

Multiple choice (5 pts each) – circle the correct answers.

- Suppose the 4 electrons in Be all have the same $m_s = 1/2$. Which is the lowest energy allowed configuration under this condition?
 - $1s^4$
 - $1s^2 2s^2$
 - $1s 2s 2p^2$
 - $1s 2s 2p 3s$
- A particle moves in 1D. For $x < 0$, the potential energy is 0. For $x > 0$, the potential energy is 1 eV. If the particle has a total energy of 10 eV, which statement(s) are correct?
 - the wave function is discontinuous at $x = 0$.
 - the de Broglie wavelength for $x < 0$ is greater than that for $x > 0$.
 - for $x > 0$, the wave function exponentially decreases towards ∞ : $\psi(x) = Ae^{-\kappa x}$.
 - for $x < 0$, the wave function exponentially decreases towards $-\infty$: $\psi(x) = Ae^{\kappa x}$.
 - none of the above are correct.
- For this question, don't worry about the spin of the electron. Circle all of the *impossible* states of hydrogen (n, l, m_l).

(a) (5, 0, 0)	(b) (2, 1, -1)	(c) (5, 5, 2)
(d) (3, 2, 3)	(e) (1, 0, 0)	(f) (50, 3, 0)
- The ${}^3_1\text{H}_2$ nucleus has a half life of 12 years. You make one ${}^3_1\text{H}_2$ when you are 25 years old. How old will you be when it radioactively decays?
 - 37 years old
 - 49 years old
 - 61 years old
 - When it decays can not be predicted.
 - none of the above
- A hydrogen atom in its ground state absorbs a photon and goes to a free electron and proton. The minimum photon energy for this process is 13.6 eV. For the same process in muonic hydrogen (a bound muon and proton in the ground state absorbs a photon and goes to a free muon and proton), what is the minimum photon energy required?
 - more than 13.6 eV.
 - less than 13.6 eV.
 - 13.6 eV.

Problems: Show *all* work to receive full credit.

Fundamental constants (e , h , masses, etc) are top of the equation page.

- 10 pts 1. ${}^8_4\text{Be}_4$ decays by emission of an alpha particle. a) What is the total energy released? b) What is the speed of the alpha particle?
- 10 pts 2. A hydrogen atom in the ground state starts at rest where the magnetic field points in the z -direction with magnitude 2.00 T. The magnetic field gives an interaction energy of $U = -\vec{\mu} \cdot \vec{B}$. It accelerates to a spot 1 cm away where the magnetic field points in the z -direction with a magnitude 2.60 T. a) What is the value of the spin magnetic quantum number? b) What is the kinetic energy of the hydrogen atom?

- 10 pts 3. In an atom, there are two electrons with $m_s = 1/2$. One is in the $2p$ state and the other is in the $3d$ state. a) What is the total number of two electron states with these conditions? b) Suppose the two angular momenta add to give a total orbital angular momentum L . What are the allowed values of L ? c) How many states are there for each L ?

- 10 pts 4. A proton is fired directly at a ${}^{235}_{92}\text{U}_{143}$ nucleus. What minimum energy does the proton need to reach the nucleus?

- 15 pts 5. A particle of mass m experiences a potential which is $U(x) = U_0 > 0$ for $x < 0$ and $U(x) = -U_0$ for $x > 0$. For $-U_0 < E < U_0$, determine the wave function for all x . (Hint: you should probably break it up into a solution for $x < 0$ and a solution for $x > 0$; then use the correct rules to connect the two pieces.)

$$\begin{aligned}
g &= 9.80 \frac{m}{s^2} & h &= 6.63 \times 10^{-34} \text{ J s} = 4.14 \times 10^{-15} \text{ eV s} & \hbar &= \frac{h}{2\pi} & c &= 3.00 \times 10^8 \frac{m}{s} \\
M_{elec} &= 9.11 \times 10^{-31} \text{ kg} & M_{prot} &= 1.67 \times 10^{-27} \text{ kg} & M_{muon} &= 1.88 \times 10^{-28} \text{ kg} \\
1.60 \times 10^{-19} \text{ J} &= 1 \text{ eV} & e &= 1.60 \times 10^{-19} \text{ C} & 1/(4\pi\epsilon_0) &= 8.99 \times 10^9 \text{ N m}^2/\text{C}^2
\end{aligned}$$

