Question #1

How does wavelength of a photon change with momentum?

(a) The wavelength is proportional to the momentum.

(b) The wavelength is proportional to the square of the momentum.

(c) The wavelength is proportional to the inverse of the momentum.

(d) The wavelength is proportional to the inverse of the square of the momentum.

(e) This is a trick question; the photon is a particle and does not have a wavelength.

Question #2

How does wavelength of an electron change with momentum?

(a) The wavelength is proportional to the momentum.

(b) The wavelength is proportional to the square of the momentum.

(c) The wavelength is proportional to the inverse of the momentum.

(d) The wavelength is proportional to the inverse of the square of the momentum.

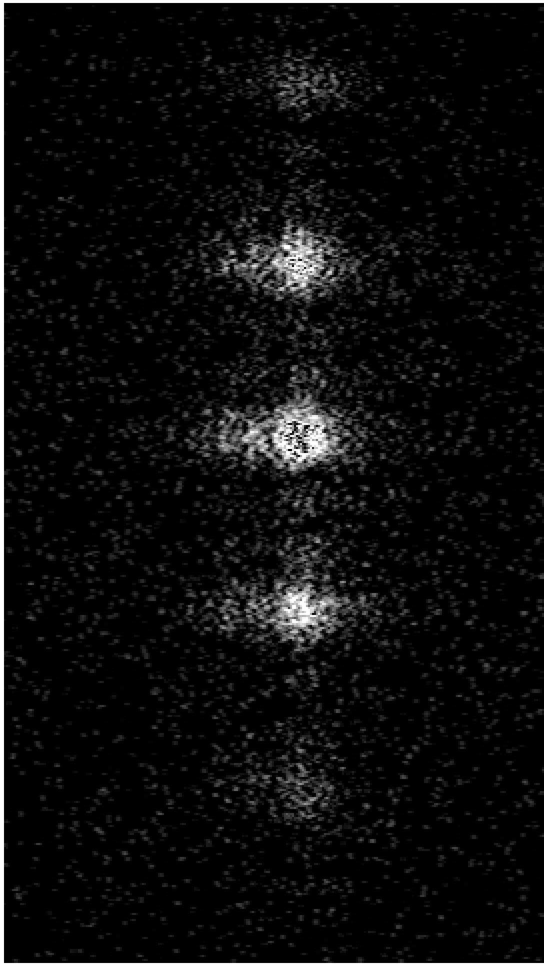
(e) This is a trick question; the electron is a particle and does not have a wavelength.

How two slit interference works

Experiment by Robert Austin & Lyman Page (Princeton Univ)

1/30 sec (5 photons)

1 sec



100 sec

Each individual photon interferes with itself!?

How can we explain?

Information

A single photon goes through at a time.

If you cover one of the holes, get a different pattern.

If you put perpendicular polarizers behind each hole, then you don't get interference.

Assumption is that each photon (particle) has a wave associated with it. The wavelength of the wave is given by $\lambda = h/p$ and the frequency of the wave is $f = E/h$.

de Broglie waves

ALL particles have a wave associated with them!!!

The wavelength is given by $\lambda = h/p$ and the frequency is given by $f = E/h$.

Why don't we notice?

An object has $v = 1$ m/s and $M = 1$ kg. Compute the wavelength and frequency.

$\lambda = h/p = 6.6 \text{ X } 10^{-34}$ J s/(1 kg X 1 m/s) = $6.6 \text{ X } 10^{-34}$ m

$f = E/h = (\frac{1}{2}\ 1 \text{ kg } 1 \text{ m}^2/\text{s}^2)/6.6\text{X}10^{-34}$ J s= $1.5 \text{ X } 10^{33}$ Hz

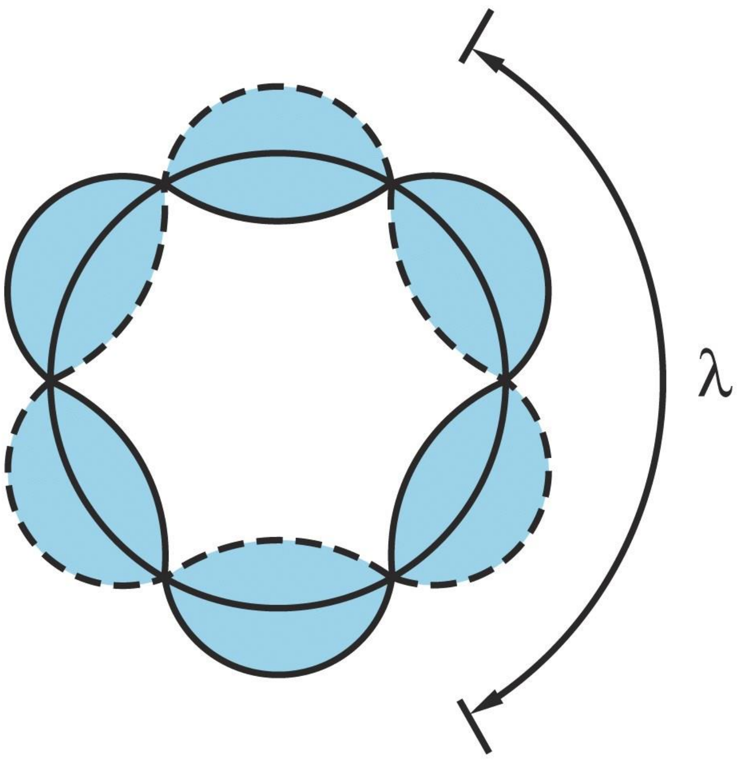
Evidence

Bohr condition:

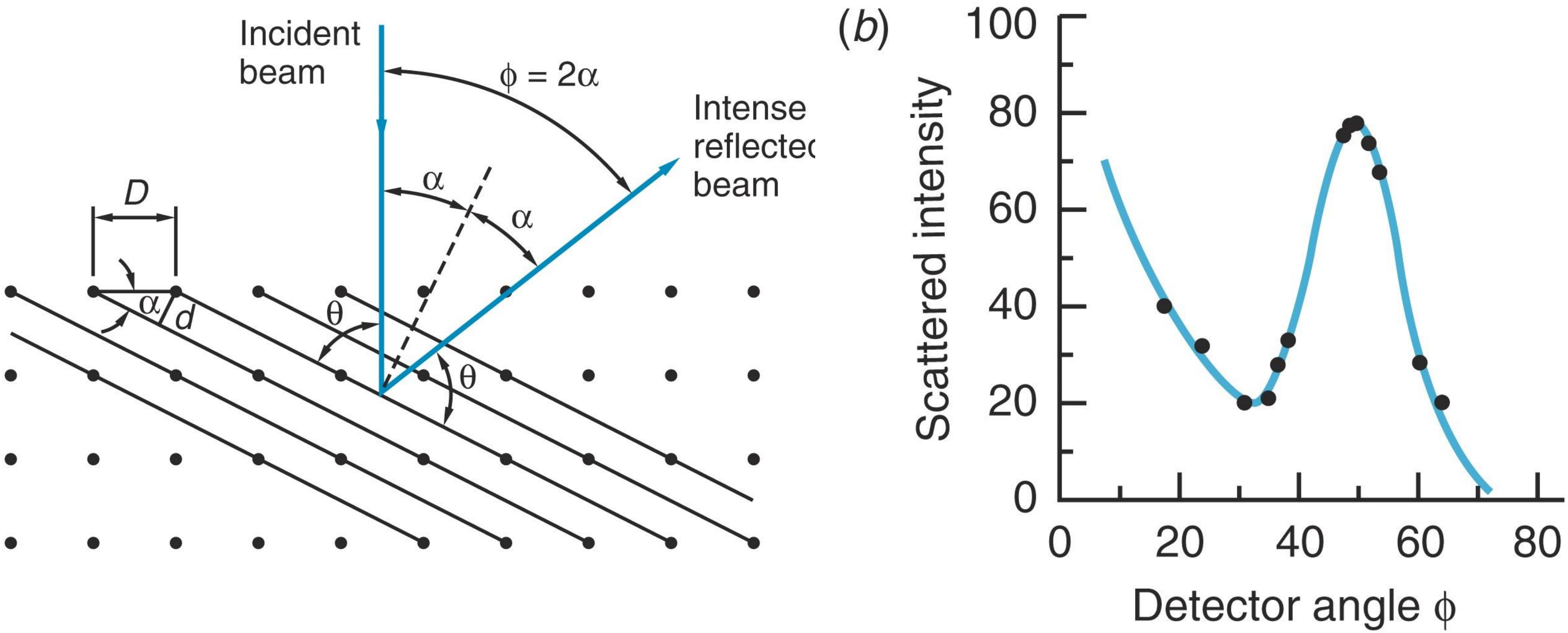
$L_n = m\,v\,r = n\,h/2\,\pi \;\;\Rightarrow\;\; 2\,\pi\,r = n\,h\,/\,m\,v = n\,\lambda$

