PHYS 416 (Fall 2026) Test 1 Name: _____

Show work to receive full credit. Problems 1, 2, 4, 6 require numerical answers.

5 pts 1. System 1 has an internal energy of 10^6 J and a temperature of 25 °C. It is in thermal contact with System 2 which has an internal energy of 3×10^6 J and a temperature of 15 °C. I tell you "0.02 J goes from System 2 to System 1 with no other changes". a) What is the change in entropy of the combined system? b) By what factor did the multiplicity of the combined system go up (or down)? c) Did I lie to you? Explain.

5 pts 2. You have 5.0×10^{-3} m^3 of Ne atoms at one atmosphere pressure. a) Calculate the total thermal energy. b) Repeat the calculation except use N_2 instead of Ne.

5 pts 3. You have two identical, isotropic, 3D harmonic oscillators. The energies for one oscillator are $E_{n_x,n_y,n_z} = hf(n_x + n_y + n_z + 3/2)$ and each n can independently be $0, 1, 2, ...$ The total energy of the two oscillators is $E = hf(q + 3)$. For the following, take $q = 3$. a) List the macrostates if you only know the energy in each oscillator? b) How many microstates compose each macrostate? c) What is the probability that oscillator 1 is in its ground state?

5 pts 4. You have one million coins that are randomly flipped. What is the probability that exactly half are tails and the other half heads?

10 pts 5. System **a** is a classical gas of N_a He atoms in a volume V_a with a temperature T_a and pressure, P_a. Its thermal energy is U_a. System **b** consists of N_b harmonic oscillators with frequency f. The energy in System **b**, $E_b = hf(q_b + N_b/2)$ with $q_b \gg N_b$. These two systems are in thermal contact so energy can be exchanged and both N_a and N_b are large numbers. a) What is the multiplicity of System **a**? b) What is the multiplicity of System **b**? c) What is the total multiplicity of the combined systems? d) When the total energy is E, how much energy is in each system?

10 pts 6. A Robon can only have two energies: $E_1 = 0$ and $E_2 = \varepsilon$. You have 2000 Robons with 1300 in state 1 and 700 in state 2. a) You add one ε of energy to the system. What is the change in entropy? b) You add one Robon to the system keeping the total energy constant. What is the change in entropy? c) What is the value of the chemical potential of this system in units of ε? (Hint: $(n+1)! = (n+1) \times n!$ and $(n-1)! = n!/n$.)

Example from class Sec 3.2

Prob 1 The idea behind this is Eq 3.57

a) $\Delta S = \frac{0.02J}{298K} + \frac{-0.02J}{288K} = -2.33\times10^{-6}\ J/K$

b) $\Omega_f/\Omega_i = e^{\Delta S/k} = e^{-1.69\times10^{17}}$

c) Ω_f is <u>much</u> smaller than Ω_i which is not possible. I <u>lied</u>.

Prob 1.23 HWK 1

Prob 2 Use Equipartition $U = \frac{f}{2}NkT$ and ideal gas $NkT = PV$

a) For N_e $f=3$ $U = \frac{3}{2}\ 1.01\times10^5 Pa\ 5\times10^{-3} m^3 = 757.5\ J$

b) For N_2 $f=5$ $U = 5/2$ " " $= 1260\ J$

Prob 2.8 HWK 2

Prob 3 a) List V_a = # of quanta in a and V_b = # qu. in b

(3,0), (2,1), (1,2), (0,3) 4 macrostates

b) The # of microstates for $V=0$ is 1, $V=1$ is 3 $\overset{n_x n_y n_z}{(100)(010)(001)}$

$V=2$ is 6 (200)(020)(002)(110)(101)(011), $V=3$ is 10 $(\overset{3x}{300})(\overset{6x}{210})(\overset{1x}{111})$

(3,0) = 10 · 1 = 10 microstates

(2,1) = 6 · 3 = 18 "

(1,2) = 3 · 6 = 18 "

(0,3) = 1 · 10 = 10 "

c) $\text{Prob} = \frac{\#\text{ microstates in gnd}}{\text{total}} = \frac{10}{10+18+18+10} = \frac{10}{56} = 0.178$

