

Numerical modeling of SGRBs: from the ground up

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Plan of the talk

- * Our present understanding of merging binary NSs
- * Anatomy of the GW signal
- * Role of B-fields and EM counterparts

The equations of numerical relativity

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = \cancel{8\pi T_{\mu\nu}}, \quad (\text{field equations})$$

$$\cancel{\nabla_{\mu}T^{\mu\nu} = 0}, \quad (\text{cons. energy/momentum})$$

$$\cancel{\nabla_{\mu}(\rho u^{\mu}) = 0}, \quad (\text{cons. rest mass})$$

$$\cancel{p = p(\rho, \epsilon, Y_e, \dots)}, \quad (\text{equation of state})$$

$$\cancel{\nabla_{\nu}F^{\mu\nu} = I^{\mu}, \quad \nabla_{\nu}^*F^{\mu\nu} = 0}, \quad (\text{Maxwell equations})$$

$$\cancel{T_{\mu\nu} = T_{\mu\nu}^{\text{fluid}} + T_{\mu\nu}^{\text{EM}} + \dots} \quad (\text{energy – momentum tensor})$$

In vacuum space times the theory is complete and the truncation error is the only error made: **“CALCULATION”**

The equations of numerical relativity

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$$T_{\mu\nu} = T_{\mu\nu}^{\text{fluid}} + T_{\mu\nu}^{\text{EM}} + \dots \quad (\text{energy - momentum tensor})$$

In **non-vacuum space times** the truncation error is the only error that is **measurable**: **“SIMULATION”**

It's our **approximation** to **“reality”**: improvable via microphysics, magnetic fields, viscosity, radiation transport, ...

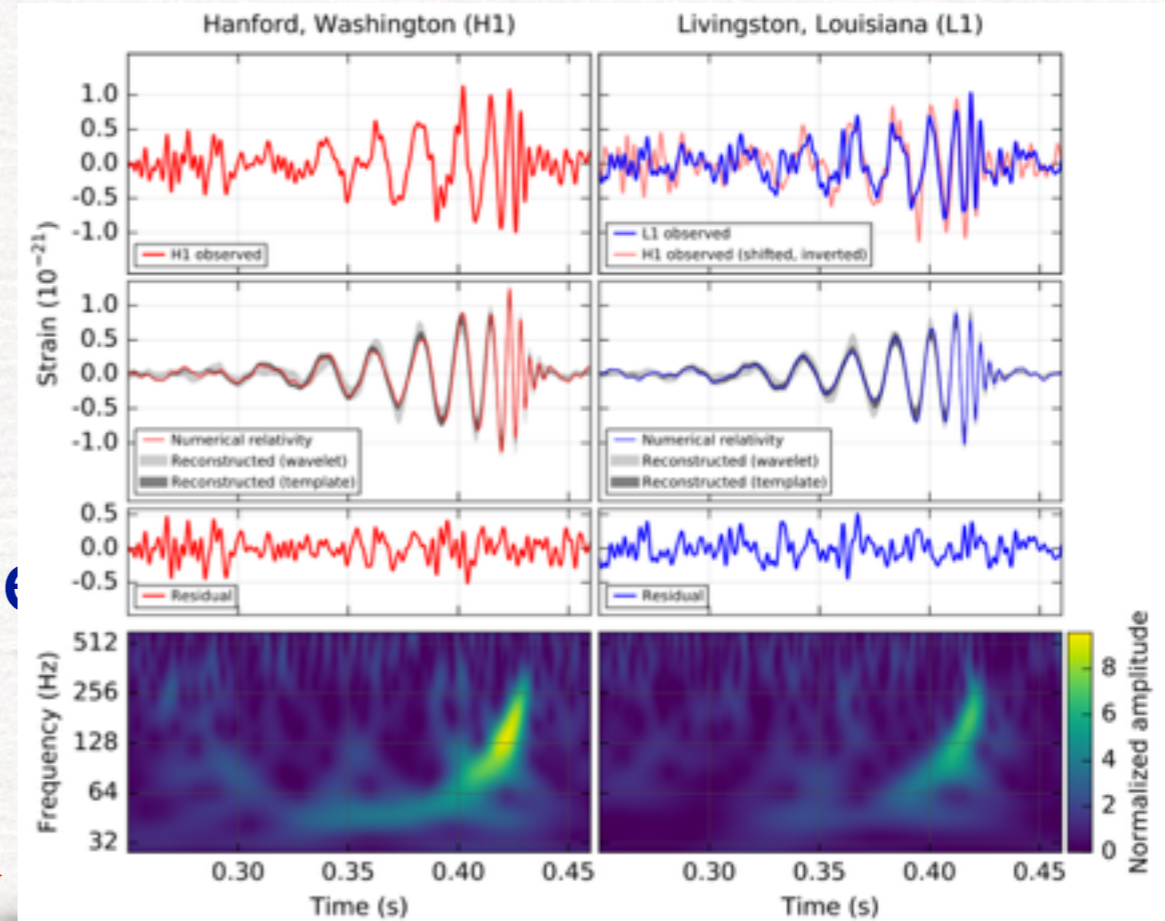
The two-body problem in GR

- For BHs we know what to **expect**:

$BH + BH \longrightarrow BH + GWs$

- For NSs the question is more **subtle**
hyper-massive neutron star (HMNS),

$NS + NS \longrightarrow HMNS + \dots ? \longrightarrow$



Abbott+ 2016

- **HMNS** phase can provide strong and clear information on **EOS**
- **BH+torus** system may tell us on the central engine of **GRBs**

