PHYS 342 (Spring 2025) Test 2, V1 Name: ________________________

Multiple choice (5 pts each) – circle the correct answers.

1. For a hydrogen atom, the electron is in the state with $n = 7$, $l = 4$, $m_l = -2$. Determine the number of nodes in the r part of the wave function, the θ part of the wave function, and the ϕ part of the wave function: (n_r, n_θ, n_ϕ)

   (a) $(7, 4, -2)$ (b) $(7, 4, 2)$ (c) $(7, 2, 2)$
   (d) $(2, 2, 2)$ (e) $(2, 4, -2)$ (f) $(2, 2, -2)$

2. A particle moves in a 2D potential energy with the form $U(x, y) = (1/2)k(x^2 + y^2)$. An eigenstate has the form $\psi_{n_x,n_y}(x, y) = \psi_{n_x}(x)\psi_{n_y}(y)$. What is the eigenenergy?

   (a) $E = (h^2/[8ML^2])(n_x^2 + n_y^2)$
   (b) $E = h^2n^2/(8ML^2)$
   (c) $E = (n_x + n_y + 1)\hbar\omega_0$
   (d) $E = (n + 1/2)\hbar\omega_0$
   (e) none of the above are correct.

3. A particle experiences a step potential energy which is $U(x) = 0$ for $x < 0$ and $U(x) = U_0 > 0$ for $x > 0$. The particle is incident on the step from the left. If it has an energy $E < U_0$, it reflects from the step but can tunnel a distance L into the $x > 0$ region with probability 0.01. If the mass of the particle is increased by a factor of 4, but U_0 and E are the same, the probability to tunnel a distance L into the $x > 0$ region is

   (a) 0.01×4 (b) $0.01/4$ (c) 0.01×10
   (d) $0.01/10$ (e) $0.01/100$ (f) $0.01/1000$

4. The ^{3}H nucleus has a half life of 12.3 years. You make exactly 10,000 ^{3}H when you are 20 years old. When you are 56.9 years old, predict how many ^{3}H are left?

   (a) exactly 1250
   (b) exactly 2500
   (c) exactly 0
   (d) a 12.5% chance of 10,000 and 87.5% chance of 0
   (e) a 25% chance of 10,000 and 75% chance of 0
   (f) There will be approximately 1250 left but exactly how many can't be predicted.
   (g) There will be approximately 2500 left but exactly how many can't be predicted.
   (h) None of the above are correct.

5. There's a potential energy which has the eigenstate $\psi_n(x) = A(L^2 - x^2)e^{-\alpha x^2}$ which is defined for all x. The parameters $L > 0$ and $\alpha > 0$. Which eigenstate is it?

   (a) The lowest energy state: $n = 1$.
   (b) The first excited state: $n = 2$.
   (c) The second excited state: $n = 3$.
   (d) An eigenstate with $n \geq 4$.
   (e) This is a trick question because this function can't be an eigenstate.

Problems: Show *all* work to receive full credit.
Fundamental constants (e, h, masses, etc) are top of the equation page.

10 pts 1. The potential energy, $U(x)$, is finite everywhere. An energy eigenstate is $\psi(x) = Ae^{-\alpha x} + Be^{\alpha x}$ for $x < 0$ and $\psi(x) = Ce^{ikx} + De^{-ikx}$ for $x > 0$. The parameters $\alpha > 0$ and $k > 0$. a) Do any of A, B, C, D have to be 0? Explain why. b) Give the two equations that determine C and D in terms of the other parameters.

10 pts 2. A hydrogen atom is in the 3p state at a spot where the magnetic field points in the z-direction with magnitude 2.00 T. The magnetic field gives an interaction energy of $U = -\vec{\mu} \cdot \vec{B}$ from the spin of the electron. Find all the energies that come from the magnetic interaction. Are there any energies that have more than 1 possible state?

10 pts 3. An electron with a kinetic energy of 25.0 eV hits a stationary hydrogen atom in its ground state. The atom makes a transition to an $n = 2$ state but hardly changes it's center of mass velocity. What is the electron's kinetic energy after the collision?