Prob 2.24a HWK3

Prob 4 $\text{Prob} = \frac{\#\text{ microstates}}{\text{total}} = \frac{N!}{(N/2!)(\frac{N}{2}!)}\frac{1}{2^N}$ $N = 10^6$

Use Stirling's approx Eq 2.14

$\text{Prob} = \sqrt{2\pi N}\ N^N e^{-N} / [(2\pi N/2)(N/2)^N e^{-N}] / 2^N = \sqrt{\frac{2}{\pi N}} = 7.98\times10^{-4}$

Problem worked in class

Prob 5

a) For System a $\Omega_a = \frac{1}{N_a!} \frac{V^{3N_a}}{h^{3N_a}} \frac{\pi^{3N/2}}{(3N/2)!} (2m)^{3N/2} U_a^{\frac{3N_a}{2}}$ Eq 2.40

b) For System b $\Omega_b = \left(\frac{e}{N_b}\right)^{N_b} q^{N_b} = \left(\frac{e}{N_b \hbar\omega}\right)^{N_b} U_b^{N_b}$ Eq 2.21

c) $\Omega = \Omega_a \Omega_b$ with $U_b = U - U_a$ or $U_a = U - U_b$

d) The product is a bunch of constants times a simple function of U_a or U_b

$\Omega = \text{const}\ U_a^{3N_a/2} (U - U_a)^{N_b}$

Need to find where is a maximum

$$\frac{\partial \Omega}{\partial U_a} = \Omega \left(\frac{3N_a}{2} \frac{1}{U_a} - N_b/(U - U_a) \right) = 0 \Rightarrow (U - U_a) \frac{3N_a}{2} = N_b \cdot U_a$$

$$U_a = \frac{\frac{3N_a}{2}}{\left(\frac{3N_a}{2} + N_b\right)} U \qquad U_b = U - U_a = \frac{N_b}{\left(\frac{3N_a}{2} + N_b\right)} U$$

d') Some clever students used a different method to answer this question. The He gas is classical and monatomic $\Rightarrow$ $U_a = \frac{3}{2} N_a k T_a$. The harmonic oscillators are also classical because $q_b \gg N_b \Rightarrow U_b = \frac{2}{2} N_b k T_b$. These systems are in thermal equil. $\Rightarrow$ $T_a = T_b = T$

Solve for kT $\qquad U_a + U_b = E = \left(\frac{3}{2} N_a + N_b\right) kT$

$kT = \frac{E}{\frac{3}{2} N_a + N_b}$ Plug into the energy expressions

$$U_a = \frac{\frac{3}{2} N_a}{\frac{3}{2} N_a + N_b} E \qquad U_b = \frac{N_b}{\frac{3}{2} N_a + N_b} E$$

HWK 5 Prob 9 and HWK 4 Prob 3.1

Prob 6 From Eq. 2.6 $\Omega = \frac{N!}{N_1!\,N_2!}$ and from Eq 2.45 $S = k \ln \Omega$

a) $S_f - S_i = k(\ln \Omega_f - \ln \Omega_i) = k \ln\left(\frac{\Omega_f}{\Omega_i}\right)$

$\Omega_i = \frac{2000!}{1300!\,700!}$ $\quad \Omega_f = \frac{2000!}{1299!\,701!}$ because $N = \text{const}$ and $N_2 \to N_2 + 1$

$\Delta S = k \ln\left(\frac{1300!\,700!}{1299!\,701!}\right) = k \ln\left(\frac{1300}{701}\right) = k\, 0.618 = 8.53 \times 10^{-24}\ J/K$

b) $\Omega_f = \frac{2001!}{1301!\,700!}$ because $N \to N+1$ and $N_2 = \text{const}$

$\Delta S = k \ln\left(\frac{2001!}{1301!}\,\frac{1300!}{2000!}\right) = k \ln\left(\frac{2001}{1301}\right) = k\, 0.431 = 5.95 \times 10^{-24}\ J/K$

c) Eq 3.41 $\quad T = \frac{1}{\Delta S/\Delta N} = \frac{1}{k/\varepsilon\, 0.618}$

Eq 3.59 $\quad \mu = -T \frac{\Delta S}{\Delta N} = -\frac{1}{k/\varepsilon\, 0.618}\, k\, 0.431 = -0.697\,\varepsilon$