Animations: Breu, Radice, LR



$$M = 2 \times 1.35 M_{\odot}$$

LS220 EOS

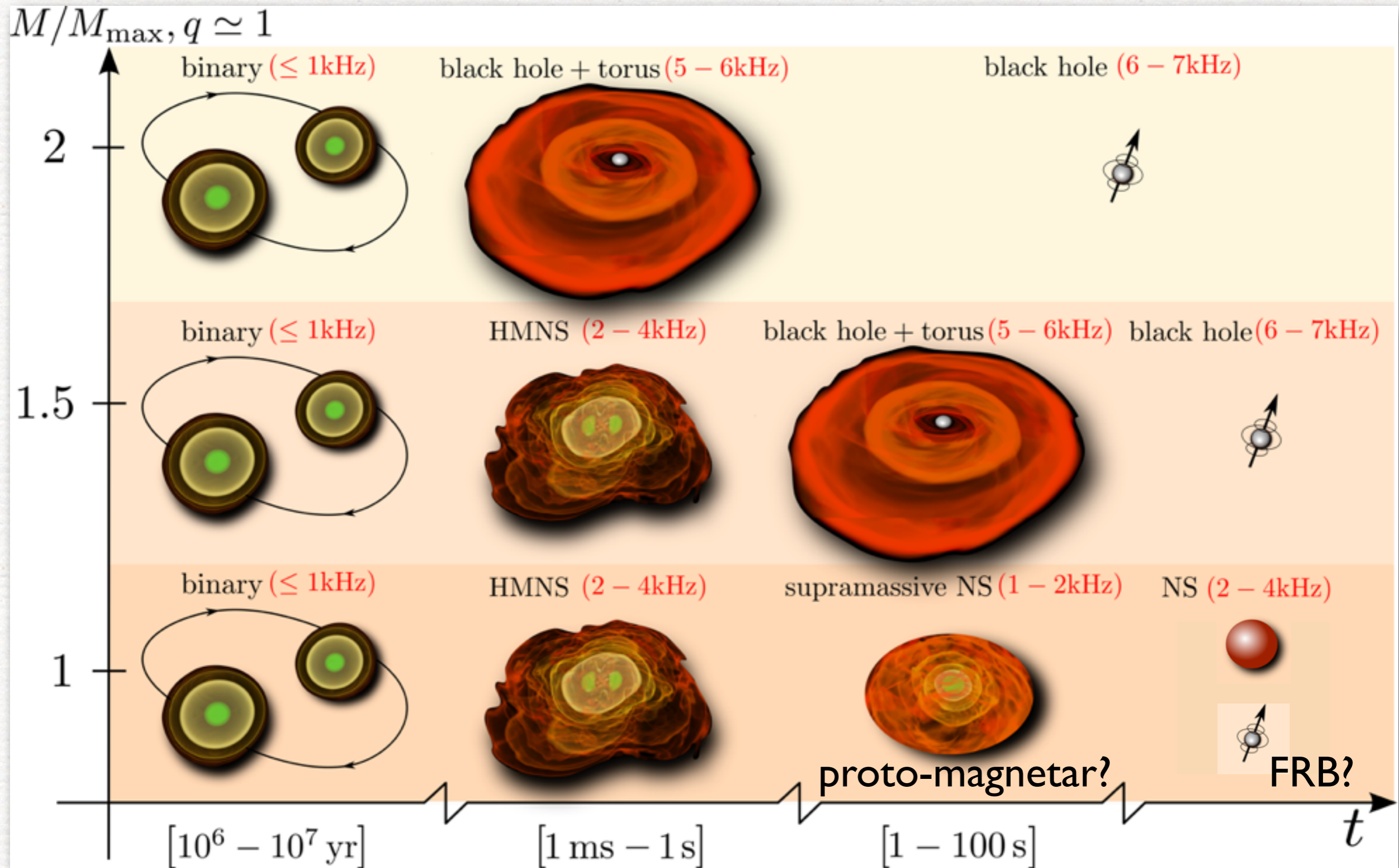
“merger → HMNS → BH + torus”

Quantitative differences are produced by:

- differences induced by the gravitational **MASS**:

a binary with smaller mass will produce a HMNS further away from the stability threshold and will collapse at a later time

Broadbrush picture

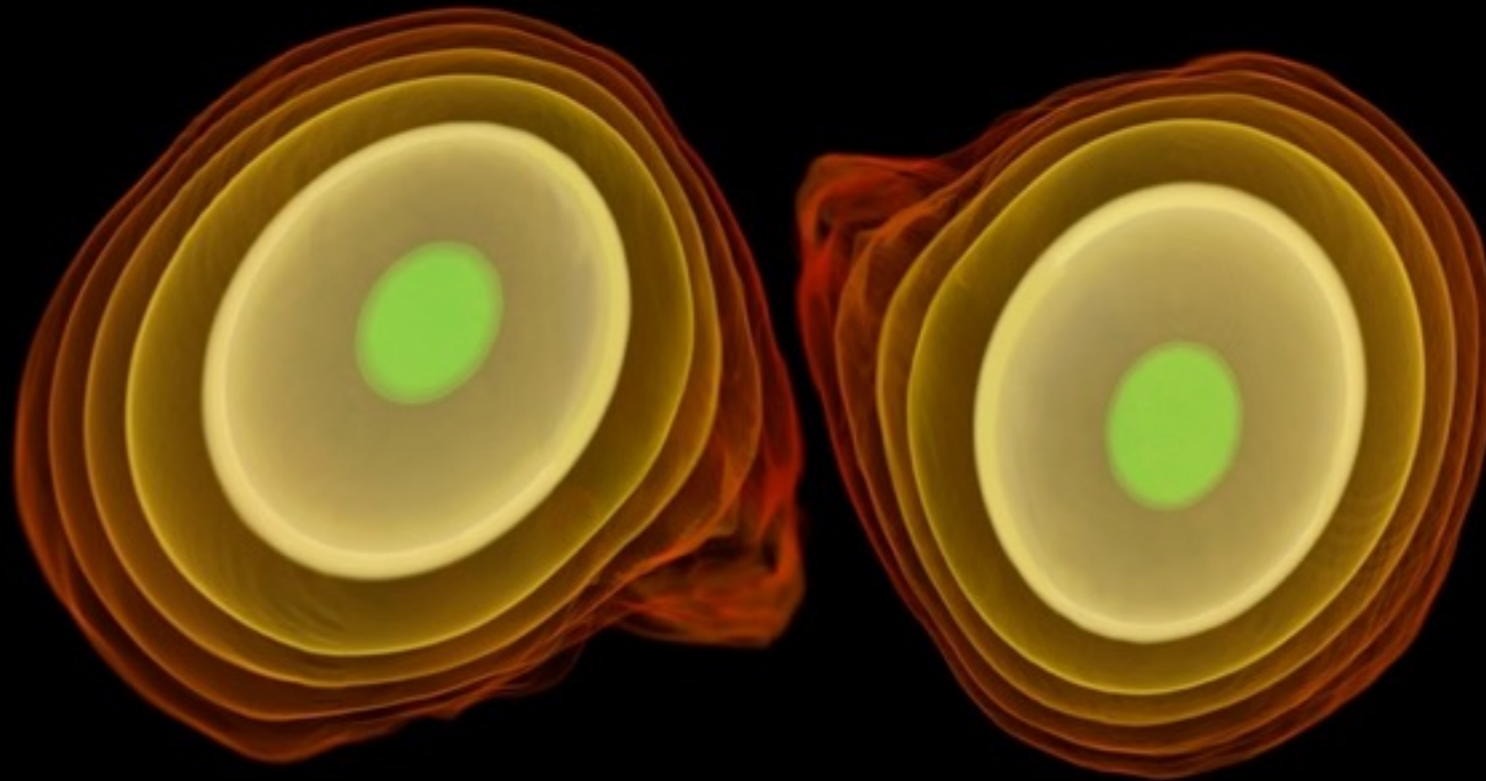


“merger → HMNS → BH + torus”