10 pts 4. The nucleus $^{236}_{92}\mathrm{U}$ fissions into $^{92}_{36}\mathrm{Kr}$ and $^{142}_{56}\mathrm{Ba}$ plus neutrons. a) How many neutrons? b) Estimate the total kinetic energy of the Kr and Ba by approximating that they start from rest with a center-to-center separation of 2/3 the U diameter.

15 pts 5. As a part of your evil plot to take over the world, you have stolen 120 kg of of tritium, ^3H. The tritium decays into a ^3He nucleus, an electron, and a neutrino. The mass of neutrinos are so small they have not yet been measured. a) How many tritium nuclei decay in the first minute you have it? b) After 4.0 years, how many tritium nuclei decay in a minute? c) In the first minute, how much kinetic energy is released?

$g = 9.80 \frac{m}{s^2}$ $h = 6.63 \times 10^{-34}\ J\ s = 4.14 \times 10^{-15}\ eV\ s$ $\hbar = \frac{h}{2\pi}$ $c = 3.00 \times 10^8 \frac{m}{s}$

$M_{elec} = 9.11 \times 10^{-31}\ kg$ $M_{prot} = 1.67 \times 10^{-27}\ kg$ $M_{muon} = 1.88 \times 10^{-28}\ kg$

$1.60 \times 10^{-19}\ J = 1\ eV$ $e = 1.60 \times 10^{-19}$ C $1/(4\pi\varepsilon_0) = 8.99 \times 10^9$ N m^2/C^2

Galilean relativity	$x' = x - ut, v'_x = v_x - u$	2.1
Einstein's postulates	(1) The laws of physics are the same in all inertial frames. (2) The speed of light has the same value c in all inertial frames.	2.3
Time dilation	$\Delta t = \dfrac{\Delta t_0}{\sqrt{1 - u^2/c^2}}$ (Δt_0 = proper time)	2.4
Length contraction	$L = L_0\sqrt{1 - u^2/c^2}$ (L_0 = proper length)	2.4
Velocity addition	$v = \dfrac{v' + u}{1 + v'u/c^2}$	2.4
Doppler effect (source and observer separating)	$f' = f\sqrt{\dfrac{1 - u/c}{1 + u/c}}$	2.4
Lorentz transformation	$x' = \dfrac{x - ut}{\sqrt{1 - u^2/c^2}}$, $y' = y$, $z' = z$, $t' = \dfrac{t - (u/c^2)x}{\sqrt{1 - u^2/c^2}}$	2.5
Lorentz velocity transformation	$v'_x = \dfrac{v_x - u}{1 - v_x u/c^2}$, $v'_y = \dfrac{v_y\sqrt{1 - u^2/c^2}}{1 - v_x u/c^2}$, $v'_z = \dfrac{v_z\sqrt{1 - u^2/c^2}}{1 - v_x u/c^2}$	2.5
Clock synchronization	$\Delta t' = \dfrac{uL/c^2}{\sqrt{1 - u^2/c^2}}$	2.5
Relativistic momentum	$\vec{\mathbf{p}} = \dfrac{m\vec{\mathbf{v}}}{\sqrt{1 - v^2/c^2}}$	2.7
Relativistic kinetic energy	$K = \dfrac{mc^2}{\sqrt{1 - v^2/c^2}} - mc^2$	2.7
Rest energy	$E_0 = mc^2$	2.7
Relativistic total energy	$E = K + E_0 = \dfrac{mc^2}{\sqrt{1 - v^2/c^2}}$	2.7
Momentum-energy relationship	$E = \sqrt{(pc)^2 + (mc^2)^2}$	2.7
Extreme relativistic approximation	$E \cong pc$	2.7
Conservation laws	In an isolated system of particles, the total momentum and the relativistic total energy remain constant.	2.8

