Chapter 6 - The Rutherford Bohr model of the atom

A bit over 110 years ago was the first experiments and understanding of what atoms are actually like: heavy positively charged with light electrons bound to it.

Sketch out the basic idea (Chap 6 slides 1-2)

Basic behavior: Prob → θ proportional to $\frac{z^2 Z^2}{K^2} \frac{1}{\sin^4(\frac{\theta}{2})}$

Although Rutherford's formula exactly matched experiment, it was almost universally ignored.

1) Light should be emitted continuously until the electron spiralled in to the nucleus.
2) Not stable against perturbations. How can atoms/molecules/solids with more than 1 electron prevent collapse of matter.

Rutherford's group measured the size of a nucleus! (Chap 6 slides 3-5)

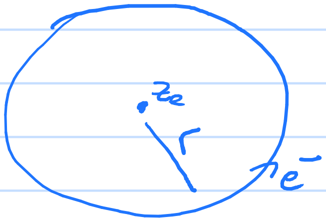
Problem from slide 5 $\quad r_d = 10^{-14}\,m$, $\quad PE = \frac{8.99\times10^{9}\, e^2\, 2\cdot 13}{10^{-14}\,m} = 6.0\times10^{-13}\,J$
$= 3.74\,M_e V$

$$v = \sqrt{\frac{2\,PE}{m}} = \sqrt{\frac{2\cdot 6\times10^{-13}}{4\cdot 1.66\times10^{-27}}} = 1.3\times10^{7}\,m/s$$

The radius of nuclei are very well measured, but there's controversy for the simples nucleus! Will discuss at end of chapter.

A couple years later Bohr came up with a kluge that gave the correct frequencies of light from hydrogen, solved problem 1), and hinted at fixing 2). (Chap 6 slide 6)

Bohr's idea is a weird mix of classical and quantum.
Be clear → WRONG!
But it gives the right sizes for almost everything



Force $F = kZe^2/r^2$
Centripetal force mv^2/r } same

$$mv^2 = \frac{kZe^2}{r} = -PE$$

$$E = \frac{1}{2}mv^2 - \frac{Zke^2}{r} = -\frac{1}{2}mv^2 = -\frac{Zke^2}{2r}$$

$L_n = m v_n r_n = n\hbar$ Two eqs and don't know v_n, r_n

$$mv^2 r \cdot mr = kZe^2 m r_n = n^2\hbar^2 \Rightarrow r_n = \frac{\hbar^2}{mke^2}\frac{n^2}{Z} = a_0 \frac{n^2}{Z}$$

$$E_n = -\frac{Zke^2}{2r} = -\frac{ke^2}{2a_0}\left(\frac{Z^2}{n^2}\right) = -13.60\,\text{eV}\,\frac{Z^2}{n^2}$$ (Chap 6 slide 7)

$$= -\left(\frac{ke^2}{\hbar}\right)^2 \frac{m}{2}\left(\frac{Z^2}{n^2}\right)$$

The spectrum of light from hydrogen $\frac{1}{\lambda_{mn}} = R\left(\frac{1}{m^2} - \frac{1}{n^2}\right)$
$0 < m < n$ are integers. Why?
The formula above gives $R_\infty = 1.097373 \times 10^7\ m^{-1}$ but this is slightly wrong because we took the nucleus to have ∞ mass.
$R_H = 1.096776 \times 10^7\ m^{-1}$ because reduced mass

Deuterium discovered because R slightly larger than hydrogen.

This theory does give the $E_{n+1} - E_n = h f_{classical}$

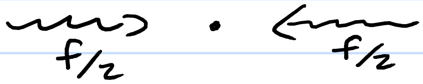
$$E_{n+1} - E_n \approx \frac{dE}{dn} = \frac{ke^2}{a_0}\frac{Z^2}{n^3} \stackrel{?}{=} h\frac{v_n}{2\pi r_n} = h\left(\frac{ke^2}{\hbar}\frac{Z}{n}\right)\frac{Z}{2\pi a_0 n^2} = \frac{ke^2}{a_0}\frac{Z^2}{n^3}\ \checkmark$$

There are many fundamental issues with Bohr's "theory"
(Chap 6 slides 8,9)

The simplicity of the H atom spurred accurate measurements

So many Nobel prizes! Bohr, Urey (deuterium), Lamb (Lamb shift)
Feynman - Schwinger - Tomonaga (QED), Ramsey (H maser), Hänsch (precision)
others!

One of the $n=2$ states (2S) has a ~ 1/8 second lifetime.
This gives a narrow spectral line



Two photon absorb
Most of Doppler shift gone.

Chap 6 slide 10 Hydrogen and antihydrogen

Know how much energies change from finite size of proton
From energy shift → proton radius
Measurements in regular hydrogen and muonic hydrogen give slightly different answers (but don't agree) Chap 6 slide 11