Circumference equals integer number of wavelengths.

Standing wave!



Scattering of Electron by Crystal

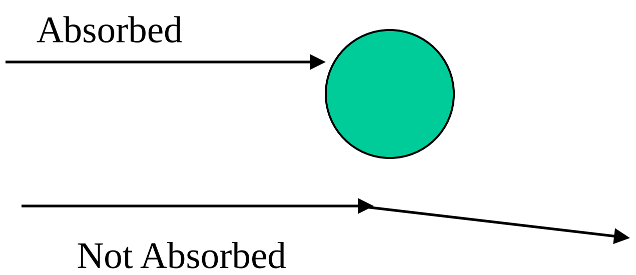


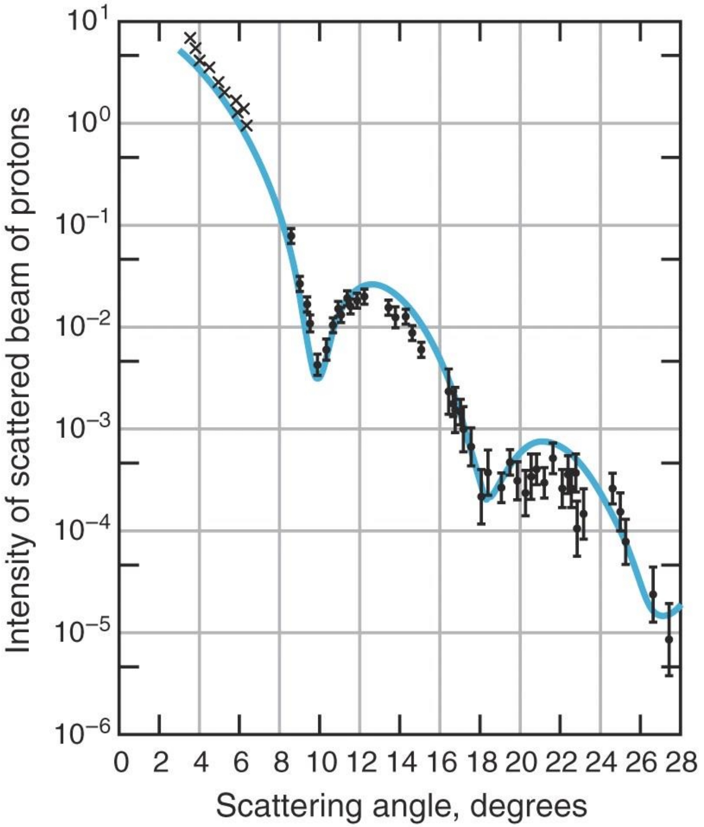
Why does a regular array of scatters give constructive interference?

Proton-Nuclei Scattering

From Rutherford, shouldn't we get simple decreasing function?

What are the dips from?





Problem

Light with a wavelength of 450 nm is incident on solid Na. The sodium absorbs the light and electrons are ejected. What is the minimum wavelength of one of these electrons? (Why minimum?)

What are the basic ideas in this problem?

Photo-electric effect & de Broglie waves

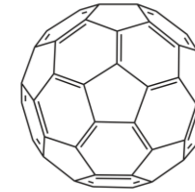
What do you need to know to compute the wavelength?

An electron's momentum = mass X speed

How do you find the electron's speed?

Conservation of energy: $KE = \frac{1}{2} M v^2_{max} = h\, c / \lambda - \phi$

The C_{60} Molecule Diffracts



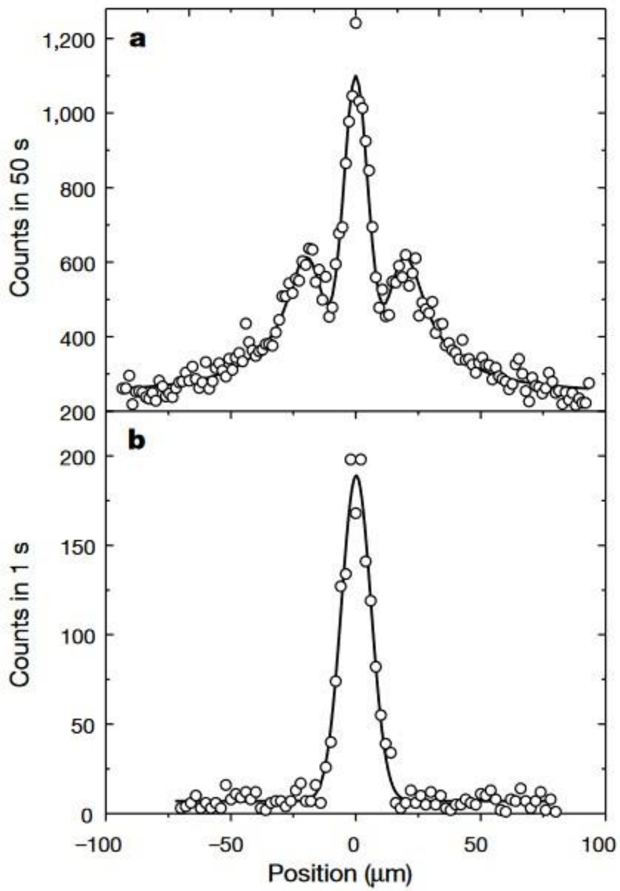


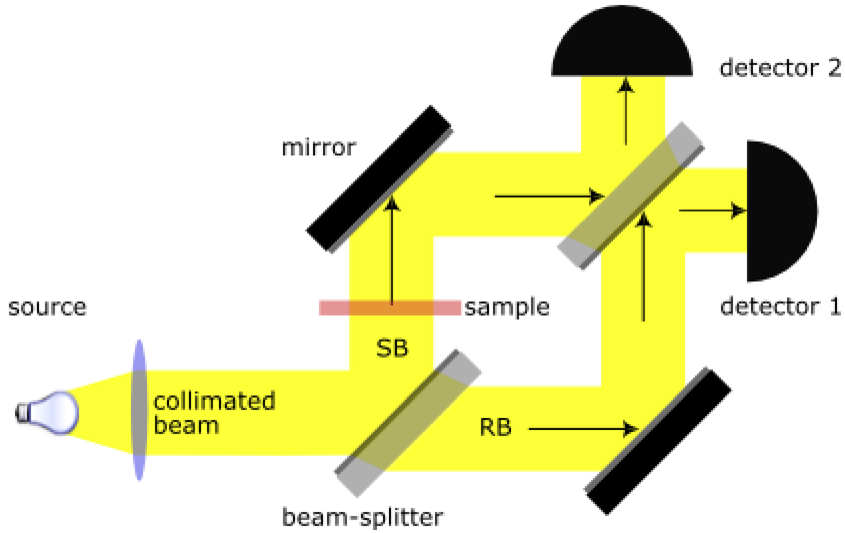
Figure 2 Interference pattern produced by C_{60} molecules. **a**, Experimental recording (open circles) and fit using Kirchhoff diffraction theory (continuous line). The expected zeroth and first-order maxima can be clearly seen. Details of the theory are discussed in the text. **b**, The molecular beam profile without the grating in the path of the molecules.

