 Name: _____

Show work to receive full credit. Problems 1, 2, 4, 6 require numerical answers.

Possibly useful $\int_{-\infty}^{\infty} e^{-\alpha x^2} dx = \sqrt{\pi/\alpha}$.

5 pts 1. You have an engine that uses a piston and 1 mole of He gas as the main component. Suppose the hot reservoir is at 100 °C and the cold reservoir is at 10 °C. In one cycle, 20 J is taken in from the hot reservoir. a) If operating at the maximum possible efficiency, how much work can be done in 1 cycle? b) If operating at the maximum possible efficiency, what is the change in entropy of the universe in one cycle? Show work. c) Suppose the efficiency is 1/2 the maximum possible. What is the change in entropy of the universe in one cycle?

5 pts 2. The Al atom has a ground electronic level with multiplicity 2. The next electronic level has an energy 2.229×10^{-21} J above the ground state with multiplicity 4. The next electronic level has an energy 5.043×10^{-19} J above the ground state with multiplicity 2. If the atom is in contact with a reservoir at $T = 300$ K, what is the probability to be in the ground electronic *level*?

5 pts 3. The energy levels for a particle constrained to go in a circle of radius R are $E_j = \hbar^2 j^2/(2MR^2)$ for $j = ... -2, -1, 0, 1, 2, ...$ a) For the case where $kT \gg \hbar^2/(2MR^2)$, compute the partition function for this particle. b) In this limit, compute the entropy.

5 pts 4. In a container held at $T = 298$ K and $P = 1$ bar, you put 2 moles of CO and one mole of O_2. a) Does this mixture convert to 2 moles of CO_2? Give the reason for your answer. b) If yes, how much energy do you get out? If no, how much energy do you have to put in to make the conversion? c) For this reaction, how much work does the container do on the gas (or vice versa)?

10 pts 5. a) You have He, near room temperature, in a piston and taken through the sequence: A) P_1, V_1 goes to P_2, V_1, B) P_2, V_1 goes to P_2, V_2, C) P_2, V_2 goes to P_1, V_2, and D) P_1, V_2 goes to P_1, V_1. For this cycle, $P_1 < P_2$ and $V_1 < V_2$. a) Draw this cycle on a PV diagram and label each step and the various P_i, V_i points. b) For each step of the sequence, determine the work done on the gas, the change in internal energy of the gas, and the heat added to the gas. c) Did this cycle do work on the gas or did you get work out of this sequence? d) Is this a refrigerator or an engine? e) What is the efficiency of this device?

10 pts 6. The atomic masses of Na and Cl are 23.0 and 35.5 and the atomic mass of water is 18.0. There is 4.0 mg of NaCl in 2.0 kg of water. In liquid water, NaCl becomes Na^+ and Cl^-. What change in pressure would keep the temperature of the water-ice phase transition unchanged? Assume the ice does not contain Na or Cl. The latent heat of fusion of water is 333.55 kJ/kg and latent heat of vaporization is 40.65 kJ/kg. The density of water is 1000 kg/m^3 and the density of ice is 917 kg/m^3.

1) HWK Prb 4.1

We know $Q_h = 20\,J$ $\quad W = e\,Q_h$ (Eq 4.1)

a) $e = 1 - T_c/T_h = 1 - \frac{283}{373} = 0.241 \Rightarrow W = 4.826\,J$ $\quad (Q_c = Q_h - W)$

b) $\Delta S = -Q_h/T_h + Q_c/T_c = -20\,J/373\,K + 15.174\,J/283\,K = 0$ $\quad \left(Q_c = \frac{T_c}{T_h} Q_h\right)$

A heat engine at max theoretical efficiency has $\Delta S = 0$

c) $W = 2.413\,J$, $Q_c = 17.587\,J$ $\quad \Delta S = -20\,J/373\,K + 17.387\,J/283\,K = 8.53\times10^{-3}\,\frac{J}{K}$

2) Class notes bottom pg 2

At 300 K $\quad kT = 4.143\times10^{-21}$

$Z = 2e^{0} + 4e^{-2.229\times10^{-21}/4.143\times10^{-21}} + 2e^{-5.043\times10^{-19}/4.143\times10^{-21}}$

$= 2 + 4 \cdot 0.5839 + \cancel{2e^{-121.7}} \approx 0 = 4.336$

Prob to be in ground state level $= 2/4.336 = 0.461$

3) Similar to derivation Z_{rot}

a) $Z = \sum_m e^{-\beta \hbar^2 j^2/2MR^2} \cong \int_{-\infty}^{\infty} e^{-\beta \frac{\hbar^2}{2MR^2} j^2} dj$ $\quad$ because $\beta \frac{\hbar^2}{2MR^2} \ll 1$

$= \sqrt{\pi 2MR^2/\beta\hbar^2}$ $\quad$ Equation at top of test

b) Eq 6.92

$S = \left(\frac{\partial F}{\partial T}\right)_{VN} = \frac{\partial}{\partial T}(kT \ln Z) = k \ln Z + kT \frac{\partial}{\partial T} \ln Z$

$\ln Z = \text{const} + \frac{1}{2} \ln T \Rightarrow \frac{\partial}{\partial T} \ln Z = \frac{1}{2T}$

$S = k \ln Z + kT \frac{1}{2} \frac{1}{T} = k\left(\ln Z + \frac{1}{2}\right)$

4) HWK Prb 5.5, Class Prb 5.2, Example Fig 5.4, ...