Hubble's law	$v = H_0 d$	15.1
Number density of photons	$N/V = (2.03 \times 10^7 \text{photons/m}^3 \cdot \text{K}^3)T^3$	15.2
Energy density of photons	$U = (4.72 \times 10^3\ \text{eV/m}^3 \cdot \text{K}^4)T^4$	15.2
Gravitational frequency change	$\Delta f/f = gH/c^2$	15.4
Deflection of starlight	$\theta = 2GM/Rc^2$	15.5
Perihelion precession	$\Delta\phi = \dfrac{6\pi GM}{c^2 r_{\text{min}}(1 + e)}$	15.5
Schwarzschild radius	$r_S = 2GM/c^2$	15.6
Age of matter-dominated universe	$t = 1/\sqrt{6\pi G\rho_{\text{m}}}$	15.7
Age of radiation-dominated universe	$t = \sqrt{3/32\pi G\rho_{\text{r}}}$	15.7
Temperature of universe at age t	$T = \dfrac{1.5 \times 10^{10}\ \text{s}^{1/2} \cdot \text{K}}{t^{1/2}}$	15.8
Fraction of photons above E_0	$f = 0.42e^{-E_0/kT} \times \left[\left(\frac{E_0}{kT}\right)^2 + 2\left(\frac{E_0}{kT}\right) + 2\right]$	15.9
Critical density of universe	$\rho_{\text{cr}} = \dfrac{3H^2}{8\pi G} = 0.97 \times 10^{-26}\ \text{kg/m}^3$	15.10

Double-slit maxima	$y_n = n\frac{\lambda D}{d} \quad n = 0, 1, 2, 3, \ldots$	3.1
Bragg's law for X-ray diffraction	$2d\sin\theta = n\lambda \quad n = 1, 2, 3, \cdots$	3.1
Energy of photon	$E = hf = hc/\lambda$	3.2
Maximum kinetic energy of photoelectrons	$K_{max} = eV_s = hf - \phi$	3.2
Cutoff wavelength	$\lambda_c = hc/\phi$	3.2
Stefan's law	$I = \sigma T^4$	3.3
Wien's displacement law	$\lambda_{max}T = 2.8978 \times 10^{-3}\ \mathrm{m \cdot K}$	3.3
Rayleigh-Jeans formula	$I(\lambda) = \frac{2\pi c}{\lambda^4}kT$	3.3
Planck's blackbody distribution	$I(\lambda) = \frac{2\pi hc^2}{\lambda^5}\frac{1}{e^{hc/\lambda kT} - 1}$	3.3
Compton scattering	$\frac{1}{E'} - \frac{1}{E} = \frac{1}{m_e c^2}(1 - \cos\theta)$, $\lambda' - \lambda = \frac{h}{m_e c}(1 - \cos\theta)$	3.4
Bremsstrahlung	$\lambda_{min} = hc/K = hc/e\Delta V$	3.5
Pair production	$hf = E_+ + E_- = (m_e c^2 + K_+) + (m_e c^2 + K_-)$	3.5
Electron-positron annihilation	$(m_e c^2 + K_+) + (m_e c^2 + K_-) = E_1 + E_2$	3.5

De Broglie wavelength	$\lambda = h/p$	4.1
Single slit diffraction	$a\sin\theta = n\lambda \quad n = 1, 2, 3, \ldots$	4.2
Classical position-wavelength uncertainty	$\Delta x \Delta\lambda \sim \varepsilon\lambda^2$	4.3
Classical frequency-time uncertainty	$\Delta f\, \Delta t \sim \varepsilon$	4.3
Heisenberg position-momentum uncertainty	$\Delta x \Delta p_x \sim \hbar$	4.4
Heisenberg energy-time uncertainty	$\Delta E \Delta t \sim \hbar$	4.4
Statistical momentum uncertainty	$\Delta p_x = \sqrt{(p_x^2)_{av} - (p_{x,av})^2}$	4.4
Wave packet (discrete k)	$y(x) = \sum A_i \cos k_i x$	4.5
Wave packet (continuous k)	$y(x) = \int A(k) \cos kx\, dk$	4.5
Group speed of wave packet	$v_{group} = \frac{d\omega}{dk}$	4.6