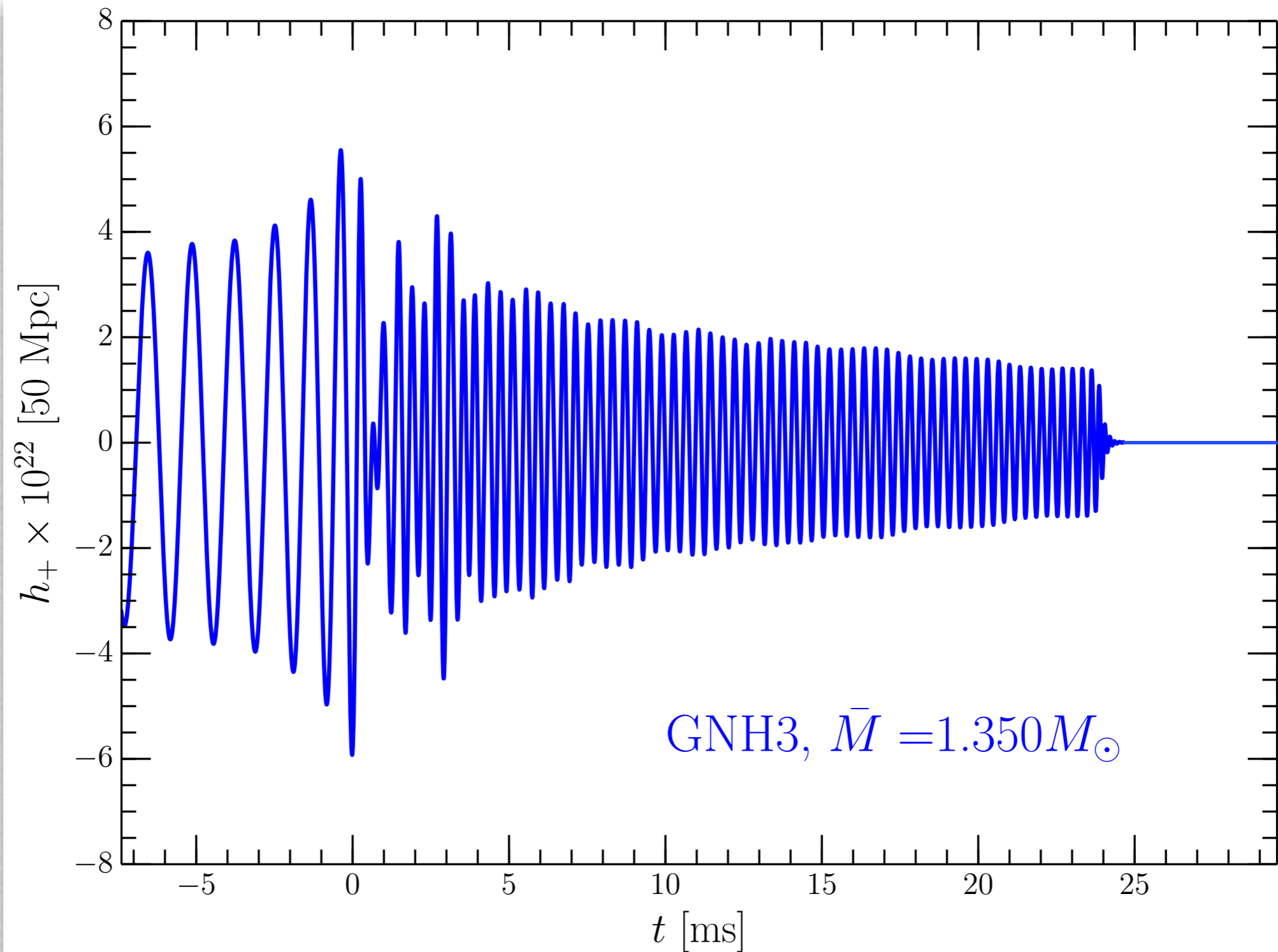
Quantitative differences are produced by:

- differences induced by the gravitational **MASS**:
a binary with smaller mass will produce a HMNS further away from the stability threshold and will collapse at a later time
- differences induced by **MASS ASYMMETRIES**:
tidal disruption before merger; may lead to prompt BH
- differences induced by the **EOS**:
stiff/soft OESs will have different compressibility and deformability, imprinting on the GW signal
- differences induced by **MAGNETIC FIELDS**:
the angular momentum redistribution via magnetic braking or MRI can increase/decrease time to collapse; EM counterparts!
- differences induced by **RADIATIVE PROCESSES**:
radiative losses will alter the equilibrium of the HMNS

