Case 3: Quantum Waves (v<<c)

Use the relationships for de Broglie waves ($\lambda = h/p$ and $f = E/h$) to get connection between k & ω.

What is relation between p and k?

$k = 2\pi / \lambda = 2\pi / (h/p) = 2\pi p / h = p / \hbar \quad \Rightarrow \quad \mathbf{p = \hbar k}$

What is the relation between E & ω?

$\omega = 2\pi f = 2\pi E / h = E / \hbar \quad \Rightarrow \quad \mathbf{E = \hbar \omega}$

What is the relationship between k and ω?

$E = p^2 / 2M \quad \Rightarrow \quad \hbar\omega = (\hbar k)^2 / 2M \quad \Rightarrow \quad \omega = \hbar k^2 / 2M$

Determine the phase and group velocity.

$v_p = \omega/k = \hbar k / 2M = p / 2M$

$v_g = d\omega/dk = \hbar k / M = p / M$

Calculated Electron Dynamics in a Strong Electric Field

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(Received 6 August 1996)

The *dynamics* of an electron wave attached to Rb^+ is calculated when the atom is in a strong electric field. The dynamic motion of the electron is generated by exciting Rb from its ground state using a weak, pulsed laser. We compare the quantum results for $m = 0$ to recent experiments. The comparison requires a calculation of the electron flux at a macroscopic distance from the atom. We discuss some of the interesting aspects of this problem including trimming autoionizing states, interference patterns downfield, and suppression of downfield scattering. [S0031-9007(96)01637-7]

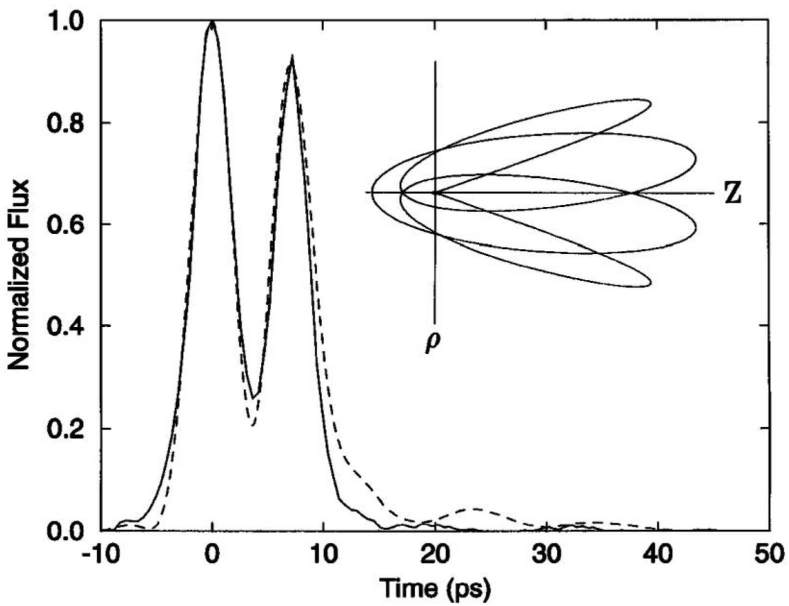


FIG. 2. The calculated (dashed line) and experimental (solid line) time dependent flux of ejected electrons (normalized to 1 at the first peak) for the parameters in Fig. 1; $t = 0$ is set at the peak of the first pulse. The inset shows the classical orbit mainly responsible for the flux ejected near 7 ps; continuation of the orbit to negative ρ is shown to ease the visualization of this trajectory.

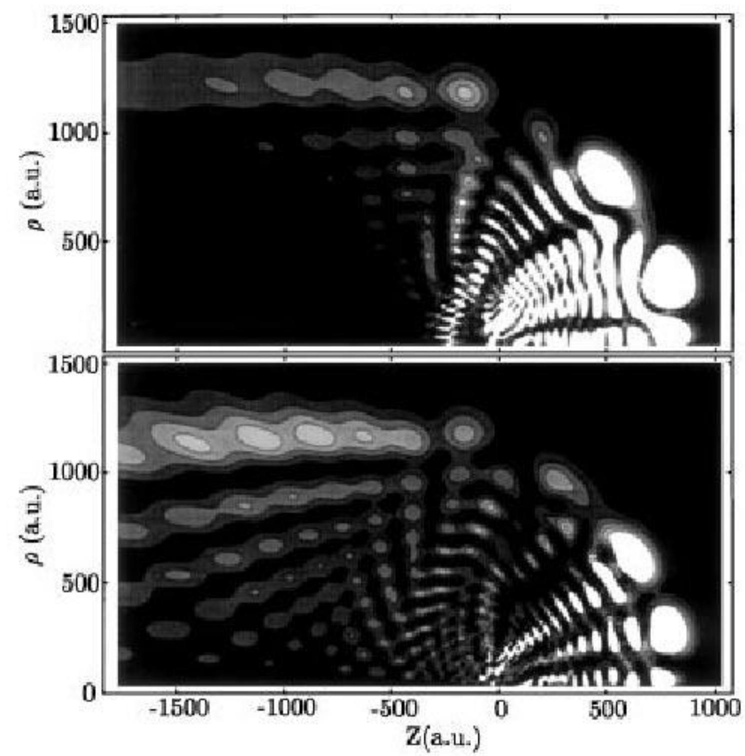


FIG. 3. Contour plot of the wave function (white at maxima) for the parameters of Figs. 1 and 2 with the upper figure at $t = 3.5$ ps and the lower figure at $t = 7.5$ ps.

Question #1

For an object that can only move in 1-dimension, the units of the wave function, Ψ, is

(a) m.

(b) J.

(c) $m^{-1/2}$.

(d) kg m/s.

(e) this is a trick question; Ψ is a complex function so it doesn't have units.

Question #2

You confine an electron to a box that extends from $x = -L$ to $x = L$. The electron is in its lowest possible energy state. The probability for finding the electron in a small region Δx at the center of the box

(a) increases linearly with L.

(b) increases with $L^{1/2}$.

(c) is inversely proportional to L.

(d) is inversely proportional to $L^{1/2}$.

(e) does not depend on L.

Problem 1

If $\exp[i(kx - \omega t)]$ is a solution of the matter wave equation, which of the 3 following are also solutions: $\exp[i(kx + \omega t)]$, $\exp[i(-kx - \omega t)]$, $\exp[i(-kx + \omega t)]$? Can the original solution be written as
$\exp[i(px - Et)/\hbar]$? Can the solution be written as cos or $\sin(kx - \omega t)$? Can the solution be written using cos or sin in the form
$\cos(kx)\exp(-i\omega t)$ or in the form
$\cos(kx)\exp(i\omega t)$? Which direction does the wave move for the solution $\exp[i(-kx - \omega t)]$?

Substitute the form into the matter wave equation and check if solution.

Electron hitting a wall (hrdwall.mpg)

The wave function at $t = 0$ has the form

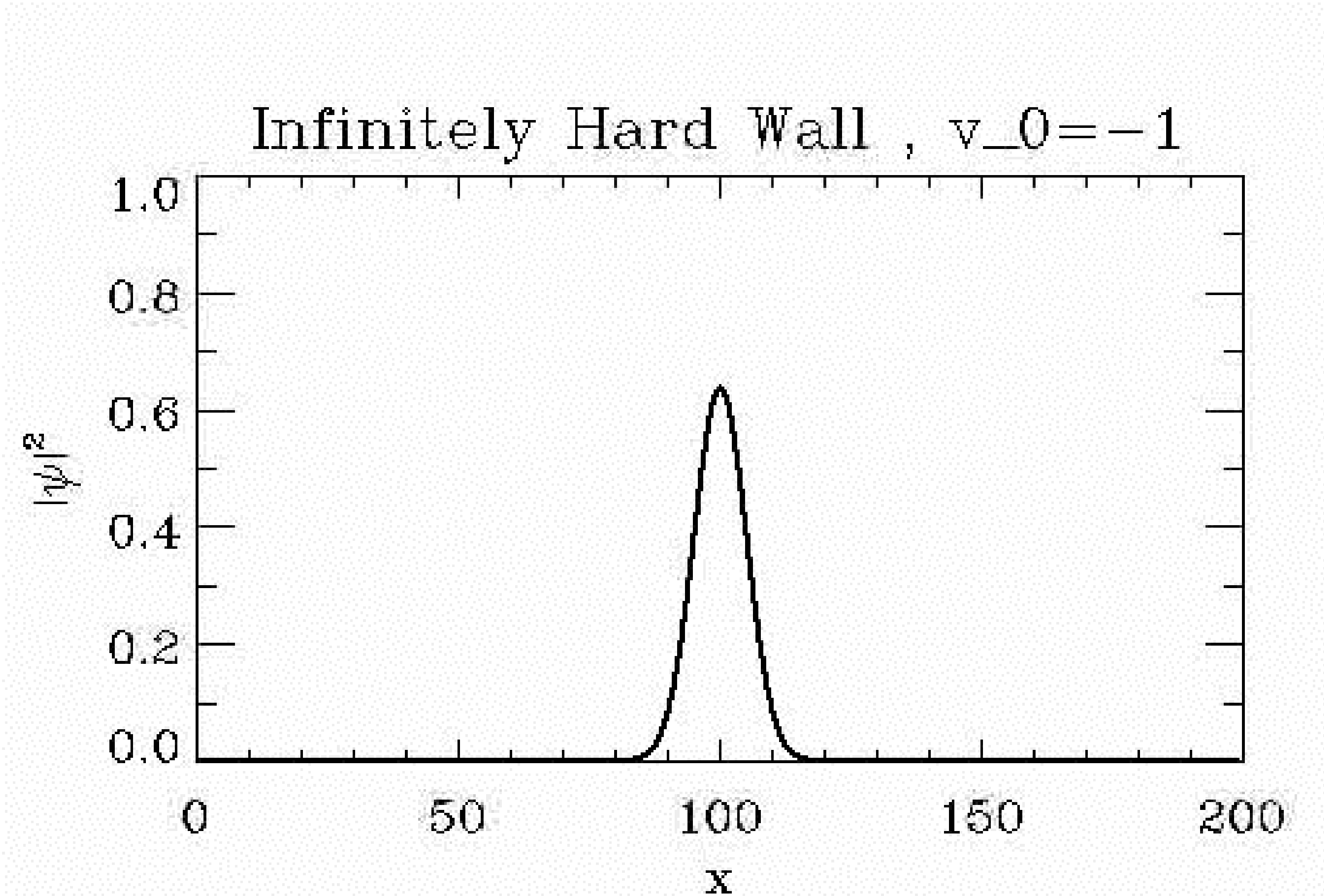
$\Psi(x,0) = A \exp[-(x-100)^2/10^2] \exp[-i\, x]$

There is an infinitely hard wall at $x = 0$.

Sketch $|\Psi|^2$ at $t = 0$.

What will the wave function do?

Electron hitting a wall (hrdwall.mpg)



Electron hitting a wall (hrdwall.mpg)

Why does the packet get broader?

Why does the peak of the packet get smaller?

What is the fast oscillation at the wall?

Schrodinger Equation w/ Forces

$$i\,\hbar\,\frac{\partial\Psi(\mathrm{x,t})}{\partial \mathrm{t}} = -\frac{\hbar^2}{2\mathrm{M}}\frac{\partial^2\Psi(\mathrm{x,t})}{\partial \mathrm{x}^2} + U(\mathrm{x,t})\Psi(\mathrm{x,t})$$

!!!!!!!MEMORIZE THIS EQUATION!!!!!!!

U(x,t) is the classical potential energy of the object at position x, time t. Equation is not relativistically correct: error is roughly $(\hbar\ \mathrm{k/M\ c})^2$.

Schrodinger Equation w/ Forces

$$i\hbar \frac{\partial \Psi(x,t)}{\partial t} = -\frac{\hbar^2}{2M}\frac{\partial^2 \Psi(x,t)}{\partial x^2} + U(x,t)\Psi(x,t)$$

Does this equation have consistent units?

????? E = KE + PE ?????

What do you do for the case of non-conservative forces?

If $\Psi(x,t)$ is a solution, is 8 $\Psi(x,t)$ a solution?

What do you need to know at t=0 to get Ψ for all t?

Normalization Condition

$$\int_{-\infty}^{\infty} \left|\Psi(x,t)\right|^2 dx = 1$$

!!!!!!!MEMORIZE THIS EQUATION!!!!!!!

