On to the Planck scale

- SOME new physics should exist at the TeV scale, to explain why the Higgs is light, i.e., why the EWSB happens at ~ 246 GeV and not at a much higher energy scale (shorter distance scale).
- Whatever it is (SUSY, technicolor, …), that new physics may demand newer physics at a somewhat higher energy scale, in order to regularize its own short-distance divergences. And so on…
- It may be turtles all the way down to the Planck mass…
- There is still an enormous gap (the “desert”) between the 1000 GeV scale and the GUT scale @ $10^{16}$ GeV, or the Planck scale at $10^{19}$ GeV when
  - gravity becomes strong
  - a theory of quantum gravity takes over
  - shorter distances (higher energy scales) become meaningless due to strong quantum fluctuations in space-time itself
- Is that desert populated by new physics (laws, particles)?
But is the Planck scale really that high?

- Maybe the Planck mass scale is much lower (e.g., the weak scale ~ 1 TeV), and the Planck length much bigger, because $G$ is much larger.

$$L_P = \sqrt{\frac{G\hbar}{c^3}} \quad M_P = \sqrt{\frac{\hbar c}{G}} = \frac{\hbar}{cL_P}$$

- This can happen if gravity actually lives in more than 3 spatial directions.

- In string theory, open strings (which can represent all the SM particles, including photons, fermions, gauge bosons, Higgs…) must remain attached to a three-spatial-dimensionhyperplane on the boundary of a d-dimensional (d>3) “bulk” space. Since gravity is curved space, it sees, and acts in, the bulk, while the SM particles live only on the hypersurface and don’t see the bulk.

- So the possibility exists that gravity lives in d (>3) dimensions, making the “shortest” distance and “highest” energy much closer to the weak scale.
Extra dimensions

Primary sources:

Thomas G. Rizzo, *SLAC Summer Institute (SSI04)*,
http://www.slac.stanford.edu/econf/C040802/papers/L013.PDF

Greg Landsberg, *SLAC Summer Institute (SSI04)*,
http://www.slac.stanford.edu/econf/C040802/lec_notes/Landsberg/default.htm

Why extra dimensions?

• Come on, if there were extra dimensions, we would have noticed!
• Unless they are “compactified”. Origins in the work of Kaluza and Klein, in the 1920’s.
• But what do they for us? Extra dimensions at the TeV scale can address some of the deepest problems in modern physics:
  – addressing the hierarchy problem (ADD, 1999; Randall-Sundrum 1999)
  – producing electroweak symmetry breaking without a Higgs boson
  – the generation of the ordinary fermion and neutrino mass hierarchy, the CKM matrix and new sources of CP violation
  – TeV scale grand unification or unification without SUSY while suppressing proton decay
  – new Dark Matter candidates and a new cosmological perspective
  – black hole production at future colliders as a window on quantum gravity (Giddings et al, 2002)…
• And, thinking about extra dimensions is fun, and almost always lead to surprising and unanticipated results.
Compact dimensions

• If there are more than 3 spatial dimensions (+ time), the additional ones must be too small for us to see, and/or inaccessible to SM particles such as the photon (ie, only gravity, the graviton)
• Compact dimensions can be curled up on themselves (in GR, space is curved), finite in extent, obeying Dirichlet boundary conditions: any field \( \psi \) satisfies \( \psi(x^4) = \psi(x^4+2\pi R) \) where \( R \) is the radius and \( L_C = 2 \pi R \) is the extent of the “4\text{th} spatial dimension”.
• A particle travelling in that dimension would return to where it started in a very short time.
• At each point in ordinary 3-space \( (x^1, x^2, x^3) \) there would be a curled-up circle, or 2-ball, or (d-3)-dimensional-ball.
Kaluza-Klein theory

- Kaluza-Klein (KK) theory (1926) attempts to unify gravity and electromagnetism (the only known forces at the time) by including one extra dimension.

- The theory actually works at some level, but in the end it failed to give a realistic theory, including matter particles.

- Quantizing non-relativistic motion in one extra compact dimension (square well, bead on ring) yields standing waves with quantized energies $E_n = (2\pi\hbar)^2/2mL_C^2$ in a “Kaluza-Klein (KK) tower”.

- In a relativistic theory, $m_n c^2 = (2\pi\hbar c/L_C)n$ is an equally-spaced tower of excitations, including a lowest, massless one.

- The smaller the space, the larger the mass of the KK excitations.