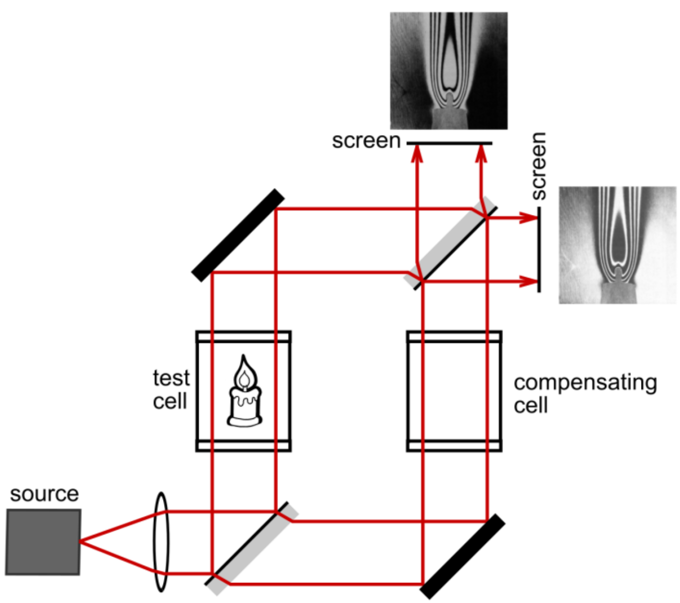
The buckyballs go through mechanical grating and give interference pattern
https://quantumnano.at/research/universal-matter-waves/far-field-diffraction/the-origin-c60-diffraction/

https://www.nature.com/articles/44348
Wave-particle duality of C_{60} molecules
M. Arndt, et al Nature **401**, 608 (1999)

https://quantumnano.at/research/quantum-classical/record-quantum-macroscopicity/
Largest molecule interference had about 2000 atoms (in 2020)
https://physics.aps.org/articles/v13/s93

Mach-Zehnder Interferometer



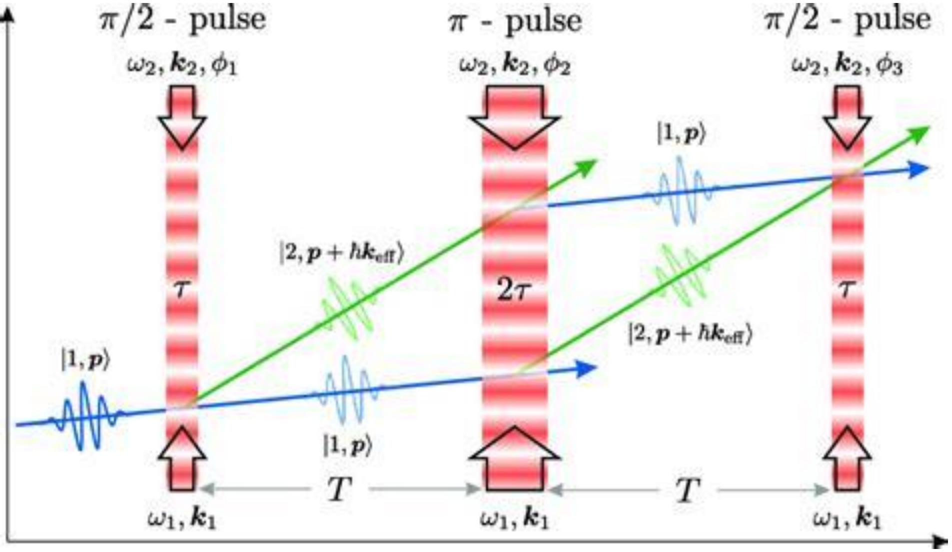


Device that uses light
The different path lengths lead to interference in the two possible ouputs
https://en.wikipedia.org/wiki/Mach%E2%80%93Zehnder_interferometer

The sample adds extra wavelengths

The amount in Det 1 vs 2 depends on wavelength difference in sample

Atom Interferometer



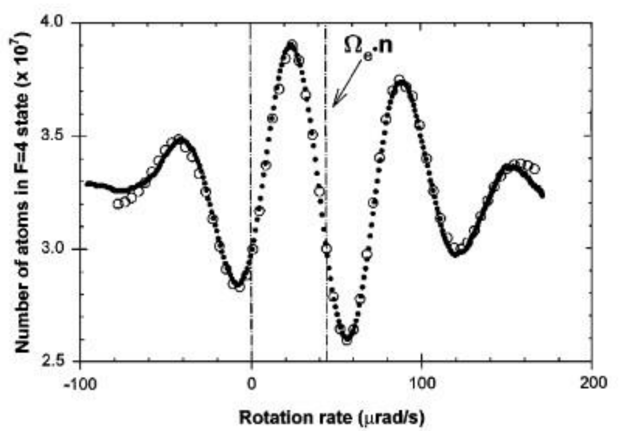


FIG. 3. Measurement of an atom interference pattern versus rotation rate. Black dots: experimental data. Open circles: calculated signal. The shift of the contrast envelope provides a measurement of the Earth's rotation rate.

Use the momentum of light and the internal states of atoms to make beam splitters and mirrors
https://en.wikipedia.org/wiki/Atom_interferometer

Can use it as a really accurate gyroscope
Precision rotation measurements with an atom interferometer gyroscope
T.L. Gustavson et al, Phys. Rev. Lett **78**, 2046 (1997).
https://journals.aps.org/prl/abstract/10.1103/PhysRevLett.78.2046
Rotation rate accuracy 20 part per billion in 1 sec of measurement
(measured rotation of earth: 44 μrad/s)

Also accurately measure gravity!

Question #3

The velocity of the wave can be found from

(a) $v = d\omega / dk$.

