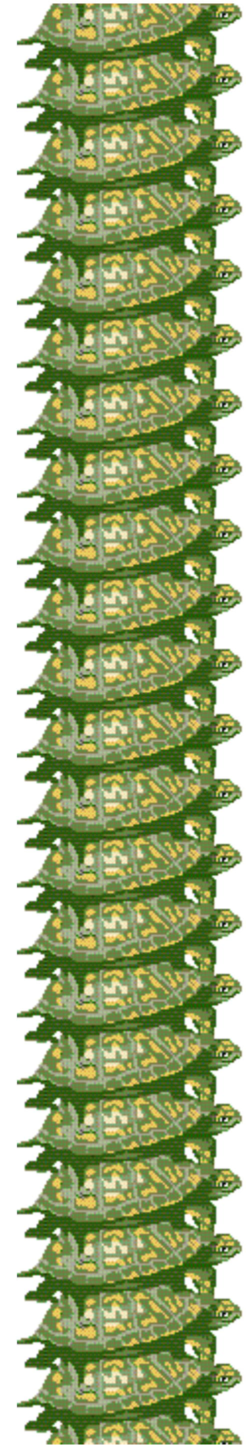
On to the Planck scale

- SOME new physics should exist at the TeV scale, to explain why the Higgs is light, ie, why the EWSB happens at ~ 246 GeV and not at a much higher energy scale (shorter distance scale).
- Whatever it is (SUSY, technicolor, …), that new physics may demand newer physics at a somewhat higher energy scale, in order to regularize its own short-distance divergences. And so on…
- It may be turtles all the way down to the Planck mass…
- There is still an enormous gap (the "desert") between the 1000 GeV scale and the GUT scale @ 10^{16} GeV, or the Planck scale at 10^{19} GeV when
  - gravity becomes strong
  - a theory of quantum gravity takes over
  - shorter distances (higher energy scales) become meaningless due to strong quantum fluctuations in space-time itself
- Is that desert populated by new physics (laws, particles)?



But is the Planck scale really that high?

- Maybe the Planck mass scale is much lower (eg, the weak scale ~ 1 TeV), and the Planck length much bigger, because G is much larger

$$L_P = \sqrt{\frac{G\hbar}{c^3}} \qquad M_P = \sqrt{\frac{\hbar c}{G}} = \frac{\hbar}{cL_P}$$

- This can happen if gravity actually lives in more than 3 spatial directions.
- In string theory, open strings (which can represent all the SM particles, including photons, fermions, gauge bosons, Higgs…) must remain attached to a three-spatial-dimension hyperplane on the boundary of a d-dimensional (d>3) "bulk" space. Since gravity is curved space, it sees, and acts in, the bulk, while the SM particles live only on the hypersurface and don't see the bulk.
- So the possibility exists that gravity lives in d (>3) dimensions, making the "shortest" distance and "highest" energy much closer to the weak scale.

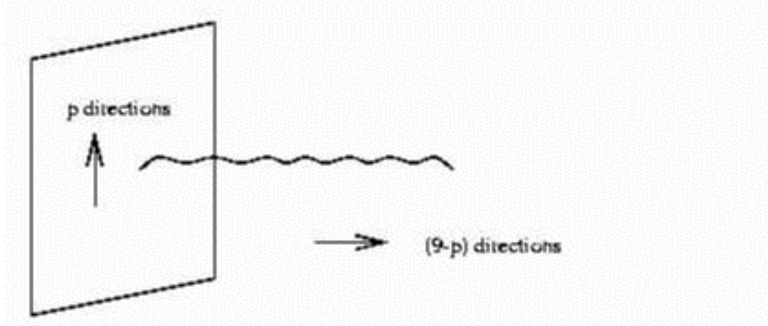


Fig.5: Open string ending on a Dirichlet p-brane.