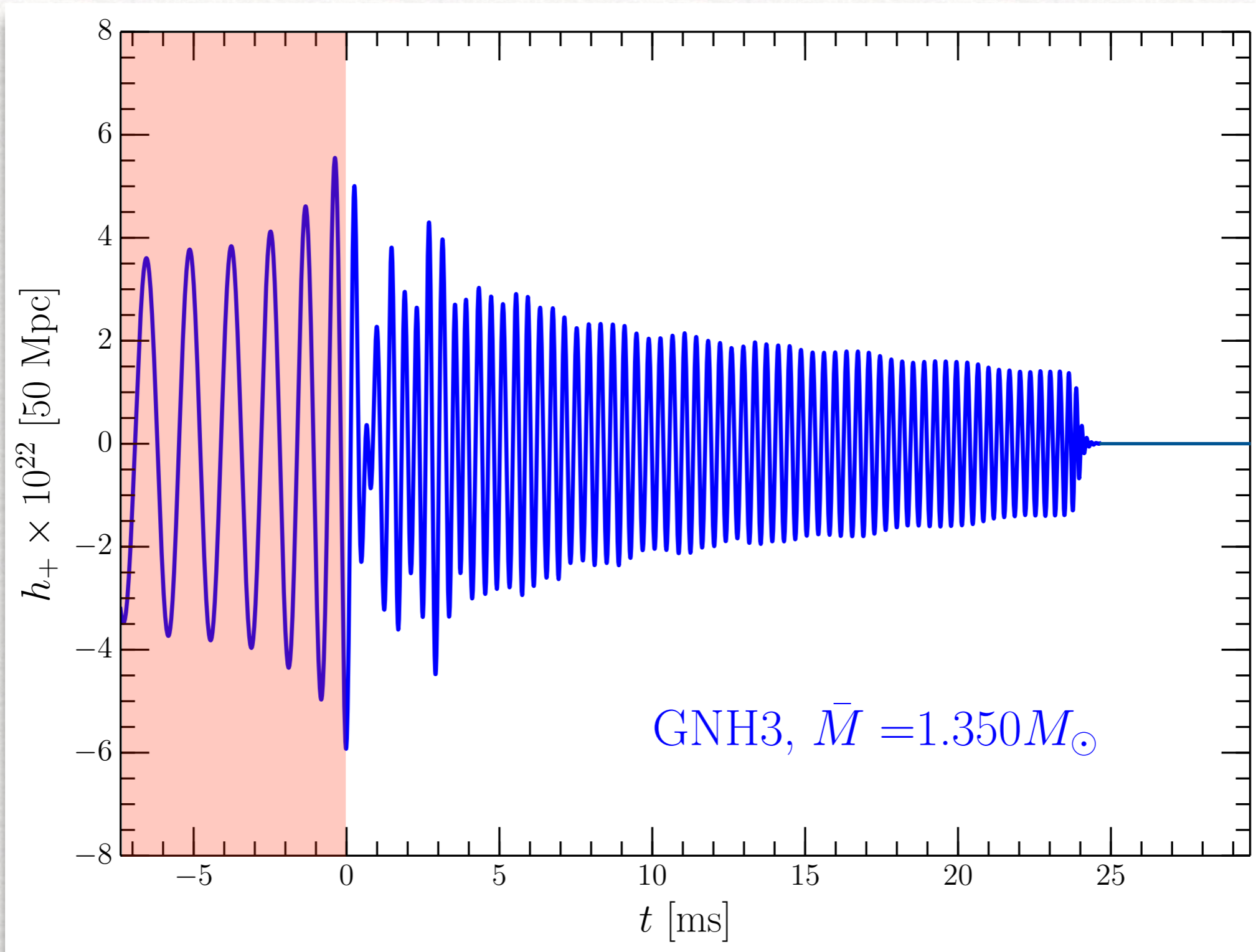
How to constrain the EOS



Anatomy of the GW signal

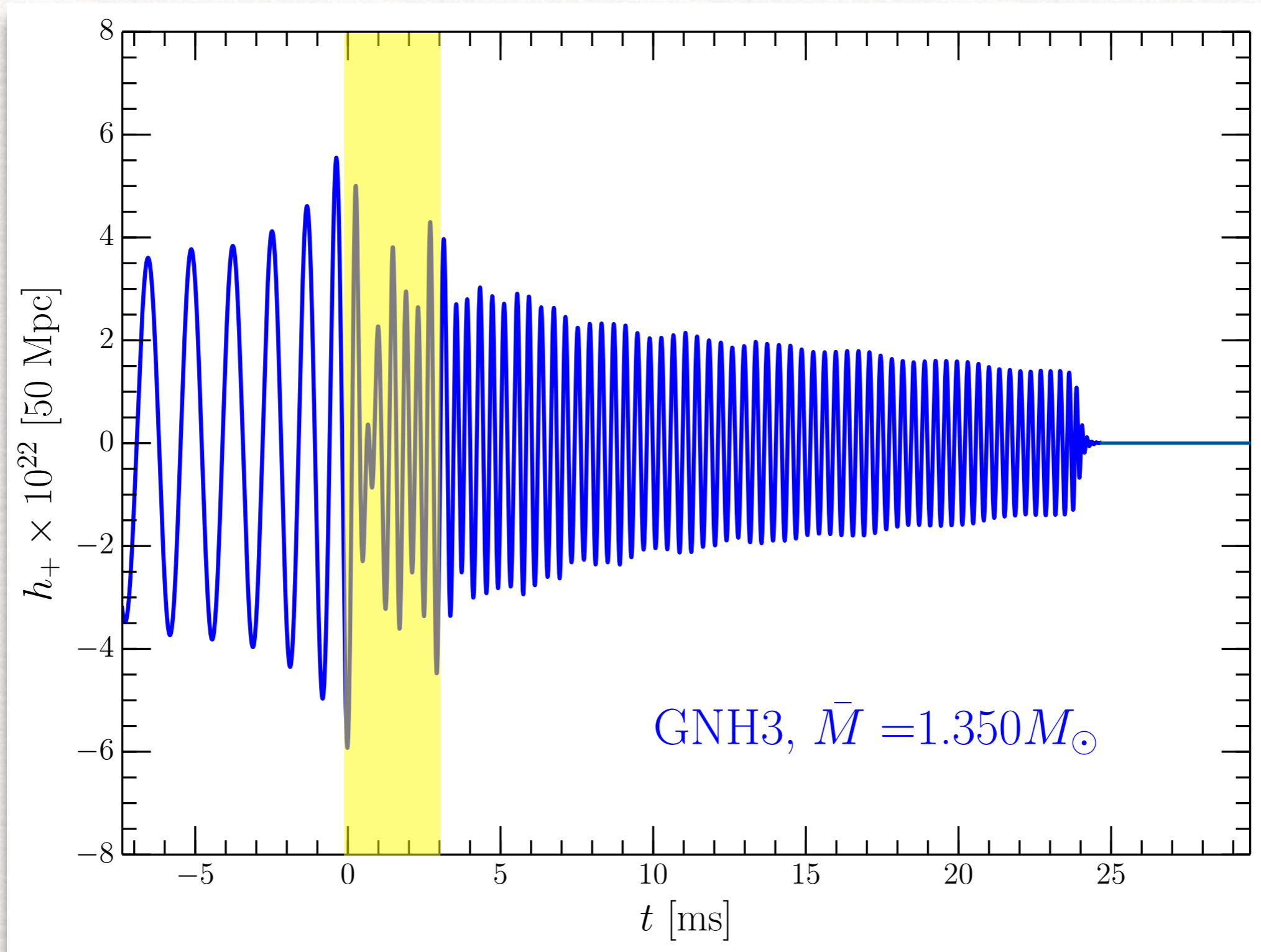


Anatomy of the GW signal



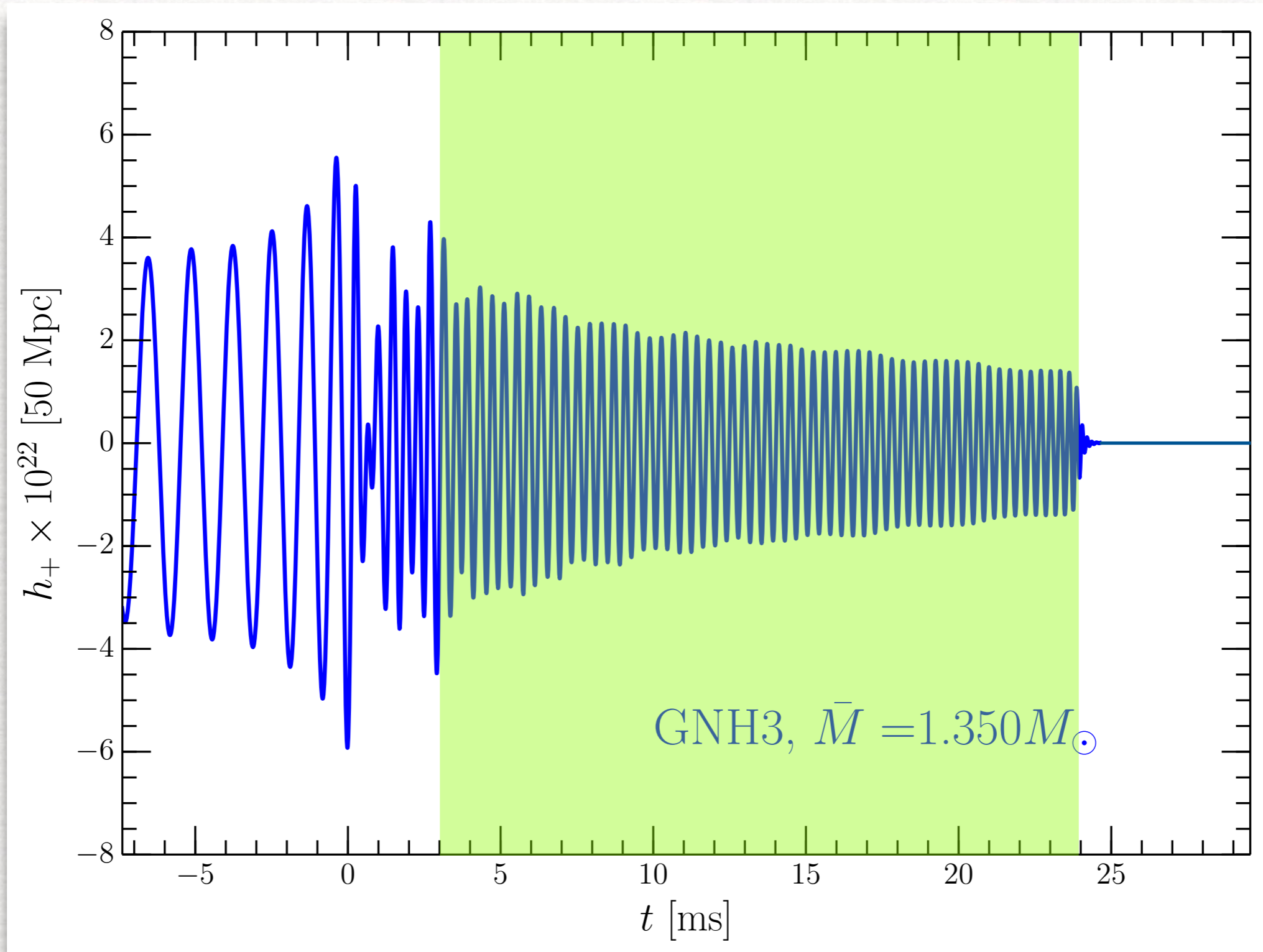