Why must this condition hold?

Does this limit the range of x where the wave function is nonzero?

Can $\Psi(x,0)$ be proportional to $1/(i + x\,k)$?
$|x|/(i + [k\,x]^2)$? $\exp(i\,k\,x - x^2/\Delta x^2)$?
$x^{1/2}/(1 + [k\,x]^4)$? 0?

If $\Psi(x,0)$ is normalized, will it be normalized for all t?

Electron in Constant F (linpot.mpg)

The wave function at t = 0 has the form

$\Psi(x,0) = A\ \exp[-(x-100)^2/10^2]\ \exp[-i\ x]$

$U(x) = -x/100$ & starting $v = -1$

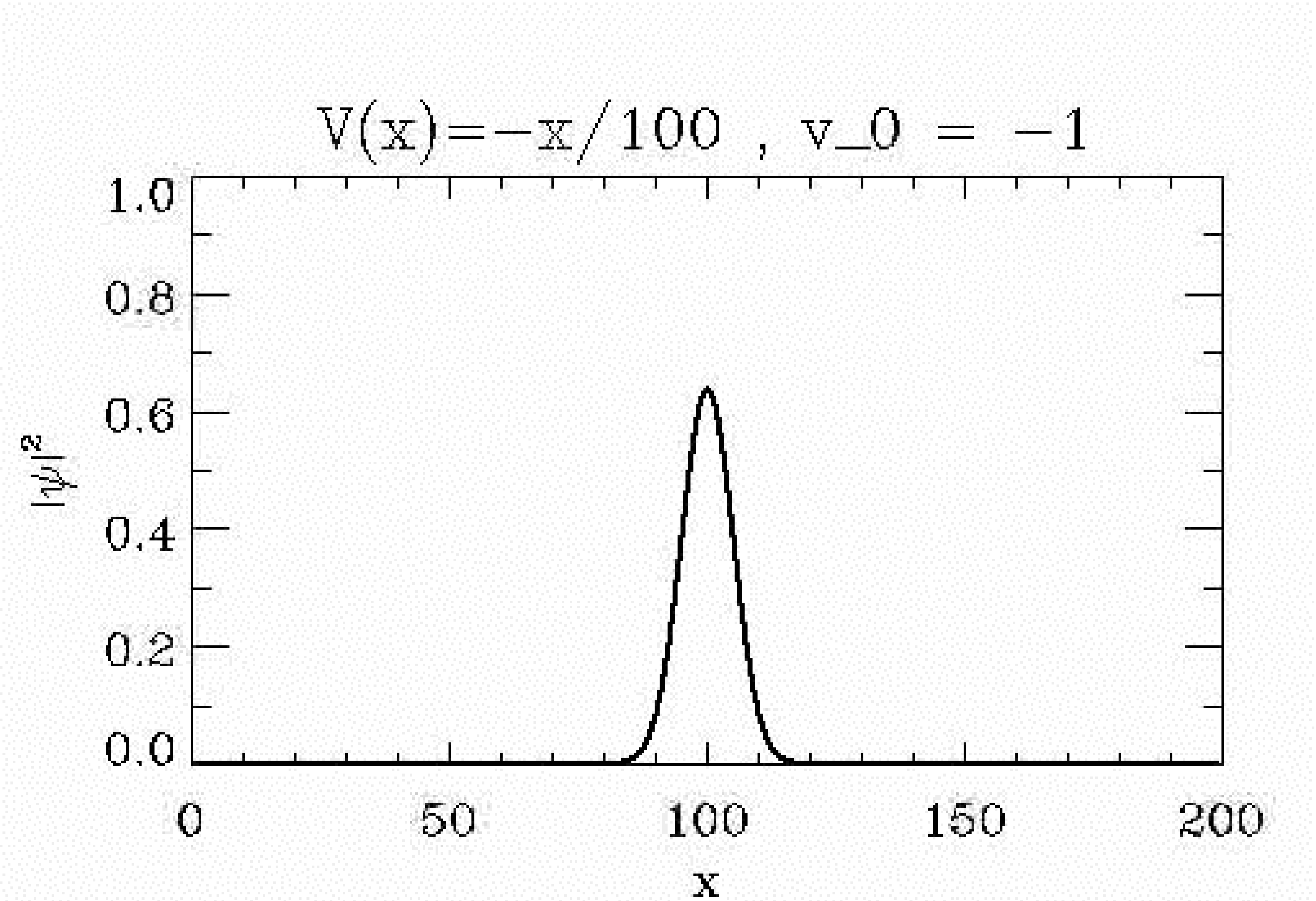
Which direction is the force?

What will the wave function do?

Estimate E.

Where would a classical electron turn around?

Electron in Constant F (linpot.mpg)



Electron in Constant F (linpot.mpg)

Why is there no interference structure where the packet turns around?

Harmonic Oscillator (harmosc2.mpg)

The wave function at $t = 0$ has the form

$\Psi(x,0) = A \exp[-x^2/10^2] \exp[i\, x]$

$U(x) = \frac{1}{2} (x/50)^2$ & starting $v = 1$

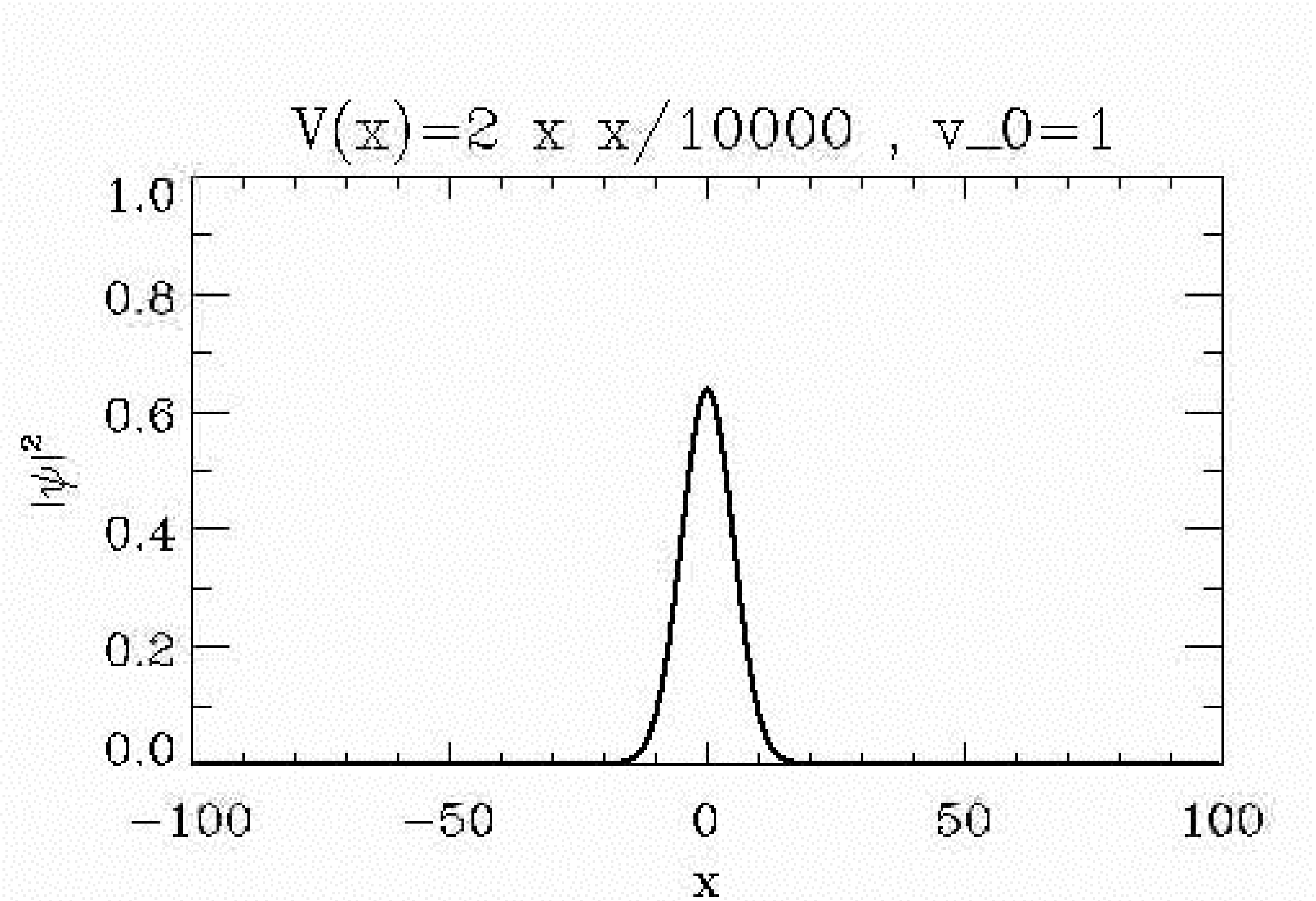
Which direction is the force?

What will the wave function do?

Estimate E.

Where would a classical electron turn around?

Harmonic Oscillator (harmosc2.mpg)



Harmonic Oscillator (harmosc2.mpg)

Why no spreading?

Down Sloping Potential (downstp.mpg)

The wave function at t = 0 has the form

$\Psi(x,0) = A \exp[-(x+50)^2/10^2] \exp[i\, x/2]$

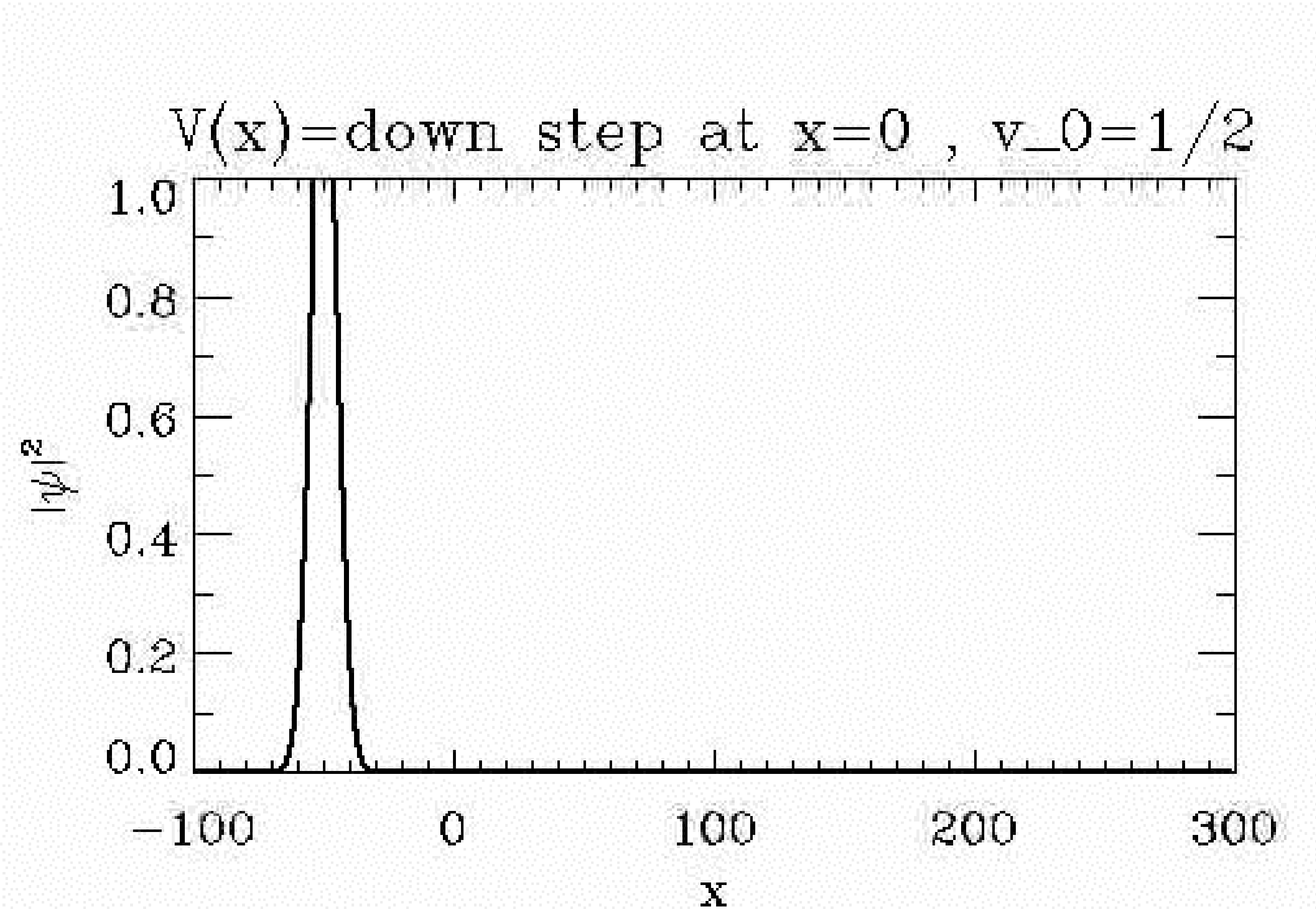
$U(x) = 1/[1+\exp(x^3)]$ & starting $v = 1/2$

Which direction is the force?

