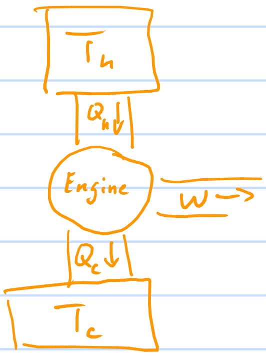
Chapter 4 Engines and Refrigerators

This chapter uses the thermodynamic relations to understand the fundamental limits of heat engines and refrigerators.

Section 4.1 Idealized Heat Engine

Heat engine: take heat from a hot reservoir at temperature T_h and turn part into useful work and dump the rest of the heat into the cold reservoir.

Reservoir: so large, the temperature doesn't substantially change during process.



Conservation of energy $Q_h = W + Q_c$

Important: Change in book convention, Q_h, Q_c, W are all positive

W = work done by the engine (not the work done on the engine)

Efficiency ≡ fraction of Q_h turned into useful work

$e = W/Q_h$

From 1st law $W = Q_h - Q_c \Rightarrow e = 1 - Q_c/Q_h$

Important question from the industrial revolution: Is there a limit on e?

There is!! From $\Delta S_{TOT} \geq 0$

Assuming no work on reservoirs, $\Delta N = 0$, $\Delta S = Q/T$

$$0 \leq \Delta S_{TOT} \leq \Delta S_{\substack{cold\\res}} + \Delta S_{\substack{hot\\res}} = \frac{Q_c}{T_c} - \frac{Q_h}{T_h} \Rightarrow \frac{Q_c}{T_c} \geq \frac{Q_h}{T_h} \Rightarrow \frac{Q_c}{Q_h} \geq \frac{T_c}{T_h}$$

$$e = 1 - Q_c/Q_h \leq 1 - T_c/T_h$$

There's a fundamental limit to the efficiency of a heat engine!! No matter what you do $e \leq 1 - T_c/T_h = \frac{T_h - T_c}{T_h}$

The smaller the ratio T_c/T_h, the higher is limit on effic. but usually not practical to artificially lower T_c.

Prob 4.3 1 GW at $e = 0.4$ $T_c = 300\,K$ $T_h = ? = \frac{300K}{0.4} \simeq 500K$

a) Rate → cold $Q_c = Q_h - W = (1-e)Q_h = (1-e)W/e = 1.5\ GW$

b) ΔT for $100\ m^3/s = 10^5\ kg/s = 10^8\ g/s \Rightarrow 4.2\ \frac{J}{g}\ 10^8\ g/s\ \Delta T = 1.5\times 10^9\ J/s \Rightarrow \Delta T = 3.6°C$

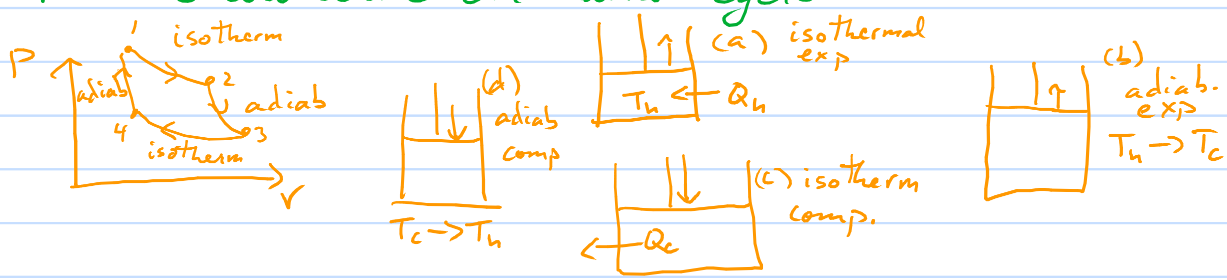
Prob 4.4 $T_c = 4°C$ $T_h = 22°C$

a) $e = 1 - \frac{277}{295} = 0.061$

b) Want $10^9\,J = W$ efficiency is only $\sim \frac{1}{2}e$ because warm water cools and cool warms $\Delta T \sim 9°C$ (not $18°C$)

$4.2\frac{J}{°C\,g}\, M\, 9°C\; 0.03 = 10^9\,J$ in 1 sec $\Rightarrow \frac{dM}{dt} \simeq 8.8\times10^5\,kg/s \simeq 880\,m^3/s$

Prob 4.5 Show work on Carnot cycle



Eq 1.30 Isothermal $W = NkT \ln(V_i/V_f)$

Eq 1.40 Adiabat $V^\gamma P = \text{const} \Rightarrow P = P_i V_i^\gamma / V^\gamma$

$\Rightarrow W = -\int_{V_i}^{V_f} P(V)\,dV = (P_f V_f - P_i V_i)/(\gamma - 1)$

$\Rightarrow W_{TOT} = NkT_h \ln(V_2/V_1) - \frac{P_3V_3 - P_2V_2}{\gamma-1} + NkT_c \ln(V_4/V_3) - \frac{P_1V_1 - P_4V_4}{\gamma - 1}$

From $PV = NkT$ $\quad P_1V_1 = P_2V_2 \quad P_3V_3 = P_4V_4$

$\Rightarrow W_{TOT} = NkT_h \ln(V_2/V_1) + NkT_c \ln(V_4/V_3)$

Eq 1.31 $Q_h = NkT_h \ln(V_2/V_1) \Rightarrow W_{TOT}/Q_h = e = 1 + \frac{T_c}{T_h}\frac{\ln(V_4/V_3)}{\ln(V_2/V_1)}$

Eq 1.38 adiabat $V_f T_f^{f/2} = V_i T_i^{f/2} \Rightarrow V_3/V_2 = \left(\frac{T_h}{T_c}\right)^{f/2} = V_4/V_1 \Rightarrow \frac{V_4}{V_3} = \frac{1}{(V_2/V_1)}$

$\Rightarrow e = 1 - T_c/T_h$

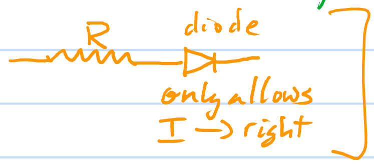
Carnot engines are not practical because the isothermal expansion and compression must take place slowly to keep T of gas approx. equal to that of the reservoir.

