PHYS 342 (Spring 2025) Test 1, V1 Name: ______________________

Multiple choice (5 pts each) – circle the correct answers.

1. Sagitarrius A* is a black hole with mass 4.3×10^6 solar masses while Messier 108 is a black hole with mass 2.4×10^7 solar masses. What is the diameter of Sagitarrius A* relative to Messier 108? *HWK2 prob 5*

   (a) 5.6× larger (V2) (b) 5.6× smaller (V1) (c) 31× larger
   (d) 31× smaller (e) 174× larger (f) 174× smaller

2. Light from a laser with wavelength λ and intensity 1 μJ/m^2 has 3.2×10^8 photons per second go through an area of 1 cm^2. For a laser with the same intensity but wavelength $\lambda/10$, how many photons go through an area of 1 cm^2? *HWK3 prob 5*

   (a) 3.2×10^7 photons per second (V1)
   (b) 3.2×10^8 photons per second
   (c) 3.2×10^9 photons per second (V2)

3. You have found two solutions of Schrodinger's equation, $\Psi_1(x,t)$ and $\Psi_2(x,t)$. Which one is also a possible solution? *HWK5 prob 4*

   (a) $\Psi(x,t) = C_1\Psi_1(x,t) + C_2\Psi_2(x,t)$ where C_1 and C_2 don't depend on x or t
   (b) $\Psi(x,t) = C_1(x)\Psi_1(x,t) + C_2(x)\Psi_2(x,t)$ where C_1 and C_2 must depend on x
   (c) $\Psi(x,t) = C_1(t)\Psi_1(x,t) + C_2(t)\Psi_2(x,t)$ where C_1 and C_2 must depend on t
   (d) $\Psi(x,t) = C_1(x,t)\Psi_1(x,t) + C_2(x,t)\Psi_2(x,t)$ where C_1 and C_2 must depend on x and t

4. A beam of photons with an energy of 6.78 eV is incident on aluminum and absorbed. Will electrons be ejected? If yes, give their maximum kinetic energy. *HWK4 prob 1*

   (a) no (b) yes, 4.50 eV (c) yes, 2.70 eV (V1)
   (d) yes, 2.08 (V2) eV (e) yes, 6.78 eV (f) yes, 10.86 eV

5. Astronauts leave earth and travel to a star 4 light-years away and immediately come back. The space ship they were in traveled at 0.8 c. How much did the astronauts age? *HWK1 prob 2*

   (a) less than 10 years
   (b) 10 years
   (c) more than 10 years

6. You measure that 2 galaxies are moving away from the Milky Way. The first moves away with speed 10^7 m/s and the second moves away with speed 2×10^7 m/s. The distance to the second galaxy is *HWK3 prob 1*

   (a) roughly 1/2 the distance to the first. (V2)
   (b) roughly the same distance to the first.
   (c) roughly 2 times the distance to the first. (V1)

Problems: Show *all* work to receive full credit.
Fundamental constants (e, h, masses, etc) are top of the equation page.

10 pts 1. A star with radius 9.0×10^8 m radiates light like an ideal thermal source at a temperature of 7200 K. a) Keeping the temperature fixed, how would the rate that energy is radiated change if the radius changes to 7.7×10^8 m? b) Keeping the radius fixed, how would the rate that energy is radiated change if the temperature increases to 8100 K? *HWK4 prob 3*

a) The rate that light is radiated from an ideal thermal source is proportional to the surface area. The surface area of a sphere is proportional to R^2. b) The rate that light is radiated from an ideal thermal source is proportional to T^4.

$$a)\ ratio = \left(\frac{7.7\times10^8}{9.0\times10^8}\right)^2 = 0.73 \qquad b)\ ratio = \left(\frac{8100}{7200}\right)^4 = 1.60 \quad (V1)$$

$$a)\ ratio = \left(\frac{7.1\times10^8}{9.5\times10^8}\right)^2 = 0.56 \qquad b)\ ratio = \left(\frac{8800}{6400}\right)^4 = 3.57 \quad (V1)$$

10 pts 2. A spaceship is traveling directly at you at a speed of 0.500 c. They fire at you a proton beam. You measure the protons in the beam to have a speed of 0.950 c. a) What is the speed of the protons as measured by the creatures on the spaceship? b) In firing one proton, what is the *change* in momentum of the spaceship as measured by you? *a) HWK1 prob 3, b) HWK2 prob 1*

a) Use the velocity addition formula or the Lorentz transformation formula. Using the latter with v_x the velocity of the proton and u the velocity of the space ship. b) The momentum change of the space ship is opposite the momentum change of the proton where the initial speed of the proton is the speed of the ship, $\Delta p = -(p_f - p_i)$.