Inspiral: well approximated by PN/EOB; tidal effects important

Anatomy of the GW signal



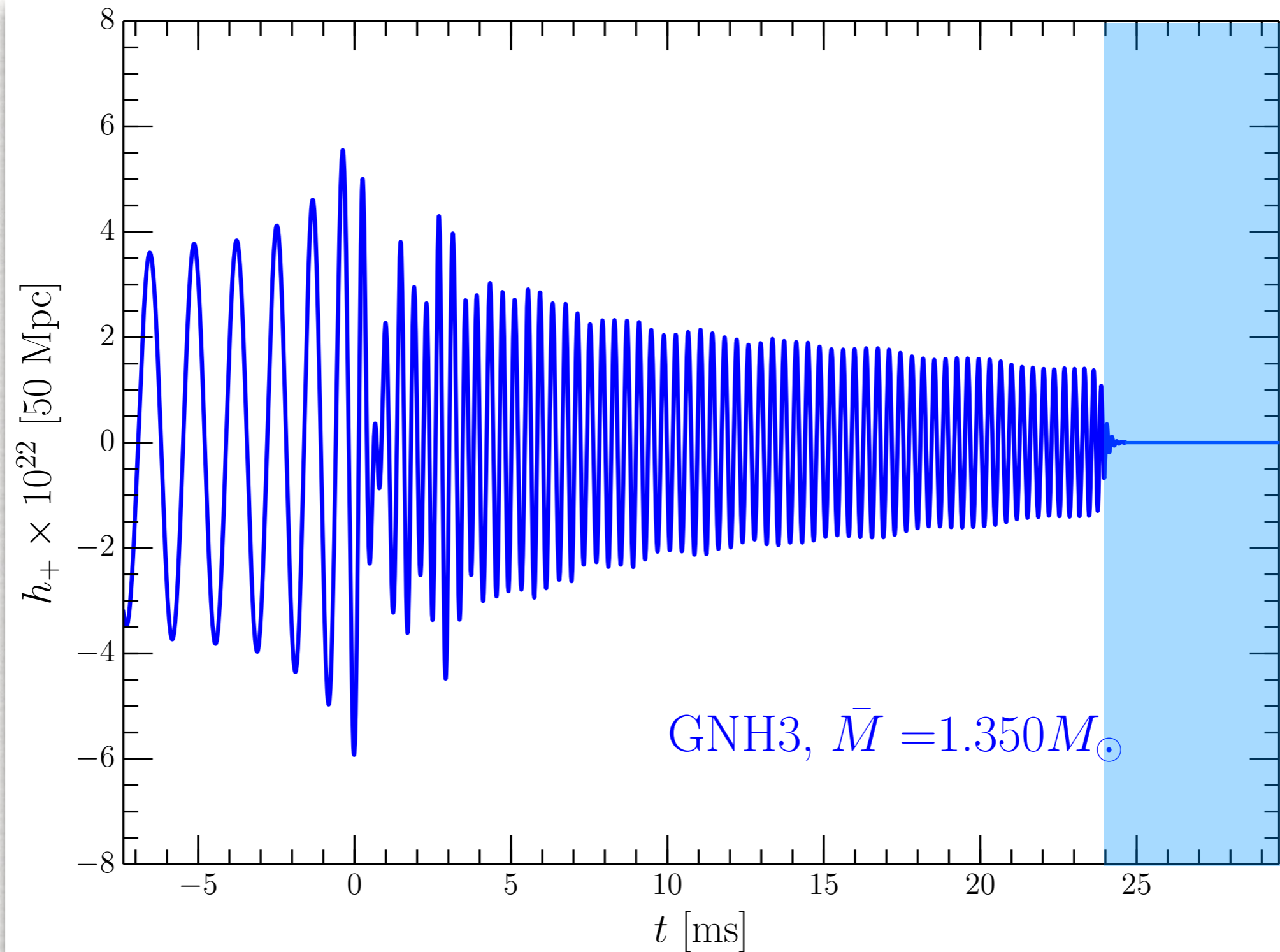
Merger: highly nonlinear but analytic description possible