Galilean relativity	$x' = x - ut, v'_x = v_x - u$	2.1	Lorentz velocity transformation	$v'_x = \frac{v_x - u}{1 - v_x u/c^2},$	2.5
Einstein's postulates	(1) The laws of physics are the same in all inertial frames. (2) The speed of light has the same value c in all inertial frames.	2.3		$v'_y = \frac{v_y \sqrt{1 - u^2/c^2}}{1 - v_x u/c^2},$	
				$v'_z = \frac{v_z \sqrt{1 - u^2/c^2}}{1 - v_x u/c^2}$	
Time dilation	$\Delta t = \frac{\Delta t_0}{\sqrt{1 - u^2/c^2}}$ (Δt_0 = proper time)	2.4	Clock synchronization	$\Delta t' = \frac{uL/c^2}{\sqrt{1 - u^2/c^2}}$	2.5
Length contraction	$L = L_0 \sqrt{1 - u^2/c^2}$ (L_0 = proper length)	2.4	Relativistic momentum	$\vec{p} = \frac{m\vec{v}}{\sqrt{1 - v^2/c^2}}$	2.7
Velocity addition	$v = \frac{v' + u}{1 + v'u/c^2}$	2.4	Relativistic kinetic energy	$K = \frac{mc^2}{\sqrt{1 - v^2/c^2}} - mc^2$	2.7
Doppler effect (source and observer separating)	$f' = f \sqrt{\frac{1 - u/c}{1 + u/c}}$	2.4	Rest energy	$E_0 = mc^2$	2.7
Lorentz transformation	$x' = \frac{x - ut}{\sqrt{1 - u^2/c^2}},$ $y' = y, z' = z,$ $t' = \frac{t - (u/c^2)x}{\sqrt{1 - u^2/c^2}}$	2.5	Relativistic total energy	$E = K + E_0 = \frac{mc^2}{\sqrt{1 - v^2/c^2}}$	2.7
			Momentum-energy relationship	$E = \sqrt{(pc)^2 + (mc^2)^2}$	2.7
			Extreme relativistic approximation	$E \cong pc$	2.7
			Conservation laws	In an isolated system of particles, the total momentum and the relativistic total energy remain constant.	2.8
Hubble's law	$v = H_0 d$	15.1	Age of matter-dominated universe	$t = 1/\sqrt{6\pi G \rho_m}$	15.7
Number density of photons	$N/V = (2.03 \times 10^7 \text{ photons/m}^3 \cdot \text{K}^3) T^3$	15.2	Age of radiation-dominated universe	$t = \sqrt{3/32\pi G \rho_r}$	15.7
Energy density of photons	$U = (4.72 \times 10^3 \text{ eV/m}^3 \cdot \text{K}^4) T^4$	15.2	Temperature of universe at age t	$T = \frac{1.5 \times 10^{10} \text{ s}^{1/2} \cdot \text{K}}{t^{1/2}}$	15.8
Gravitational frequency change	$\Delta f/f = gH/c^2$	15.4	Fraction of photons above E_0	$f = 0.42e^{-E_0/kT} \times \left[\left(\frac{E_0}{kT} \right)^2 + 2 \left(\frac{E_0}{kT} \right) + 2 \right]$	15.9
Deflection of starlight	$\theta = 2GM/Rc^2$	15.5	Critical density of universe	$\rho_{cr} = \frac{3H^2}{8\pi G}$ $= 0.97 \times 10^{-26} \text{ kg/m}^3$	15.10
Perihelion precession	$\Delta\phi = \frac{6\pi GM}{c^2 r_{\min}(1 + e)}$	15.5			
Schwarzschild radius	$r_s = 2GM/c^2$	15.6			