$$a)\ v'_x = \frac{0.950 - 0.500}{1 - 0.950\times0.500}c = 0.857\ c$$

$$b)\Delta p = -M_{prot}c\left(\frac{0.95}{\sqrt{1-0.95^2}} - \frac{0.5}{\sqrt{1-0.5^2}}\right) = -2.465M_{prot}c = 1.235\times10^{-18}\ kg\ m/s \quad (V1)$$

$$a)\ v'_x = \frac{0.910 - 0.600}{1 - 0.910\times0.600}c = 0.683\ c$$

$$b)\Delta p = -M_{prot}c\left(\frac{0.91}{\sqrt{1-0.91^2}} - \frac{0.6}{\sqrt{1-0.6^2}}\right) = -1.445M_{prot}c = 7.239\times10^{-19}\ kg\ m/s \quad (V2)$$

10 pts 3. A type of Kaon with mass 497.611 MeV/c^2 decays into two identical Pions (the mass of one Pion is 134.977 MeV/c^2). If the Kaon is initially at rest, what speed does each Pion have? *HWK2 prob 2*

This is a conservation of momentum and energy problem. Because the Kaon is initially at rest, the sum of the Pions' momenta must be 0. Because the pions have the same mass, they must have the same velocity. For conservation of energy, the total energy before is the Kaon rest energy. The total energy after is 2× the total energy of one Pion.

$$M_K c^2 = 2\frac{M_\pi c^2}{\sqrt{1-(v_\pi/c)^2}} \quad \rightarrow \quad \sqrt{1-(v_\pi/c)^2} = \frac{2M_\pi}{M_K} \quad \rightarrow \quad v_\pi = c\sqrt{1-(2M_\pi/M_K)^2}$$

$$v_\pi = c\sqrt{1-(2\times 134.977/497.611)^2} = 0.840\ c \quad (V1)$$

$$v_\pi = c\sqrt{1-(2\times 139.570/497.611)^2} = 0.828\ c \quad (V2)$$

10 pts 4. An electron travels in a region with no forces. It has speed of 3.0×10^6 m/s. Determine the electron's wavelength and frequency. *Chap4 notes pg 1*

The wavelength of an object with mass is $\lambda = h/p$ and its frequency is $f = E/h$. Because the speed of the electron is non relativistic, use $p = mv$ and $E = (1/2)mv^2$.

$$\lambda = \frac{6.63\times 10^{-34}}{9.11\times 10^{-31}\times 3.0\times 10^6}\ m = 2.43\times 10^{-10}\ m$$

$$f = \frac{9.11\times 10^{-31}\times (3.0\times 10^6)^2}{2\times 6.63\times 10^{-34}}\ Hz = 6.18\times 10^{15}\ Hz \quad (V1)$$

$$\lambda = \frac{6.63\times 10^{-34}}{9.11\times 10^{-31}\times 2.0\times 10^6}\ m = 3.64\times 10^{-10}\ m$$

$$f = \frac{9.11\times 10^{-31}\times (2.0\times 10^6)^2}{2\times 6.63\times 10^{-34}}\ Hz = 2.75\times 10^{15}\ Hz \quad (V2)$$

15 pts 5. Light with a wavelength of 633 nm and intensity of 1.30 W/m^2 is incident on a spherical microparticle. Assume the light travels in the $+y$-direction and the microparticle is in space with no other interactions. The radius of the microparticle is 4.5×10^{-6} m and it has a density of 1800 kg/m^3. Assume that all of the photons that hit the microparticle are absorbed. a) Does the mass of the microparticle increase with time? If so, what is the rate of change of the mass (give your answer in kg/s). b) Does the microparticle accelerate? If so, give the acceleration vector. *HWK3 prob 4*

a) This is a conservation of energy question. The rate that the microparticle absorbs energy from the light beam is $I \times \pi R^2$. The microparticle gets very little kinetic energy from the light so this energy must go into increasing the rest mass: $c^2 \Delta M/\Delta t = I \times \pi R^2$. In the answers below there is less than one proton mass added to the microparticle each second (another comparison: the mass changes by about a part per 10^{15} each second)

$$\frac{\Delta M}{\Delta t} = \frac{1.30 \times \pi \times (4.5 \times 10^{-6})^2}{(3 \times 10^8)^2} \; kg/s = 9.19 \times 10^{-28} \; kg/s \quad (V1)$$