1st Law $W = Q_h - Q_c \longrightarrow$ You only get out what you put in.

2nd Law $W \leq \left(1 - \frac{T_h}{T_c}\right)Q_h \to$ You can't get out what you put in.

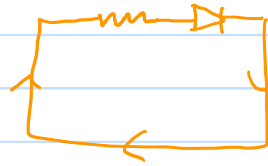
Example: Mr Lanier's diode/amplifier

Resistor at temp T has fluctuating current



only allows I → right

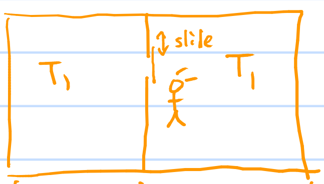
⟹



get net current

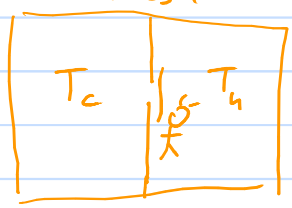
Example: Maxwell's demon

start



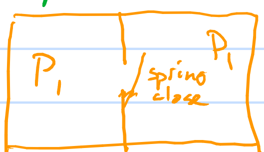
demon does no work
fast atoms allow left to right
slow atoms allow right to left

finish

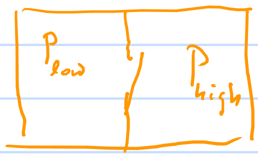


once get T difference run heat engine

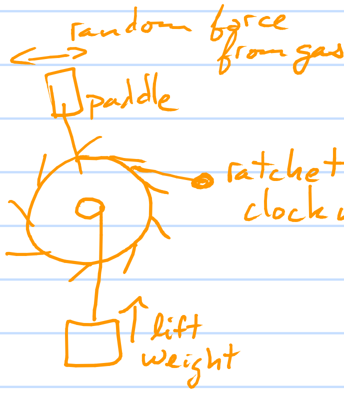
Example: Pressure diff



atoms hit spring door from left get through

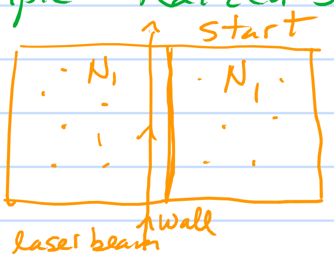


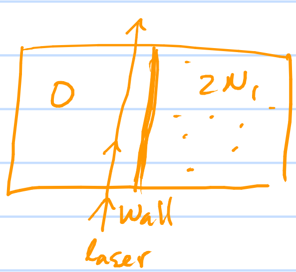
Example: Feynman's ratchet



Example: Raizen's 1 way wall

start



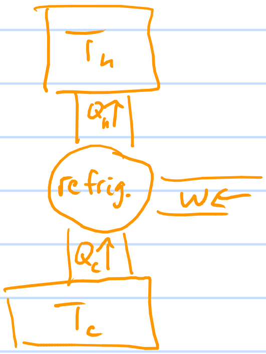


all atoms on right side!
allow wall to move left to get free work

excited atoms get through wall; ground state atoms bounce off wall; laser excites atoms just before wall

Section 4.2 Refrigerators

A refrigerator is a heat engine with all arrows reversed. You put in work to move heat from the cold reservoir to the hot reservoir. Also how heat pump works.



From 1st law $Q_h = W + Q_c$

Instead of efficiency, measure coefficient of performance

$$COP = Q_c/W = \frac{Q_c}{Q_h - Q_c} = \frac{Q_c/Q_h}{1 - Q_c/Q_h}$$

As with heat engine there are limits due to entropy

$$0 \le \Delta S_{TOT} \le \Delta S_{hot} + \Delta S_{cold} = Q_h/T_h - Q_c/T_c \Rightarrow Q_c/Q_h \le T_c/T_h$$

$$COP \le \frac{T_c/T_h}{1 - Q_c/Q_h} \le \frac{T_c/T_h}{1 - T_c/T_h} = \frac{T_c}{T_h - T_c}$$

As $T_h - T_c \to 0$ the limit on COP can be bigger. "Easier" to move heat from cold to hot when ΔT is small.

As with heat engine, Carnot cycle gives equality.

Prob 4.7 explain

Prob 4.8 explain

Prob 4.9 $T_h - T_c = 15\,K$ $T_c \sim 285\,K$ $COP \sim 20$

Prob 4.14

a) Q_h = what you want W = what you supply

b) $Q_h = Q_c + W$ yes $COP = Q_h/(Q_h - Q_c)$

c) $COP \le T_h/(T_h - T_c) = 1/(1 - T_c/T_h)$

d) Class? for furnace $Q_h = W$, for heat pump $Q_h = COP * W$

why do heat pumps work poorly when T is really cold outside?