Anatomy of the GW signal



post-merger: quasi-periodic emission of bar-deformed HMNS

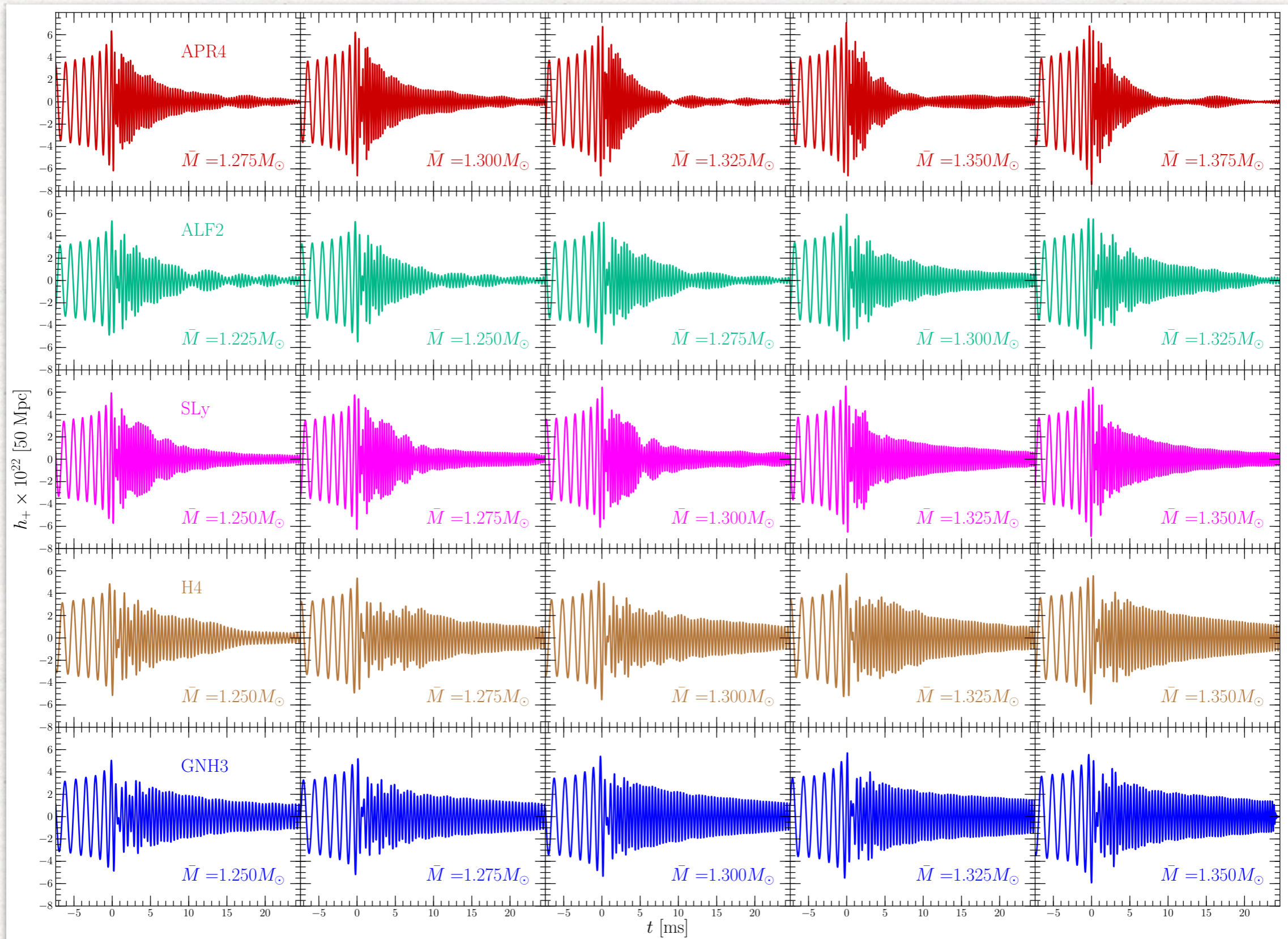
Anatomy of the GW signal



Collapse-ringdown: signal essentially shuts off.