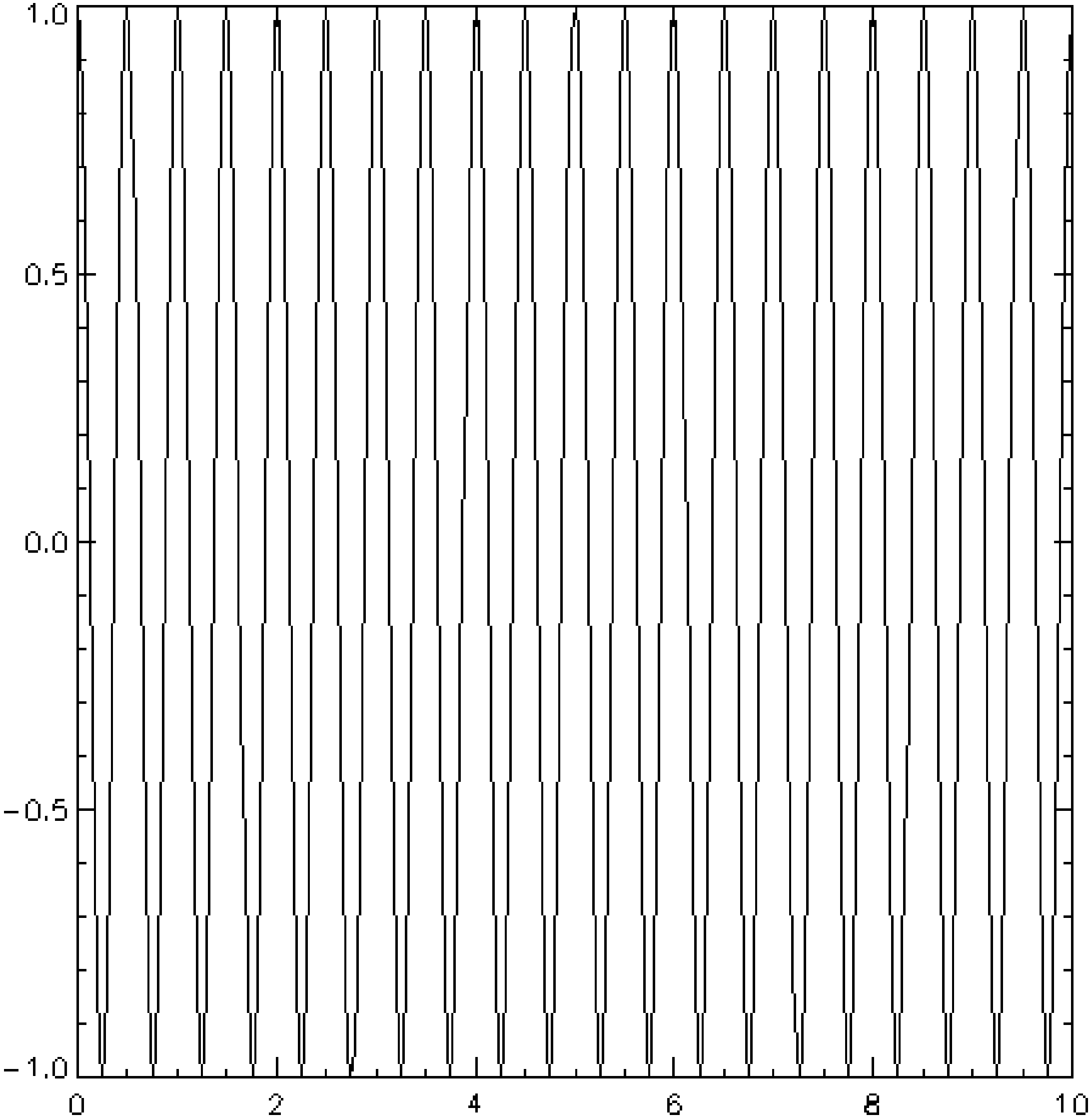
(b) $v = \omega / k$.

(c) $v = dk / d\omega$.

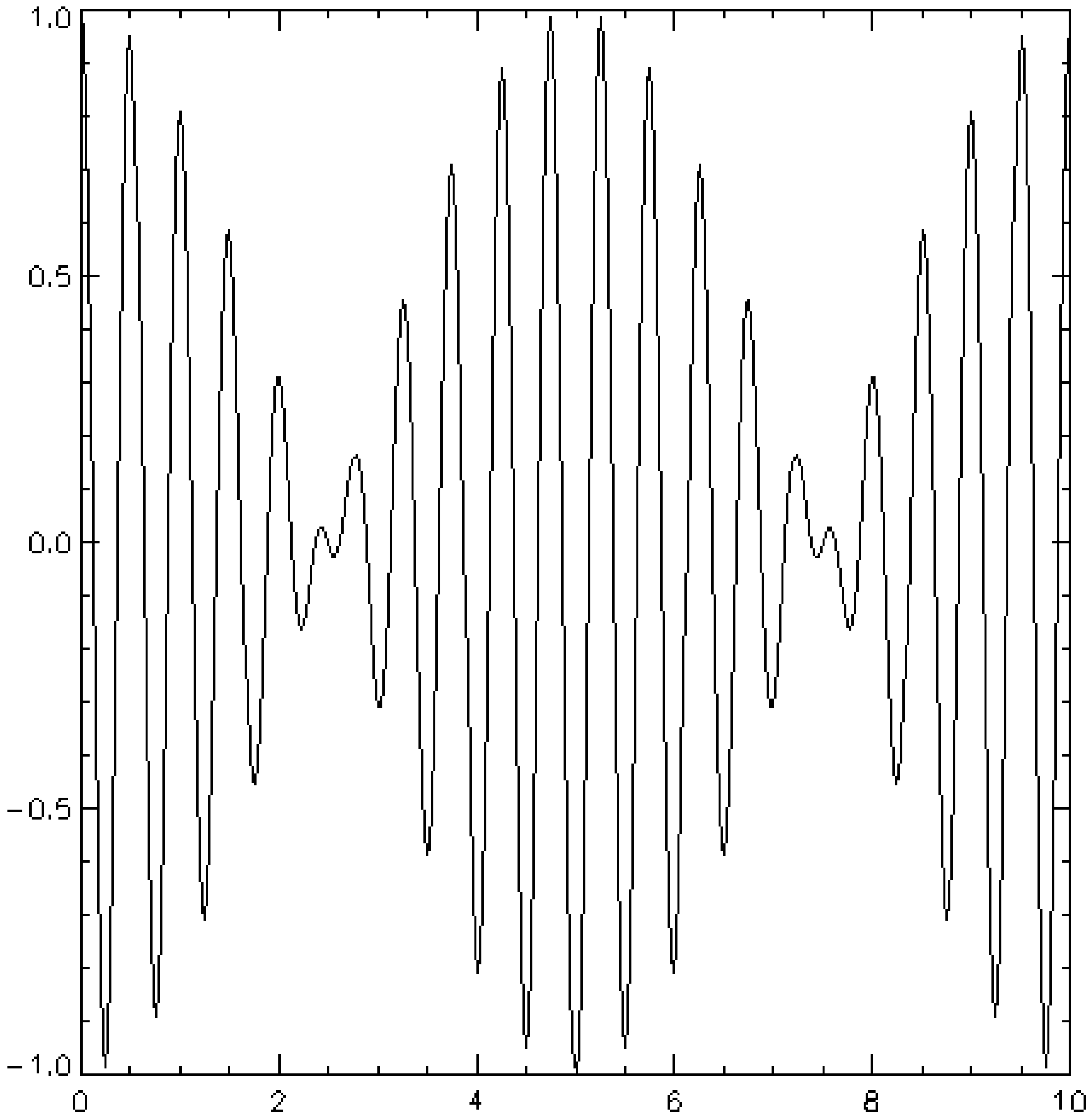
(d) $v = k / \omega$.