Extra dimensions

Primary sources:

Thomas G. Rizzo, *SLAC Summer Institute (SSI04),*
http://www.slac.stanford.edu/econf/C040802/papers/L013.PDF

Greg Landsberg, *SLAC Summer Institute (SSI04),*
http://www.slac.stanford.edu/econf/C040802/lec_notes/Landsberg/default.htm

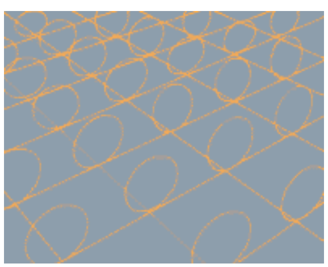
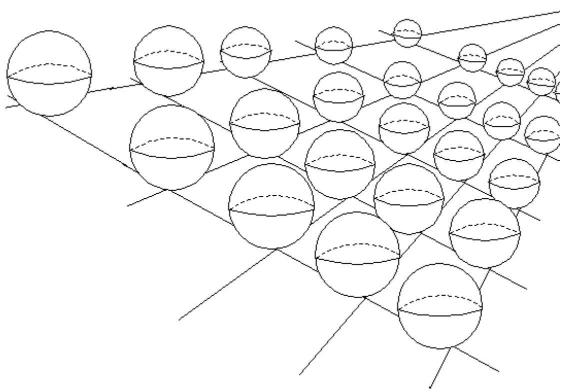
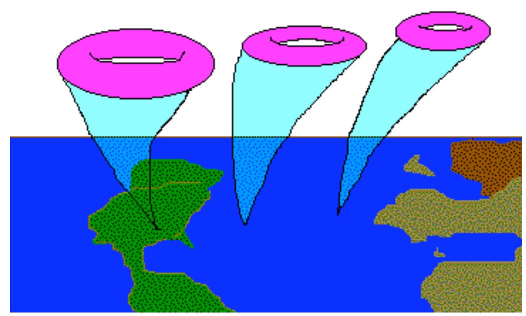
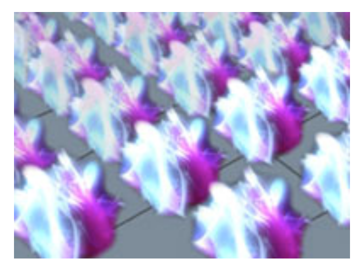
Barton Zweibach, A First Course in String Theory, Cambridge U Press (2004)

Why extra dimensions?

- Come on, if there were extra dimensions, we would have noticed!
- Unless they are "compactified". Origins in the work of Kaluza and Klein, in the 1920's.
- But what do they for us? Extra dimensions at the TeV scale can address some of the deepest problems in modern physics:
  - addressing the hierarchy problem (ADD, 1999; Randall-Sundrum 1999)
  - producing electroweak symmetry breaking without a Higgs boson
  - the generation of the ordinary fermion and neutrino mass hierarchy, the CKM matrix and new sources of CP violation
  - TeV scale grand unification or unification without SUSY while suppressing proton decay
  - new Dark Matter candidates and a new cosmological perspective
  - black hole production at future colliders as a window on quantum gravity (Giddings et al, 2002)…
- And, thinking about extra dimensions is fun, and almost always lead to surprising and unanticipated results.

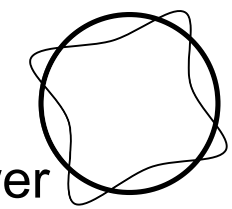
Compact dimensions

- If there are more than 3 spatial dimensions (+ time), the additional ones must be too small for us to see, and/or inaccessible to SM particles such as the photon (ie, only gravity, the graviton)
- Compact dimensions can be curled up on themselves (in GR, space is curved), finite in extent, obeying Dirichlet boundary conditions: any field ψ satisfies $\psi(x^4) = \psi(x^4+2\pi R)$ where R is the radius and $L_C = 2\,\pi R$ is the extent of the "4th spatial dimension".
- A particle travelling in that dimension would return to where it started in a very short time.
- At each point in ordinary 3-space (x^1, x^2, x^3) there would be a curled-up circle, or 2-ball, or (d-3)-dimensional-ball.

