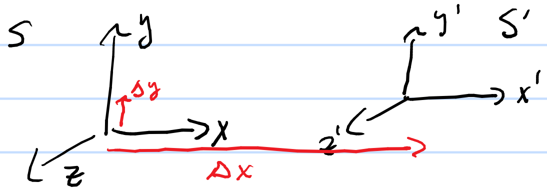
Chapter 2 - The Special Theory of Relativity

Assumptions about how space/time behave imply conservation laws.
Laws of Physics don't depend on:
Position in space ⟹ conservation of momentum
Point in time ⟹ conservation of energy
Direction ⟹ conservation of angular momentum
Emmy Noether ~1915 Noether's Theorem

For quantitative description of where/when things happen, you need to define coordinate system. (Chap 2 slides 1-3)



Galileo
$x' = x - \Delta x - u_x t - a_x t^2/2$
$y' = y - \Delta y$
$z' = z - \Delta z$
$t' = t$ ⟵ note

u_x, a_x = velocity and acceleration of S' relative to S

What is the velocity in S' if velocity in S is $\vec{u}$?
$v_x' = v_x - u_x - a_x t$; $v_y' = v_y$; $v_z' = v_z$

Inertial frame $\vec{a} = 0$

Galilean relativity is a good approximation if things don't move very fast and gravity not that large. (Chap 2 slides 4,5)

Special relativity
1) The laws of physics are the same in all inertial frames
(Noether: can't tell that you have constant velocity!)
2) The laws of Electricity + magnetism are correct ⟹ speed of light
$c = 299{,}792{,}458$ m/s exactly how can we measure exactly??

Several important implications
1) Nothing physical (or info) faster than c.
2) Simultaneity of spatially separated events can be uncertain.
3) Duration depends on who measures
4) Size depends on who measures

Suppose two things happen x_1, t_1 and x_2, t_2

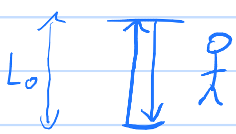
If $|x_2 - x_1| < c\,|t_2 - t_1|$ everyone agrees: not simultaneous

If $|x_2 - x_1| > c\,|t_2 - t_1|$ some say 1 earlier, some say 2 earlier, some say at same time (Chap 2 slide 6)

This implies disagree about whether separated clocks are synchronized (chap 2 slide 7)

The clocks in S and S' progress differently!!

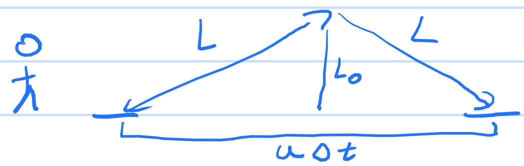
Time Dilation (chap 2 slide 8,9)



$\Delta t_0 = \frac{2L_0}{c}$

Clock not moving with respect to you

$\gamma = 1/\sqrt{1 - u^2/c^2}$



clock moving speed u

$\Delta t = \frac{2L}{c} = \frac{2}{c}\sqrt{L_0^2 + \frac{1}{4}u^2\Delta t^2}$

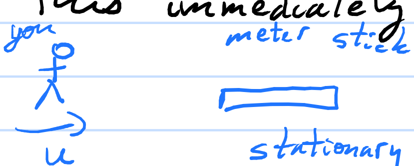
solve for Δt

$\Delta t = \frac{2L_0}{c} / \sqrt{1 - u^2/c^2}$

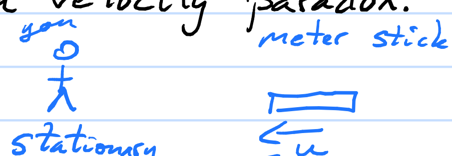
$= \Delta t_0 / \sqrt{1 - u^2/c^2} = \gamma \Delta t_0$

Chap 2 slide 10,11

This immediately leads to a velocity paradox.



implies



$u = \frac{1\,m}{\Delta t} = \frac{1\,m\,\sqrt{1 - u^2/c^2}}{\Delta t_0}$

$u \overset{?}{=} \frac{1\,m}{\Delta t_0} \neq \frac{1\,m}{\Delta t}$

Lengths contract in the direction of motion

$L = L_0\sqrt{1 - u^2/c^2} = L_0/\gamma$ Chap 2 slide 12, 13, 14

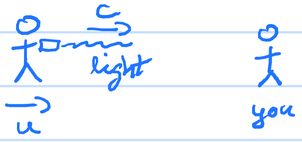
Lorentz Transformation: You can use the time dilation and length contraction to derive the position, time variables in the inertial frame S' from that of S