(e) Either (a) or (b) depending on the situation

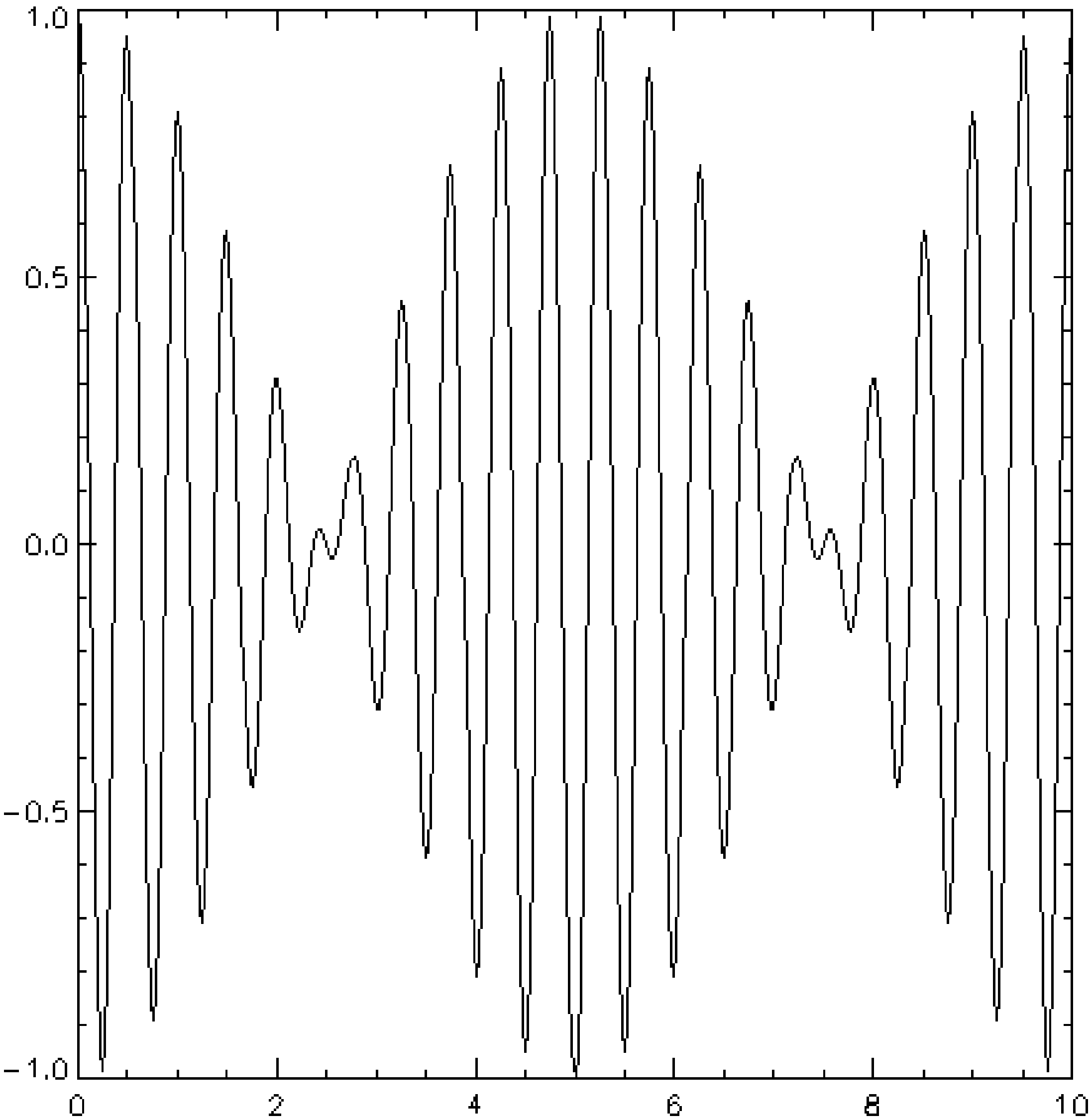
cos(kx – ωt) (phase velocity)



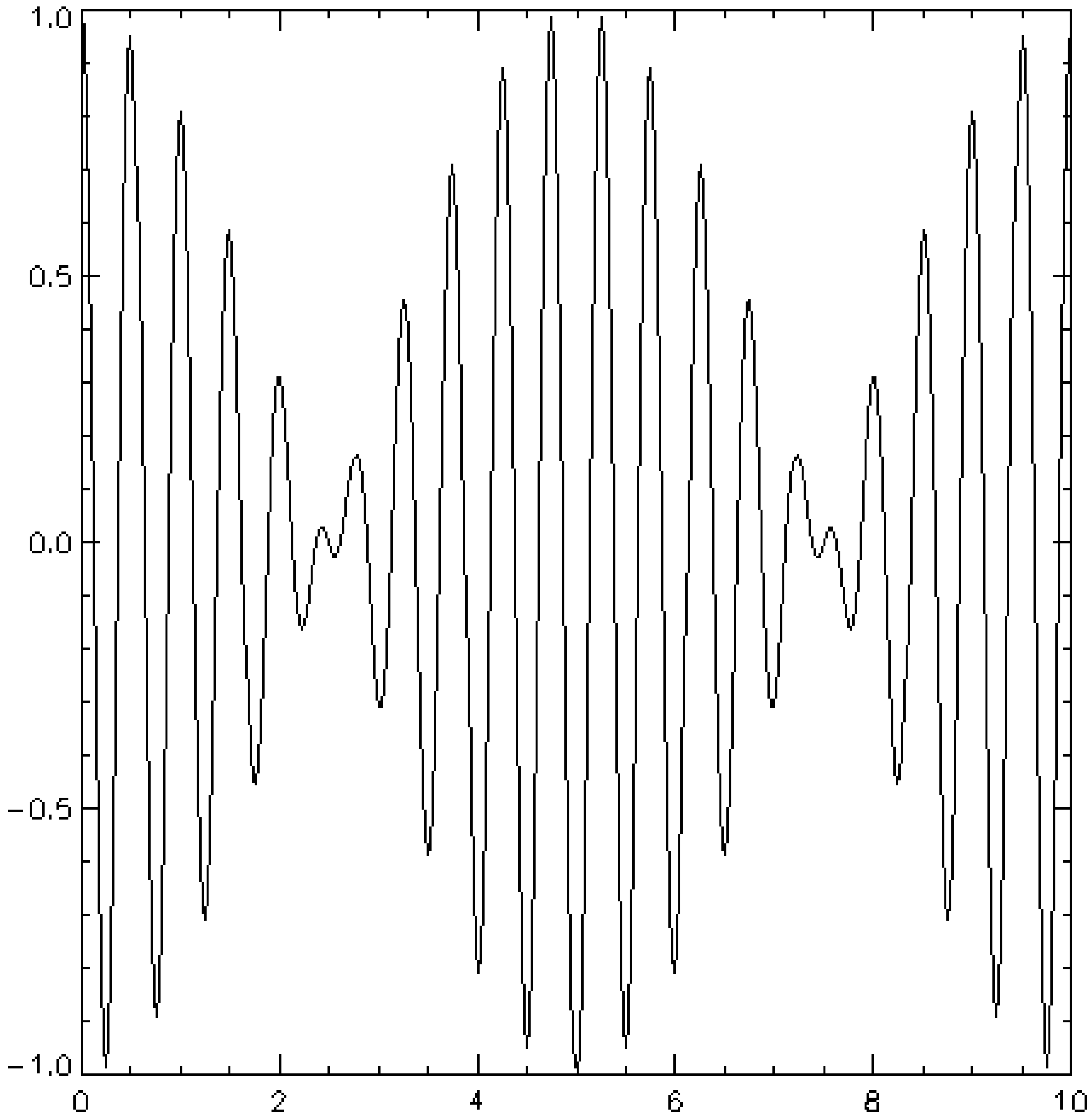
Group velocity = Phase velocity



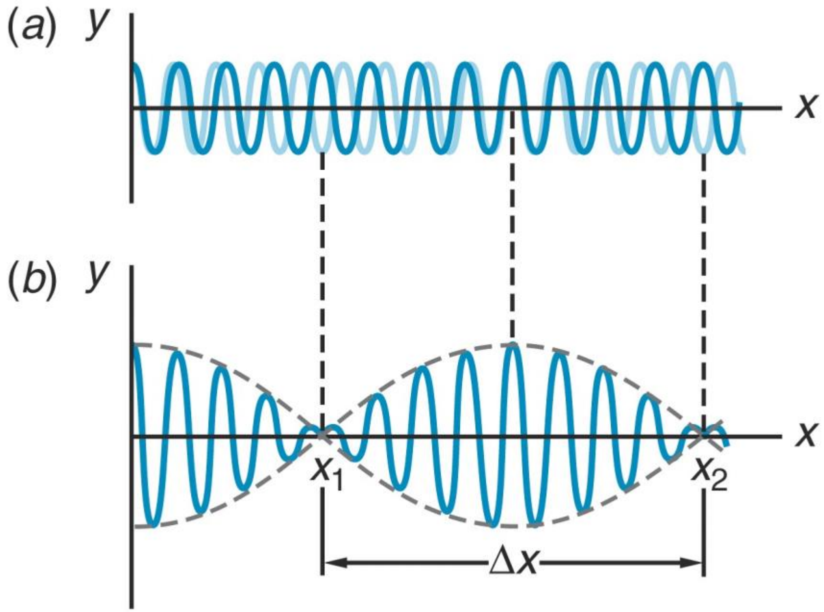
Group velocity > Phase velocity



Group velocity < Phase velocity



Case 2: Adding 2 waves



What is relation between Δx & Δk?