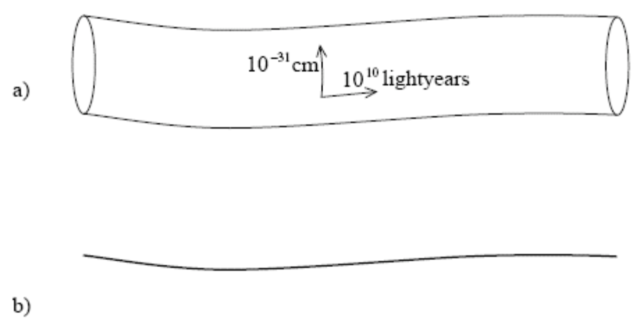
Kaluza-Klein theory

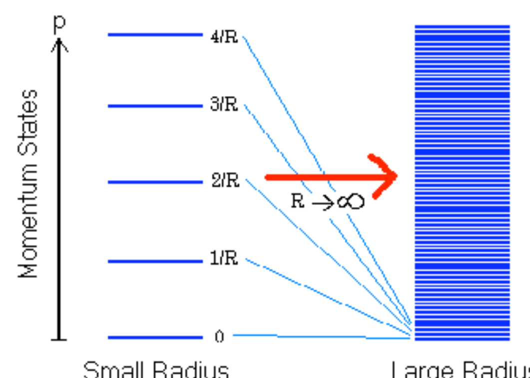
- Kaluza-Klein (KK) theory (1926) attempts to unify gravity and electromagnetism (the only known forces at the time) by including one extra dimension.
- The theory actually works at some level, but in the end it failed to give a realistic theory, including matter particles.
- Quantizing non-relativistic motion in one extra compact dimension (square well, bead on ring) yields standing waves with quantized energies $E_n = (2\pi\hbar)^2/2mL_C^2$ in a "Kaluza-Klein (KK) tower".



- In a relativistic theory, $m_n c^2 = (2\pi\hbar c/L_C)n$ is an equally-spaced tower of excitations, including a lowest, massless one.
- The smaller the space, the larger the mass of the KK excitations.

$$\left[\begin{array}{c|c} g_{\mu\nu} & A_\mu \\ \hline A_\nu & \phi \end{array}\right]$$





The scalar "radion" field ϕ plays the role of a Higgs, giving mass to the KK tower of A_μ.
The lowest state remains massless and preserves gauge invariance.
In theories that include gravity, one radion field ϕ remains, describing fluctuations in the size of the extra dimension.

E fields in different dimensions

- Gauss' law in 3-D … and d-spatial dimensions

$$\nabla \cdot \vec{E} = \rho$$

$$\int_{V^3} dV \; \nabla \cdot \vec{E} = \int_{V^3} dV \; \rho$$

$$\int_{S^2} d\vec{S} \cdot \vec{E} = q$$

$$4\pi \, r^2 \, E(r) = q$$

$$E(r) = \frac{q}{4\pi \, r^2}$$

$$area(S^{d-1}) \, E(r) = q$$

$$E(r) = \frac{\Gamma(d/2)}{2\pi^{d/2}} \frac{q}{r^{d-1}}$$

Surfaces and volumes in d spatial dimensions

"volume" of a ball of radius R in d dimensions

$$vol(V^d) = \frac{\pi^{d/2}}{\Gamma(1+d/2)} r^d$$

$$vol(V^2) = \pi r^2$$

$$vol(V^3) = \frac{4}{3}\pi r^3$$

$$vol(V^4) = \frac{1}{2}\pi^2 r^4$$

"area" of surface of that ball of radius R in d dimensions

$$area(S^{d-1}) = \frac{2\pi^{d/2}}{\Gamma(d/2)} r^{d-1}$$

$$area(S^1) = 2\pi r$$

$$area(S^2) = 4\pi r^2$$

$$area(S^3) = 2\pi^2 r^3$$

What is M_P in d dimensions?