Time-independent Schrödinger equation	$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} + U(x)\psi(x) = E\psi(x)$	5.3
Time-dependent Schrödinger equation	$\Psi(x, t) = \psi(x)e^{-i\omega t}$	5.3
Probability density	$P(x) = \lvert\psi(x)\rvert^2$	5.3
Normalization condition	$\int_{-\infty}^{+\infty} \lvert\psi(x)\rvert^2\, dx = 1$	5.3
Probability in interval x_1 to x_2	$P(x_1 : x_2) = \int_{x_1}^{x_2} \lvert\psi(x)\rvert^2\, dx$	5.3
Average or expectation value of $f(x)$	$[f(x)]_{av} = \int_{-\infty}^{+\infty} \lvert\psi(x)\rvert^2 f(x)\, dx$	5.3
Constant potential energy, $E > U_0$	$\psi(x) = A\sin kx + B\cos kx$, $k = \sqrt{2m(E - U_0)/\hbar^2}$	5.4
Constant potential energy, $E < U_0$	$\psi(x) = Ae^{k'x} + Be^{-k'x}$, $k' = \sqrt{2m(U_0 - E)/\hbar^2}$	5.4
Infinite potential energy well	$\psi_n(x) = \sqrt{\frac{2}{L}}\sin\frac{n\pi x}{L}$, $E_n = \frac{h^2 n^2}{8mL^2}\ (n = 1, 2, 3, \ldots)$	5.4
Two-dimensional infinite well	$\psi(x, y) = \frac{2}{L}\sin\frac{n_x \pi x}{L}\sin\frac{n_y \pi y}{L}$, $E = \frac{h^2}{8mL^2}(n_x^2 + n_y^2)$	5.4
Simple harmonic oscillator ground state	$\psi(x) = (m\omega_0/\hbar\pi)^{1/4}\, e^{-(\sqrt{km}/2\hbar)x^2}$	5.5
Simple harmonic oscillator energies	$E_n = (n + \frac{1}{2})\hbar\omega_0\ (n = 0, 1, 2, \ldots)$	5.5
Potential energy step, $E > U_0$	$\psi_0(x < 0) = A\sin k_0 x + B\cos k_0 x$, $\psi_1(x > 0) = C\sin k_1 x + D\cos k_1 x$	5.6
Potential energy step, $E < U_0$	$\psi_0(x < 0) = A\sin k_0 x + B\cos k_0 x$, $\psi_1(x > 0) = Ce^{k_1 x} + De^{-k_1 x}$	5.6

Scattering impact parameter	$b = \frac{zZ}{2K}\frac{e^2}{4\pi\varepsilon_0}\cot\frac{1}{2}\theta$	6.3
Fraction scattered at angles $> \theta$	$f_{>\theta} = nt\pi b^2$	6.3
Rutherford scattering formula	$N(\theta) = \frac{nt}{4r^2}\left(\frac{zZ}{2K}\right)^2\left(\frac{e^2}{4\pi\varepsilon_0}\right)^2\frac{1}{\sin^4\frac{1}{2}\theta}$	6.3
Distance of closest approach	$d = \frac{1}{4\pi\varepsilon_0}\frac{zZe^2}{K}$	6.3
Balmer formula	$\lambda = (364.5\,\text{nm})\frac{n^2}{n^2-4}$ $(n = 3, 4, 5, \ldots)$	6.4
Radii of Bohr orbits in hydrogen	$r_n = \frac{4\pi\varepsilon_0\hbar^2}{me^2}n^2 = a_0 n^2$ $(n = 1, 2, 3, \ldots)$	6.5
Energies of Bohr orbits in hydrogen	$E_n = -\frac{me^4}{32\pi^2\varepsilon_0^2\hbar^2}\frac{1}{n^2} = \frac{-13.60\,\text{eV}}{n^2}$ $(n = 1, 2, 3, \ldots)$	6.5
Excitation energy of level n	$E_n - E_1$	6.5
Binding (or ionization) energy of level n	$\lvert E_n\rvert$	6.5
Hydrogen wavelengths in Bohr model	$\lambda = \frac{64\pi^3\varepsilon_0^2\hbar^3 c}{me^4}\left(\frac{n_1^2 n_2^2}{n_1^2 - n_2^2}\right) = \frac{1}{R_\infty}\left(\frac{n_1^2 n_2^2}{n_1^2 - n_2^2}\right)$	6.5
Single-electron atoms with $Z > 1$	$r_n = \frac{a_0 n^2}{Z}$, $E_n = -(13.60\,\text{eV})\frac{Z^2}{n^2}$	6.5
Reduced mass of proton-electron system	$m = \frac{m_e m_p}{m_e + m_p}$	6.8

