PHYS 342 (Spring 2023) Test 1, V1 Name: ____________________

Multiple choice (5 pts each) – circle the correct answers.

1. Alice travels past you at 0.866 c. You measure that it takes 2 seconds as measured by your clock for her clock to advance by 1 second. Alice measures that it takes
   (a) 1 second by her clock for your clock to advance by 2 seconds.
   (b) 4 seconds by her clock for your clock to advance by 2 second. *Chap 2 slides pg 8*
   (c) 2 seconds by her clock for your clock to advance by 2 second.
   (d) none of the above.

2. A perfectly black sphere of radius 1 m emits 1 W of light when it is at a temperature of 34.4 K. What happens when the temperature is raised?
   (a) it emits more than 1 W of light. (V1) $I = \sigma T^4$ *inspired from Chp 3 slides pg 11*
   (b) it emits less than 1 W of light. (V2)
   (c) it emits 1 W of light.

3. When C_{60} molecules go through a pair of holes at a speed of 220 m/s, they make an interference pattern on a screen with peaks separated by 24 μm. If the only thing that changes is the speed of the molecules to 110 m/s, the peaks will be separated by
   (a) 12 μm. (V2) $\lambda = h/(mv)$ $y_n = n\lambda D/d$ *from HWK 5 prob 3*
   (b) 24 μm.
   (c) 48 μm. (V1)

4. The diameter of a nucleus is 10×10^{-15} m. Suppose you want to study the diffraction of photons by nuclei. What energy of photons would you choose? $\lambda \sim R$ $E = hc/\lambda$
   (a) 4×10^{-11} J (V1) (b) 4×10^{-14} J (c) 4×10^{-17} J (V2)
   (d) 4×10^{-20} J (e) 4×10^{-23} J (f) 4×10^{-26} J

5. For an object of mass M and charge q that can only move in one dimension, what are the units of Ψ?
   (a) meter
   (b) 1/kilogram
   (c) Coulomb
   (d) $1/\sqrt{\text{meter}}$ *Chap 5 slides pg 3*
   (e) none of the above

6. One of the pieces of evidence for the existence of dark matter is
   (a) galaxies rotate faster than expected based on the amount of visible matter. *from HWK 3 prob 2*
   (b) galaxies rotate more slowly than expected based on the amount of visible matter.
   (c) neither. The rotation of galaxies is *not* relevant.

Problems: Show *all* work to receive full credit.
Fundamental constants (e, h, masses, etc) are top of the equation page.

10 pts 1. For an electron, $\Psi(x,0) = iAx(L-x)$ for $0 < x < L$ and is otherwise equal to 0. a) Determine A. (If there are math steps you can't do, just define it to be some parameter and move on. Most of the credit will be for correct concepts.) b) Roughly, what is the uncertainty in the momentum? *a) is from HWK 6 prob 4 and b) is from HWK 6 prob 1*

This problem was the same on the two versions. a) To find A, you need to normalize the wave function:

$$\int_{-\infty}^{\infty} |\Psi(x,0)| dx = 1$$

$$= A^2 \int_0^L x^2(L^2 - 2Lx + x^2)dx = A^2 \left(\frac{1}{3}x^3L^2 - \frac{2}{4}x^4L + \frac{1}{5}x^5\right)|_0^L = A^2L^5/30$$

$$\underline{A = \sqrt{30/L^5}}$$

b) For this you can use the Heisenberg position-momentum uncertainty $\Delta p_x \sim \hbar/\Delta x$. Any reasonable value for Δx is OK. It needs to be less than or equal to L and roughly greater than $L/2$: $\underline{\Delta p_x \sim \hbar/L}$.