Where is the force large?

What will the wave function do?

Down Sloping Potential (downstp.mpg)



Down Sloping Potential (downstp.mpg)

Why is there some reflection? (Reflection total as $v \rightarrow 0$!)

Why is the packet broader $x > 0$?

Why is the interference only at $x < 0$?

Finite Barrier (barrier1.mpg)

The wave function at t = 0 has the form

$\Psi(x,0) = A \exp[-(x+50)^2/10^2] \exp[i\,x]$

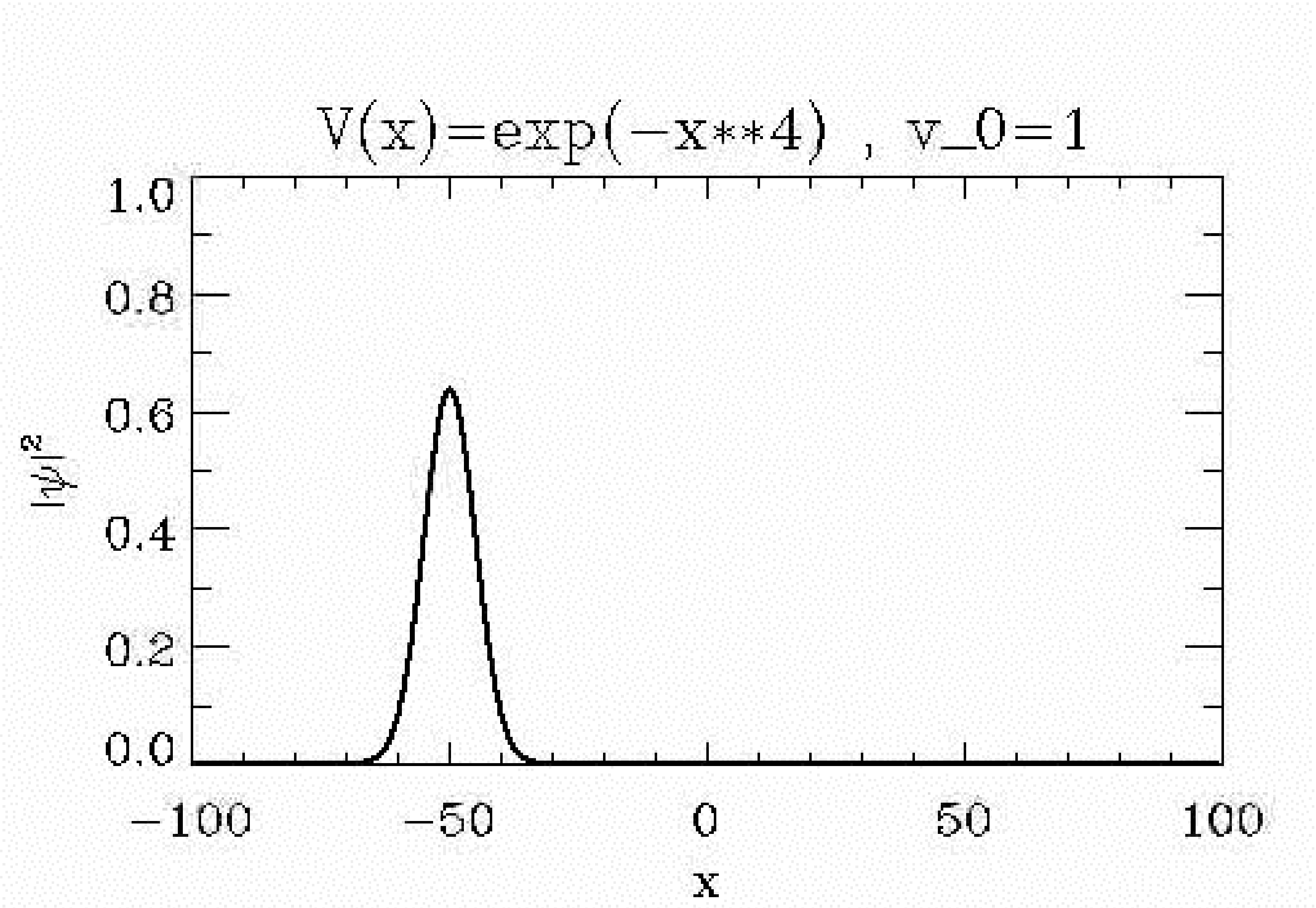
$U(x) = \exp(-x^4)$ & starting v = 1

Which direction is the force? Where is the force large?

Estimate E.

What will the wave function do?

Finite Barrier (barrier1.mpg)



Finite Barrier (barrier1.mpg)

Why is there some transmission? (Optics) (Amount of transmission $e^{-S/h}$; S depends on E & U; h is Planck's constant)

Why is the interference only at $x < 0$?

Finite Barrier (barrier2.mpg)

The wave function at $t = 0$ has the form

$\Psi(x,0) = A \exp[-(x+50)^2/10^2] \exp[i\,3\,x/2]$

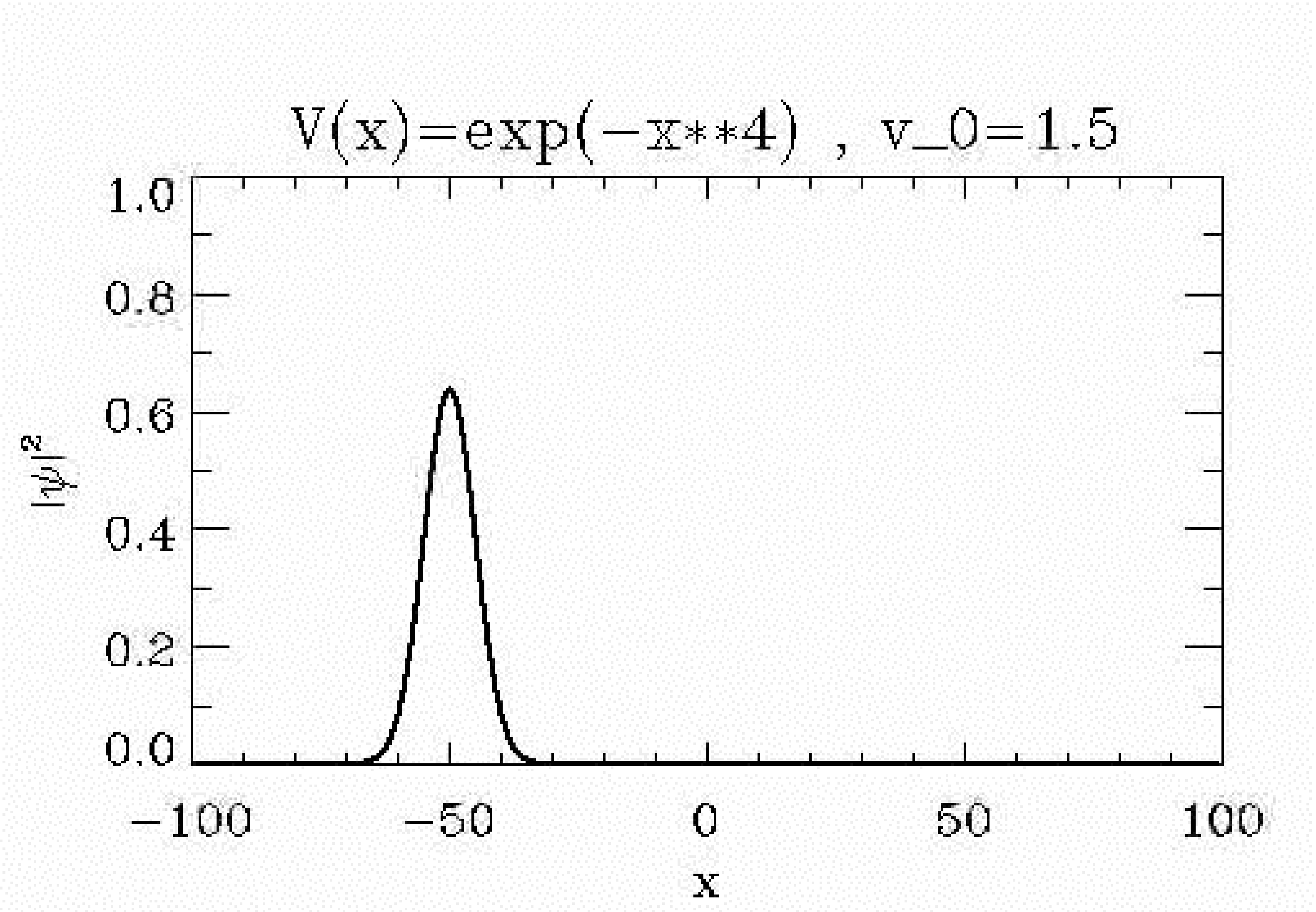
$U(x) = \exp(-x^4)$ & starting $v = 3/2$

Which direction is the force? Where is the force large?

Estimate E.

What will the wave function do?

Finite Barrier (barrier2.mpg)



Finite Barrier (barrier2.mpg)

Why is there some reflection?

Why is the interference only at $x < 0$?

Small, oscillating U (bragg2.mpg)

The wave function at t = 0 has the form

$\Psi(x,0) = A \exp[-(x+50)^2/20^2] \exp[i\ 4\ x/5]$

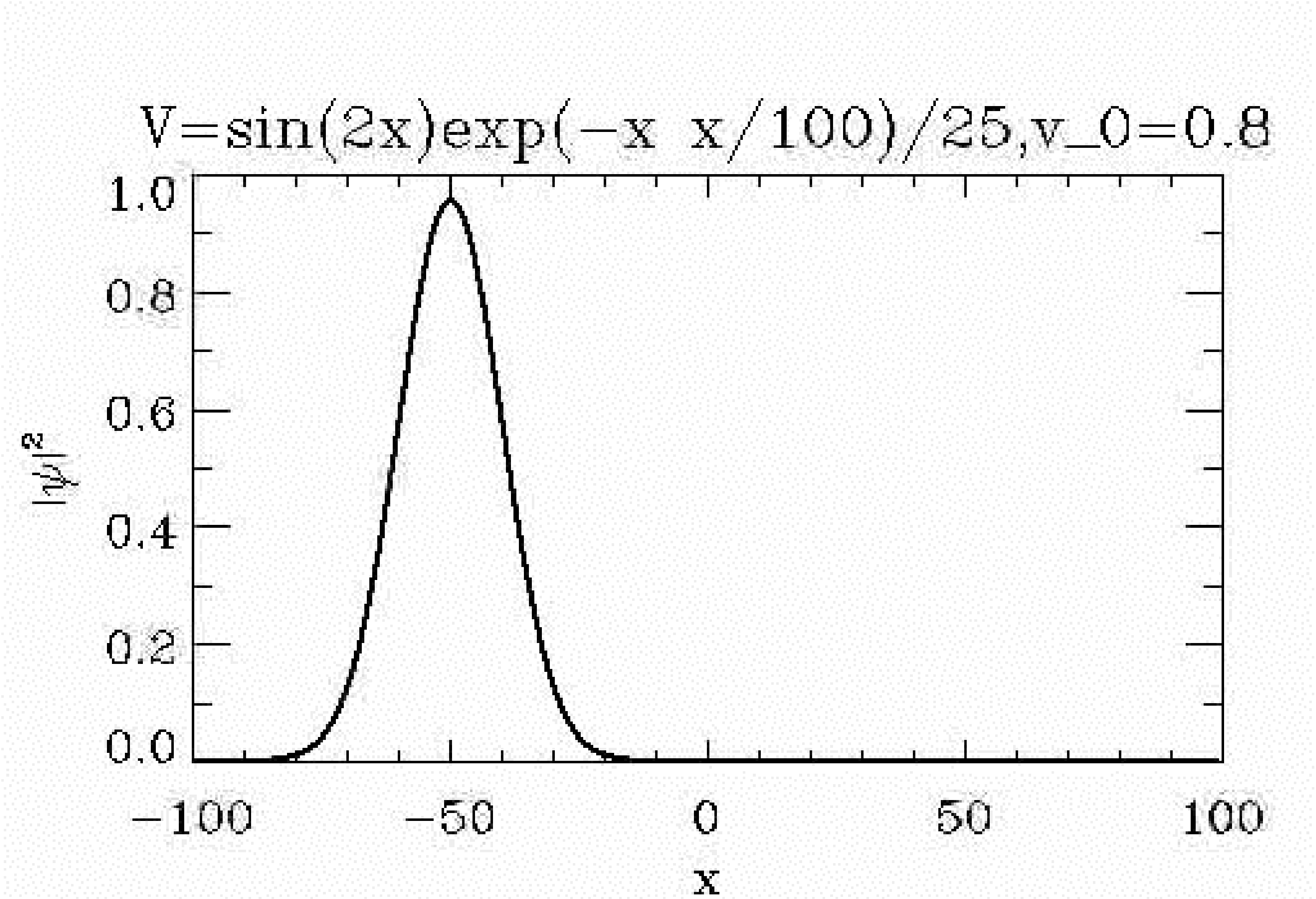
$U(x) = 0.04 \sin(2\ x) \exp(-x^2/10^2)$ & starting v = 1

Rough description of the force and potential energy.