The scalar “radion” field $\phi$ plays the role of a Higgs, giving mass to the KK tower of $A_\mu$. The lowest state remains massless and preserves gauge invariance. In theories that include gravity, one radion field $\phi$ remains, describing fluctuations in the size of the extra dimension.
E fields in different dimensions

- Gauss’ law in 3-D …

\[ \nabla \cdot \vec{E} = \rho \]
\[ \int_{V^3} dV \nabla \cdot \vec{E} = \int_{V^3} dV \rho \]
\[ \int_{S^2} dS \cdot \vec{E} = q \]
\[ 4\pi \, r^2 \, E(r) = q \]
\[ E(r) = \frac{q}{4\pi \, r^2} \]

and d-spatial dimensions

\[ \text{area}(S^{d-1}) \, E(r) = q \]
\[ E(r) = \frac{\Gamma(d/2)}{2\pi^{d/2}} \, \frac{q}{r^{d-1}} \]
Surfaces and volumes in d spatial dimensions

“volume” of a ball of radius R in d dimensions

\[
vol(V^d) = \frac{\pi^{d/2}}{\Gamma(1 + d/2)} r^d
\]

\[
vol(V^2) = \pi r^2
\]

\[
vol(V^3) = \frac{4}{3} \pi r^3
\]

\[
vol(V^4) = \frac{1}{2} \pi^2 r^4
\]

“area” of surface of that ball of radius R in d dimensions

\[
area(S^{d-1}) = \frac{2\pi^{d/2}}{\Gamma(d/2)} r^{d-1}
\]

\[
area(S^1) = 2\pi r
\]

\[
area(S^2) = 4\pi r^2
\]

\[
area(S^3) = 2\pi^2 r^3
\]
What is $M_P$ in $d$ dimensions?

- Easiest to work with the gravitational potential, analogous to the electric scalar potential.
- Gravitational potential of a mass distribution has units of Energy/mass, independent of dimensionality of space.
- It satisfies a Poisson Equation, valid in any number of spatial dimensions.
- LHS always has units of energy/mass/length$^2$.
- RHS has units of $G \times$ mass/length$^d$.
- Therefore, $G$ has units of energy/mass$^2 \times$ length$^{d-2}$, different units in different spatial dimensions.
- Planck mass and Planck length will be different as well.

\[ \nabla \cdot \vec{E} = -\rho \]
\[ \vec{E} = -\nabla \varphi \Rightarrow \nabla^2 \varphi = -\rho \]
\[ \nabla^2 \varphi_G = 4\pi G \rho_m \]
\[ \nabla^2 \varphi_G^{(d)} = 4\pi G^{(d)} \rho_m \]
The Planck scale in d dimensions

• The dimensionality of G changes: \( G^{(d)} \sim (L)^{(d-3)} G_N \)

• The Planck scale is defined uniquely by forming combinations of G, \( \hbar \), c with dimensions of \( m_P, L_P, t_P \).

• So the Planck scale will also change in d dimensions.

• Can we bring \( M_P^{(d)} \) much closer to the weak scale of \( \sim 1 \) TeV by invoking compact dimensions?

\[
\nabla^2 \varphi_G^{(d)} = 4\pi G^{(d)} \rho_m
\]
\[
\rho \sim M / L^d
\]
\[
G^{(d)} \frac{M}{L^d} \sim G_N \frac{M}{L^3}
\]
\[
G^{(d)} \sim G_N L^{d-3}
\]
\[
L_P = \sqrt{\frac{G \hbar}{c^3}}
\]
\[
(L_P^{(d)})^{d-1} = (L_P)^2 \frac{G^{(d)}}{G_N}
\]
\[
M_P = \sqrt{\frac{\hbar c}{G}} = \frac{\hbar}{c L_P}
\]
\[
\left( \frac{c M_P^{(d)}}{\hbar} \right)^{d-1} = \left( \frac{c M_P}{\hbar} \right)^2 \frac{G_N}{G^{(d)}}
\]
Gravity in d spatial dimensions

- In d spatial dimensions, the force laws change their power-law behavior;
- The dimensions of the gravitational constant $G^{(d)}$ change.
- What stays the same? The Poisson equation for the gravitational potential.

$$\nabla^2 \varphi^{(d)}_G = 4\pi G^{(d)} \rho_m$$

- So how does $G^{(d)}$ change in the presence of (d-3) small compact dimensions of length $L_C$?

$$M = 2\pi R \mu = L_C \mu$$
$$\rho^{(3)} = M \delta (x^1) \delta (x^2) \delta (x^3)$$
$$\rho^{(4)} = \mu \delta (x^1) \delta (x^2) \delta (x^3)$$
$$\rho^{(3)} = \int_0^{2\pi R} dx^4 \rho^{(4)}$$
$$M = \int_{-\infty}^{\infty} dx^1 \int_{-\infty}^{\infty} dx^2 \int_{-\infty}^{\infty} dx^3 \int_0^{2\pi R} dx^4 \rho^{(4)}$$
$$\rho^{(4)} = \frac{\rho^{(3)}}{2\pi R}$$

$$\nabla^2 \varphi^{(4)}_G = 4\pi G^{(4)} \rho^{(4)}$$
$$= 4\pi G^{(4)} \frac{\rho^{(3)}}{2\pi R} = 4\pi G^{(3)} \rho^{(3)}$$

$$G^{(4)} = 2\pi R G^{(3)} = L_C G_N$$
$$G^{(d)} = \left( L_C \right)^{d-3} G_N = V_C G_N$$
Can we bring the Planck scale down to, say 2 TeV?

