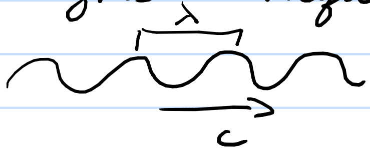
Chapter 3 - The Particle-like Properties of Light

By the end of the 19th century, it was known that light, microwaves, etc were electromagnetic waves at different wave lengths or frequencies. Maxwell, Faraday, Hertz, ...



$f = c/\lambda$ ← frequency, ← wavelength

$c^2 = \frac{1}{\varepsilon_0 \mu_0}$ ← constants in Maxwell eq.

By early 20th century, even new, weird X-rays = electromagnetic waves
Roentgen, Laue, Bragg x 2, ...

For monochromatic light of specific polarization, the electric and magnetic fields are

$\vec{E} = \vec{E}_0 \sin(\vec{k}\cdot\vec{r} - \omega t)$ $\qquad$ $\vec{B} = \vec{B}_0 \sin(\vec{k}\cdot\vec{r} - \omega t)$ (Chap 3 slide 1)

$\vec{k}$ = wave number, direction light is moving $\qquad |\vec{k}| = \frac{2\pi}{\lambda}$

ω = angular frequency $\qquad \omega = 2\pi f$ $\qquad \omega = |\vec{k}|c$

$\vec{E}_0$ = amplitude of electric field, direction of polarization

$\vec{B}_0$ = " " magnetic field

$\vec{k}, \vec{E}_0, \vec{B}_0$ are all perpendicular to each other $\qquad \hat{k} = \hat{E}_0 \times \hat{B}$

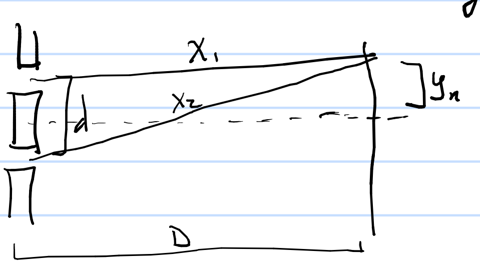
Size of $\vec{B}_0$ related to $\vec{E}_0$ $\qquad |\vec{B}_0| = B_0 = |\vec{E}_0|/c = \frac{E_0}{c}$

The average rate of energy per area (units $\frac{W}{m^2}$ ← watt, ← meter) $\qquad I = \frac{1}{2\mu_0 c}E_0^2 = \frac{1}{2}\varepsilon_0 c E_0^2$

If the polarization is the same then the waves can interfere. Since intensity proportional to E_0^2,

constructive interference → 4x bigger] if same size
destructive " → 0

For two slits with light incident normal (Chap 3 slide 2)



$|x_1 - x_2| = n\lambda$ = constructive $\qquad n = 0, 1, 2, \ldots$

$= (n + \frac{1}{2})$ = destructive $\qquad n = 0, 1, 2, \ldots$

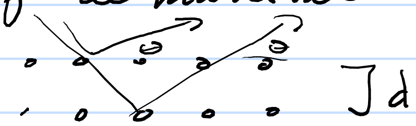
Geometry $x_1 = \sqrt{D^2 + \left(\frac{d}{2} - y_n\right)^2}$ $\qquad x_2 = \sqrt{D^2 + \left(\frac{d}{2} + y_n\right)^2}$ $\qquad D \gg \frac{d}{2}, y_n$

$\qquad\qquad \cong D + \frac{1}{2}\left(\frac{d}{2} - y_n\right)^2/D \qquad \cong D + \frac{1}{2}\left(\frac{d}{2} + y_n\right)^2/D$

$x_2 - x_1 = \frac{d\, y_n}{D}$ $\Rightarrow$ constructive $y_n = n\lambda D/d$ $\quad n = 0, \pm 1, \pm 2, \ldots$

destructive $y_n = (n + \frac{1}{2})\lambda D/d$ $\quad n = 0, \pm 1, \pm 2, \ldots$

For short wave length, the regular array of atoms in crystals gives constructive interference in spots that reflect the properties of the material



Constructive $|x_1 - x_2| = n\lambda$ (Chap 3 slide 3)

Geometry $= 2d\sin\theta$

X-ray diffraction patterns often used to measure properties of materials (Chap 3 slide 4)

Despite extensive evidence that light is a wave, the beginning of the 20th century gave many examples of particle like behavior. (Chap 3 slide 5)

Photon - particle of light $\quad E = hf = \frac{hc}{\lambda}$

Planck's constant

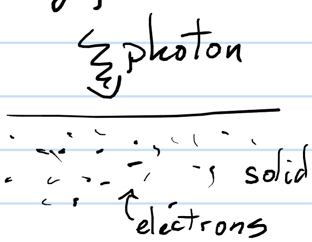
$h = 6.626\,070\,15 \times 10^{-34}$ J s

exact why?