Estimate E.

What will the wave function do?

Small, oscillating U (bragg2.mpg)



Small, oscillating U (bragg2.mpg)

Why is there interference over almost whole region of U?

Small, oscillating U (bragg1.mpg)

The wave function at t = 0 has the form

$\Psi(x,0) = A \exp[-(x+50)^2/20^2] \exp[i\, x]$

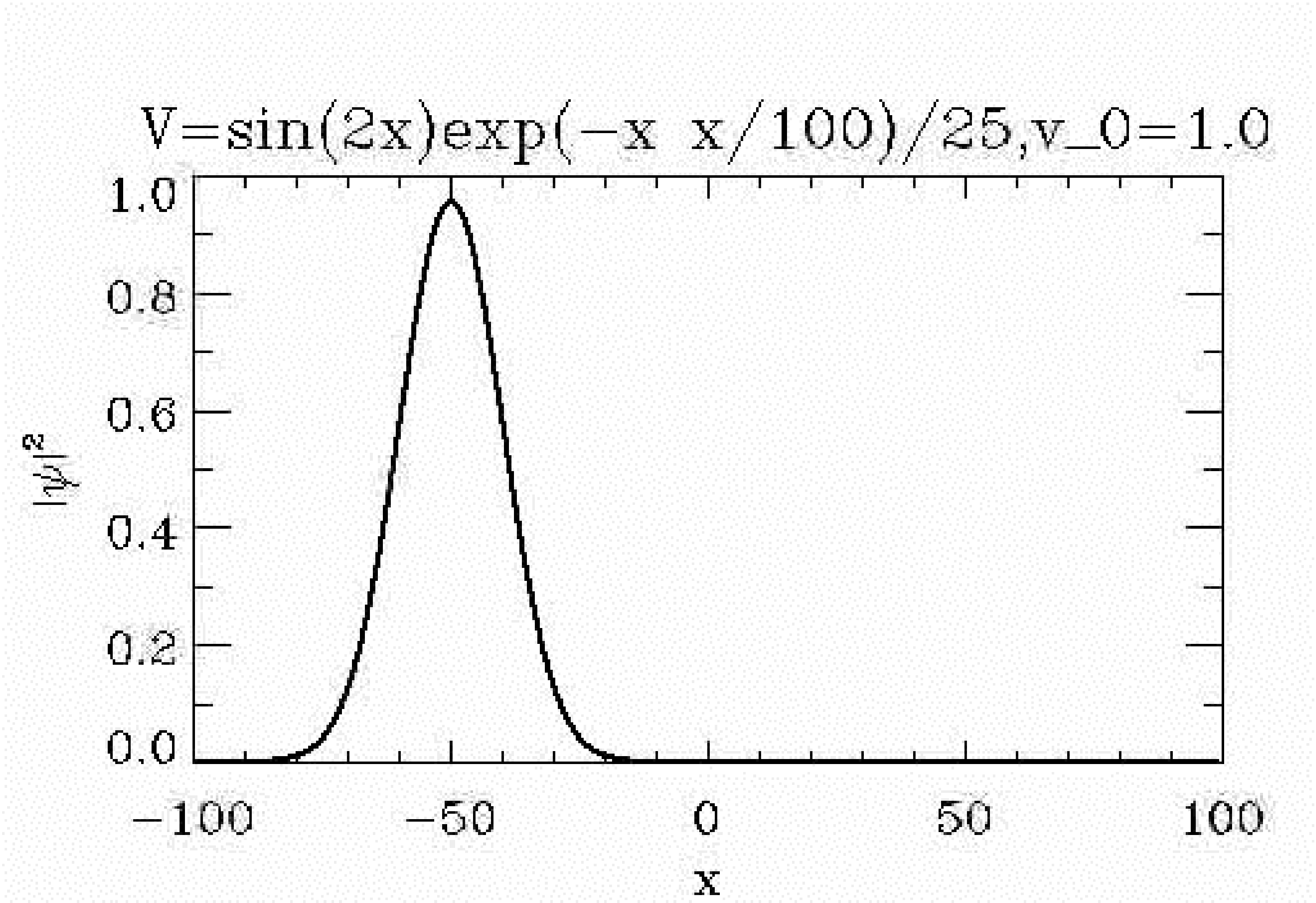
$U(x) = 0.04 \sin(2\, x) \exp(-x^2/10^2)$ & starting v = 1

Rough description of the force and potential energy.

Estimate E. (Higher than previous)

What will the wave function do?

Small, oscillating U (bragg1.mpg)



Small, oscillating U (bragg1.mpg)

Why is there substantial reflection for v~1?

Why is the interference only at $x < 0$?

Problem

A particle of mass M moves in a 1-dimensional box of length L. The particle's wave must go to zero at $x = 0$ and $x = L$. (a) Show that λ can only take specific values. (b) What are the allowed p? (c) What are the allowed energies? (d) What is the frequency of light if the particle makes a transition from n to n-1? (e) How close is this to the classical frequency of the motion?

Question #3

For a potential energy $\frac{1}{2} M \omega^2 x^2$, the number of physical solutions of the time independent Schrodinger equation (for random E), $E \psi(x) = -(\hbar^2/2M)d^2\psi/dx^2 + U(x) \psi(x)$, is

(a) 0

(b) 1

(c) 2

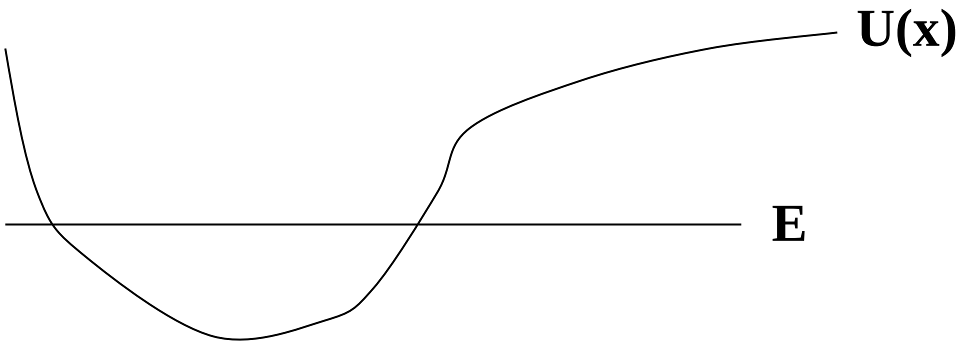
(d) 3

(e) depends on V in a complicated way.

Eigenstates & Eigenvalues of S.E.

The time independent Schrodinger equation is
$H_{op}\ \psi_n(x) = E_n\ \psi_n(x)$
What is the eigenvalue? eigenstate?

E can not take every possible value if classical motion restricted to finite range.



Question #2

In the classically forbidden region (x where $E - U(x) < 0$) the solution of $E\,\psi(x) = -(\hbar^2/2M)d^2\psi/dx^2 + U(x)\,\psi(x)$

(a) oscillates with x.

(b) is 0

(c) must be imaginary.

(d) exponentially diverges or converges with x

(e) linearly increases with x

Properties of Eigenstates of S.E.

(1) E_n are all real and increase with n.

(2) The $\psi_n(x)$ are ortho-normal (orthogonal & normalized).

(3) The eigenstates can be chosen to be real at every x.

(4) Eigenstates are continuous.

(5) Derivative of eigenstate is continuous if U(x) is finite.

(6) $<H_{op}>$ does not depend on t.

(7) $\psi_n(x) = \psi_n(-x)$ or $-\psi_n(-x)$ if $U(x) = U(-x)$

General Behavior

$d^2\psi/dx^2 = -2\ M\ [E - U(x)]\ \psi/\hbar^2$

What sort of behavior when at positions where $E > U(x)$?

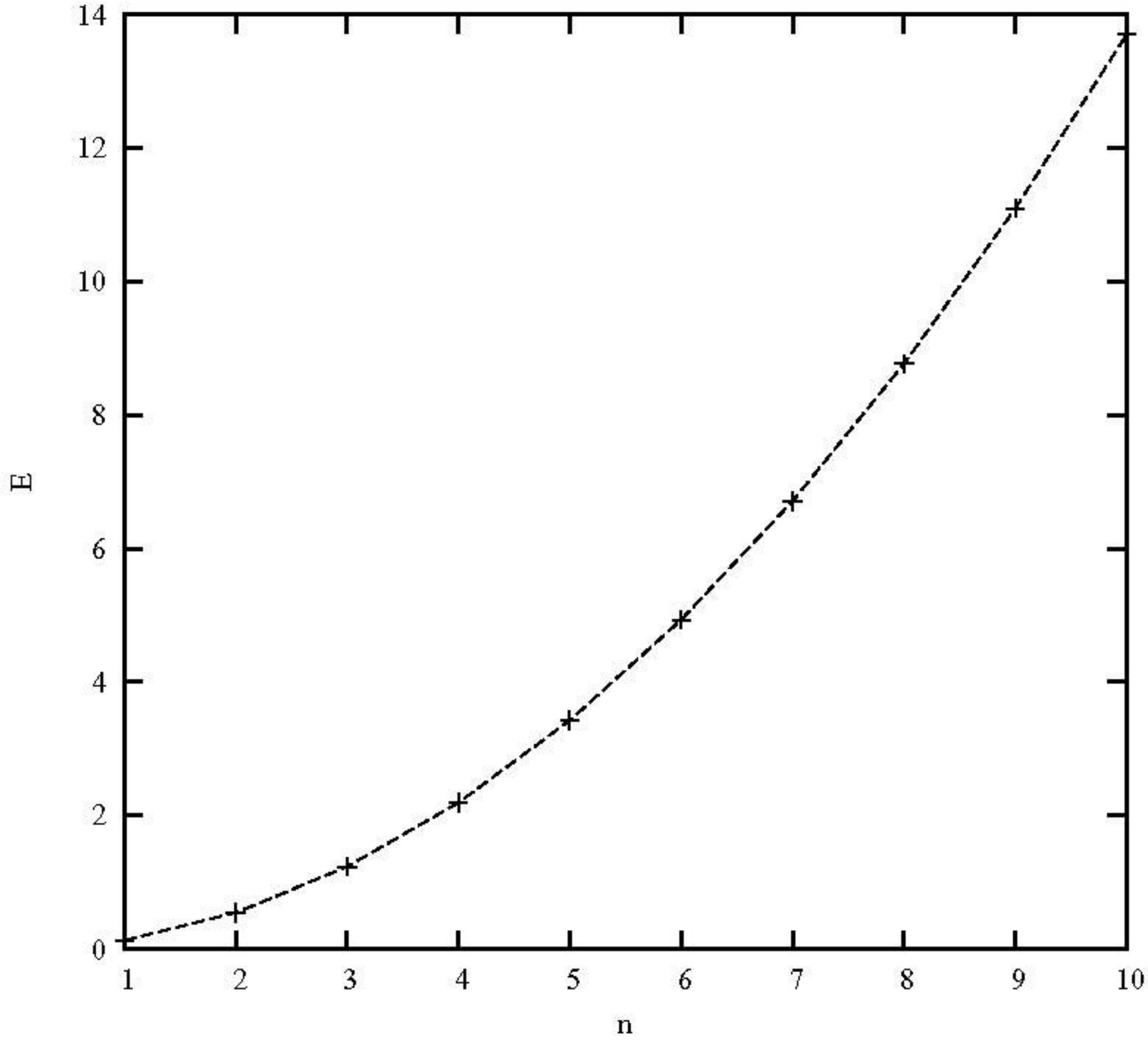
What sort of behavior when at positions where $E < U(x)$?