10 pts 2. A photon with a wavelength of 2.10×10^{-14} m is traveling in the $+x$-direction. It hits an initially stationary muon and the photon goes in the $+y$-direction. a) What is the final wavelength of the photon? b) What is the momentum vector of the muon? *HWK 4 prob 4*

This is a Compton scattering problem but the photon scatters from a muon, not an electron. This means that in the Compton equation you need to use the mass of the muon: $h/(M_\mu c) = 1.18 \times 10^{-14}$ m. In both versions, the angle is 90°.

$$a) \quad \lambda' = 2.10 \times 10^{-14}\ m + 1.18 \times 10^{-14}\ m = \underline{3.28 \times 10^{-14} m} \quad (V1)$$

$$a) \quad \lambda' = 1.80 \times 10^{-14}\ m + 1.18 \times 10^{-14}\ m = \underline{2.98 \times 10^{-14} m} \quad (V2)$$

Use conservation of momentum for b). The muon momentum in $+x$ must be the initial photon momentum and the momentum in $-y$ must be the final photon momentum.

$$p_x = \frac{6.63 \times 10^{-34}\ J\ s}{2.10 \times 10^{-14}\ m} = \underline{3.16 \times 10^{-20}\ kg\frac{m}{s}} \quad p_y = -\frac{6.63 \times 10^{-34}\ J\ s}{3.28 \times 10^{-14}\ m} = \underline{-2.02 \times 10^{-20}\ kg\frac{m}{s}} \quad (V1)$$

$$p_x = \frac{6.63 \times 10^{-34}\ J\ s}{1.80 \times 10^{-14}\ m} = \underline{3.68 \times 10^{-20}\ kg\frac{m}{s}} \quad p_y = -\frac{6.63 \times 10^{-34}\ J\ s}{2.98 \times 10^{-14}\ m} = \underline{-2.22 \times 10^{-20}\ kg\frac{m}{s}} \quad (V2)$$

10 pts 3. Astronauts journey to a star that is 10.0 light-years from earth. (A light-year is the distance light travels in 1 year.) They travel there and back at a speed of 0.80 c. a) How long does the trip take as measured by someone who stayed on earth? b) How much do the astronauts age over the trip? *HWK 1 prob 2*

For part a), you just need to divide the total distance of the trip by how fast the astronaut travels.

$$t = \frac{20\ yr\ c}{0.8\ c} = \underline{25.0\ yr} \quad (V1) \qquad t = \frac{24\ yr\ c}{0.75\ c} = \underline{32.0\ yr} \quad (V2)$$

For part b), you need to account for time dilation. The astronaut ages more slowly by the factor of $\sqrt{1 - v^2/c^2}$.

$$age = 25\ yr\ \sqrt{1 - 0.8^2} = \underline{15.0\ yrs} \quad (V1) \qquad age = 32\ yr\ \sqrt{1 - 0..75^2} = \underline{21.2\ yrs} \quad (V2)$$

10 pts 4. In 1960, Pound and Rebka sent $\lambda = 8.63 \times 10^{-11}$ m photons vertically upward 22.5 m in a lab at Harvard. What was the change in frequency of these photons? Make sure to be clear whether the frequency increased, decreased, or remained the same. *HWK 2 prob 4*

This problem was the same on the two versions. This is a problem about the gravitational frequency change (red shift): $\Delta f = fgH/c^2$. Because the photon is going up, the frequency will decrease. The frequency can be found from $f = c/\lambda$.

$$\Delta f = -\frac{3.00 \times 10^8\ m/s}{8.63 \times 10^{-11}\ m} \frac{9.8\frac{m}{s^2}\ 22.5\ m}{(3.00 \times 10^8\ m/s)^2} = \underline{-8520\ Hz}$$

15 pts 5. Light with an intensity of 1.30 W/m^2 and wavelength 450 nm is incident normal on solid Na. Electrons are ejected from the Na. a) What is the minimum wavelength of the electrons ejected? b) If the Na surface is a square with edge length 1.90 cm and all of the incident light is absorbed, what is the maximum rate that the electrons are ejected? For the last question, the answer should be a number per second. *a) HWK 4 prob 1 with HWK 5 prob 4 and b) HWK 3 prob 5*