- Easiest to work with the gravitational potential, analogous to the electric scalar potential.
- Gravitational potential of a mass distribution has units of Energy/mass, independent of dimensionality of space
- It satisfies a Poisson Equation, valid in any number of spatial dimensions.
- LHS always has units of energy/mass/length^2
- RHS has units of G×mass/length^d.
- Therefore, G has units of energy/mass^2 ×length^{d-2}, different units in different spatial dimensions.
- Planck mass and Planck length will be different as well.

$$\vec{\nabla} \cdot \vec{E} = -\rho$$

$$\vec{E} = -\vec{\nabla}\varphi \Rightarrow \nabla^2\varphi = -\rho$$

$$\nabla^2\varphi_G = 4\pi G\rho_m$$

$$\nabla^2\varphi_G^{(d)} = 4\pi G^{(d)}\rho_m$$

The Planck scale in d dimensions

- The dimensionality of G changes: $G^{(d)} \sim (L)^{(d-3)} G_N$
- The Planck scale is defined uniquely by forming combinations of G, ħ, c with dimensions of m_P, L_P, t_P .
- So the Planck scale will also change in d dimensions.
- Can we bring $M_P^{(d)}$ much closer to the weak scale of ~ 1 TeV by invoking compact dimensions?

$$\nabla^2 \varphi_G^{(d)} = 4\pi G^{(d)} \rho_m$$

$$\rho \sim M / L^d$$

$$G^{(d)} \frac{M}{L^d} \sim G_N \frac{M}{L^3}$$

$$G^{(d)} \sim G_N L^{d-3}$$

$$L_P = \sqrt{\frac{G\hbar}{c^3}}$$

$$\left(L_P^{(d)}\right)^{d-1} = \left(L_P\right)^2 \frac{G^{(d)}}{G_N}$$

$$M_P = \sqrt{\frac{\hbar c}{G}} = \frac{\hbar}{c L_P}$$

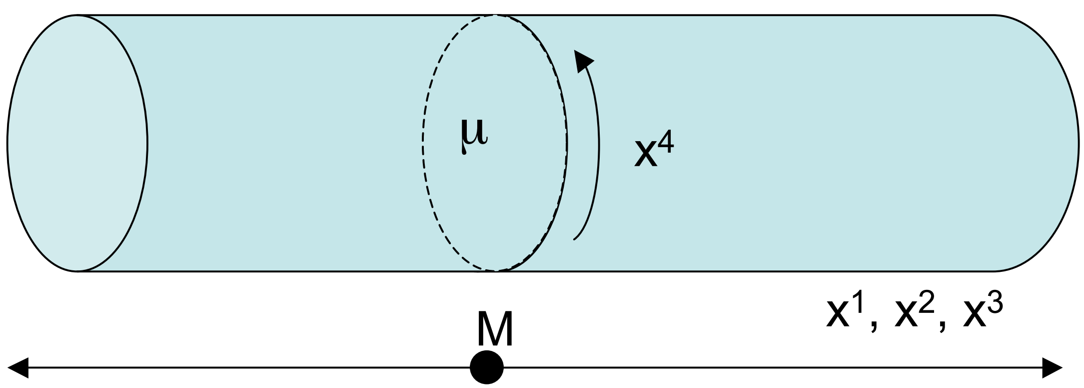
$$\left(\frac{c M_P^{(d)}}{\hbar}\right)^{d-1} = \left(\frac{c M_P}{\hbar}\right)^2 \frac{G_N}{G^{(d)}}$$

Gravity in d spatial dimensions

- In d spatial dimensions, the force laws change their power-law behavior;
- The dimensions of the gravitational constant $G^{(d)}$ change.
- What stays the same? The Poisson equation for the gravitational potential.

$$\nabla^2 \varphi_G^{(d)} = 4\pi G^{(d)} \rho_m$$

- So how does $G^{(d)}$ change in the presence of (d-3) small compact dimensions of length L_C?