For a given potential where does ψ oscillate fastest w/ x?

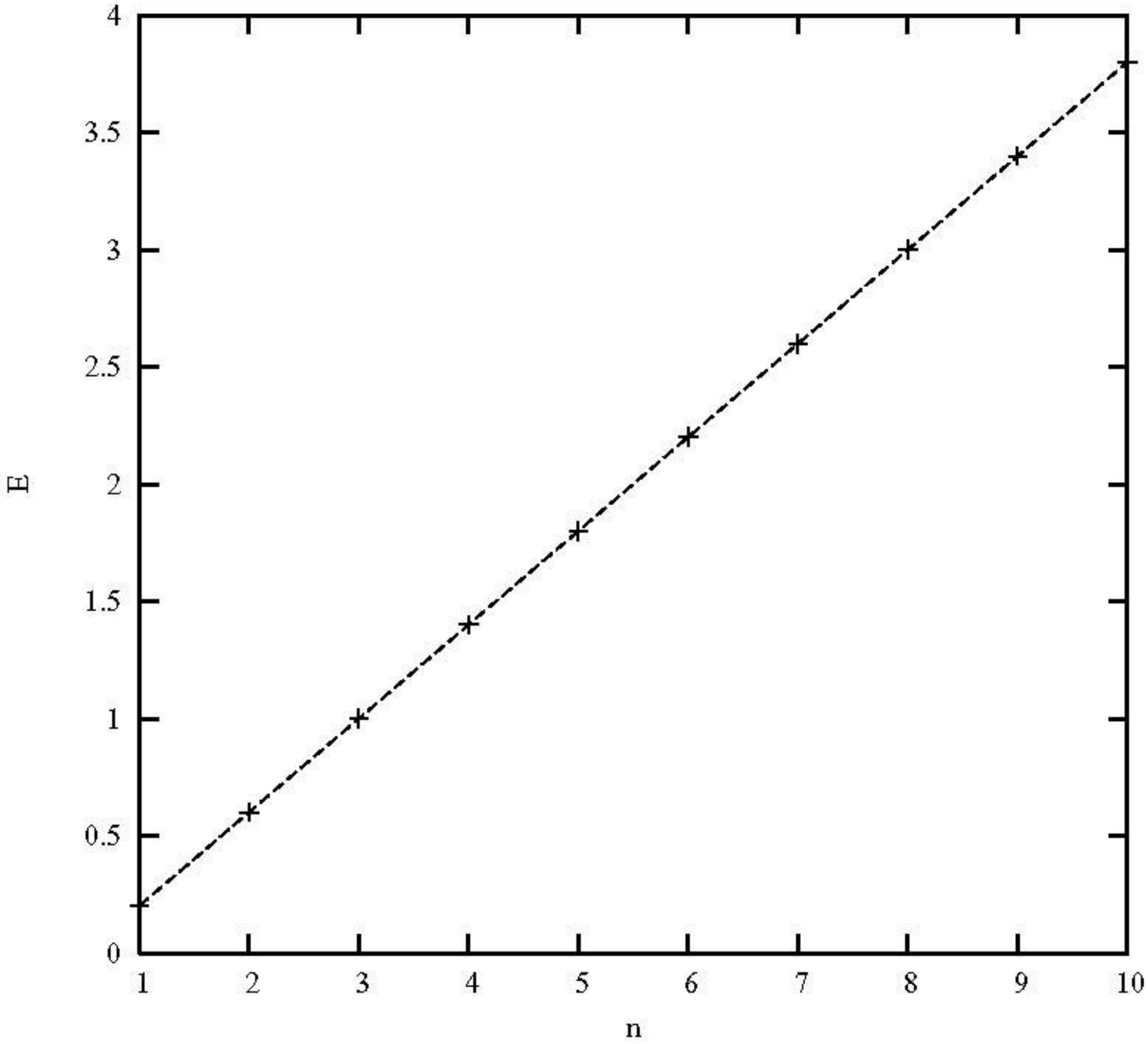
Where are the positions where curvature of ψ is 0?

Correspondence principle: period = $h/(E_{n+1} - E_n)$ should be approximately the classical period. If we plot E vs n, how should the curve look if classical period increases with E? decreases with E? is independent of E?