For part a), the basic idea is that the photo-electric effect will determine the maximum kinetic energy: $K_{max} = (hc/\lambda) - \phi$ with ϕ the work function. From the maximum kinetic energy, you can compute the minimum de Broglie by using $p = \sqrt{2\,m\,K}$ and $\lambda = h/p$.

$$K_{max} = \frac{4.14 \times 10^{-15}\ eV\ s\ 3 \times 10^{8}\ m/s}{450 \times 10^{-9}\ m} - 2.28\ eV = 2.76 - 2.28\ eV = 0.48\ eV = 7.68 \times 10^{-20}\ J$$

$$p = \sqrt{2\ \times\ 9.11 \times 10^{-31}\ kg\ 7.68 \times 10^{-20}\ J} = 3.74 \times 10^{-25}\ kg\ m/s$$

$$\lambda = \frac{6.63 \times 10^{-34}\ J\ s}{3.74 \times 10^{-25}\ kg\ m/s} = \underline{1.77 \times 10^{-9}\ m} = 1.77\ nm \quad (V1)$$

$$K_{max} = \frac{4.14 \times 10^{-15}\ eV\ s\ 3 \times 10^{8}\ m/s}{210 \times 10^{-9}\ m} - 4.70\ eV = 5.91 - 4.70\ eV = 1.21\ eV = 1.94 \times 10^{-19}\ J$$

$$p = \sqrt{2\ \times\ 9.11 \times 10^{-31}\ kg\ 1.94 \times 10^{-19}\ J} = 5.95 \times 10^{-25}\ kg\ m/s$$

$$\lambda = \frac{6.63 \times 10^{-34}\ J\ s}{5.95 \times 10^{-25}\ kg\ m/s} = \underline{1.12 \times 10^{-9}\ m} = 1.12\ nm \quad (V2)$$

The idea for part b) is that at most every photon that hits the material will be absorbed *and* eject an electron. For b), you need to figure out the rate that photons hit the material. The rate that light energy enters the material is the intensity times the area: $P = I\ A$. The rate that photons hit the material is the rate that energy enters divided by the energy of 1 photon.

$$P = 1.30\ \frac{W}{m^2}\ (1.9 \times 10^{-2}\ m)^2 = 4.69 \times 10^{-4}\ \frac{J}{s} \qquad E = \frac{6.63 \times 10^{-34}\ J\ s\ 3 \times 10^{8}\ m/s}{450 \times 10^{-9}\ m} = 4.42 \times 10^{-19}\ J$$

$$max\ rate = \frac{4.69 \times 10^{-4}\ \frac{J}{s}}{4.42 \times 10^{-19}\ J} = \underline{1.06 \times 10^{15}\ electrons/sec}$$

$$P = 1.60\ \frac{W}{m^2}\ (1.4 \times 10^{-2}\ m)^2 = 3.14 \times 10^{-4}\ \frac{J}{s} \qquad E = \frac{6.63 \times 10^{-34}\ J\ s\ 3 \times 10^{8}\ m/s}{210 \times 10^{-9}\ m} = 9.47 \times 10^{-19}\ J$$

$$max\ rate = \frac{3.14 \times 10^{-4}\ \frac{J}{s}}{9.47 \times 10^{-19}\ J} = \underline{3.32 \times 10^{14}\ electrons/sec}$$

15 pts 6. A wejy is a fundamental particle with mass 2.34 kg. In your lab, you measure a wejy traveling at a speed of 0.888 c. It hits a stationary wejy and converts to a nugy. The nugy is moving at a speed v_n. a) What is the mass of the nugy? b) What is the speed of the nugy? *HWK 2 prob 2*

Before saying how to work the problem, note that conservation of mass is not a thing.