$x' = \gamma(x - ut)$ $y' = y$ $z' = z$ $t' = \gamma(t - xu/c^2)$

Velocity Transformation Chap 2 slide 15, 16

$v_x' = \Delta x'/\Delta t' = \gamma(\Delta x - u\Delta t)/[\gamma(\Delta t - \Delta x\, u/c^2)] = (v_x - u)/(1 - \frac{v_x u}{c^2})$

$v_x = (v_x' + u)/(1 + \frac{v_x' u}{c^2})$



$$v_x = \frac{u+c}{1+\frac{u}{c}} = \underline{c}$$

light measured by you

$$v_x = \frac{\frac{3}{4}c+\frac{3}{4}c}{1+\frac{\frac{3}{4}c\,\frac{3}{4}c}{c^2}} = \underline{0.96\,c}$$

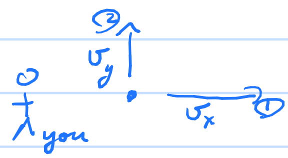
measured by you

What if velocity not in the same direction as $\vec{u}$?

$v_x' = (v_x - u_x)/(1 - v_x u_x/c^2)$ $\quad v_y' = \frac{1}{\gamma}\, v_y/(1-\frac{u_x v_x}{c^2})$ $\quad$ similar $y \to z$

$v_x = (v_x' + u_x)/(1 + \frac{v_x' u_x}{c^2})$ $\quad v_y = \frac{1}{\gamma}\, v_y'/(1+\frac{u_x v_x'}{c^2})$

Chap 2 slide 17



This is a hard problem

Ship 1 stationary $\quad$ Equation: $u_x = -v_x$, $v_x' = 0$, $v_y' = v_y$

$v_{x1} = -v_x$, $\quad v_{y1} = \sqrt{1 - v_x^2/c^2}\; v_y$

Ship 2 stationary $\quad u_y = -v_y$ $\quad$ No equations! $\quad$ Swap y & x

$v_{y2} = -v_y$ $\quad v_{x2} = \sqrt{1 - v_y^2/c^2}\; v_x$

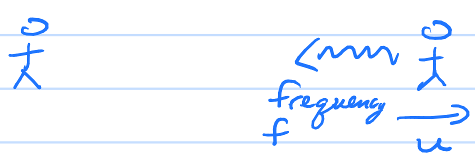
Is something messed up? The velocities aren't opposite each other?

The speeds are the same

$$v_{x1}^2 + v_{y1}^2 = v_x^2 + v_y^2 - v_y^2 v_x^2/c^2 = v_{x2}^2 + v_{y2}^2$$

You can show that the relative speed is always less than c

The frequency of light depends on the velocity of the observer relative to the emitter



$$f' = f\sqrt{\frac{1-u/c}{1+u/c}}$$

What is fractional change in frequency if $u = 1\,m/s$? $\quad \frac{f'-f}{f} \sim 3.3\times10^{-9}$

Chap 2 slide 18

Red shift can measure that whole galaxies are moving away from us. chap 2 slide 19

$$1+z = f/f' = \sqrt{(1+u/c)/(1-u/c)}$$

If you know z, you can find u

Expansion of universe and gravity affects z

If you use the usual definition of momentum, $\vec{p} = m\vec{v}$, you can show that momentum conservation depends on frame. Chap 2 slides 20, 21

Fix by defining

$$\vec{p} = \frac{m\vec{v}}{\sqrt{1 - v^2/c^2}}$$ ← not γ which is the factor between frames

Can solve for $\vec{v}$ in terms of $\vec{p}$

$$\frac{\vec{v}}{c} = \frac{\vec{p}}{mc} \Big/ \sqrt{1 + (p^2/m^2c^2)}$$

Show momentum conserved for example Chap 2, slide 21

Before $$p_x = -2mV/(1+v^2/c^2) \big/ \left[1 - 4v^2/(c^2(1+v^2/c^2)^2)\right]^{1/2}$$
$$= -2mV \big/ \left[(1+v^2/c^2)^2 - \frac{4v^2}{c^2}\right]^{1/2} = -2mv/(1 - v^2/c^2)$$

same

After $$p_x = -2mv \big/ \left[1 - [v^2 + v^2(1 - v^2/c^2)]/c^2\right]^{1/2}$$
$$= -2mv/(1 - 2v^2/c^2 + v^4/c^4)^{1/2} = -2mv/(1 - v^2/c^2)$$