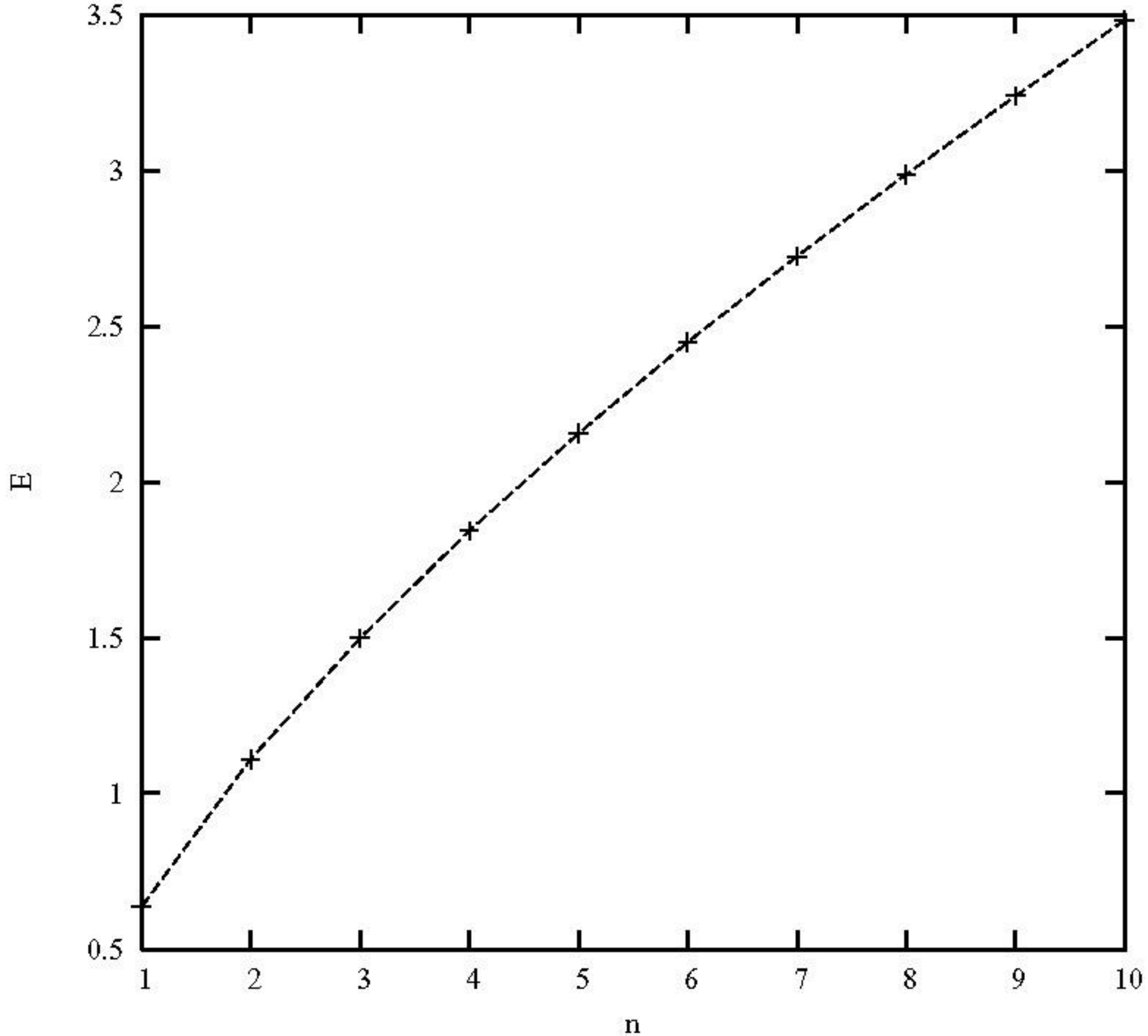
Infinite Square Well



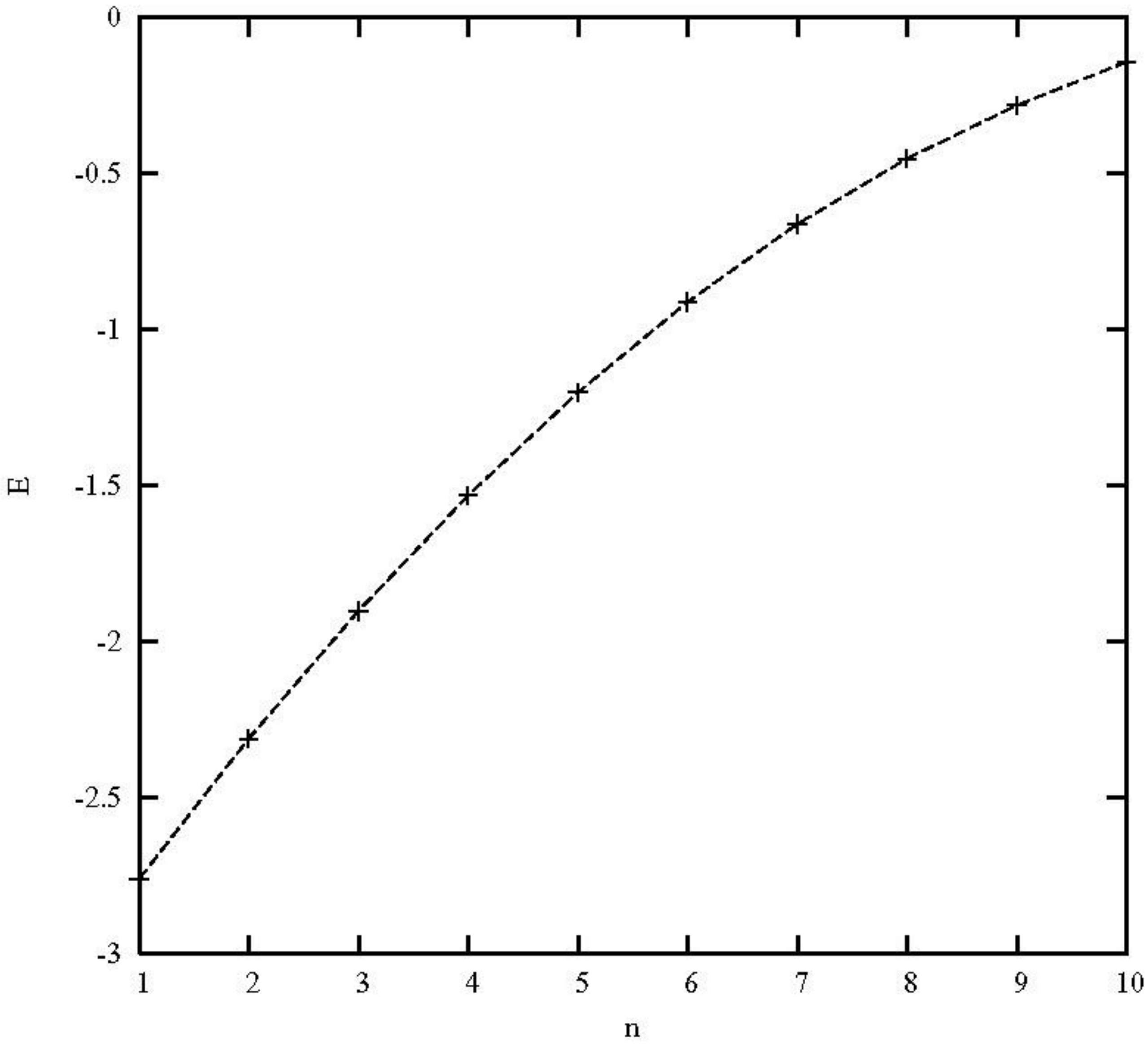
Harmonic Oscillator



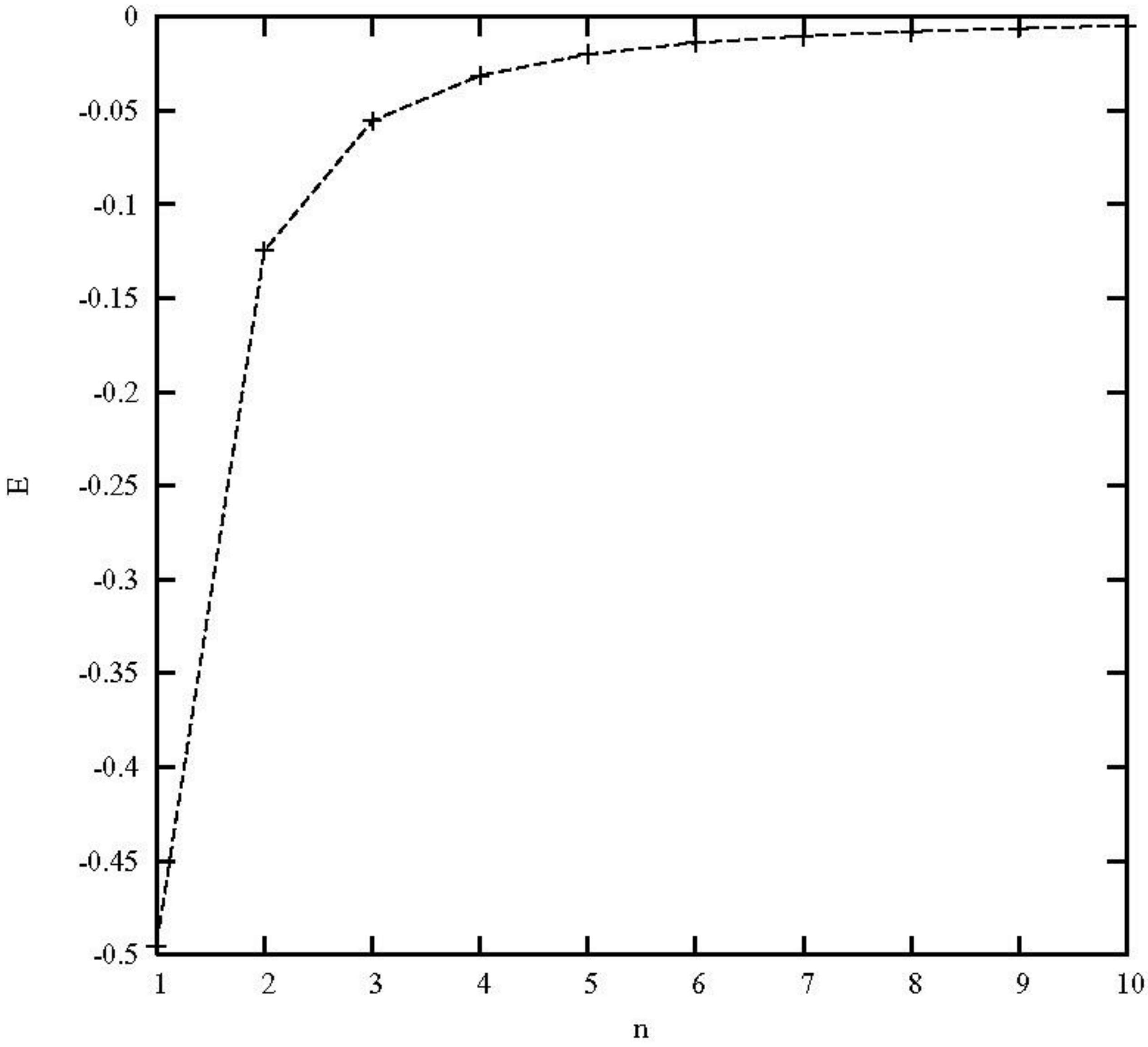
Linear Potential, Wall x=0



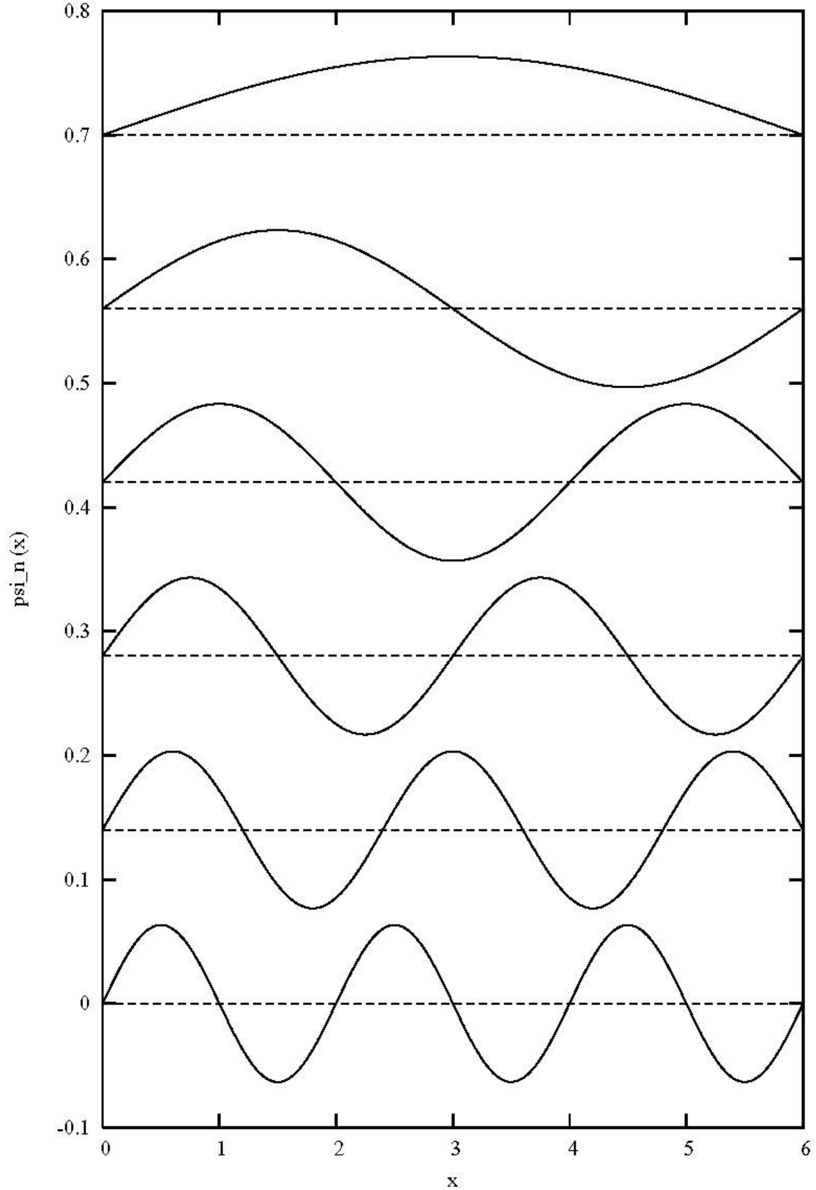
"Molecule" Vibration



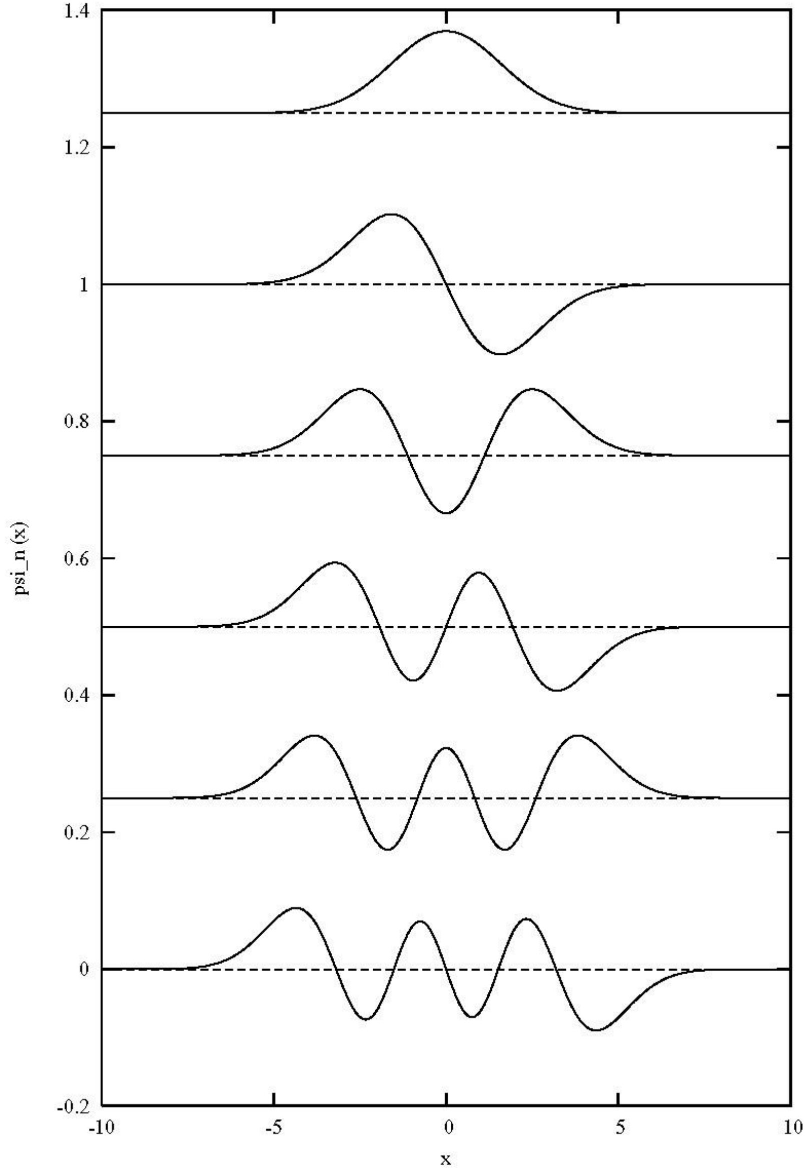
Coulomb Potential (-1/r)