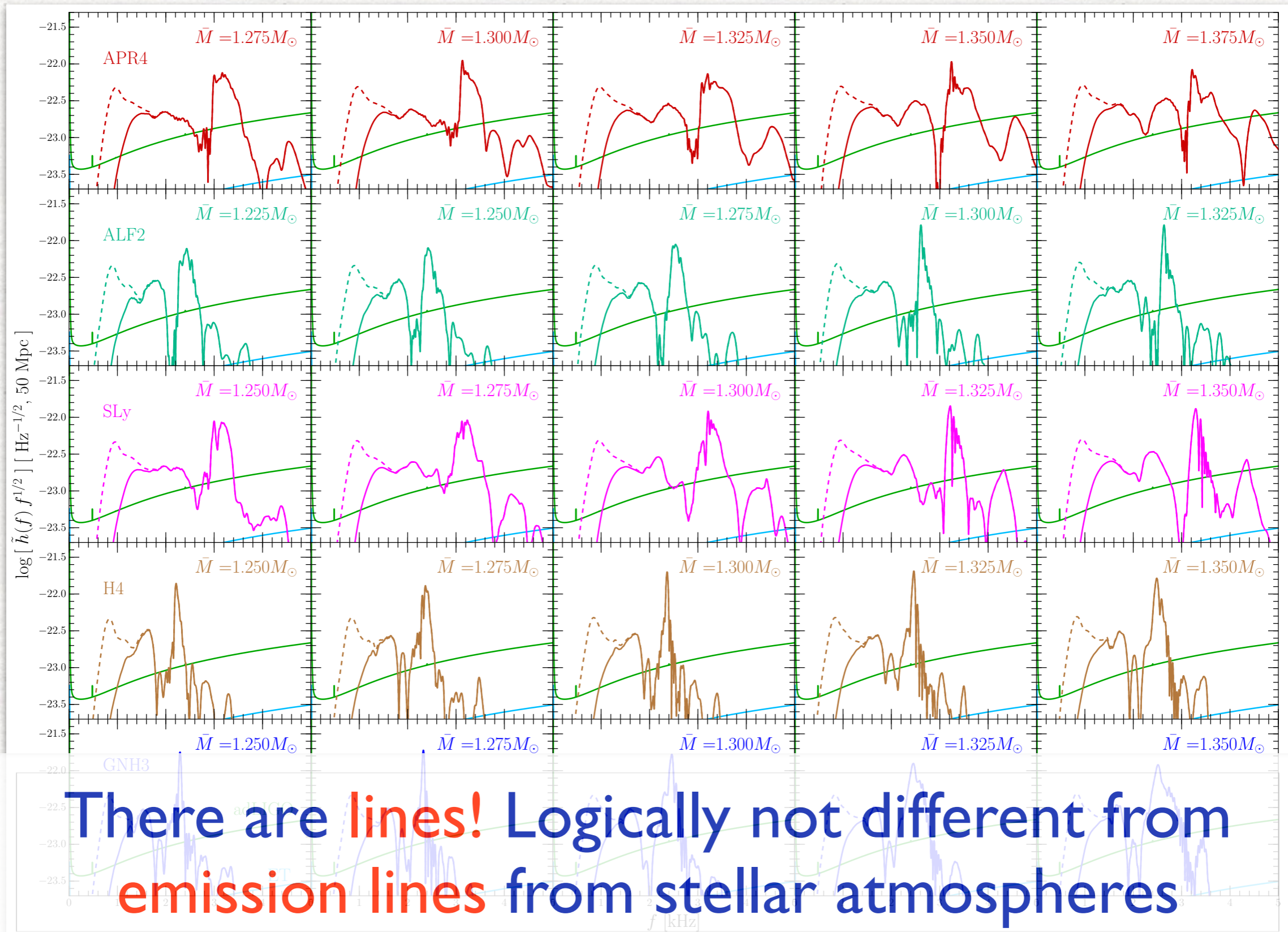
Extracting information from the EOS

Takami, LR, Baiotti (2014, 2015), LR+ (2016)



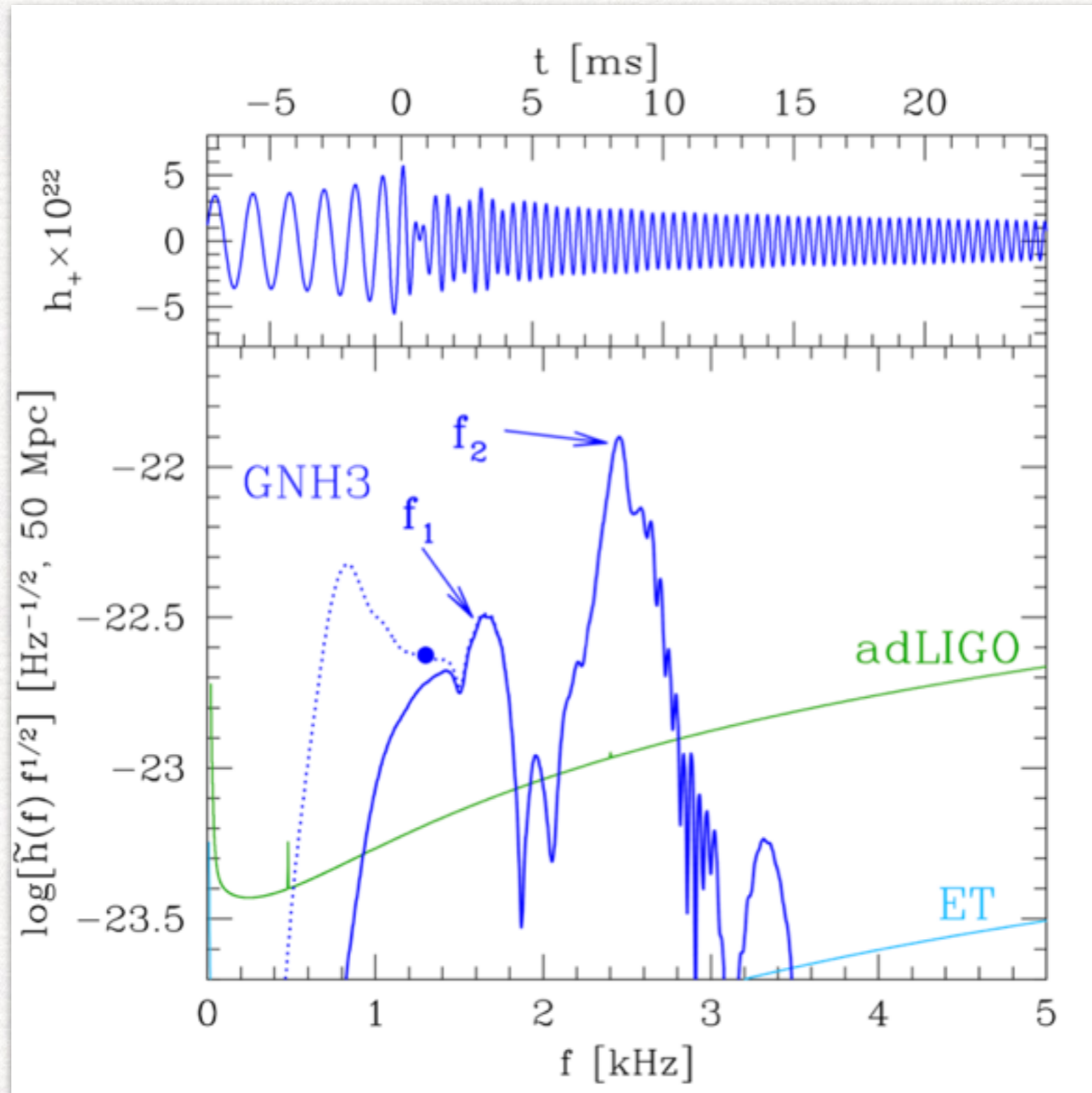
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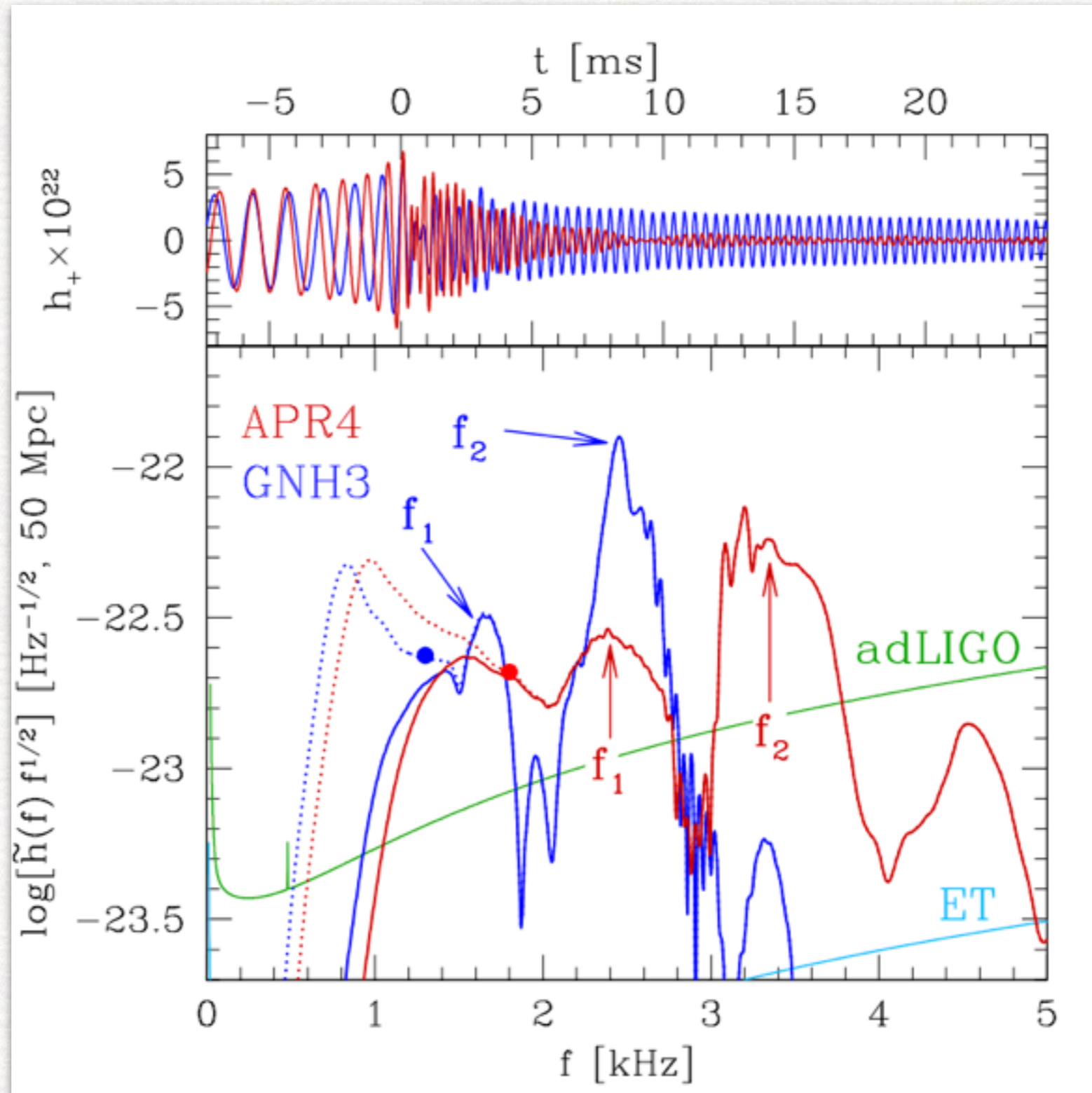
A new approach to constrain the EOS