Orbital angular momentum	$\lvert\vec{\mathbf{L}}\rvert = \sqrt{l(l+1)}\hbar$ $(l = 0, 1, 2, \ldots)$	7.2
Orbital magnetic quantum number	$L_z = m_l\hbar$ $(m_l = 0, \pm 1, \pm 2, \ldots, \pm l)$	7.2
Spatial quantization	$\cos\theta = \frac{L_z}{\lvert\mathbf{L}\rvert} = \frac{m_l}{\sqrt{l(l+1)}}$	7.2
Angular momentum uncertainty relationship	$\Delta L_z \Delta\phi \geqslant \hbar$	7.2
Hydrogen quantum numbers	$n = 1, 2, 3, \ldots$ $l = 0, 1, 2, \ldots, n-1$ $m_l = 0, \pm 1, \pm 2, \ldots, \pm l$	7.3
Hydrogen energy levels	$E_n = -\frac{me^4}{32\pi^2\varepsilon_0^2\hbar^2}\frac{1}{n^2}$	7.3
Hydrogen wave functions	$\psi_{n,l,m_l}(r,\theta,\phi) = R_{n,l}(r)\Theta_{l,m_l}(\theta)\Phi_{m_l}(\phi)$	7.3
Radial probability density	$P(r) = r^2\lvert R_{n,l}(r)\rvert^2$	7.4
Angular probability density	$P(\theta,\phi) = \lvert\Theta_{l,m_l}(\theta)\Phi_{m_l}(\phi)\rvert^2$	7.5
Orbital magnetic dipole moment	$\vec{\boldsymbol{\mu}}_L = -(e/2m)\vec{\mathbf{L}}$	7.6
Spin magnetic dipole moment	$\vec{\boldsymbol{\mu}}_S = -(e/m)\vec{\mathbf{S}}$	7.6
Spin angular momentum	$\lvert\vec{\mathbf{S}}\rvert = \sqrt{s(s+1)}\hbar = \sqrt{3/4}\hbar$ (for $s = 1/2$)	7.6
Spin magnetic quantum number	$S_z = m_s\hbar$ $(m_s = \pm 1/2)$	7.6
Spectroscopic notation	$s\ (l = 0), p\ (l = 1), d\ (l = 2), f\ (l = 3), \ldots$	7.7
Selection rules for photon emission	$\Delta l = \pm 1 \quad \Delta m_l = 0, \pm 1$	7.7, 7.8
Normal Zeeman effect	$\Delta\lambda = \frac{\lambda^2}{hc}\Delta E = \frac{\lambda^2}{hc}\mu_B B$	7.8
Fine-structure estimate	$\Delta E = mc^2\alpha^4/n^5 \quad (\alpha \approx 1/137)$	7.9

Pauli exclusion principle	*No two electrons in a single atom can have the same set of quantum numbers* (n, l, m_l, m_s).	8.1
Filling order of atomic subshells	$1s, 2s, 2p, 3s, 3d, 4s, 3d, 4p, 5s, 4d, 5p, 6s, 4f, 5d, 6p, 7s, 5f, 6d$	8.2
Capacity of subshell nl	$2(2l+1)$	8.2
Energy of screened electron	$E_n = (-13.6\,\text{eV})\frac{Z_{\text{eff}}^2}{n^2}$	8.3
Moseley's law for K_α X rays	$\Delta E = (10.2\,\text{eV})(Z-1)^2$	8.5
Adding angular momenta l_1, m_{l1} and l_2, m_{l2}	$L_{\text{max}} = l_1 + l_2$, $L_{\text{min}} = \lvert l_1 - l_2\rvert$, $M_L = m_{l1} + m_{l2}$	8.6
Hund's rules for ground state	First $S = M_{S,\text{max}}$, then $L = M_{L,\text{max}}$	8.6