For this problem, you need to use conservation of momentum and conservation of energy. Before the collision, the momentum is $p_0 = m_w v_w / \sqrt{1-(v_w/c)^2}$ and the energy is $E_0 = m_w c^2 + m_w c^2/\sqrt{1-(v_w/c)^2}$ (the first term is from the stationary wejy and the second is from the moving one). After the collision the momentum and energy is only from the moving nugy. This gives $p_n = m_n v_n/\sqrt{1-(v_n/c)^2}$ and $E_n = m_n c^2/\sqrt{1-(v_n/c)^2}$. The conservation of momentum and energy mean $p_n = p_0$ and $E_n = E_0$ which give two equations for two unknowns: v_n and m_n. If you do the ratio, you get $v_n/c = (cp_n)/E_n$. Once you have v_n/c you can solve for m_n using either the energy or momentum equation. I used the energy equation: $m_n = (E_n/c^2)\sqrt{1-(v_n/c)^2}$.

$$p_0 = 2.34\ kg\ 0.888\ c/\sqrt{1-0.888^2} = 4.519\ kg\ c$$

$$E_0 = 2.34\ kg\left(1+\frac{1}{\sqrt{1-0.888^2}}\right)c^2 = 7.429\ kg\ c^2$$

$$\frac{v_n}{c} = \frac{4.519\ kg\ c^2}{7.429\ kg\ c^2} = \underline{0.608} \qquad m_n = 7.429\ kg\ \sqrt{1-0.608^2} = \underline{5.90\ kg} \quad (V1)$$

$$p_0 = 1.23\ kg\ 0.777\ c/\sqrt{1-0.777^2} = 1.518\ kg\ c$$

$$E_0 = 1.23\ kg\left(1+\frac{1}{\sqrt{1-0.777^2}}\right)c^2 = 3.184\ kg\ c^2$$

$$\frac{v_n}{c} = \frac{1.518\ kg\ c^2}{3.184\ kg\ c^2} = \underline{0.477} \qquad m_n = 3.184\ kg\ \sqrt{1-0.477^2} = \underline{2.80\ kg} \quad (V2)$$

$g = 9.80\ \frac{m}{s^2}$ $h = 6.63 \times 10^{-34}\ J\ s = 4.14 \times 10^{-15}\ eV\ s$ $\hbar = \frac{h}{2\pi}$ $c = 3.00 \times 10^{8}\ \frac{m}{s}$

$M_{elec} = 9.11 \times 10^{-31}\ kg$ $M_{prot} = 1.67 \times 10^{-27}\ kg$ $M_{muon} = 1.88 \times 10^{-28}\ kg$

$1.60 \times 10^{-19}\ J = 1\ eV$ Work functions: Na 2.28 eV, Al 4.08 eV, Cu 4.70 eV

Galilean relativity	$x' = x - ut, v'_x = v_x - u$	2.1
Einstein's postulates	(1) The laws of physics are the same in all inertial frames. (2) The speed of light has the same value c in all inertial frames.	2.3
Time dilation	$\Delta t = \dfrac{\Delta t_0}{\sqrt{1-u^2/c^2}}$ (Δt_0 = proper time)	2.4
Length contraction	$L = L_0\sqrt{1-u^2/c^2}$ (L_0 = proper length)	2.4
Velocity addition	$v = \dfrac{v'+u}{1+v'u/c^2}$	2.4
Doppler effect (source and observer separating)	$f' = f\sqrt{\dfrac{1-u/c}{1+u/c}}$	2.4
Lorentz transformation	$x' = \dfrac{x-ut}{\sqrt{1-u^2/c^2}}$, $y' = y,\ z' = z,$ $t' = \dfrac{t-(u/c^2)x}{\sqrt{1-u^2/c^2}}$	2.5
Lorentz velocity transformation	$v'_x = \dfrac{v_x - u}{1-v_x u/c^2}$, $v'_y = \dfrac{v_y\sqrt{1-u^2/c^2}}{1-v_x u/c^2}$, $v'_z = \dfrac{v_z\sqrt{1-u^2/c^2}}{1-v_x u/c^2}$	2.5
Clock synchronization	$\Delta t' = \dfrac{uL/c^2}{\sqrt{1-u^2/c^2}}$	2.5
Relativistic momentum	$\vec{\mathbf{p}} = \dfrac{m\vec{\mathbf{v}}}{\sqrt{1-v^2/c^2}}$	2.7
Relativistic kinetic energy	$K = \dfrac{mc^2}{\sqrt{1-v^2/c^2}} - mc^2$	2.7
Rest energy	$E_0 = mc^2$	2.7
Relativistic total energy	$E = K + E_0 = \dfrac{mc^2}{\sqrt{1-v^2/c^2}}$	2.7
Momentum-energy relationship	$E = \sqrt{(pc)^2 + (mc^2)^2}$	2.7
Extreme relativistic approximation	$E \cong pc$	2.7
Conservation laws	In an isolated system of particles, the total momentum and the relativistic total energy remain constant.	2.8