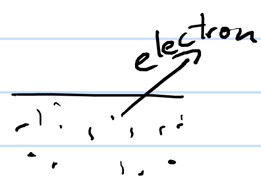
The photoelectric effect is an exercise in conservation of energy

photon "absorbed" = disappears

Before



after



$K_{max} = hf - \phi$

max KE outside $\quad$ photon energy $\quad$ work function

In actual experiment, used a stopping potential V_s to determine K_{max}

$K_{max} = eV_s$

(Chap 3 slides 6-9)

In the history of physics, the thermal behavior of light was important. How do you know light is at a temperature T? Objects at temperature T radiate "light" and can absorb light. Eventually goes to same temperature. (Chap 3 slides 10 - 12)

Black body → all light that hits it is absorbed.

Estimate Earth's temperature treating Sun and Earth as black bodies

$T_{sun} \sim 6000K$

$$\text{Rate emit} = \sigma T_{sun}^4 \, 4\pi R_{sun}^2 = 5.67\times10^{-8} \frac{W}{m^2K^4} (6\times10^3 K)^4 \, 4\pi (7\times10^8 m)^2 = 4.5\times10^{26} W$$

$$\text{Fraction hit earth} = \frac{\pi R_{Earth}^2}{4\pi R_{orbit}^2} = \frac{1}{4} \frac{(6.4\times10^6 m)^2}{(1.5\times10^{11} m)^2} = 4.6\times10^{-10}$$

$$\text{Rate Earth absorb} = 4.5\times10^{26} W \; 4.6\times10^{-10} = 2.1\times10^{17} W$$

$$\text{Rate Earth emit} = \sigma T_{Earth}^4 \, 4\pi R_{Earth}^2 = 2.4\times10^7 \frac{W}{K^4} T_{Earth}^4$$

$$T_{earth} = \left(\frac{2.1\times10^{17} W}{2.4\times10^7 W/K^4}\right)^{1/4} = \boxed{306 K}$$ actual $14°C + 273 = 287 K$

10% error

Can derive the distribution of light energy for thermal distribution using the photon idea. For a given $\vec{k}$ and polarization direction

average energy $\bar{E} = \frac{hf}{e^{hf/k_BT} - 1}$

Boltzman's const $k_B = 1.38\times10^{-23} J/K$

The energy between λ and $\lambda + d\lambda$, x and $x + dx$, ...

$$E = u(\lambda)\, d\lambda\, dx\, dy\, dz \qquad u(\lambda) = \frac{8\pi hc}{\lambda^5} \frac{1}{e^{\frac{hc}{k_BT\lambda}} - 1}$$

Intensity between λ and $\lambda + d\lambda = I(\lambda) d\lambda$ $\quad I(\lambda) = u(\lambda)\frac{c}{4}$

(Chap 3 slide 15)

In 1915, Einstein added the last piece to the photon idea

Relativity $\quad E^2 = p^2c^2 + (m_0c^2)^2 \qquad m_0 = ?$

$E = pc$ if $m_0 = 0$

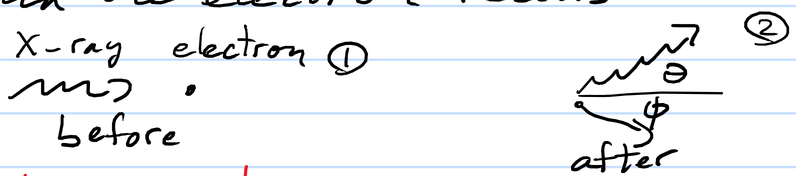
For photon $E = hf = hc/\lambda$ $\qquad p = E/c = h/\lambda$

For visible light (say) $\lambda = 600 \times 10^{-9}\,m$ $\qquad p = \dfrac{6.626 \times 10^{-34}\,J\,s}{600 \times 10^{-9}\,m} = 1.1 \times 10^{-27}\,kg\,m/s$

Classical small thing $1\,mg\ 1\,m/s = 10^{-6}\,kg\ 1\,m/s = 10^{-6}\,kg\,m/s$

For X-ray (say) $\lambda = 10^{-10}\,m$ $\qquad p = \dfrac{6.626 \times 10^{-34}\,J\,s}{10^{-10}\,m} \cong 6.6 \times 10^{-24}\,kg\,m/s$