$$M = 2\pi R \mu = L_C \mu$$

$$\rho^{(3)} = M\delta(x^1)\delta(x^2)\delta(x^3)$$

$$\rho^{(4)} = \mu\delta(x^1)\delta(x^2)\delta(x^3)$$

$$\rho^{(3)} = \int_0^{2\pi R} dx^4 \rho^{(4)}$$

$$M = \int_{-\infty}^{\infty} dx^1 \int_{-\infty}^{\infty} dx^2 \int_{-\infty}^{\infty} dx^3 \int_0^{2\pi R} dx^4 \rho^{(4)}$$

$$\rho^{(4)} = \frac{\rho^{(3)}}{2\pi R}$$

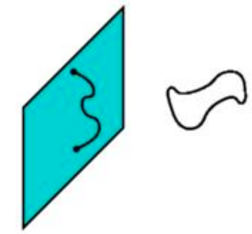
$$\nabla^2 \varphi_G^{(4)} = 4\pi G^{(4)} \rho^{(4)}$$

$$= 4\pi G^{(4)} \frac{\rho^{(3)}}{2\pi R} = 4\pi G^{(3)} \rho^{(3)}$$

$$G^{(4)} = 2\pi R G^{(3)} = L_C G_N$$

$$G^{(d)} = \left(L_C\right)^{d-3} G_N = V_C G_N$$

Can we bring the Planck scale down to, say 2 TeV?



- The "true" Planck length in d spatial dimensions is $L_P^{(d)}$
- The larger the size of the compact dimensions L_c, the larger is $G^{(d)}$ and thus $L_P^{(d)}$, and the smaller is $M_P^{(d)}$, reducing the severity of the hierarchy problem, bringing the scale of quantum gravity closer to the weak scale.
- Can we bring $M_P^{(d)}$ much closer to the weak scale of ~ 1 TeV by invoking compact dimensions?
- Let's try $M_P^{(d)}c^2$ ~ 2 TeV, or $L_P^{(d)}$ ~ 10^{-19} m.
- With one extra spatial dimension, $L_C = 10^{13}$ m. This does not work!
- With two, $L_C \sim 10^{-3}$ m.
  Surely we would have noticed that by now!
- Not if photons and fermions are constrained to live in the large 3 spatial dimensions we know and love.
- Only gravity sees these extra dimensions, and the $1/r^2$ law of gravity has not been tested accurately down to such short distances!

$$G^{(d)} = \left(L_C\right)^{d-3} G_N$$

$$\left(L_P^{(d)}\right)^{d-1} = \left(L_P\right)^2 \frac{G^{(d)}}{G_N} = \left(L_P\right)^2 \left(L_C\right)^{d-3}$$

$$L_C = L_P^{(d)} \left(\frac{L_P^{(d)}}{L_P}\right)^{\frac{2}{d-3}}$$

$$L_C = L_P^{(4)} \left(\frac{L_P^{(4)}}{L_P}\right)^2 = 10^{-19} \left(\frac{10^{-19}}{10^{-35}}\right)^2 = 10^{13}\, m$$