Double-slit maxima	$y_n = n \frac{\lambda D}{d} \quad n = 0, 1, 2, 3, \dots$	3.1	Rayleigh-Jeans formula	$I(\lambda) = \frac{2\pi c}{\lambda^4} kT$	3.3
Bragg's law for X-ray diffraction	$2d \sin \theta = n\lambda \quad n = 1, 2, 3, \dots$	3.1	Planck's blackbody distribution	$I(\lambda) = \frac{2\pi hc^2}{\lambda^5} \frac{1}{e^{hc/\lambda kT} - 1}$	3.3
Energy of photon	$E = hf = hc/\lambda$	3.2	Compton scattering	$\frac{1}{E'} - \frac{1}{E} = \frac{1}{m_e c^2} (1 - \cos \theta),$ $\lambda' - \lambda = \frac{h}{m_e c} (1 - \cos \theta)$	3.4
Maximum kinetic energy of photoelectrons	$K_{\max} = eV_s = hf - \phi$	3.2	Bremsstrahlung	$\lambda_{\min} = hc/K = hc/e\Delta V$	3.5
Cutoff wavelength	$\lambda_c = hc/\phi$	3.2	Pair production	$hf = E_+ + E_- = (m_e c^2 + K_+) + (m_e c^2 + K_-)$	3.5
Stefan's law	$I = \sigma T^4$	3.3	Electron-positron annihilation	$(m_e c^2 + K_+) + (m_e c^2 + K_-) = E_1 + E_2$	3.5
Wien's displacement law	$\lambda_{\max} T = 2.8978 \times 10^{-3} \text{ m} \cdot \text{K}$	3.3			

De Broglie wavelength	$\lambda = h/p$	4.1	Statistical momentum uncertainty	$\Delta p_x = \sqrt{(p_x^2)_{\text{av}} - (p_{x,\text{av}})^2}$	4.4
Single slit diffraction	$a \sin \theta = n\lambda \quad n = 1, 2, 3, \dots$	4.2	Wave packet (discrete k)	$y(x) = \sum A_i \cos k_i x$	4.5
Classical position-wavelength uncertainty	$\Delta x \Delta \lambda \sim \varepsilon \lambda^2$	4.3	Wave packet (continuous k)	$y(x) = \int A(k) \cos kx \, dk$	4.5
Classical frequency-time uncertainty	$\Delta f \Delta t \sim \varepsilon$	4.3	Group speed of wave packet	$v_{\text{group}} = \frac{d\omega}{dk}$	4.6
Heisenberg position-momentum uncertainty	$\Delta x \Delta p_x \sim \hbar$	4.4			
Heisenberg energy-time uncertainty	$\Delta E \Delta t \sim \hbar$	4.4			