Hubble's law	$v = H_0 d$	15.1
Number density of photons	$N/V = (2.03 \times 10^7 \text{photons/m}^3 \cdot \text{K}^3)T^3$	15.2
Energy density of photons	$U = (4.72 \times 10^3\ \text{eV/m}^3 \cdot \text{K}^4)T^4$	15.2
Gravitational frequency change	$\Delta f/f = gH/c^2$	15.4
Deflection of starlight	$\theta = 2GM/Rc^2$	15.5
Perihelion precession	$\Delta\phi = \dfrac{6\pi GM}{c^2 r_{\text{min}}(1+e)}$	15.5
Schwarzschild radius	$r_S = 2GM/c^2$	15.6
Age of matter-dominated universe	$t = 1/\sqrt{6\pi G\rho_{\text{m}}}$	15.7
Age of radiation-dominated universe	$t = \sqrt{3/32\pi G\rho_{\text{r}}}$	15.7
Temperature of universe at age t	$T = \dfrac{1.5 \times 10^{10}\ \text{s}^{1/2} \cdot \text{K}}{t^{1/2}}$	15.8
Fraction of photons above E_0	$f = 0.42e^{-E_0/kT} \times \left[\left(\frac{E_0}{kT}\right)^2 + 2\left(\frac{E_0}{kT}\right) + 2\right]$	15.9
Critical density of universe	$\rho_{\text{cr}} = \dfrac{3H^2}{8\pi G} = 0.97 \times 10^{-26}\ \text{kg/m}^3$	15.10

Double-slit maxima	$y_n = n\dfrac{\lambda D}{d} \quad n = 0, 1, 2, 3, \ldots$	3.1
Bragg's law for X-ray diffraction	$2d \sin\theta = n\lambda \quad n = 1, 2, 3, \cdots$	3.1
Energy of photon	$E = hf = hc/\lambda$	3.2
Maximum kinetic energy of photoelectrons	$K_{\max} = eV_s = hf - \phi$	3.2
Cutoff wavelength	$\lambda_c = hc/\phi$	3.2
Stefan's law	$I = \sigma T^4$	3.3
Wien's displacement law	$\lambda_{\max} T = 2.8978 \times 10^{-3}\ \mathrm{m \cdot K}$	3.3

Rayleigh-Jeans formula	$I(\lambda) = \dfrac{2\pi c}{\lambda^4} kT$	3.3
Planck's blackbody distribution	$I(\lambda) = \dfrac{2\pi hc^2}{\lambda^5} \dfrac{1}{e^{hc/\lambda kT} - 1}$	3.3
Compton scattering	$\dfrac{1}{E'} - \dfrac{1}{E} = \dfrac{1}{m_e c^2}(1 - \cos\theta)$, $\lambda' - \lambda = \dfrac{h}{m_e c}(1 - \cos\theta)$	3.4
Bremsstrahlung	$\lambda_{\min} = hc/K = hc/e\Delta V$	3.5
Pair production	$hf = E_+ + E_- =$ $(m_e c^2 + K_+) + (m_e c^2 + K_-)$	3.5
Electron-positron annihilation	$(m_e c^2 + K_+) + (m_e c^2 + K_-)$ $= E_1 + E_2$	3.5