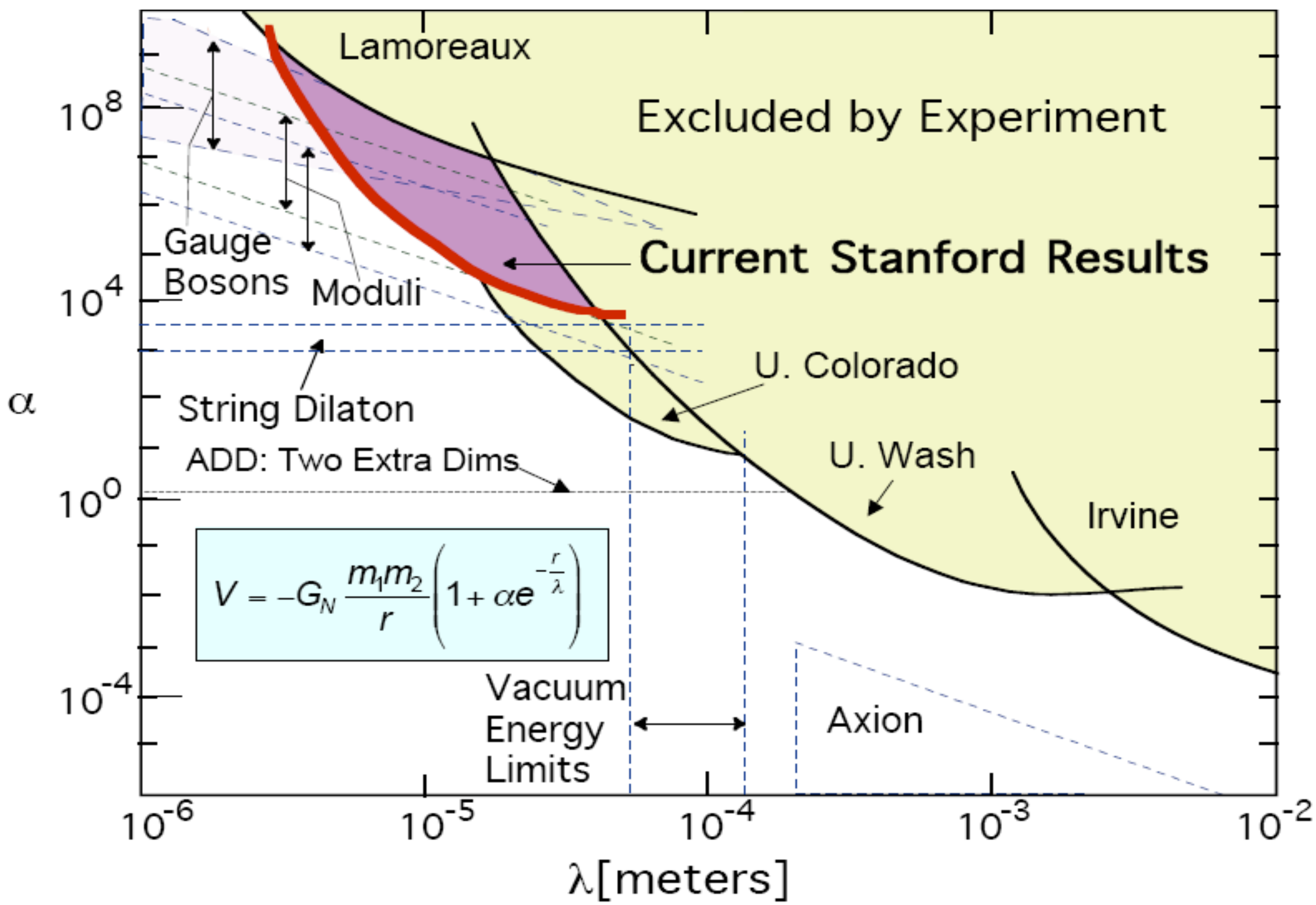
$$L_C = L_P^{(5)} \left(\frac{L_P^{(5)}}{L_P}\right) = 10^{-19} \left(\frac{10^{-19}}{10^{-35}}\right) = 10^{-3}\, m$$

$$M_P = \sqrt{\frac{\hbar c}{G}} = \frac{\hbar}{cL_P}$$

$$\left(\frac{cM_P^{(d)}}{\hbar}\right)^{d-1} = \left(\frac{cM_P}{\hbar}\right)^2 \frac{G_N}{G^{(d)}} = \left(\frac{cM_P}{\hbar}\right)^2 \frac{1}{L_C^{d-3}}$$

$$M_P^2 = V_C \left(M_P^{(d)}\right)^{d-1}$$

Tests of deviations of Grav potential at short distances



$M_P^{(5)}$ could be as small as a few TeV in the ADD model!