$\Delta x \, \Delta k = 2\pi$ General ⇨ $\Delta x \, \Delta k \sim 1$

The wave has width in time. What is relation between Δt & $\Delta\omega$?

$\Delta t \, \Delta\omega = 2\pi$ General ⇨ $\Delta t \, \Delta\omega \sim 1$

Question #4

You make a wave by adding simple waves (t=0) with the form $\cos[(k+\Delta k*j)x]$, $j = 0,1,2,\ldots j_{max}$. As you increase the number of waves that you add, the width of the wave in x

(a) increases linearly with j_{max}.

(b) increases quadratically with j_{max}.

(c) decreases proportional to $1 / j_{max}$.

(d) decreases quadratically with to $1 / j_{max}$.

(e) This is a trick question; there is no relationship between j_{max} and the width of the wave in x.

Adding Many Waves

Make a wave at (t=0) by adding

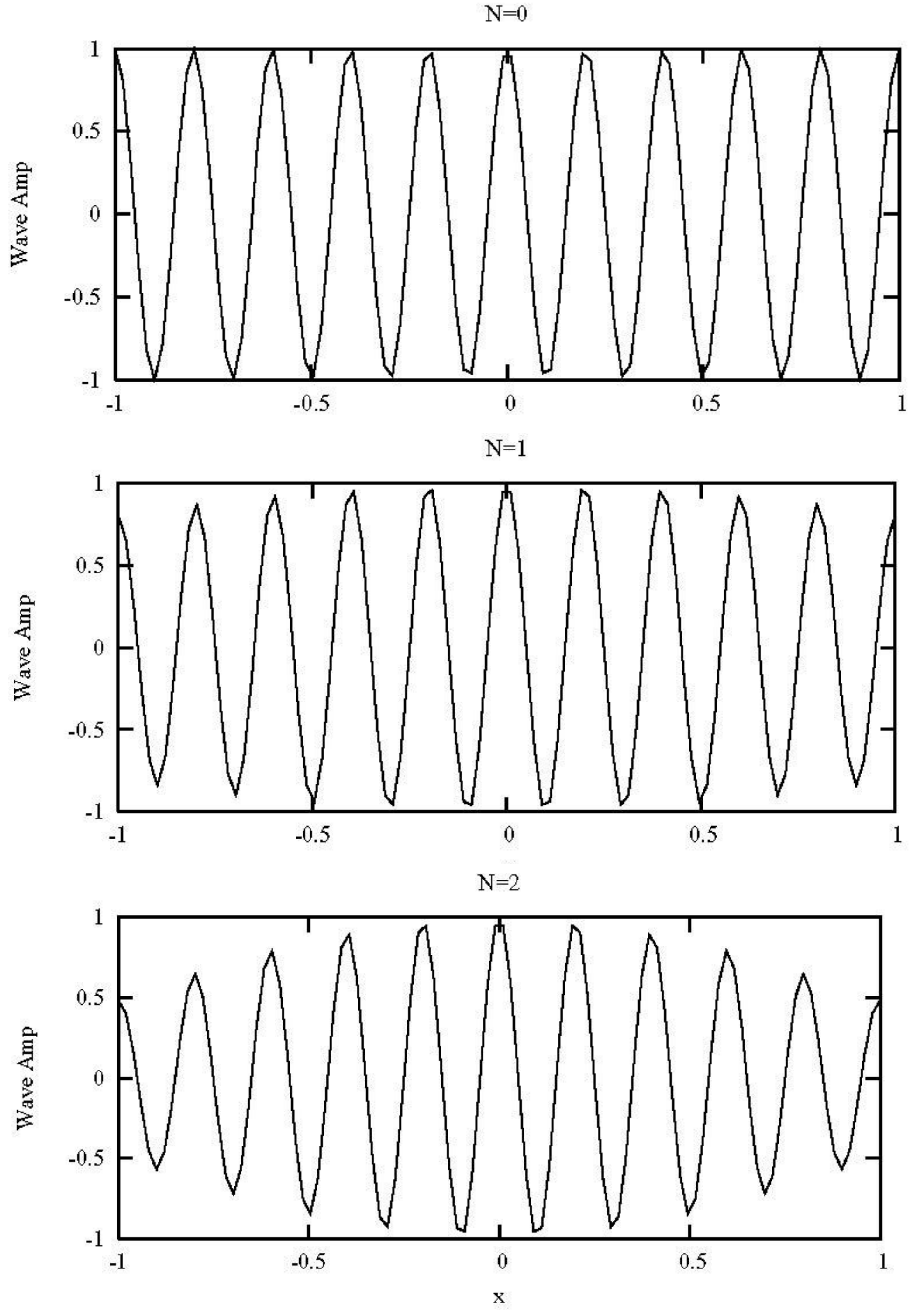
$\{\cos[k\,x] + \cos[(k+\Delta k)x] + \cos[(k-\Delta k)x] + \cos[(k+2\Delta k)x] + \cos[(k-2\Delta k)x] + \ldots\}/N$

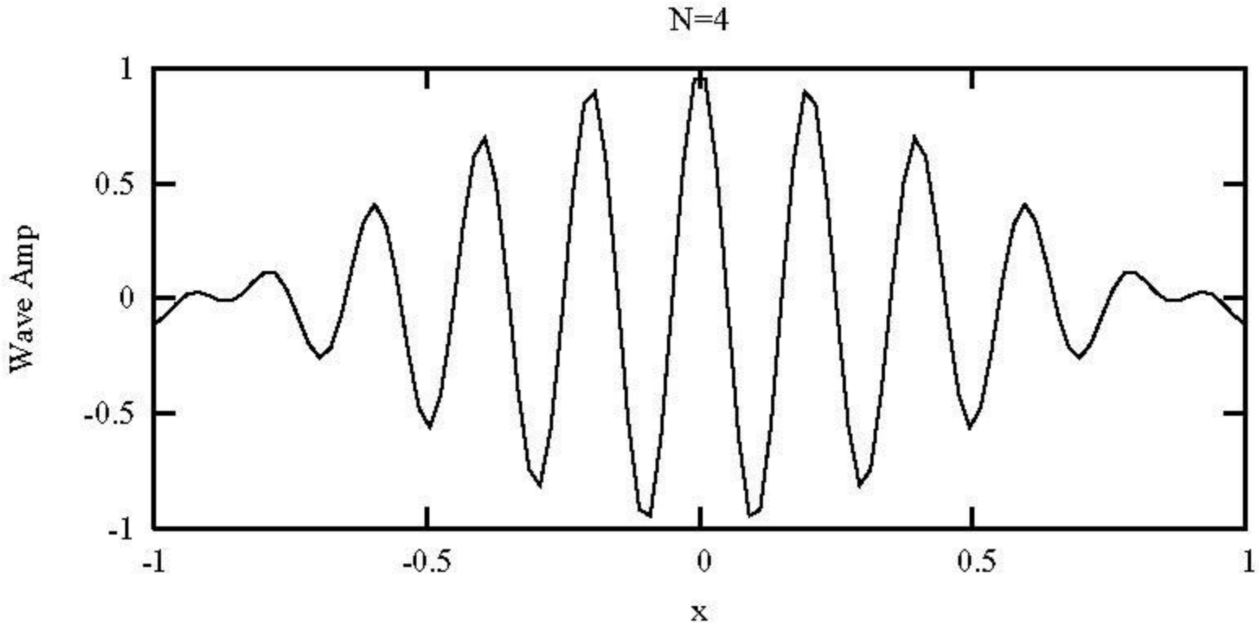
What will this look like as number of terms increase?