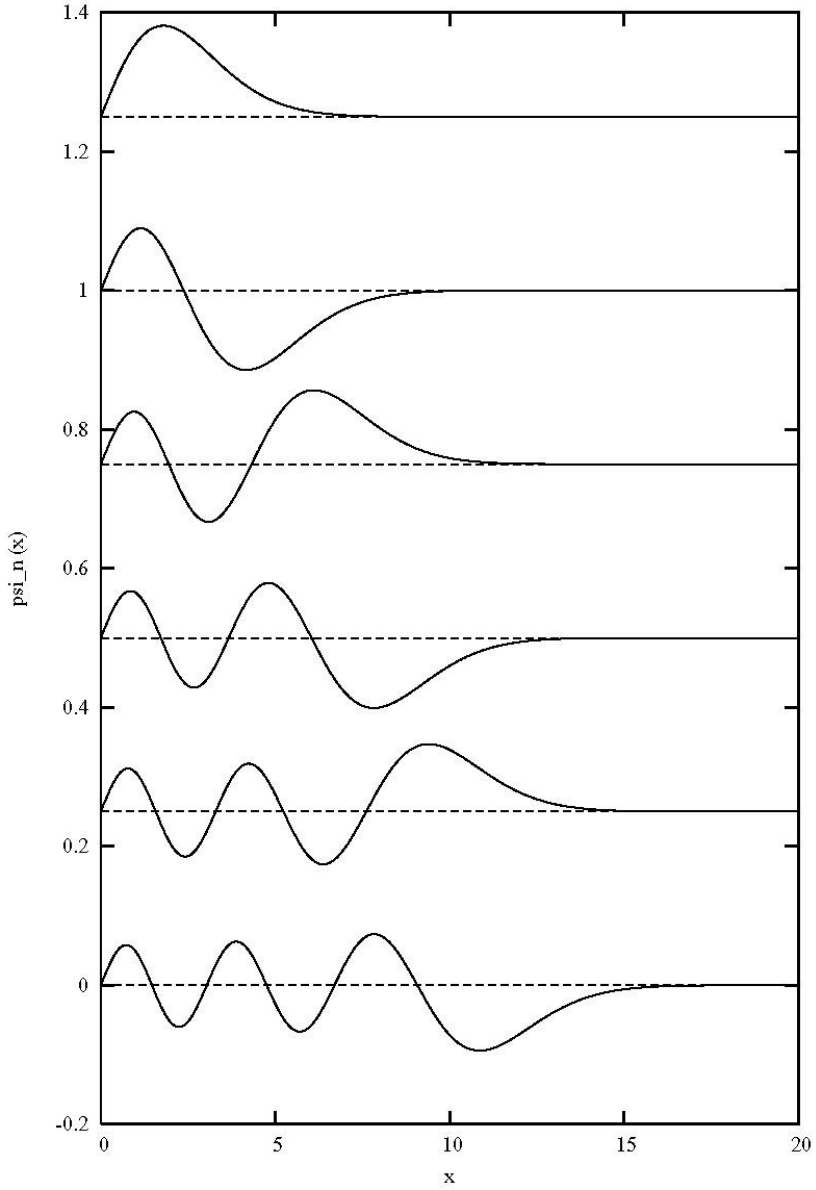
Infinite Square Well



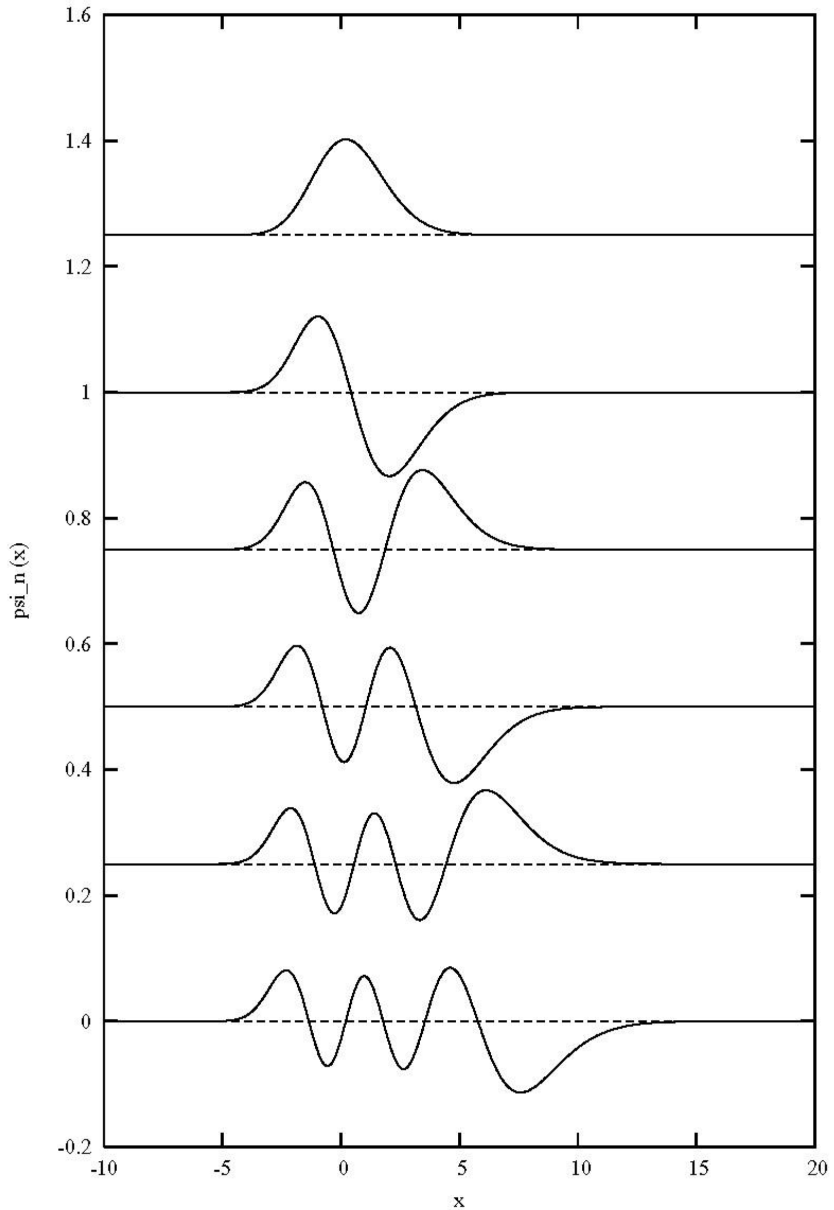
Harmonic Oscillator



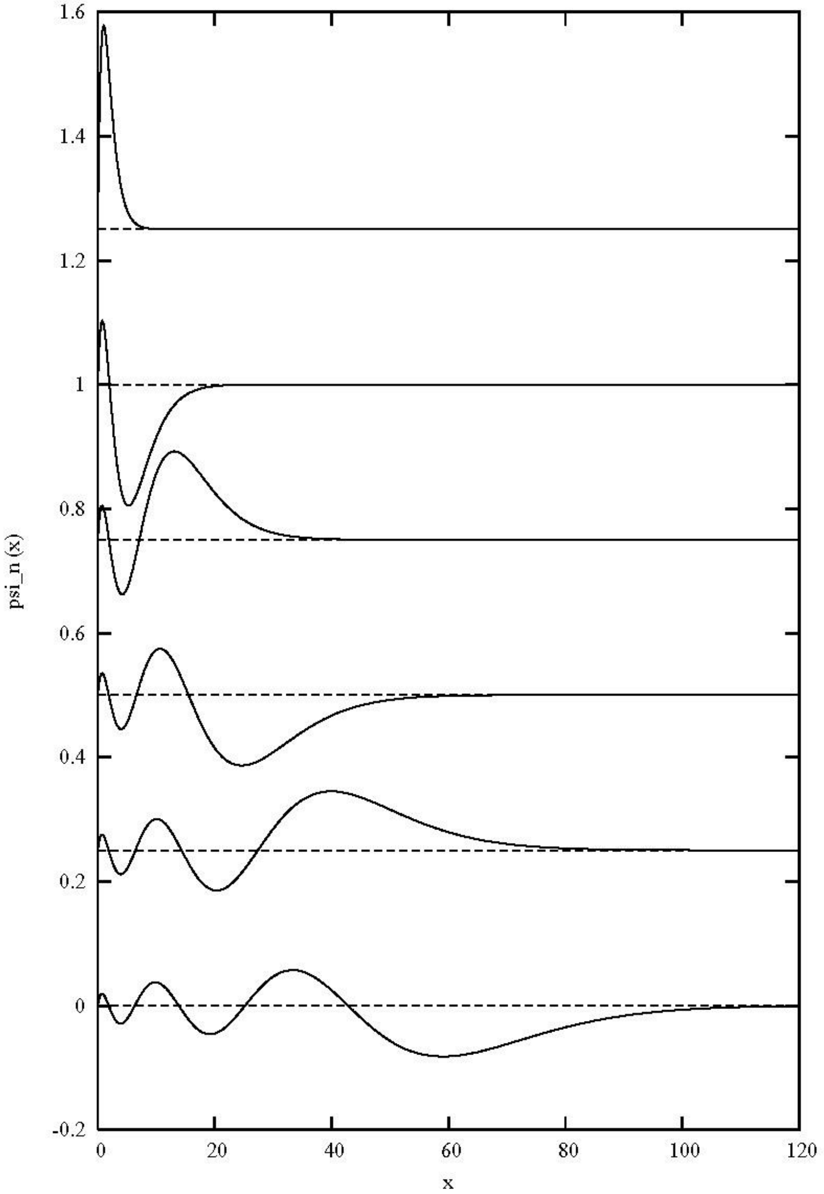
Linear Potential, Wall x=0



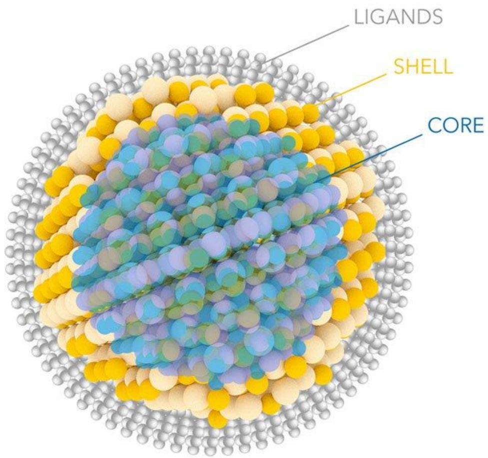
“Molecule” Vibration

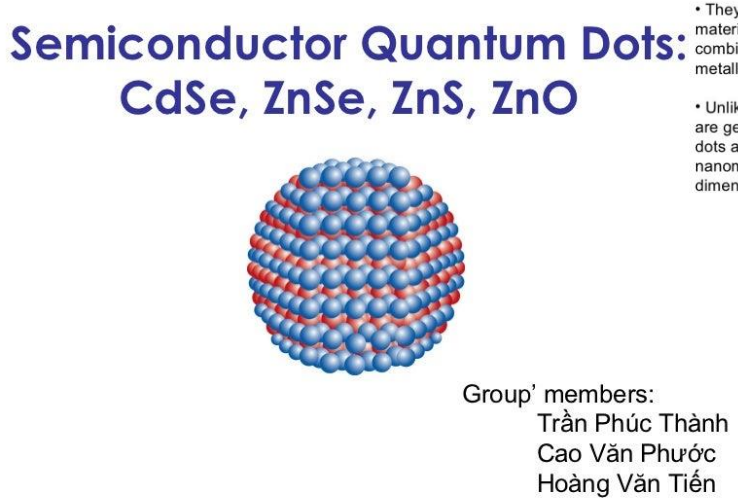


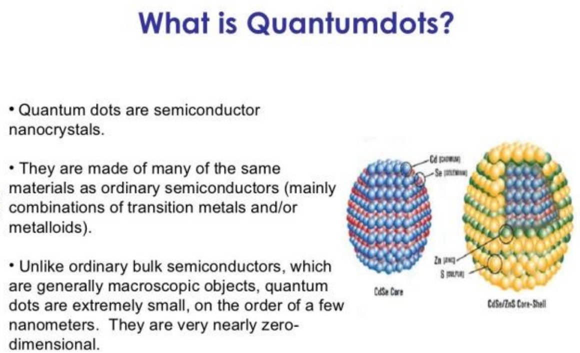
Coulomb Potential (-1/r)



Quantum Dots



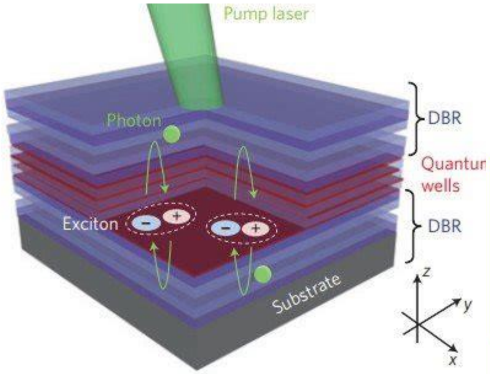


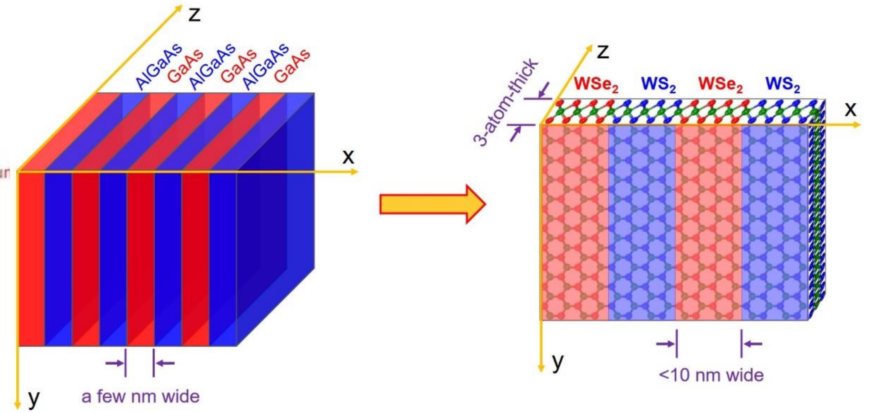


Can make small structures that look like tiny parts of a crystal
Leads to quantized energy levels for free electrons inside

https://www.avsforum.com/threads/electroluminescent-quantum-dots-are-coming-sooner-than-you-think.3164041/
https://www.slideshare.net/hoangtienbk/quantum-dots-15108239

