Chapter 12 Nuclear Structure + Radioactivity

The type of nuclei is specified as $^A_Z X_N$ or $^A X$

$^{14}_{8}C_6$ or ^{14}C $\quad$ $^4_2He_2$ or ^4He $\quad$ $^{235}_{92}U_{143}$ or ^{235}U

The density of nucleons is roughly constant (Chap 12 slide 1)

$$\frac{A}{\frac{4\pi}{3}R^3} \simeq \text{const} \rightarrow R \cong R_0 A^{1/3} \qquad R_0 \simeq 1.2 \times 10^{-15}\,m \pm 0.2 \times 10^{-15}\,m$$

Combining protons and neutrons into a nucleus can give off a lot of energy (Chap 12 slide 2)

$M_p = 1.672622 \times 10^{-27}$ kg $\qquad M_n = 1.674927 \times 10^{-27}$

$M_p + M_n = 3.347549 \times 10^{-27}$ kg $\qquad M_d = 3.343584 \times 10^{-27}$ kg

$$(M_p + m_n - m_D)c^2 = 3.965 \times 10^{-30}\,\text{kg}\,(3 \times 10^8\,m/s)^2 = 3.57 \times 10^{-13}\,J = 2.23\,MeV$$

This is the amount of energy you get when $p + n \rightarrow d$

In general, the binding energy of the nucleus

$$B = (N m_n + Z m_p - \underbrace{m_X}_{\text{nuclear mass}})c^2$$

Sometimes you have the mass of the atom (Table D)
To a good approximation

$$B \cong (N m_n + Z \underbrace{M(^1_1H_0)}_{\text{hydrogen atom}} - \underbrace{M(^A_Z X_N)}_{\text{atom mass}})c^2$$

Nuclei with the same Z are isotopes $\quad ^{12}_{6}C_6$ and $^{14}_{6}C_8$
" " " " N " isotones $\quad ^2_1H_1$ and $^3_2He_1$
" " " " A " isobars $\quad ^3_1H_2$ and $^3_2He_1$

Isotope interesting because chemistry mostly from Z
Isobar interesting because some nuclear decay $p \rightarrow n$ or $n \rightarrow p$ keeps A the same

The calculation of binding energy from first principles is very difficult but can be done for small nuclei. However, a phenomenologically motivated formula gives pretty decent estimate. (Chap 12 slides 3-6)

$$B = a_v A - a_s A^{2/3} - a_c \frac{Z^2}{A^{1/3}} - a_A \frac{(N-Z)^2}{A} \pm a_5 \quad \text{(Chap 12 slide 7)}$$

volume — surface — Coulomb — asymmetry — pairing

Volume: each nucleon has ~same nearby nucleons, each contribute
Surface: nuclei at surface are missing neighbors
Coulomb: from electric repulsion between nuclei
Asymmetry: from Pauli exclusion princ.
Pairing: nucleons have energy cost when going from even → odd
(see Table of calculated binding energies on web page + chap 12 slide 8)

Can find the Z that gives the strongest binding for a fixed $A \Rightarrow \partial B / \partial Z = 0$

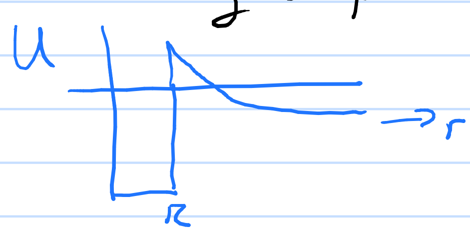
$$Z = A / \left(2 + \frac{a_c}{2 a_A} A^{2/3}\right) = A / (2 + 0.015 A^{2/3})$$

Just because $B > 0$ does <u>not</u> mean that nucleus is stable. For example you might have

$M({}^A_Z X_N) > M({}^{A-4}_{Z-2} X'_{N-2}) + M({}^4_2 He_2)$ decay by emitting α

Could also emit a proton or neutron.

Emitting a proton or α, need to tunnel!



Lower energy α tunnel farther
⇒ longer half-life

$Q = [M({}^A X) - M({}^{A-4} X') - M({}^4 He)] c^2$ is the energy released in the decay.

Decay because of the weak force

$n \rightarrow p + e^- + \bar{\nu}_e$ — very little mass   Return to neutrino below

$Q = (m_n - m_p - m_e) c^2$

For nuclei overly rich in neutrons $\;{}^{A}_{Z}X_N \rightarrow {}^{A}_{Z+1}X'_{N-1} + e^- + \bar{\nu}_e$ (nuc. → nuc)

" " " " protons can convert one of the electrons and protons $\;{}^{A}_{Z}X_N \rightarrow {}^{A}_{Z-1}X'_{N+1} + \nu_e$ (atom → atom) or emit positron $\;{}^{40}_{19}K \rightarrow {}^{40}_{18}Ar + e^+ + \nu_e$

Nuclei that decay behaves like spontaneous photon emission from atoms. All quantum systems with continuous final energy decay the same way: exponential decrease in population and Lorentzian distribution of final energies.

$N(t)$ = # of radioactive nuclei at time t

$$\frac{dN(t)}{dt} = -\lambda N(t) \implies N(t) = N(0)\, e^{-\lambda t} = N_0\, e^{-\lambda t}$$

Activity $a(t) = \lambda N(t) = a_0\, e^{-\lambda t}$

λ = decay constant = 1/(mean lifetime) $= \dfrac{\ln(2)}{t_{1/2}} = \dfrac{0.693}{t_{1/2}}$ ($t_{1/2}$: half life)

(Chap 12 slides 9-11)

Radioactivity can be used to date various rocks, fossils, ... Basic idea: 1) (example Uranium-Lead dating) know that it starts with 100% ${}^{A}X$ and 0% ${}^{A'}X'$ but radioactivity gives $(1-f)$ ${}^{A}X$ and f ${}^{A'}X'$ 2) (example ${}^{14}C$ dating) method to estimate initial fractions

Some basics of nuclear energy production (Chap 12 slides 12-15)