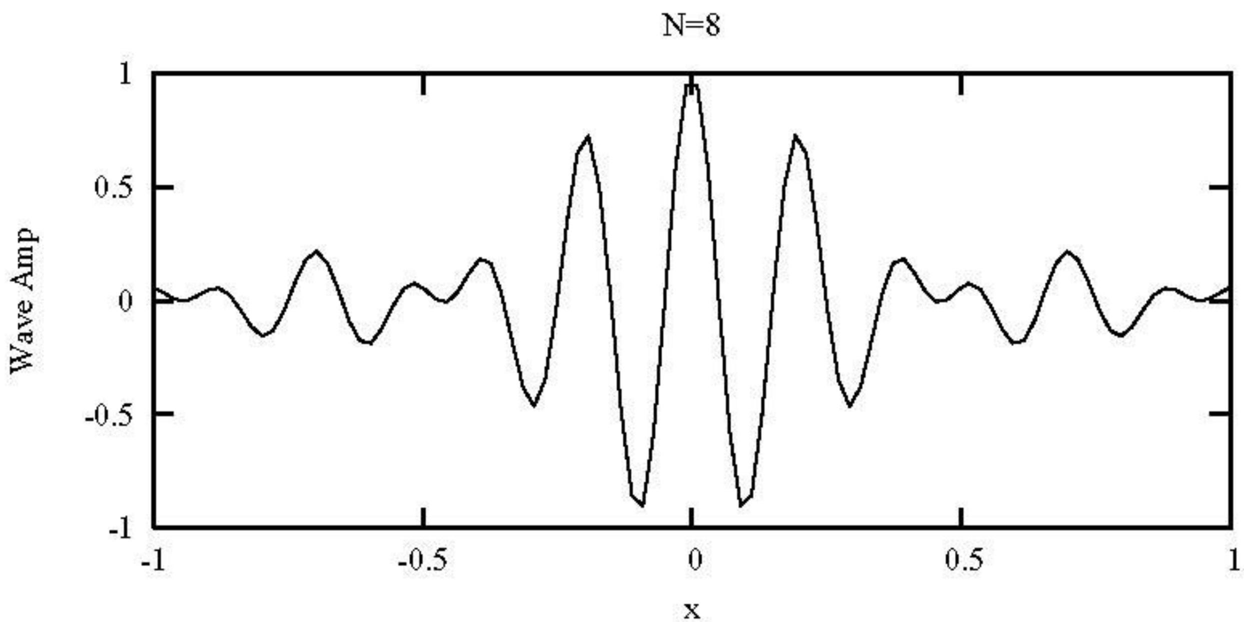
Specific calculation with $k = 10\,\pi$ and $\Delta k = \pi/4$

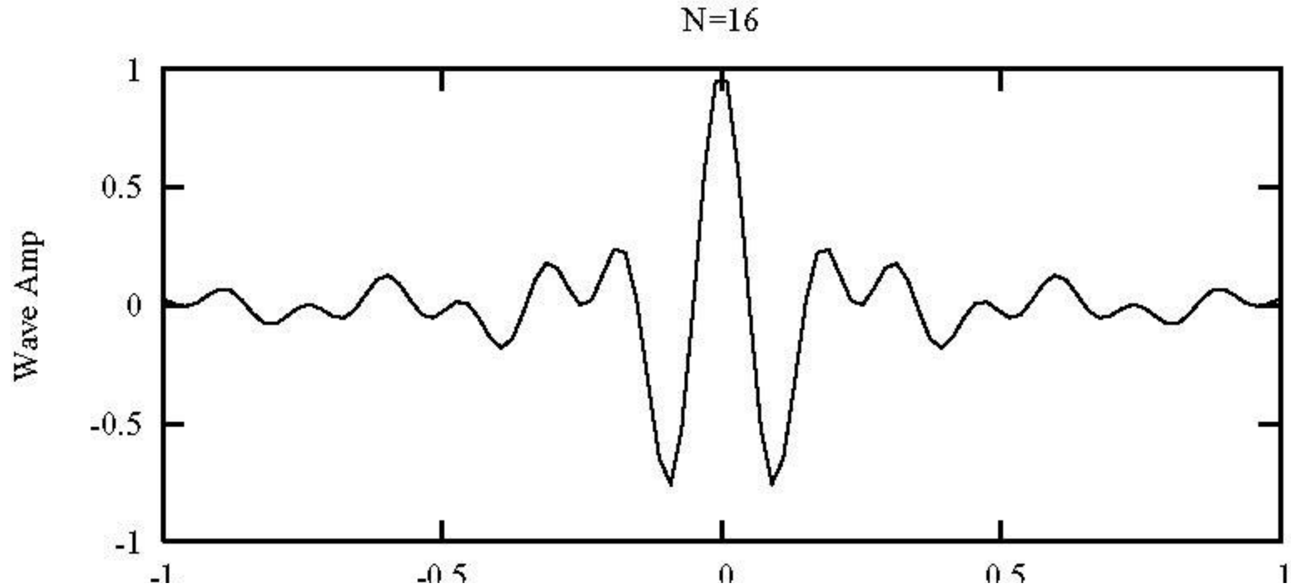
Not important but can get simple expression for sum

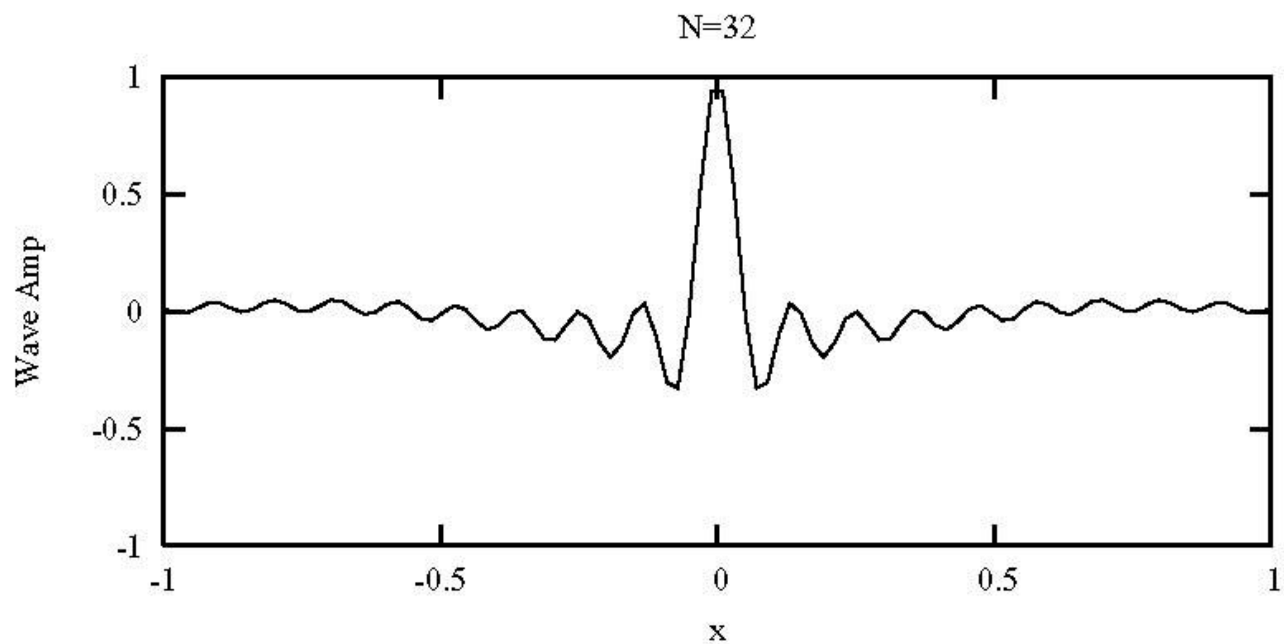
$\text{Sum} = \cos[kx]\,\sin[(2N+1)\,\Delta k\,x/2]/\{(2N+1)\,\sin[\Delta k\,x/2]\}$


N=0
Wave Amp
1
0.5
0
-0.5
-1
-1
-0.5
0
0.5
1
N=1
Wave Amp
N=2
Wave Amp
x


N=4
Wave Amp
x
1
0.5
0
-0.5
-1
-1
-0.5
0
0.5
1


N=8
Wave Amp
x
1
0.5
0
-0.5
-1
-1
-0.5
0
0.5
1


N=16
Wave Amp
1
0.5
0
-0.5
-1
-1
-0.5
0
0.5
1


N=32
Wave Amp
1
0.5
0
-0.5
-1
-1
-0.5
0
0.5
1
x

Case 3: Quantum Waves ($v \ll c$)

Use the relationships for de Broglie waves ($\lambda = h/p$ and $f = E/h$) to get connection between k & ω.