Compton effect: when an X-ray scatters off an initially stationary electron, the X-ray changes direction and energy and the electron recoils



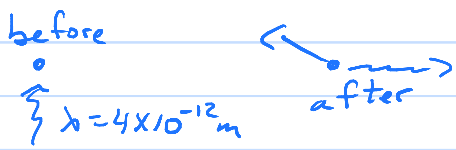
(Chap 3 slides 16–19)

$\lambda_2 = \lambda_1 + \frac{h}{mc}(1 - \cos\theta)$ $\qquad E_{e,2} - E_{e,1} = E_{p,1} - E_{p,2}$

Book $\lambda' = \lambda + \frac{h}{mc}(1 - \cos\theta)$ $\qquad K = E - E'$

prime = after

Problem



$\lambda_{after} = ?$
$E_{e,2} = ?$
$\vec{P}_{e,2} = ?$

$\lambda = 4 \times 10^{-12}\,m + 2.426 \times 10^{-12}\,m\,(1 - \cos 90°)$
$= 6.426 \times 10^{-12}\,m$

$mc^2 = 9.109 \times 10^{-31}\,kg\,(2.998 \times 10^{8}\,m/s)^2 = 8.187 \times 10^{-14}\,J$

$E_{p,1} = 6.626 \times 10^{-34}\,J\,s\ 2.998 \times 10^{8}\,m/s \,/\, 4 \times 10^{-12}\,m = 4.966 \times 10^{-14}\,J$

$E_{p,2} =$ " " $/\, 6.426 \times 10^{-12}\,m = 3.091 \times 10^{-14}\,J$

$E_{e,2} = mc^2 + E_{p,1} - E_{p,2} = 10.06 \times 10^{-14}\,J$

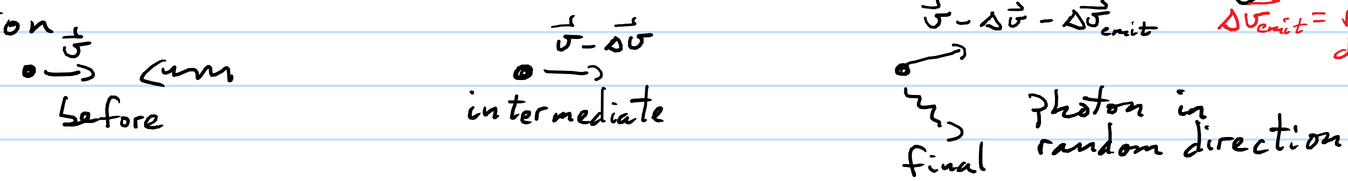
$P_{e,2y} = P_{p,1y} = 6.626 \times 10^{-34}\,J\,s \,/\, 4 \times 10^{-12}\,m = 1.657 \times 10^{-22}\,kg\,m/s$

$P_{e,2x} = -P_{p,2x} = -6.626 \times 10^{-34}\,J\,s \,/\, 6.426 \times 10^{-12}\,m = -1.031 \times 10^{-22}\,kg\,m/s$

Variety of interesting things can be done with photon properties and atomic spectra. (Chap 3 slides 20, 21)

Blackbody spectrum of photons plus spectra of molecules has implications for the Greenhouse effect. (Chap 3 slide 22)

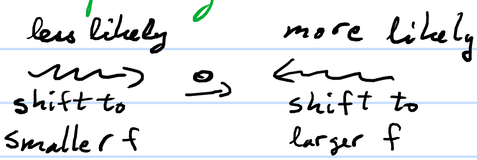
Can use lasers to cool atoms. This is possible because of the specific absorption wave lengths and the momentum of photon



$\Delta\vec{v}_{emit}$ = random direction

The change in energy of atom is <u>much</u> less (less than 10^{-5} ratio)
You can ignore the change in wave length of the photon for calculating the effect on the atom. (Chap 3 slides 23, 24)

If you don't know the direction of the atom, how to know what direction to have laser oppose the atom velocity?
All directions, tune the laser to slightly low frequency, let the Doppler shift pick the correct direction: optical molasses.
For simplicity think about 1D



(Chap 3 slides 25, 26)

Wave vs. Particle picture ((Chap 3 slide 27)
The current understanding is that each photon interferes with itself. In some sense, each photon goes through both slits. (Chap 3 slide 28)

A variety of experiments test this idea: "quantum eraser"
Chap 3 slides 29, 30

There's a wide variety of quantum eraser experiments with the same basic idea. Set up interference by having two or more paths. Label one of the paths somehow. Interference goes away. Somehow undo the label. Interference returns