- The “true” Planck length in d spatial dimensions is $L_P^{(d)}$.
- The larger the size of the compact dimensions $L_c$, the larger is $G^{(d)}$ and thus $L_P^{(d)}$, and the smaller is $M_P^{(d)}$, reducing the severity of the hierarchy problem, bringing the scale of quantum gravity closer to the weak scale.
- Can we bring $M_P^{(d)}$ much closer to the weak scale of ~ 1 TeV by invoking compact dimensions?
- Let’s try $M_P^{(d)}c^2 \sim 2$ TeV, or $L_P^{(d)} \sim 10^{-19}$ m.
- With one extra spatial dimension, $L_C = 10^{13}$ m. This does not work!
- With two, $L_C \sim 10^{-3}$ m. Surely we would have noticed that by now!
- Not if photons and fermions are constrained to live in the large 3 spatial dimensions we know and love.
- Only gravity sees these extra dimensions, and the $1/r^2$ law of gravity has not been tested accurately down to such short distances!
Tests of deviations of Grav potential at short distances

\[ V = -G_N \frac{m_1 m_2}{r} \left( 1 + \frac{\alpha}{2} e^{\frac{-r}{\lambda}} \right) \]

\( \lambda \) [meters]

\( M_P^{(5)} \) could be as small as a few TeV in the ADD model!
A Test of Physics Beyond the Standard Model

\[ V = -G_N \frac{m_1 m_2}{r} \left( 1 + \alpha e^{-\frac{r}{\lambda}} \right) \]

Experiment tests many theories:

- **New particles:** Exotic particles with Compton wavelength \( \lambda \sim 0.01 - 1 \) mm predicted in string theories, etc.

- **Extra dimensions:** Proposed hierarchy solution involves large, compact spatial dimensions accessed only by gravitons.
  Radius of compactification \( \sim \lambda < 1 \) mm

- **Cosmological Constant:** Small value of \( \Lambda \) could be stabilized by particles of wavelength \( \lambda \sim 0.1 \) mm
ADD Model:
- "Eliminates" the hierarchy problem by stating that physics ends at a TeV scale
- Only gravity lives in the "bulk" space
- Size of ED's (n=2-7) between ~100 μm and ~1 fm
- Doesn't explain how to make ED large

TeV-1 Scenario:
- Lowers GUT scale by changing the running of the couplings
- Only gauge bosons (g/γ/W/Z) propagate in a single ED; gravity is not in the picture
- Size of the ED ~1 TeV^{-1} or ~10^{-19} m

RS Model:
- A rigorous solution to the hierarchy problem via localization of gravity
- Gravitons (and possibly other particles) propagate in a single ED, w/ special metric
- Size of this ED as small as ~1/M_{Pl} or ~10^{-35} m
Kaluza-Klein excitations

- If the SM fields are not fixed to our (3+1)-brane (as in string theory), but can live in the compact dimensions, they will have KK towers.
- If we haven’t seen their KK excitations, they must be too high in mass for us to see them; the first excitations are > 100 GeV and $L_C < 10^{-18}$ m.
- To get $M_P^{(d)}$ of order ~ 1 TeV, need $d \geq 3+10$ dimensions.
- In any case, the graviton lives in the compact dimensions, and its KK excitations should be of order 100 GeV or less.
- These fields couple to standard model particles with (the larger, d-dim) gravitational coupling strength.
- They will be produced at high energy accelerators. The coupling is weak, but if the KK tower contains many particles with mass < $E_{CM}$, the total cross section may be large enough to observe.
- Except, they interact only weakly, so they won’t register in the particle detectors, except as missing energy/momentum.
Kaluza-Klein Spectrum

ADD Model:
- Winding modes with energy spacing $\sim 1/r$, i.e. $1 \text{ meV} - 100 \text{ MeV}$
- Can’t resolve these modes – they appear as continuous spectrum

TeV$^{-1}$ Scenario:
- Winding modes with nearly equal energy spacing $\sim 1/r$, i.e. $\sim \text{TeV}$
- Can excite individual modes at colliders or look for indirect effects

$$M_i = \sqrt{M_0^2 + i^2/r^2}$$

RS Model:
- “Particle in a box” with a special metric
- Energy eigenvalues are given by zeroes of Bessel function $J_1$
- Light modes might be accessible at colliders

$$M_i = M_0 x_i / x_0 \approx M_0, 1.83 M_0, 2.66 M_0, 3.48 M_0, 4.30 M_0, ...$$
Missing energy signature at LHC

• Signals may be visible above background for “true” Plank mass scales in the 4-8 TeV range, with up to 4 extra compact dimensions.

• At a high-energy e+e- linear collider (ILC), the backgrounds are much smaller, and the beam energy can be varied; one can easily distinguish different models of extra dimensions from each other and from other new physics sources of missing energy (like SUSY).

• Graviton exchange as SM particles scatter at the ILC will also produce unique signatures.