$$\frac{\Delta M}{\Delta t} = \frac{2.40 \times \pi \times (3.6 \times 10^{-6})^2}{(3 \times 10^8)^2} \; kg/s = 1.09 \times 10^{-27} \; kg/s \quad (V2)$$

b) This is a conservation of momentum question. The rate that the microparticle absorbs momentum from the beam is $I \times \pi R^2/c$ using the momentum of light with energy E is $p = E/c$. This must be the rate of change of the microparticle momentum so the total momentum of light plus microparticle doesn't change. To get the rate of change of velocity (this is acceleration), divide the rate of change of momentum by the mass of the microparticle. The mass of the microparticle is its density times volume.

$$M = \frac{4}{3}\pi(4.5 \times 10^{-6})^3 \times 1800 \; kg = 6.87 \times 10^{-13} \; kg$$

$$a = \frac{1.30 \times \pi \times (4.5 \times 10^{-6})^2}{3 \times 10^8 \times 6.87 \times 10^{-13}} \; \frac{m}{s^2} = 4.01 \times 10^{-7} \; \frac{m}{s^2} \text{ in the + y direction} \quad (V1)$$

$$M = \frac{4}{3}\pi(3.6 \times 10^{-6})^3 \times 1600 \; kg = 3.13 \times 10^{-13} \; kg$$

$$a = \frac{2.40 \times \pi \times (3.6 \times 10^{-6})^2}{3 \times 10^8 \times 3.13 \times 10^{-13}} \; \frac{m}{s^2} = 1.04 \times 10^{-6} \; \frac{m}{s^2} \text{ in the + z direction} \quad (V1)$$

15 pts 6. An electron experiences a potential energy which is $U(x) = 0$ for $x_0 < x < x_f$ and $U(x) = \infty$ for $x < x_0$ or $x > x_f$. The x_0, x_f are constants. a) What are the allowed wavelengths for the electron? b) What are the allowed momenta? c) If you **don't** have to worry about relativity, what are the allowed energies? d) If you **do** have to worry about relativity, what are the allowed energies? e) For the nonrelativistic case, the electron starts in the 5th state then emits a photon and ends in the 4th state. What is the frequency of the photon? *HWK5 prob2*

a) There must be an integer number of (1/2) wavelengths from x_0 to x_f meaning $n\lambda_n/2 = (x_f - x_0)$. The combination $x_f - x_0$ comes up a lot so define it to be $\Delta x = x_f - x_0$ The allowed wavelengths are $\lambda_n = 2\Delta x/n$ with $n = 1, 2, 3, ...$

b) The allowed momenta from de Broglie is $p_n = h/\lambda_n = nh/(2\Delta x)$ with $n = 1, 2, 3, ...$

c) If you don't have to worry about relativity, you can use the $E = p^2/(2M_{elec})$ to get the allowed energies as $E_n = n^2h^2/(8M_{elec}\Delta x^2)$ with $n = 1, 2, 3, ...$

d) If you do have to worry about relativity, then use the relativistic energy expression. You can do it by giving either the allowed total energy or the allowed kinetic energy. For the allowed total energy, $E^2 = c^2[p^2 + (M_{elec}c)^2]$ to get $E_n^2 = c^2[(nh/[2\Delta x])^2 + (M_{elec}c)^2]$ with $n = 1, 2, 3, ...$

e) To conserve energy, the energy of the photon, $E = hf$, must equal the energy decrease of the electron. This means $f = (E_5 - E_4)/h$. For the nonrelativistic case this is $f = 9h/(8M_{elec}\Delta x^2)$.

$g = 9.80\ \frac{m}{s^2}$ $h = 6.63 \times 10^{-34}\ J\ s = 4.14 \times 10^{-15}\ eV\ s$ $\hbar = \frac{h}{2\pi}$ $c = 3.00 \times 10^{8}\ \frac{m}{s}$
$M_{elec} = 9.11 \times 10^{-31}\ kg$ $M_{prot} = 1.67 \times 10^{-27}\ kg$ $M_{muon} = 1.88 \times 10^{-28}\ kg$
$1.60 \times 10^{-19}\ J = 1\ eV$ Work functions: Na 2.28 eV, Al 4.08 eV, Cu 4.70 eV
$V_{sph} = \frac{4}{3}\pi r^3$