A Test of Physics Beyond the Standard Model

$$V = -G_N \frac{m_1 m_2}{r}\left(1 + \alpha e^{-\frac{r}{\lambda}}\right)$$

Experiment tests many theories:

- **New particles:** Exotic particles with Compton wavelength λ~0.01 - 1 mm predicted in string theories, etc.
  S. Dimopoulos and G. F. Giudice, Phys. Lett. B **379**, 105 (1996)

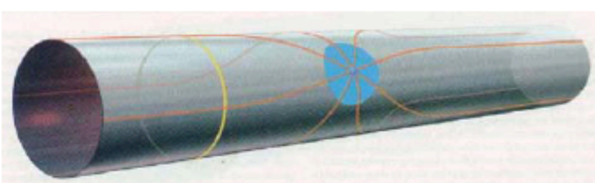
- **Extra dimensions:** Proposed hierarchy solution involves large, compact spatial dimensions accessed only by gravitons. Radius of compactification ~ λ < 1 mm
  N. Arkani-Hamed, S. Dimopoulos, and G. Dvali, Phys Rev. D **59**, 086004 (1999)

- **Cosmological Constant:** Small value of Λ could be stabilized by particles of wavelength λ~ 0.1 mm
  R. Sundrum, J. High Energy Phys, **9907**, 001(1999)

Difference Between the Models

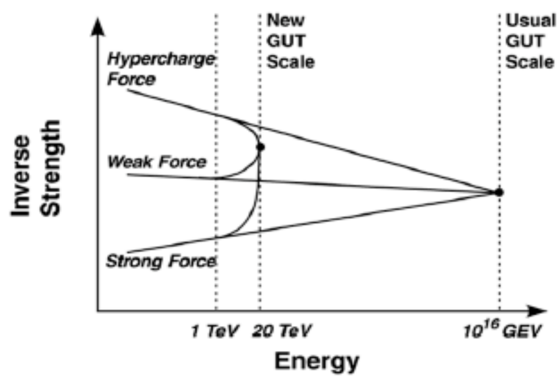
ADD Model:

- "Eliminates" the hierarchy problem by stating that physics ends at a TeV scale
- Only gravity lives in the "bulk" space
- Size of ED's (n=2-7) between ~100 μm and ~1 fm
- Doesn't explain how to make ED large



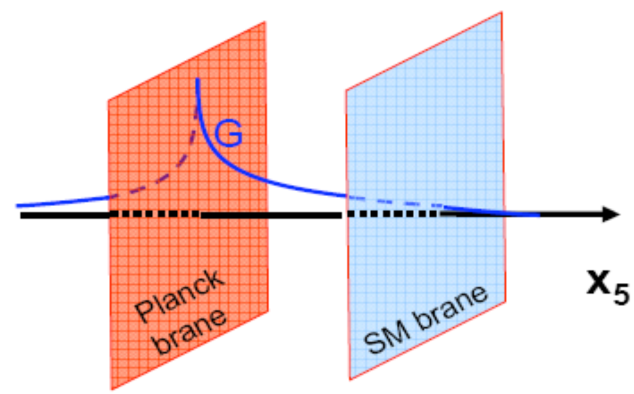
TeV-1 Scenario:

- Lowers GUT scale by changing the running of the couplings
- Only gauge bosons (g/γ/W/Z) propagate in a single ED; gravity is not in the picture
- Size of the ED ~1 TeV^{-1} or ~10^{-19} m



RS Model:

- A rigorous solution to the hierarchy problem via localization of gravity
- Gravitons (and possibly other particles) propagate in a single ED, w/ special metric
- Size of this ED as small as ~$1/M_{Pl}$ or ~10^{-35} m