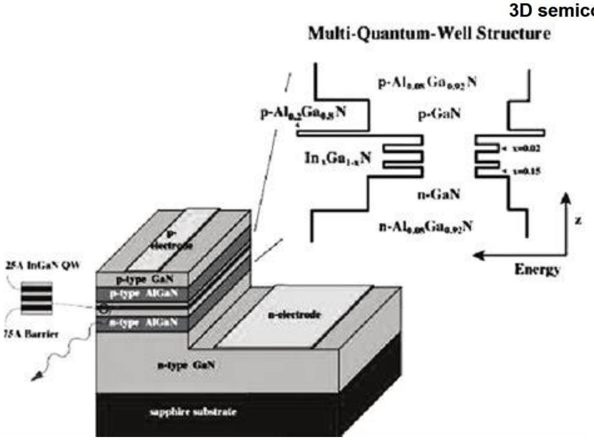
Semiconductor Quantum Wells





3D semiconductor superlattice

2D semiconductor quantum well superlattice



Layer the materials in a semiconductor to get quantum wells

Manipulate the properties

https://www.researchgate.net/publication/314772334_The_theoretical_explanation_of_GaN-based_laser_anomalous_electrical_properties

https://english.cas.cn/newsroom/archive/research_archive/rp2018/201803/t20180329_191167.shtml

https://www.researchgate.net/publication/224830788_Characterization_Parameters_of_InGaNInGaN_and_InGaNGaN_Quantum_Well_Laser_Diode

Question #1

A gwoster travels to the left with an energy, E. It is in a potential that is 0 for $x>0$ and $V_0 > E$ for $x<0$. The probability it will be reflected and travel to the right is

(a) 0%

(b) between 0% and 100%

(c) 100%

Question #2

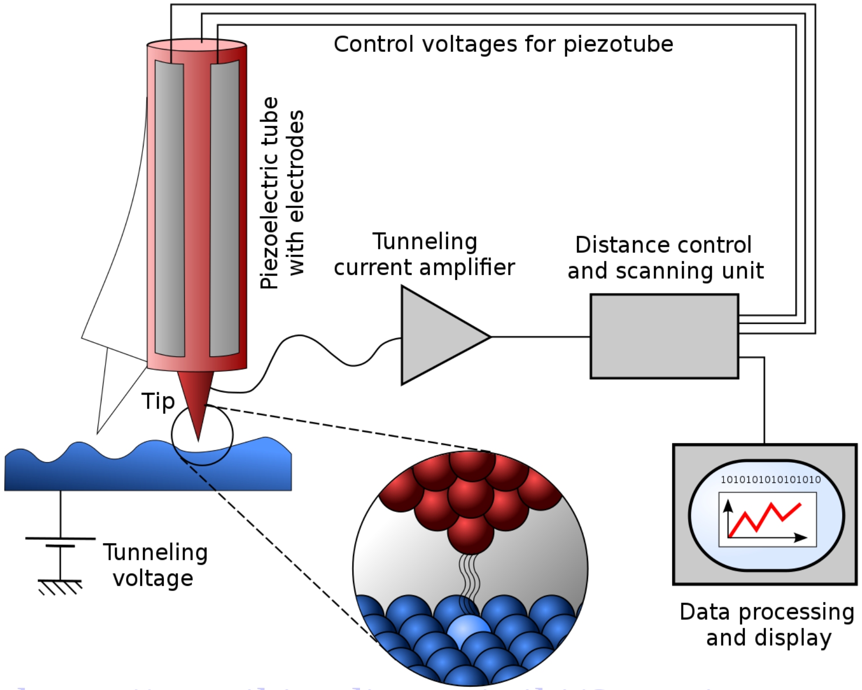
A gwoster travels to the left with an energy, E. It is in a potential that is 0 for $|x|>L$ and $V_0 > E$ for $|x|<L$. The probability it will be reflected and travel to the right is

(a) 0%

(b) between 0% and 100%

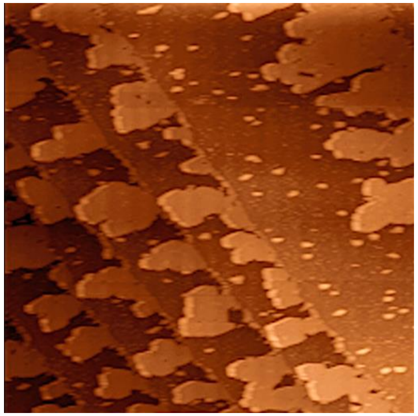
(c) 100%

Scanning Tunneling Microscope



Electrons can tunnel from the sharp metal tip into the sample

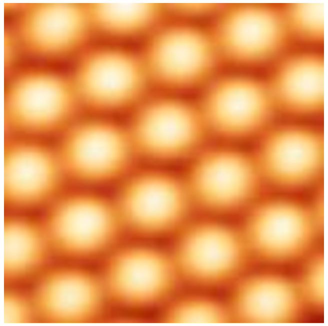
Exponential sensitivity



1 atom thick silver islands on palladium

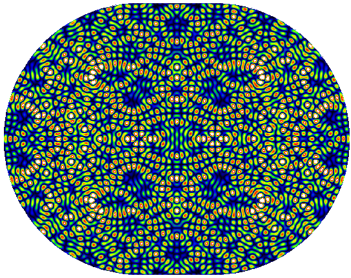
https://en.wikipedia.org/wiki/Scanning_tunneling_microscope

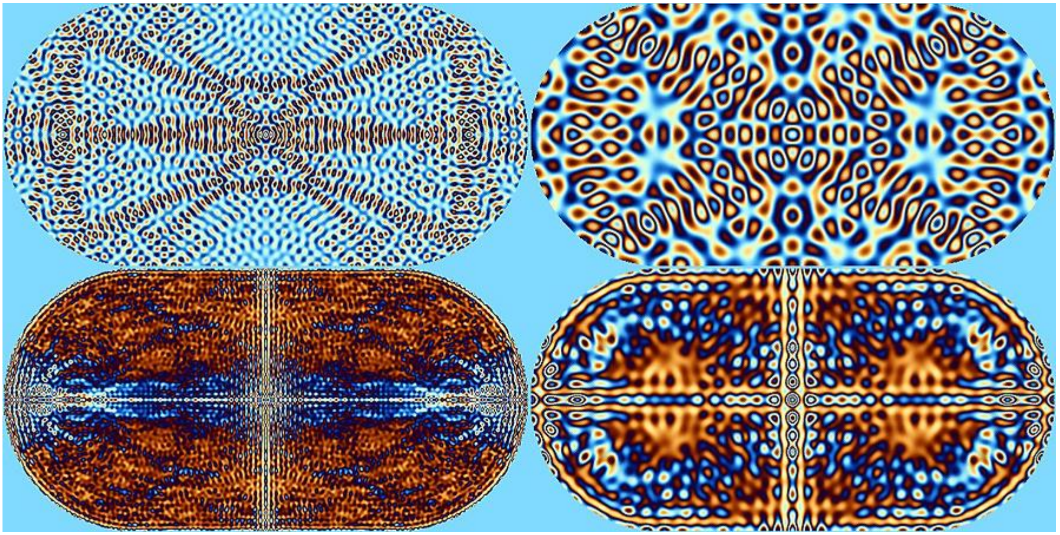
https://www.nist.gov/pml/scanning-tunneling-microscope/scanning-tunneling-microscope-introduction

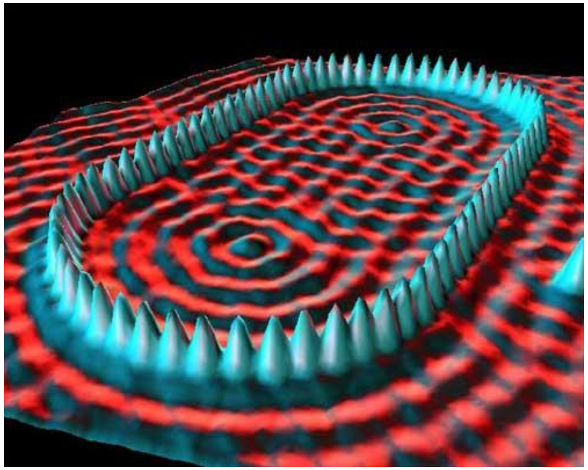


Atoms on the surface of silicon carbide (0.3 nm separation)

Quantum Corrals & Scars



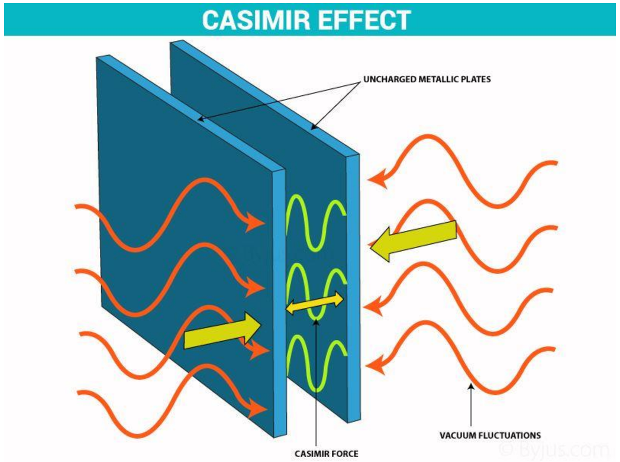




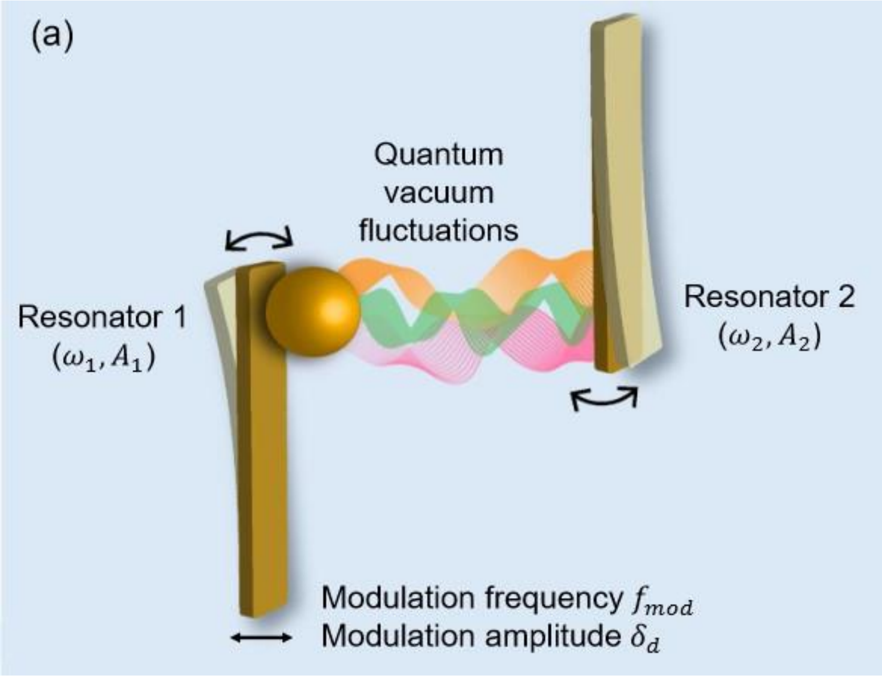
https://journals.aps.org/rmp/abstract/10.1103/RevModPhys.75.933
https://en.wikipedia.org/wiki/Quantum_mirage
https://en.wikipedia.org/wiki/Quantum_scar

Nonlinear 2D (or higher) potentials can lead to classical chaos
Unstable periodic classical orbits can lead to "scars" on the resulting energy eigenstates

Casimir effect (vacuum force)



Two uncharged metal plates in the vacuum feel a force at small separations
There is an energy associated with 0 photons
The metal plates don’t allow all possible 0 photon states



https://en.wikipedia.org/wiki/Casimir_effect
https://byjus.com/physics/casimir-effect/
https://www.nature.com/articles/s41565-021-01026-8