Galilean relativity	$x' = x - ut,\ v_x' = v_x - u$	2.1
Einstein's postulates	(1) The laws of physics are the same in all inertial frames. (2) The speed of light has the same value c in all inertial frames.	2.3
Time dilation	$\Delta t = \dfrac{\Delta t_0}{\sqrt{1 - u^2/c^2}}$ (Δt_0 = proper time)	2.4
Length contraction	$L = L_0\sqrt{1 - u^2/c^2}$ (L_0 = proper length)	2.4
Velocity addition	$v = \dfrac{v' + u}{1 + v'u/c^2}$	2.4
Doppler effect (source and observer separating)	$f' = f\sqrt{\dfrac{1 - u/c}{1 + u/c}}$	2.4
Lorentz transformation	$x' = \dfrac{x - ut}{\sqrt{1 - u^2/c^2}}$, $y' = y,\ z' = z,$ $t' = \dfrac{t - (u/c^2)x}{\sqrt{1 - u^2/c^2}}$	2.5
Lorentz velocity transformation	$v_x' = \dfrac{v_x - u}{1 - v_x u/c^2}$, $v_y' = \dfrac{v_y\sqrt{1 - u^2/c^2}}{1 - v_x u/c^2}$, $v_z' = \dfrac{v_z\sqrt{1 - u^2/c^2}}{1 - v_x u/c^2}$	2.5
Clock synchronization	$\Delta t' = \dfrac{uL/c^2}{\sqrt{1 - u^2/c^2}}$	2.5
Relativistic momentum	$\vec{\mathbf{p}} = \dfrac{m\vec{\mathbf{v}}}{\sqrt{1 - v^2/c^2}}$	2.7
Relativistic kinetic energy	$K = \dfrac{mc^2}{\sqrt{1 - v^2/c^2}} - mc^2$	2.7
Rest energy	$E_0 = mc^2$	2.7
Relativistic total energy	$E = K + E_0 = \dfrac{mc^2}{\sqrt{1 - v^2/c^2}}$	2.7
Momentum-energy relationship	$E = \sqrt{(pc)^2 + (mc^2)^2}$	2.7
Extreme relativistic approximation	$E \cong pc$	2.7
Conservation laws	In an isolated system of particles, the total momentum and the relativistic total energy remain constant.	2.8

Hubble's law	$v = H_0 d$	15.1
Number density of photons	$N/V = (2.03 \times 10^7 \text{photons/m}^3 \cdot \text{K}^3)T^3$	15.2
Energy density of photons	$U = (4.72 \times 10^3\ \text{eV/m}^3 \cdot \text{K}^4)T^4$	15.2
Gravitational frequency change	$\Delta f/f = gH/c^2$	15.4
Deflection of starlight	$\theta = 2GM/Rc^2$	15.5
Perihelion precession	$\Delta\phi = \dfrac{6\pi GM}{c^2 r_{\text{min}}(1 + e)}$	15.5
Schwarzschild radius	$r_S = 2GM/c^2$	15.6
Age of matter-dominated universe	$t = 1/\sqrt{6\pi G\rho_m}$	15.7
Age of radiation-dominated universe	$t = \sqrt{3/32\pi G\rho_r}$	15.7
Temperature of universe at age t	$T = \dfrac{1.5 \times 10^{10}\ \text{s}^{1/2} \cdot \text{K}}{t^{1/2}}$	15.8
Fraction of photons above E_0	$f = 0.42e^{-E_0/kT} \times \left[\left(\frac{E_0}{kT}\right)^2 + 2\left(\frac{E_0}{kT}\right) + 2\right]$	15.9
Critical density of universe	$\rho_{cr} = \dfrac{3H^2}{8\pi G} = 0.97 \times 10^{-26}\ \text{kg/m}^3$	15.10

Double-slit maxima	$y_n = n\frac{\lambda D}{d} \quad n = 0, 1, 2, 3, \ldots$	3.1
Bragg's law for X-ray diffraction	$2d \sin\theta = n\lambda \quad n = 1, 2, 3, \cdots$	3.1
Energy of photon	$E = hf = hc/\lambda$	3.2
Maximum kinetic energy of photoelectrons	$K_{\text{max}} = eV_s = hf - \phi$	3.2
Cutoff wavelength	$\lambda_c = hc/\phi$	3.2
Stefan's law	$I = \sigma T^4$	3.3
Wien's displacement law	$\lambda_{\text{max}} T = 2.8978 \times 10^{-3}\ \text{m}\cdot\text{K}$	3.3