Now do the relativistic kinetic energy

$$K = \int F\,dx = \int \frac{dp}{dt}\,dx = \int_0^{p_f} v\,dp = \int_0^{p_f} \frac{1}{m}\frac{p}{\sqrt{1 + p^2/m^2c^2}}\,dp = mc^2\sqrt{1 + \frac{p^2}{m^2c^2}}\,\Bigg|_0^{p}$$

$$= mc^2\sqrt{1 + p^2/m^2c^2} - mc^2 = \frac{mc^2}{\sqrt{1 - v^2/c^2}} - mc^2 = E - E(v=0) = E - E_0$$

The Kinetic energy is the energy because of motion.

The rest energy

$E_0 = mc^2$

The total energy $E = \dfrac{mc^2}{\sqrt{1 - v^2/c^2}}$

Don't get tied up in direction $E_0 = mc^2 \Rightarrow m = E_0/c^2$

Mass ^{4}He nucleus 6.646477×10^{-27} kg $- 2 \cdot 9.109 \times 10^{-31}$ kg $= 6.644655 \times 10^{-27}$ kg

Mass 2 proton + 2 neutron $2(1.672622 \times 10^{-27} + 1.674927 \times 10^{-27})$ kg $= 6.695098 \times 10^{-27}$ kg

$\Delta M = -5.04 \times 10^{-29}$ kg $\Rightarrow$ $\Delta E = -4.54 \times 10^{-12}$ J $= -28.2$ MeV

Chap 2 slides 22, 23, 24

There is a Lorentz transformation of $\vec{p}, E$

$p_x' = \gamma\left(p_x - \frac{u}{c}\frac{E}{c}\right)$ $\quad p_y' = p_y$ $\quad \frac{E'}{c} = \gamma\left(\frac{E}{c} - \frac{u}{c} p_x\right)$

$p_x = \gamma\left(p_x' + \frac{u}{c} E'/c\right)$ $\quad p_y = p_y'$ $\quad E/c = \gamma\left(\frac{E'}{c} + \frac{u}{c} p_x'\right)$

Substituting can show $E^2 = (\vec{p}c)^2 + (mc^2)^2$

Since the rest mass doesn't depend on the frame

$(mc^2)^2 = E^2 - (\vec{p}c)^2 = (E')^2 - (\vec{p}\,'c)^2$

Suppose the rest mass is 0 $\quad E = |\vec{p}|c$

Problem Chap 2 slide 25

$\frac{dp}{dt} = F \Rightarrow p = Ft \qquad v = \frac{p}{m}\frac{1}{\sqrt{1+\frac{p^2}{m^2c^2}}} = \frac{Ft/m}{\sqrt{1+\left(\frac{Ft}{mc}\right)^2}}$

$x = \int_0^t v(t')\,dt' = \frac{mc^2}{F}\int_0^t \frac{Ft'/mc}{\sqrt{1+\left(\frac{Ft'}{mc}\right)^2}}\, d\left(\frac{Ft'}{mc}\right) = \frac{mc^2}{F}\left[\sqrt{1+\left(\frac{Ft}{mc}\right)^2} - 1\right]$

$\frac{v}{c} = 0.9 = \frac{Ft/mc}{\sqrt{1+\left(\frac{Ft}{mc}\right)^2}} \Rightarrow t = \frac{mc}{F}\sqrt{\frac{0.81}{0.19}} = \frac{9.11\times10^{-31}\,kg\; 3\times10^8\,m/s}{1.6\times10^{-13}\,N}\,2.06$

$= 3.53\times10^{-9}\,s$

$x = \frac{9.11\times10^{-31}\,kg\,(3\times10^8\,m/s)^2}{1.6\times10^{-13}\,N}\left[\sqrt{1+\frac{0.81}{0.19}} - 1\right]$

$= 0.66\,m$

Nonrelativistic $\quad v = Ft/m \Rightarrow t = 0.9\,c\,m/F = 1.5\times10^{-9}\,s$

$x = \frac{1}{2}at^2 = \frac{1}{2}v_f \cdot t = \frac{1}{2}\,0.9\,c \cdot 1.5\times10^{-9}\,s = 0.2\,m$

When objects are accelerating, they behave as if their mass increases as $v \to c$ $\quad m(v) = m_0 / \sqrt{1 - v^2/c^2}$