Nuclear radius	$R = R_0 A^{1/3}, R_0 = 1.2$ fm	12.2
Nuclear binding energy	$B = [Nm_n + Zm(^1_1H_0) - m(^A_ZX_N)]c^2$	12.3
Proton separation energy	$S_p = [m(^{A-1}_{Z-1}X'_N) + m(^1H) - m(^A_ZX_N)]c^2$	12.3
Neutron separation energy	$S_n = [m(^{A-1}{}_ZX_{N-1}) + m_n - m(^A_ZX_N)]c^2$	12.3
Range of exchanged particle	$mc^2 = \hbar c/x$	12.4
Activity	$a = \lambda N, \lambda = \ln 2/t_{1/2} = 0.693/t_{1/2}$	12.5
Radioactive decay law	$N = N_0 e^{-\lambda t}, a = a_0 e^{-\lambda t}$	12.5
Q value of decay $X \rightarrow X' + x$	$Q = [m_X - (m_{X'} + m_x)]c^2$	12.6
Q value of alpha decay	$Q = [m(X) - m(X') - m(^4He)]c^2$	12.7
Kinetic energy of alpha particle	$K_\alpha \cong Q(A-4)/A$	12.7
Q values of beta decay	$Q_{\beta^-} = [m(^AX) - m(^AX')]c^2$, $Q_{\beta^+} = [m(^AX) - m(^AX') - 2m_e]c^2$	12.8
Recoil in gamma decay	$K_R = E_\gamma^2/2Mc^2$	12.9

Forces	Strong, electromagnetic, weak, gravitational	14.1
Field particles	Gluon (g), photon (γ), weak boson ($W^\pm, Z^0$), graviton	14.1
Leptons	$e^-, \nu_e, \mu^-, \nu_\mu, \tau^-, \nu_\tau$	14.2
Mesons	$\pi^\pm, \pi^0, K^\pm, K^0, \overline{K}^0, \eta, \rho^\pm, \eta'$, $D^\pm, \psi, B^\pm, \Upsilon, \ldots$	14.2
Baryons	$p, n, \Lambda^0, \Sigma^{\pm,0}, \Xi^{-,0}, \Omega^-, \ldots$	14.2
Conservation of lepton number L	*In any process, L_e, L_μ, and L_τ remain constant.*	14.3
Conservation of baryon number B	*In any process, B remains constant.*	14.3
Conservation of strangeness S	*In strong and electromagnetic processes, S remains constant; in weak processes,* $\Delta S = 0$ or ± 1.	14.3
Q value in decays or reactions	$Q = (m_i - m_f)c^2$	14.5, 14.6
Threshold energy in reactions	$K_{th} = -Q(m_1 + m_2 + m_3 + m_4 + m_5 + \cdots)/2m_2$	14.6
Quarks	u, d, c, s, t, b	14.7

SOME PARTICLE MASSES

	kg	u	MeV/c^2
Electron	$9.1093829 \times 10^{-31}$	$5.485799095 \times 10^{-4}$	0.51099893
Proton	$1.67262178 \times 10^{-27}$	1.0072764668	938.27205
Neutron	$1.67492735 \times 10^{-27}$	1.0086649160	939.56538
Deuteron	$3.3435835 \times 10^{-27}$	2.0135532127	1875.61286
Alpha	$6.6446568 \times 10^{-27}$	4.001506179	3727.3792

	Z	A	Atomic mass (u)	*Abundance* or Half-life
H	1	1	1.0078250	*99.985%*
		2	2.014102	*0.015%*
		3	3.016049	12.3 y
He	2	3	3.016029	*0.000137%*
		4	4.002603	*99.999863%*
Li	3	6	6.015123	*7.59%*
		7	7.016005	*92.41%*
		8	8.022487	0.84 s
Be	4	7	7.016930	53.2 d
		8	8.005305	0.07 fs
		9	9.012182	*100%*
		10	10.013534	1.5 My
		11	11.021658	13.8 s
B	5	8	8.024607	0.77 s
		9	9.013329	0.85 as
		10	10.012937	*19.8%*
		11	11.009305	*80.2%*
		12	12.014352	20.2 ms

CONVERSION FACTORS

$1 \text{ eV} = 1.60217657 \times 10^{-19}$ J

$1 \text{ u} = 931.49406 \text{ MeV}/c^2$

$= 1.66053892 \times 10^{-27}$ kg

$1 \text{ y} = 3.156 \times 10^7 \text{ s} \cong \pi \times 10^7$ s

1 barn (b) $= 10^{-28}$ m^2

1 curie (Ci) $= 3.7 \times 10^{10}$ decays/s

1 light-year $= 9.46 \times 10^{15}$ m

1 parsec $= 3.26$ light-year