Rayleigh-Jeans formula	$I(\lambda) = \frac{2\pi c}{\lambda^4} kT$	3.3
Planck's blackbody distribution	$I(\lambda) = \frac{2\pi hc^2}{\lambda^5} \frac{1}{e^{hc/\lambda kT} - 1}$	3.3
Compton scattering	$\frac{1}{E'} - \frac{1}{E} = \frac{1}{m_e c^2}(1 - \cos\theta)$, $\lambda' - \lambda = \frac{h}{m_e c}(1 - \cos\theta)$	3.4
Bremsstrahlung	$\lambda_{\text{min}} = hc/K = hc/e\Delta V$	3.5
Pair production	$hf = E_+ + E_- = (m_e c^2 + K_+) + (m_e c^2 + K_-)$	3.5
Electron-positron annihilation	$(m_e c^2 + K_+) + (m_e c^2 + K_-) = E_1 + E_2$	3.5

De Broglie wavelength	$\lambda = h/p$	4.1
Single slit diffraction	$a \sin\theta = n\lambda \quad n = 1, 2, 3, \ldots$	4.2
Classical position-wavelength uncertainty	$\Delta x \Delta\lambda \sim \varepsilon\lambda^2$	4.3
Classical frequency-time uncertainty	$\Delta f\, \Delta t \sim \varepsilon$	4.3
Heisenberg position-momentum uncertainty	$\Delta x \Delta p_x \sim \hbar$	4.4
Heisenberg energy-time uncertainty	$\Delta E \Delta t \sim \hbar$	4.4

Statistical momentum uncertainty	$\Delta p_x = \sqrt{(p_x^2)_{\text{av}} - (p_{x,\text{av}})^2}$	4.4
Wave packet (discrete k)	$y(x) = \sum A_i \cos k_i x$	4.5
Wave packet (continuous k)	$y(x) = \int A(k) \cos kx\, dk$	4.5
Group speed of wave packet	$v_{\text{group}} = \frac{d\omega}{dk}$	4.6

Time-independent Schrödinger equation	$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} + U(x)\psi(x) = E\psi(x)$	5.3
Time-dependent Schrödinger equation	$\Psi(x, t) = \psi(x) e^{-i\omega t}$	5.3
Probability density	$P(x) = \lvert\psi(x)\rvert^2$	5.3
Normalization condition	$\int_{-\infty}^{+\infty} \lvert\psi(x)\rvert^2\, dx = 1$	5.3
Probability in interval x_1 to x_2	$P(x_1 : x_2) = \int_{x_1}^{x_2} \lvert\psi(x)\rvert^2\, dx$	5.3
Average or expectation value of $f(x)$	$[f(x)]_{\text{av}} = \int_{-\infty}^{+\infty} \lvert\psi(x)\rvert^2 f(x)\, dx$	5.3
Constant potential energy, $E > U_0$	$\psi(x) = A \sin kx + B \cos kx$, $k = \sqrt{2m(E - U_0)/\hbar^2}$	5.4
Constant potential energy, $E < U_0$	$\psi(x) = Ae^{k'x} + Be^{-k'x}$, $k' = \sqrt{2m(U_0 - E)/\hbar^2}$	5.4

Infinite potential energy well	$\psi_n(x) = \sqrt{\frac{2}{L}} \sin\frac{n\pi x}{L}$, $E_n = \frac{h^2 n^2}{8mL^2}\ (n = 1, 2, 3, \ldots)$	5.4
Two-dimensional infinite well	$\psi(x, y) = \frac{2}{L} \sin\frac{n_x \pi x}{L} \sin\frac{n_y \pi y}{L}$, $E = \frac{h^2}{8mL^2}(n_x^2 + n_y^2)$	5.4
Simple harmonic oscillator ground state	$\psi(x) = (m\omega_0/\hbar\pi)^{1/4}\, e^{-(\sqrt{km}/2\hbar)x^2}$	5.5
Simple harmonic oscillator energies	$E_n = (n + \frac{1}{2})\hbar\omega_0\ (n = 0, 1, 2, \ldots)$	5.5
Potential energy step, $E > U_0$	$\psi_0(x < 0) = A \sin k_0 x + B \cos k_0 x$, $\psi_1(x > 0) = C \sin k_1 x + D \cos k_1 x$	5.6
Potential energy step, $E < U_0$	$\psi_0(x < 0) = A \sin k_0 x + B \cos k_0 x$, $\psi_1(x > 0) = Ce^{k_1 x} + De^{-k_1 x}$	5.6