Chapter 15 - General Relativity and Cosmology

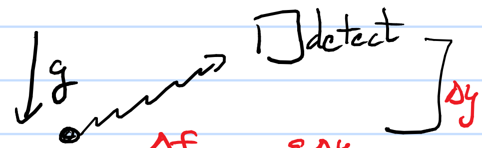
Einstein realized implications for gravity from ideas in relativity. The resulting equations have been tested many times. Too complicated for details, but interesting!!

Local accelerating frame is the same as uniform gravity: Equivalence principle (Chap 15 slide)

If true, gravitational mass exactly equals inertial mass. Experimental test (Eöt-Wash $< 10^{-13}$)

Weird implication Chap 15 slide 2, 3

For small gravity and height changes



$$\frac{\Delta f}{f} = -\frac{g \Delta y}{c^2}$$

What does this imply for clocks??

For small gravity but large distances

$$\frac{\Delta f}{f} = \frac{-GM}{Rc^2}$$

GPS satellites have period of 12 hours. Effect from GR and special relativity

$$\frac{m v^2}{R} = \frac{G M_E m}{R^2} \qquad \frac{2\pi R}{v} = 12\,\text{hr}\; 60\,\frac{\text{min}}{\text{hr}}\; 60\,\frac{\text{sec}}{\text{min}} \Rightarrow \frac{R}{v} = 6875\ \text{sec}$$

$$R = \left[G M_E \frac{R^2}{v^2}\right]^{1/3} = \left[6.67\times10^{-11}\,\frac{m^3}{kg\,s^2}\; 5.97\times10^{24}\,kg\,\left(6.88\times10^{3}\,s\right)^2\right]^{1/3} = 2.66\times10^{7}\,m$$

$$-\frac{6.67\times10^{-11}\; 5.97\times10^{24}}{2.66\times10^{7}\;(3\times10^{8})^2} = -1.66\times10^{-10}$$

$$v = \frac{2\pi\; 2.66\times10^{7}\,m}{4.32\times10^{4}\,s} = 3.87\times10^{3}\,m/s \qquad \frac{1}{2}\left(\frac{v}{c}\right)^2 = \frac{1}{2}\,1.66\times10^{-10}$$

partially compensates

In 1 Day, error $\sim \frac{1}{2}\, 1.66\times10^{-10} \cdot 24 \cdot 3600\,s \sim 7\times10^{-6}\,s \sim 7\,\mu s$

Although tiny this is important $7\,\mu s \cdot 3\times10^{8}\,m/s \sim 2\,km$

Atomic Clocks $\frac{\Delta f}{f} \sim \frac{0.1\,m\; 9.8\,m/s^2}{(3\times10^{8}\,m/s)^2} \sim 1\times10^{-17}$ (Jun Ye group Colorado)

Gravity can bend the path of light Chap 15 slide 4
The path bends toward the mass as if attracted (even though light doesn't have mass).

Proper description in general relativity – mass curves time and space. This becomes important when gravity is strong.

As stars run out of fuel, only the stiffness of matter can prevent collapse. Planets don't collapse because too small. High enough mass ⇒ electrons and protons become neutrons plus neutrinos. The neutrinos have little mass so escape.

Neutron star – example $R \sim 10^4 m$, $M \sim 1.6 M_\odot \sim 3.2 \times 10^{30} kg$
$\Delta f/f \sim -GM/Rc^2 = 6.67\times10^{-11}\ 3.2\times10^{30}/(10^4\ [3\times10^8]^2) = 0.24$

Pulsars are neutron stars with magnetic axis not aligned with rotation axis ⇒ "lighthouse" effect.

Even more massive, matter collapses into even smaller R object! Black hole – nothing (not even light) can escape
Schwarzschild radius
$r_s = 2GM/c^2$
The theoretical limit ~ 50 billion solar masses
Many measured at roughly this size (or larger?).

Sagittarius A* (the black hole at the center of Milky Way) is about 4.3 million solar masses.
$r_s = 2 \cdot 6.67\times10^{-11} (4.3\times10^6\ 2\times10^{30})/(3\times10^8)^2\ m = 1.3\times10^{10} m$
(smaller than Mercury orbit ~ $5-7\times10^{10} m$) Chap 15 slide 5

Taylor and Hulse measured binary pulsar losing orbital energy?
Gravitational waves!! Chap 15 slides 6,7

LIGO 36 solar mass $r_s = 1.1\times10^5 m = 110\ km$ (solar $R = 7\times10^5 km$)
misuse Kepler $R = (GMT^2/4\pi^2)^{1/3} = (6.67\times10^{-11}\ 36\times2\times10^{30} (0.01)^2/4\pi^2)^{1/3}\ m$
$= 2.3\times10^5 m$

General relativity can be solved for universe. It gives dynamic changes to space. All points tend to move away or move together. Experimentally seems to move away. Chap 15 slide 8

Velocity of galaxy away from us vs distance

$v = H_0 d$ $H_0 = 72$ km/s /Mpc 1 Mpc = 3.3×10^6 light-year (size Milky Way ~ 0.027 Mpc)

This suggests a starting time for the universe!
Current estimate 13.79 ± 0.02 billion years

Not just motion of galaxies! Cosmic Microwave Background.
Hot → short wave length light
Expansion means short → long
First measured by Penzias + Wilson 1965. Several satellite based measurements extended and improved

One curious feature: the rate of separation seems to be accelerating instead of slowing down. No one knows why.
Dark Energy

Chap 15 slide 9
The speed that objects orbit a (much bigger) mass, M, only depends on M and the distance between, R, and fundamental constants. It does not depend on the mass of the object.

Kepler $T = \frac{2\pi}{\sqrt{GM}} R^{3/2}$ $\frac{2\pi R}{T} = v = \sqrt{\frac{GM}{R}}$ (↑ speed)

We thought we knew where most matter is because they emit light, microwaves, x-rays.... Rotation of galaxies say we are missing a lot of mass or Gravitational law wrong
chap 15 slide 10

About 10 other reasons chap 15 slide 11 (not colliding galaxies have lensing where visible matter, colliding galaxies have lensing not with matter)
Dark Matter If it exists, no one knows what it is! Missing fundamental particle? WIMP Clump? small black hole, planet, ...