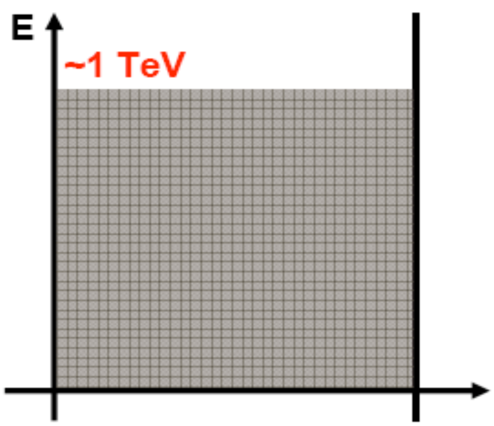
Kaluza-Klein excitations

- If the SM fields are not fixed to our (3+1)-brane (as in string theory), but can live in the compact dimensions, they will have KK towers.
- If we haven't seen their KK excitations, they must be too high in mass for us to see them; the first excitations are > 100 GeV and $L_C < 10^{-18}$ m.
- To get $M_P^{(d)}$ of order ~ 1 TeV, need d ≥ 3+10 dimensions.
- In any case, the graviton lives in the compact dimensions, and its KK excitations should be of order 100 GeV or less.
- These fields couple to standard model particles with (the larger, d-dim) gravitational coupling strength.
- They will be produced at high energy accelerators. The coupling is weak, but if the KK tower contains many particles with mass < E_{CM}, the total cross section may be large enough to observe.
- Except, they interact only weakly, so they won't register in the particle detectors, except as missing energy/momentum.

Kaluza-Klein Spectrum

ADD Model:

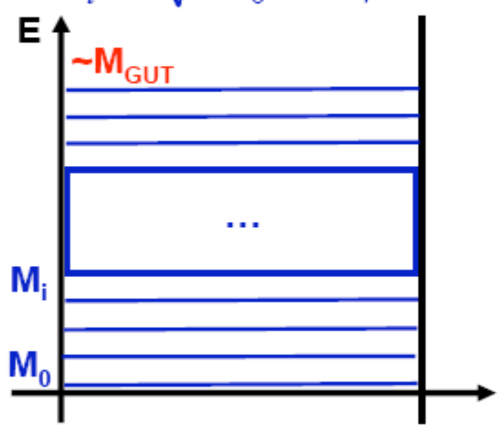
- Winding modes with energy spacing ~1/r, i.e. 1 meV – 100 MeV
- Can't resolve these modes – they appear as continuous spectrum



TeV^{-1} Scenario:

- Winding modes with nearly equal energy spacing ~1/r, i.e. ~TeV
- Can excite individual modes at colliders or look for indirect effects

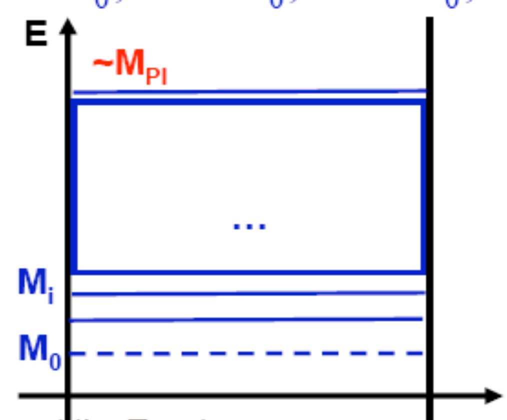
$$M_i = \sqrt{M_0^2 + i^2/r^2}$$



RS Model:

- "Particle in a box" with a special metric
- Energy eigenvalues are given by zeroes of Bessel function J_1
- Light modes might be accessible at colliders

$$M_i = M_0 x_i / x_0 \approx M_0, 1.83M_0, 2.66M_0, 3.48M_0, 4.30M_0, \ldots$$



Missing energy signature at LHC

- Signals may be visible above background for “true” Plank mass scales in the 4-8 TeV range, with up to 4 extra compact dimensions.
- At a high-energy e+e- linear collider (ILC), the backgrounds are much smaller, and the beam energy can be varied; one can easily distinguish different models of extra dimensions from each other and from other new physics sources of missing energy (like SUSY).
- Graviton exchange as SM particles scatter at the ILC will also produce unique signatures.