De Broglie wavelength	$\lambda = h/p$	4.1
Single slit diffraction	$a \sin\theta = n\lambda \quad n = 1, 2, 3, \ldots$	4.2
Classical position-wavelength uncertainty	$\Delta x \Delta\lambda \sim \varepsilon\lambda^2$	4.3
Classical frequency-time uncertainty	$\Delta f\, \Delta t \sim \varepsilon$	4.3
Heisenberg position-momentum uncertainty	$\Delta x \Delta p_x \sim \hbar$	4.4
Heisenberg energy-time uncertainty	$\Delta E \Delta t \sim \hbar$	4.4

Statistical momentum uncertainty	$\Delta p_x = \sqrt{(p_x^2)_{\mathrm{av}} - (p_{x,\mathrm{av}})^2}$	4.4
Wave packet (discrete k)	$y(x) = \sum A_i \cos k_i x$	4.5
Wave packet (continuous k)	$y(x) = \int A(k) \cos kx\, dk$	4.5
Group speed of wave packet	$v_{\mathrm{group}} = \dfrac{d\omega}{dk}$	4.6

Time-independent Schrödinger equation	$-\dfrac{\hbar^2}{2m} \dfrac{d^2\psi}{dx^2} + U(x)\psi(x) = E\psi(x)$	5.3
Time-dependent Schrödinger equation	$\Psi(x, t) = \psi(x) e^{-i\omega t}$	5.3
Probability density	$P(x) = \lvert\psi(x)\rvert^2$	5.3
Normalization condition	$\int_{-\infty}^{+\infty} \lvert\psi(x)\rvert^2\, dx = 1$	5.3
Probability in interval x_1 to x_2	$P(x_1 : x_2) = \int_{x_1}^{x_2} \lvert\psi(x)\rvert^2\, dx$	5.3
Average or expectation value of $f(x)$	$[f(x)]_{\mathrm{av}} = \int_{-\infty}^{+\infty} \lvert\psi(x)\rvert^2 f(x)\, dx$	5.3
Constant potential energy, $E > U_0$	$\psi(x) = A \sin kx + B \cos kx$, $k = \sqrt{2m(E - U_0)/\hbar^2}$	5.4
Constant potential energy, $E < U_0$	$\psi(x) = Ae^{k'x} + Be^{-k'x}$, $k' = \sqrt{2m(U_0 - E)/\hbar^2}$	5.4

Infinite potential energy well	$\psi_n(x) = \sqrt{\dfrac{2}{L}} \sin \dfrac{n\pi x}{L}$, $E_n = \dfrac{h^2 n^2}{8mL^2} \ (n = 1, 2, 3, \ldots)$	5.4
Two-dimensional infinite well	$\psi(x, y) = \dfrac{2}{L} \sin \dfrac{n_x \pi x}{L} \sin \dfrac{n_y \pi y}{L}$ $E = \dfrac{h^2}{8mL^2}(n_x^2 + n_y^2)$	5.4
Simple harmonic oscillator ground state	$\psi(x) = (m\omega_0/\hbar\pi)^{1/4}\, e^{-(\sqrt{km}/2\hbar)x^2}$	5.5
Simple harmonic oscillator energies	$E_n = (n + \frac{1}{2})\hbar\omega_0 \ (n = 0, 1, 2, \ldots)$	5.5
Potential energy step, $E > U_0$	$\psi_0(x < 0) = A \sin k_0 x + B \cos k_0 x$ $\psi_1(x > 0) = C \sin k_1 x + D \cos k_1 x$	5.6
Potential energy step, $E < U_0$	$\psi_0(x < 0) = A \sin k_0 x + B \cos k_0 x$ $\psi_1(x > 0) = Ce^{k_1 x} + De^{-k_1 x}$	5.6