a) The stable form is the one with the smaller ΔG

$2 \cdot CO_2$ $\Delta G = -394.36\,kJ \times 2 = -788.72\,kJ$ ← stable } yes

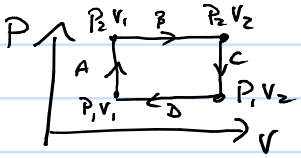
$2 \cdot CO + 1 \cdot O_2$ $\Delta G = -137.17\,kJ \times 2 + 0 = -274.34\,kJ$

b) You get out $-(-788.72\,kJ - -274.34\,kJ) = 514.38\,kJ$

c) You go from 3 moles of gas to 2 moles of gas $\Rightarrow$ volume of the gas decreases $\Rightarrow$ work is done on the gas

$W = P \cdot \Delta V = 1\,mol\ 8.31\,\frac{J}{K}\ 298\,K = 2.48\,kJ$

5) a)



$$W = -\int_{V_i}^{V_f} P(V)\,dV$$
$$\Delta U = \tfrac{3}{2} N k (T_f - T_i) = \tfrac{3}{2}(P_f V_f - P_i V_i)$$
$$Q = \Delta U - W$$

b)

	W	ΔU	Q
A)	0	$\frac{3}{2}(P_2-P_1)V_1$	$\frac{3}{2}(P_2-P_1)V_1$
B)	$-P_2(V_2-V_1)$	$\frac{3}{2}P_2(V_2-V_1)$	$\frac{5}{2}P_2(V_2-V_1)$
C)	0	$\frac{3}{2}(P_1-P_2)V_2$	$\frac{3}{2}(P_1-P_2)V_2$
D)	$-P_1(V_1-V_2)$	$\frac{3}{2}P_1(V_1-V_2)$	$\frac{5}{2}P_1(V_1-V_2)$

c) Work on gas $= -(P_2-P_1)(V_2-V_1)$ = negative $\Rightarrow$ get work out

d) Get work out $\Rightarrow$ engine

e) The heat comes into the gas in steps A + B

$$Q_h = \tfrac{3}{2}(P_2-P_1)V_1 + \tfrac{5}{2}P_2(V_2-V_1) = \tfrac{5}{2}(P_2-P_1)(V_2-V_1) + \tfrac{5}{2}P_1(V_2-V_1) + \tfrac{3}{2}(P_2-P_1)V_1$$
$$W_{out} = (P_2-P_1)(V_2-V_1)$$

$e = W/Q_h$ This is less than 1 because $Q_h = \frac{5}{2}W$ + positive

6) HWK Prob 5.81, .82, .32

The phase transition occurs when $\mu_{sol}(T,P) = \mu_{liq}(T,P)$

The chemical potential of the liquid changes because of the solute particles

$$\mu_{liq}(T,P) = \mu_{0,liq}(T,P) - \frac{N_B}{N_A}kT$$
$$\mu_{sol}(T,P) = \mu_{0,sol}(T,P)$$

$\mu_{0,liq}$ = chem pot of pure liquid, N_B = # of solute particles, N_A = # H_2O

Need to account for the changed pressure as well

$$\mu_{liq}(T_0,P) = \mu_{0liq}(T_0,P_0) - \frac{N_B}{N_A}kT_0 + \left(\frac{\partial \mu_{liq}}{\partial P}\right)_{TN}\Delta P$$
$$\mu_{sol}(T_0,P) = \underbrace{\mu_{0sol}(T_0,P_0)}_{\text{same}} + \left(\frac{\partial \mu_{sol}}{\partial P}\right)_{TN}\Delta P$$

$$\left[\left(\frac{\partial (N_A \mu_{liq})}{\partial P}\right) - \frac{\partial (N_A \mu_{sol})}{\partial P}\right]\Delta P = N_B kT = n_B RT$$
$$(V_{liq} - V_{sol})\,\Delta P = N_B kT$$

$$\left.\begin{aligned} V_{liq} &= 2\,kg/(10^3\,kg/m^3) = 2\times10^{-3}\,m^3 \\ V_{sol} &= 2\,kg/(917\,kg/m^3) = 2.181\times10^{-3}\,m^3 \end{aligned}\right\} \quad V_{liq} - V_{sol} = -1.81\times10^{-4}\,m^3$$

$$n_B = 2\,\frac{4\times10^{-3}}{58.5} = 1.37\times10^{-4}\,mol$$

$$\Delta P = 1.37\times10^{-4}\,mol\; 8.315\,\frac{J}{mol\,K}\,273\,K/(-1.81\times10^{-4}\,m^3) = -1.72\times10^{3}\,Pa$$

With the information given it's possible to solve a completely different way.
You can first find the change in temperature of the phase transition due to the presence of $NaCl$

Eq (5.90) $\Delta T = \frac{n_B R T_0^2}{L} = \frac{1.37 \times 10^{-4}\,\text{mol}\; 8.31 \frac{J}{K\,\text{mol}} (273\,K)^2}{2\,kg \quad 333.55 \times 10^3\,J/kg} = 1.27 \times 10^{-4}\,K$

Then you change the pressure to bring the temperature back to 0. This is allowed because ΔT is so small

Eq (5.47) $\Delta P = \Delta T \frac{L}{T_0 \Delta V} = 1.27 \times 10^{-4}\,K \frac{2\,kg\; 333.55 \times 10^3\,J/kg}{273\,K(-1.81 \times 10^{-4}\,m^3)} = -1.72 \times 10^3\,Pa$