Time-independent Schrödinger equation	$-\frac{\hbar^2}{2m} \frac{d^2 \psi}{dx^2} + U(x) \psi(x) = E \psi(x)$	5.3	Infinite potential energy well	$\psi_n(x) = \sqrt{\frac{2}{L}} \sin \frac{n\pi x}{L},$ $E_n = \frac{\hbar^2 n^2}{8mL^2} \quad (n = 1, 2, 3, \dots)$	5.4
Time-dependent Schrödinger equation	$\Psi(x, t) = \psi(x) e^{-i\omega t}$	5.3	Two-dimensional infinite well	$\psi(x, y) = \frac{2}{L} \sin \frac{n_x \pi x}{L} \sin \frac{n_y \pi y}{L}$ $E = \frac{\hbar^2}{8mL^2} (n_x^2 + n_y^2)$	5.4
Probability density	$P(x) = \psi(x) ^2$	5.3	Simple harmonic oscillator ground state	$\psi(x) = (m\omega_0/\hbar\pi)^{1/4} e^{-(\sqrt{km}/2\hbar)x^2}$	5.5
Normalization condition	$\int_{-\infty}^{+\infty} \psi(x) ^2 dx = 1$	5.3	Simple harmonic oscillator energies	$E_n = (n + \frac{1}{2}) \hbar \omega_0 \quad (n = 0, 1, 2, \dots)$	5.5
Probability in interval x_1 to x_2	$P(x_1 : x_2) = \int_{x_1}^{x_2} \psi(x) ^2 dx$	5.3	Potential energy step, $E > U_0$	$\psi_0(x < 0) = A \sin k_0 x + B \cos k_0 x$ $\psi_1(x > 0) = C \sin k_1 x + D \cos k_1 x$	5.6
Average or expectation value of $f(x)$	$[f(x)]_{\text{av}} = \int_{-\infty}^{+\infty} \psi(x) ^2 f(x) dx$	5.3	Potential energy step, $E < U_0$	$\psi_0(x < 0) = A \sin k_0 x + B \cos k_0 x$ $\psi_1(x > 0) = C e^{k_1 x} + D e^{-k_1 x}$	5.6
Constant potential energy, $E > U_0$	$\psi(x) = A \sin kx + B \cos kx,$ $k = \sqrt{2m(E - U_0)}/\hbar$	5.4			
Constant potential energy, $E < U_0$	$\psi(x) = A e^{k'x} + B e^{-k'x},$ $k' = \sqrt{2m(U_0 - E)}/\hbar$	5.4			

