PHYS 416 (Fall 2025) Test 2 Name: ____________________________

Show work to receive full credit. Problems 1-3, 4a, 5c,d require numerical answers.

Possibly useful $\int_0^\infty f'(x)e^{-\alpha f(x)}dx = 1/\alpha$ if $f(x) \geq 0$ and $f(0) = 0$ and $f(x \to \infty) \to \infty$.

5 pts 1. At *constant pressure* of 1 bar, you dissociate hydrogen $H_2 + energy \to 2H$. How much energy must you put in per molecule for this to occur?

5 pts 2. You have a device that takes heat from the outside which is at 5.0 °C so that 1 J heat is added to your room where the temperature is 20.0 °C. Suppose this device works at the theoretical optimum efficiency. a) How much heat came from outside? b) What is the minimum energy your device uses to accomplish this?

5 pts 3. The atomic masses of Na and Cl are 23.0 and 35.5. There is 10 g of NaCl in 2.0 kg of water at 25 °C. In water NaCl dissociates to Na^+ and Cl^-. You push the water through a barrier permeable to water but not to Na^+ and Cl^-. How much work do you need to do to get 1.0 kg of pure water?

5 pts 4. A quantum system has energies $E_n = \epsilon n^3$ with a multiplicity of $\Omega_n = n^2$ for $n = 1, 2, 3, ...$. a) Give the numerical value of the partition function to 3 significant digits when $\beta\epsilon = 1/2$. b) Evaluate the partition function when $\beta\epsilon \ll 1$. (Hint: for part b), you should obtain the form $Z = c\beta^a + d$ where a, c, d are constants.)

10 pts 5. a) At 1 bar and 298 K, which is the stable form: $NaAlSi_3O_8$ or $NaAlSi_2O_6 + SiO_2$? b) Give your reason for choosing the form in a). c) At 298 K, at what pressure is the phase transition between the two forms? d) If the temperature were at 308 K, what is the pressure of the phase transition between the two forms? (The answer to b) must be correct to get points for part a).)

10 pts 6. A single quantum system has 3 states with energies 0, ϵ, 4ϵ with ϵ a constant. This system is in contact with a reservoir at temperature T. a) What is the partition function for a single quantum system? For the rest of the questions, there are N of these *distinguishable* quantum systems in contact with a reservoir at temperature T. b) What is the partition function? c) What is the average energy? d) What is the entropy? e) What is the chemical potential?

1) See Prob 1.53 Chap 1 notes

H_2 + energy → 2H at constant P means use enthalpy info

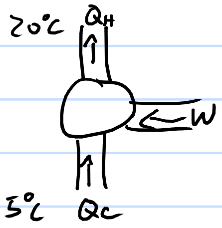
$2\Delta H_H - \Delta H_{H_2} = 2\cdot 217.97\frac{kJ}{mol} - 0 = 435.94\,kJ/mol = \Delta U + P\Delta V$ (Eq 1.53)

$P\Delta V = 10^5 Pa\,(V_H - V_{H_2}) = (2mol - 1mol)\,8.31\frac{J}{mol\,K}\,298\,K \Rightarrow 2.48\,kJ/mol$

$\Delta U = 435.94\,kJ/mol - 2.48\,kJ/mol = 433.46\,kJ/mol$

$\Delta U_1 = 433.46\times10^3 J/6.022\times10^{23} = 7.198\times10^{-19}\,J$ $(\simeq 4.49\,eV)$

2)



$Q_H = 1\,J$ See HWK Prob 4.15

1st Law $Q_H = W + Q_c$ ⇒ $W = Q_H - Q_c$

Theoretical limit $\frac{Q_c}{Q_H} = \frac{T_c}{T_H}$ $= Q_H(1 - \frac{Q_c}{Q_H})$

$W = 1\,J\,(1 - \frac{278}{293}) = 0.0512\,J$ $Q_c = Q_H - W = 0.9488\,J$

3) See HWK Prob 5.76

The pressure you have to apply needs to counterbalance the osmotic pressure Eq 5.78 $\Delta P = N_B kT/V$

$N_B = 2\ \ 10/(23+35.5)\ \ 6.022\times10^{23} = 2.06\times10^{23}$ (0.342 mol)