What is relation between p and k?

$k = 2\,\pi\,/\,\lambda = 2\,\pi\,/\,(h/p) = 2\,\pi\,p\,/\,h = p\,/\,\hbar$ ⇨ $\mathbf{p = \hbar\, k}$

What is the relation between E & ω?

$\omega = 2\,\pi\,f = 2\,\pi\,E\,/\,h = E\,/\,\hbar$ ⇨ $\mathbf{E = \hbar\,\omega}$

What is the relationship between k and ω?

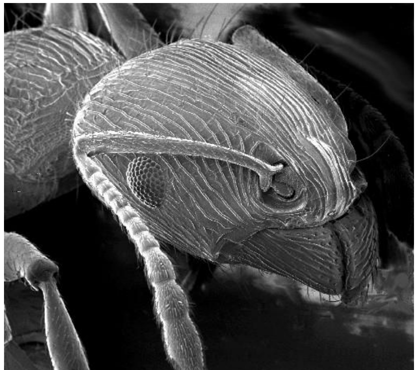
$E = p^2\,/2M$ ⇨ $\hbar\omega = (\hbar\,k)^2\,/\,2M$ ⇨ $\omega = \hbar\,k^2\,/\,2M$

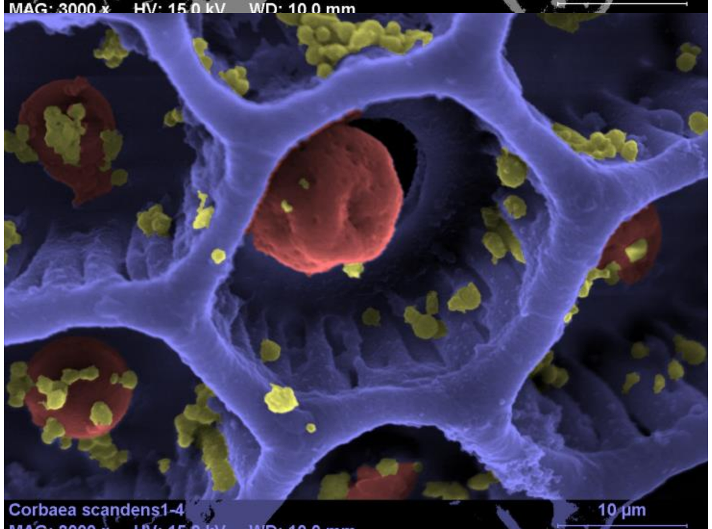
Determine the phase and group velocity.

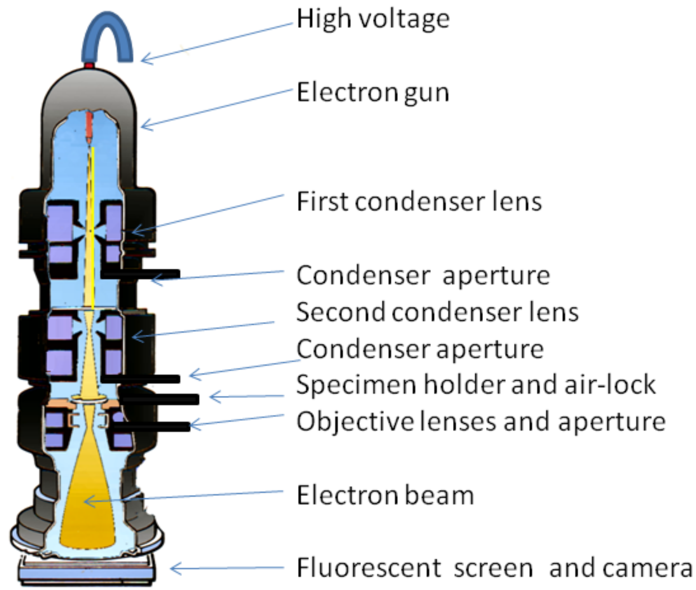
$v_p = \omega/k = \hbar\,k\,/\,2M = p\,/\,2\,M$

$v_g = d\omega/dk = \hbar\,k\,/\,M = p\,/\,M$

Electron Microscope







Transmission Electron Microscope

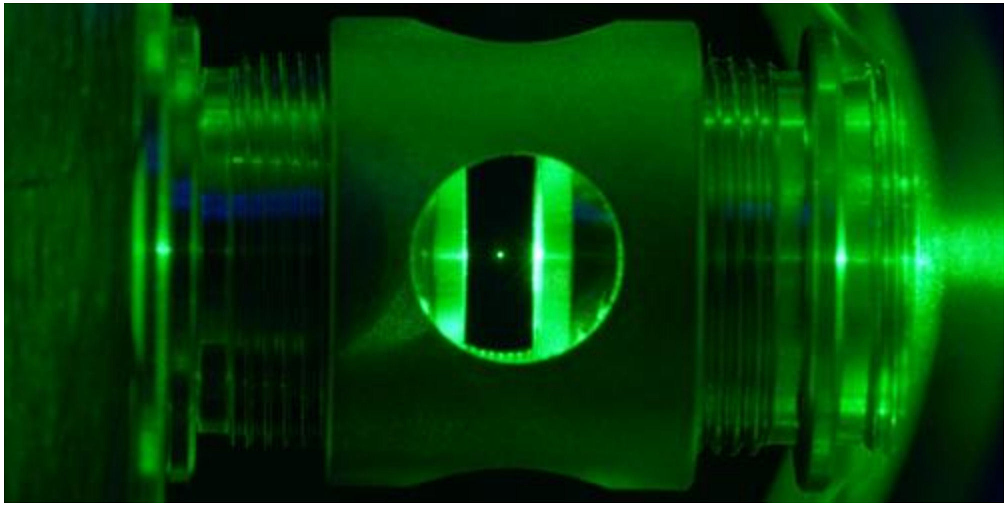
Accelerate electrons to decrease l

Resolution better than 50 pm

Magnification up to 10 millionX

Colorize by using other info

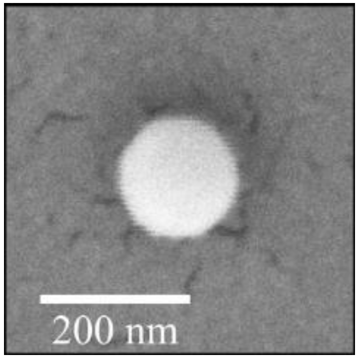
Quantum Nanoparticles



Use a laser to levitate a nanoparticle.

Nothing material touching it

Silica sphere



Cooling of a levitated nanoparticle to the motional quantum ground state, U. Delic et al, Science 367, 892 (2020). https://www.science.org/doi/10.1126/science.aba3993

Quantum control of a nanoparticle optically levitated in cryogenic free space, F. Tebbenjohanns, et al, Nature 595, 378 (2021). https://www.nature.com/articles/s41586-021-03617-w