Scattering impact parameter	$b = \frac{zZ}{2K} \frac{e^2}{4\pi\epsilon_0} \cot \frac{1}{2}\theta$	6.3	Excitation energy of level n	$E_n - E_1$	6.5
Fraction scattered at angles $> \theta$	$f_{>\theta} = n\pi b^2$	6.3	Binding (or ionization) energy of level n	$ E_n $	6.5
Rutherford scattering formula	$N(\theta) = \frac{nt}{4r^2} \left(\frac{zZ}{2K}\right)^2 \left(\frac{e^2}{4\pi\epsilon_0}\right)^2 \frac{1}{\sin^4 \frac{1}{2}\theta}$	6.3	Hydrogen wavelengths in Bohr model	$\lambda = \frac{64\pi^3\epsilon_0^2\hbar^3c}{me^4} \left(\frac{n_1^2n_2^2}{n_1^2 - n_2^2}\right) = \frac{1}{R_\infty} \left(\frac{n_1^2n_2^2}{n_1^2 - n_2^2}\right)$	6.5
Distance of closest approach	$d = \frac{1}{4\pi\epsilon_0} \frac{zZe^2}{K}$	6.3	Single-electron atoms with $Z > 1$	$r_n = \frac{a_0 n^2}{Z}, E_n = -(13.60 \text{ eV}) \frac{Z^2}{n^2}$	6.5
Balmer formula	$\lambda = (364.5 \text{ nm}) \frac{n^2}{n^2 - 4}$ ($n = 3, 4, 5, \dots$)	6.4	Reduced mass of proton-electron system	$m = \frac{m_e m_p}{m_e + m_p}$	6.8
Radii of Bohr orbits in hydrogen	$r_n = \frac{4\pi\epsilon_0\hbar^2}{me^2} n^2 = a_0 n^2$ ($n = 1, 2, 3, \dots$)	6.5			
Energies of Bohr orbits in hydrogen	$E_n = -\frac{me^4}{32\pi^2\epsilon_0^2\hbar^2} \frac{1}{n^2}$ $= \frac{-13.60 \text{ eV}}{n^2}$ ($n = 1, 2, 3, \dots$)	6.5			
Orbital angular momentum	$ \vec{L} = \sqrt{l(l+1)}\hbar$ ($l = 0, 1, 2, \dots$)	7.2	Angular probability density	$P(\theta, \phi) = \Theta_{l,m_l}(\theta)\Phi_{m_l}(\phi) ^2$	7.5
Orbital magnetic quantum number	$L_z = m_l\hbar$ ($m_l = 0, \pm 1, \pm 2, \dots, \pm l$)	7.2	Orbital magnetic dipole moment	$\vec{\mu}_L = -(e/2m)\vec{L}$	7.6
Spatial quantization	$\cos \theta = \frac{L_z}{ \vec{L} } = \frac{m_l}{\sqrt{l(l+1)}}$	7.2	Spin magnetic dipole moment	$\vec{\mu}_S = -(e/m)\vec{S}$	7.6
Angular momentum uncertainty relationship	$\Delta L_z \Delta \phi \geq \hbar$	7.2	Spin angular momentum	$ \vec{S} = \sqrt{s(s+1)}\hbar = \sqrt{3/4}\hbar$ (for $s = 1/2$)	7.6
Hydrogen quantum numbers	$n = 1, 2, 3, \dots$ $l = 0, 1, 2, \dots, n-1$ $m_l = 0, \pm 1, \pm 2, \dots, \pm l$	7.3	Spin magnetic quantum number	$S_z = m_s\hbar$ ($m_s = \pm 1/2$)	7.6
Hydrogen energy levels	$E_n = -\frac{me^4}{32\pi^2\epsilon_0^2\hbar^2} \frac{1}{n^2}$	7.3	Spectroscopic notation	s ($l = 0$), p ($l = 1$), d ($l = 2$), f ($l = 3$), \dots	7.7
Hydrogen wave functions	$\psi_{n,l,m_l}(r, \theta, \phi) = R_{n,l}(r)\Theta_{l,m_l}(\theta)\Phi_{m_l}(\phi)$	7.3	Selection rules for photon emission	$\Delta l = \pm 1$ $\Delta m_l = 0, \pm 1$	7.7, 7.8
Radial probability density	$P(r) = r^2 R_{n,l}(r) ^2$	7.4	Normal Zeeman effect	$\Delta\lambda = \frac{\lambda^2}{hc} \Delta E = \frac{\lambda^2}{hc} \mu_B B$	7.8
			Fine-structure estimate	$\Delta E = mc^2\alpha^4/n^5$ ($\alpha \approx 1/137$)	7.9
Pauli exclusion principle	<i>No two electrons in a single atom can have the same set of quantum numbers (n, l, m_l, m_s).</i>	8.1	Energy of screened electron	$E_n = (-13.6 \text{ eV}) \frac{Z_{\text{eff}}^2}{n^2}$	8.3
Filling order of atomic subshells	$1s, 2s, 2p, 3s, 3d, 4s, 3d, 4p, 5s, 4d, 5p, 6s, 4f, 5d, 6p, 7s, 5f, 6d$	8.2	Moseley's law for K_α X rays	$\Delta E = (10.2 \text{ eV})(Z-1)^2$	8.5
Capacity of subshell nl	$2(2l+1)$	8.2	Adding angular momenta l_1, m_{l1} and l_2, m_{l2}	$L_{\text{max}} = l_1 + l_2$, $L_{\text{min}} = l_1 - l_2 $, $M_L = m_{l1} + m_{l2}$	8.6
			Hund's rules for ground state	First $S = M_{S,\text{max}}$, then $L = M_{L,\text{max}}$	8.6

Nuclear radius	$R = R_0 A^{1/3}, R_0 = 1.2 \text{ fm}$	12.2	Q value of decay	$Q = [m_X - (m_{X'} + m_x)]c^2$	12.6
Nuclear binding energy	$B = [Nm_n + Zm({}^1_1\text{H}_0) - m({}^A_Z\text{X}_N)]c^2$	12.3	$X \rightarrow X' + x$		
Proton separation energy	$S_p = [m({}^{A-1}_Z\text{X}'_N) + m({}^1_1\text{H}) - m({}^A_Z\text{X}_N)]c^2$	12.3	Q value of alpha decay	$Q = [m(X) - m(X') - m({}^4_2\text{He})]c^2$	12.7
Neutron separation energy	$S_n = [m({}^{A-1}_Z\text{X}_{N-1}) + m_n - m({}^A_Z\text{X}_N)]c^2$	12.3	Kinetic energy of alpha particle	$K_\alpha \cong Q(A-4)/A$	12.7
Range of exchanged particle	$mc^2 = \hbar c/x$	12.4	Q values of beta decay	$Q_{\beta^-} = [m({}^A_Z\text{X}) - m({}^A_{Z'}\text{X}')]c^2,$ $Q_{\beta^+} = [m({}^A_Z\text{X}) - m({}^A_{Z'}\text{X}') - 2m_e]c^2$	12.8
Activity	$a = \lambda N, \lambda = \ln 2/t_{1/2} = 0.693/t_{1/2}$	12.5	Recoil in gamma decay	$K_R = E_\gamma^2/2Mc^2$	12.9
Radioactive decay law	$N = N_0 e^{-\lambda t}, a = a_0 e^{-\lambda t}$	12.5			