$V_{init} = 2.0\,kg/(1000\,kg/m^3) = 2\times10^{-3}\,m^3$

$V_{fin} = \frac{1}{2}V_{init} = 10^{-3}\,m^3$ because $\frac{1}{2}$ the water has been pushed through

Eq(1.30) $W = -\int_{V_i}^{V_f} P\,dV = -N_B kT\int_{V_i}^{V_f}\frac{1}{V}dV = N_B kT\ln(V_i/V_f)$

$= 2.06\times10^{23}\ \ 1.38\times10^{-23}\frac{J}{K}\,298\,K\ \ln 2 = 587\,J$

4) Eq (6.13) $Z = \sum_s e^{-\beta E(s)} = \sum_{n=1}^{\infty} n^2 e^{-\beta\epsilon n^3}$

a) $Z = e^{-\beta\epsilon} + 4e^{-\beta\epsilon 8} + \underbrace{9e^{-\beta\epsilon 27}}_{1.2\times10^{-5}} = e^{-1/2} + 4e^{-4} = 6.798\times10^{-1} \simeq 6.80\times10^{-1}$

b) $Z = \sum_{n=0}^{\infty} n^2 e^{-\beta\epsilon n^3} \simeq \int_0^{\infty} n^2 e^{-\beta\epsilon n^3}dn = \frac{1}{3}\int_0^{\infty}F'(n)e^{-\beta\epsilon F(n)}dn = \frac{1}{3}\frac{1}{\beta\epsilon}$

$F(n) = n^3$

5) See HWK Prob 5.28

a) $NaAlSi_3O_8$ is stable because b) $\Delta G_{albite} < \Delta G_{jadeite} + \Delta G_{quartz}$

c) $\Delta G(P,T_0) = \Delta G(P_0,T_0) + \Delta V \cdot (P-P_0) = 0$ at phase trans.

$= -3711.5\,kJ - (-2852.1\,kJ - 856.64\,kJ) + (P-P_0)(100.07\times10^{-6}\,m^3 - [60.40\times10^{-6}\,m^3 + 22.69\times10^{-6}\,m^3])$

$= -2.76\times10^3\,J + 16.98\times10^{-6}\,m^3\,(P-P_0)$

$P - P_0 = 1.62\times10^8\,Pa \Rightarrow P \cong 1.62\times10^8\,Pa$

d) $\Delta G(P, 1) = \Delta G(P_0, T_0) + \Delta V\,(P-P_0) - \Delta S\,(T-T_0)$

$\Delta S = 207.40\,J/K - (133.5\,J/K + 41.84\,J/K) = 32.1\,J/K$

$\Delta G(P,T) = 0 = -2.76\times10^3\,J + 16.98\times10^{-6}\,m^3\,(P-P_0) - 321\,J$

$P - P_0 = 1.81\times10^8\,Pa \Rightarrow P \cong 1.81\times10^8\,Pa$

6) Combines several HWK and class problems

a) Eq 6.13 $Z(\beta) = 1 + e^{-\beta\varepsilon} + e^{-4\beta\varepsilon}$

b) Eq 6.67 $Z(\beta,N) = Z^N(\beta) = (1 + e^{-\beta\varepsilon} + e^{-4\beta\varepsilon})^N$

c) Eq 6.25, 6.19 $U(\beta) = N\varepsilon(\varepsilon e^{-\beta\varepsilon} + 4\varepsilon e^{-4\beta\varepsilon})/Z(\beta)$

d) Eq 6.56, 6.58 $F = -kTN\ln(1 + e^{-\beta\varepsilon} + e^{-4\beta\varepsilon}) = -\frac{N}{\beta}\ln(1 + e^{-\beta\varepsilon} + e^{-4\beta\varepsilon})$

Eq 6.57 $S = (U - F)/T$

$= \frac{N}{T}\,\frac{\varepsilon e^{-\beta\varepsilon} + 4\varepsilon e^{-4\beta\varepsilon}}{1 + e^{-\beta\varepsilon} + 4e^{-4\beta\varepsilon}} + Nk\ln(1 + e^{-\beta\varepsilon} + e^{-4\beta\varepsilon})$

e) Eq 6.64 $\mu = \left(\frac{\partial F}{\partial N}\right)_{\beta,V} = -kT\ln(1 + e^{-\beta\varepsilon} + e^{-4\beta\varepsilon})$