Forces	Strong, electromagnetic, weak, gravitational	14.1	Conservation of baryon number B	<i>In any process, B remains constant.</i>	14.3
Field particles	Gluon (g), photon (γ), weak boson (W^\pm, Z^0), graviton	14.1	Conservation of strangeness S	<i>In strong and electromagnetic processes, S remains constant; in weak processes, $\Delta S = 0$ or ± 1.</i>	14.3
Leptons	$e^-, \nu_e, \mu^-, \nu_\mu, \tau^-, \nu_\tau$	14.2	Q value in decays or reactions	$Q = (m_i - m_f)c^2$	14.5, 14.6
Mesons	$\pi^\pm, \pi^0, K^\pm, K^0, \bar{K}^0, \eta, \rho^\pm, \eta', D^\pm, \psi, B^\pm, \Upsilon, \dots$	14.2	Threshold energy in reactions	$K_{th} = -Q(m_1 + m_2 + m_3 + m_4 + m_5 + \dots)/2m_2$	14.6
Baryons	$p, n, \Lambda^0, \Sigma^{\pm,0}, \Xi^{\pm,0}, \Omega^-, \dots$	14.2	Quarks	u, d, c, s, t, b	14.7
Conservation of lepton number L	<i>In any process, L_e, L_μ, and L_τ remain constant.</i>	14.3			

SOME PARTICLE MASSES

	kg	u	MeV/ c^2		Z	A	Atomic mass (u)	Abundance or Half-life
Electron	$9.1093829 \times 10^{-31}$	$5.485799095 \times 10^{-4}$	0.51099893	H	1	1	1.0078250	99.985%
Proton	$1.67262178 \times 10^{-27}$	1.0072764668	938.27205			2	2.014102	0.015%
Neutron	$1.67492735 \times 10^{-27}$	1.0086649160	939.56538			3	3.016049	12.3 y
Deuteron	$3.3435835 \times 10^{-27}$	2.0135532127	1875.61286	He	2	3	3.016029	0.000137%
Alpha	$6.6446568 \times 10^{-27}$	4.001506179	3727.3792			4	4.002603	99.999863%
				Li	3	6	6.015123	7.59%
						7	7.016005	92.41%
						8	8.022487	0.84 s
				Be	4	7	7.016930	53.2 d
						8	8.005305	0.07 fs
						9	9.012182	100%
						10	10.013534	1.5 My
						11	11.021658	13.8 s
				B	5	8	8.024607	0.77 s
						9	9.013329	0.85 as
						10	10.012937	19.8%
						11	11.009305	80.2%
						12	12.014352	20.2 ms

CONVERSION FACTORS

1 eV = $1.60217657 \times 10^{-19} \text{ J}$	1 barn (b) = 10^{-28} m^2
1 u = $931.49406 \text{ MeV}/c^2$	1 curie (Ci) = $3.7 \times 10^{10} \text{ decays/s}$
= $1.66053892 \times 10^{-27} \text{ kg}$	1 light-year = $9.46 \times 10^{15} \text{ m}$
1 y = $3.156 \times 10^7 \text{ s} \cong \pi \times 10^7 \text{ s}$	1 parsec = 3.26 light-year