Oechslin+2007, Baiotti+2008, Bauswein+ 2011, 2012, Stergioulas+ 2011, Hotokezaka+ 2013, Takami 2014, 2015, Bernuzzi 2014, 2015, Bauswein+ 2015, LR+2016...

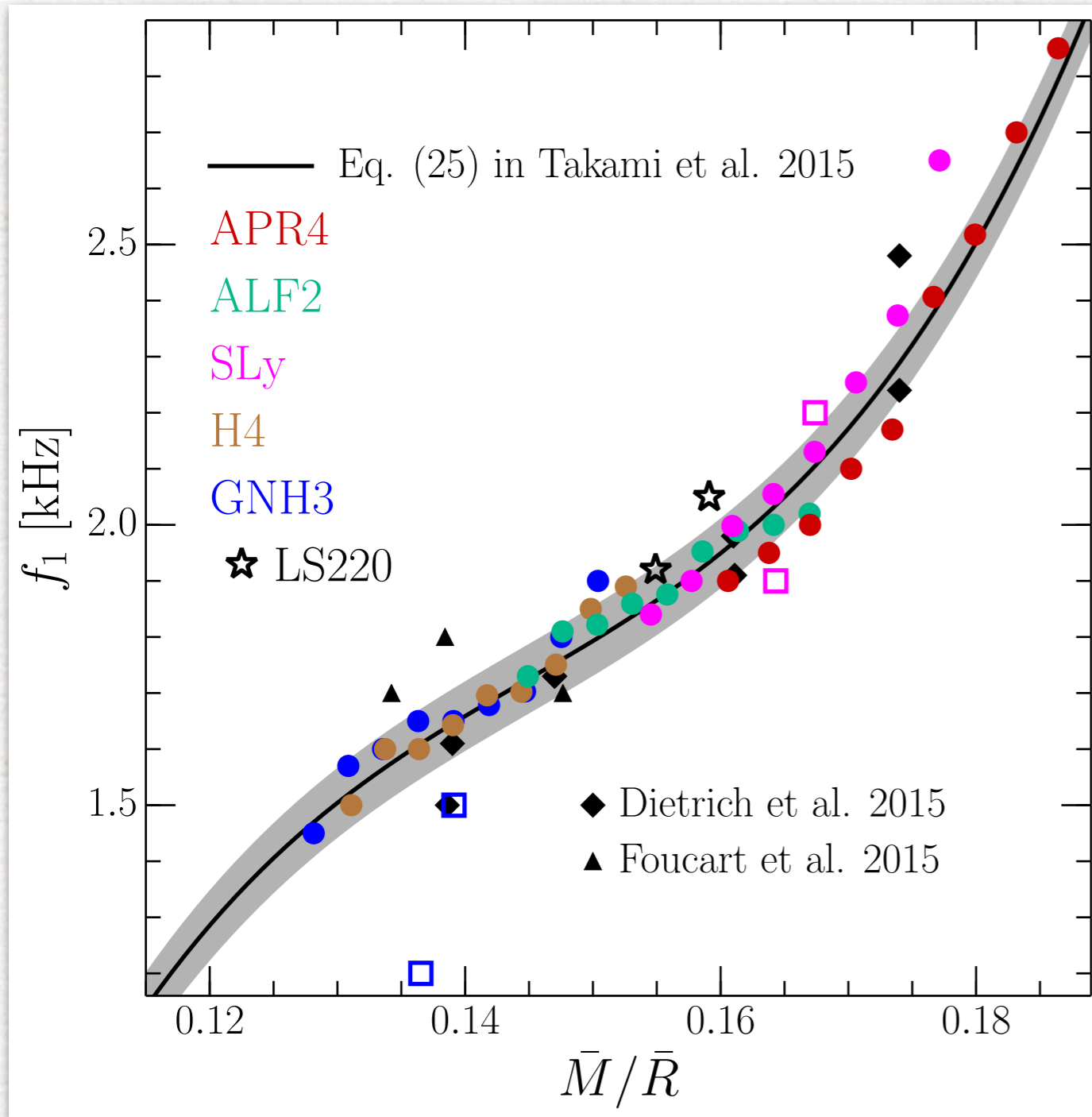


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Quasi-universal behaviour

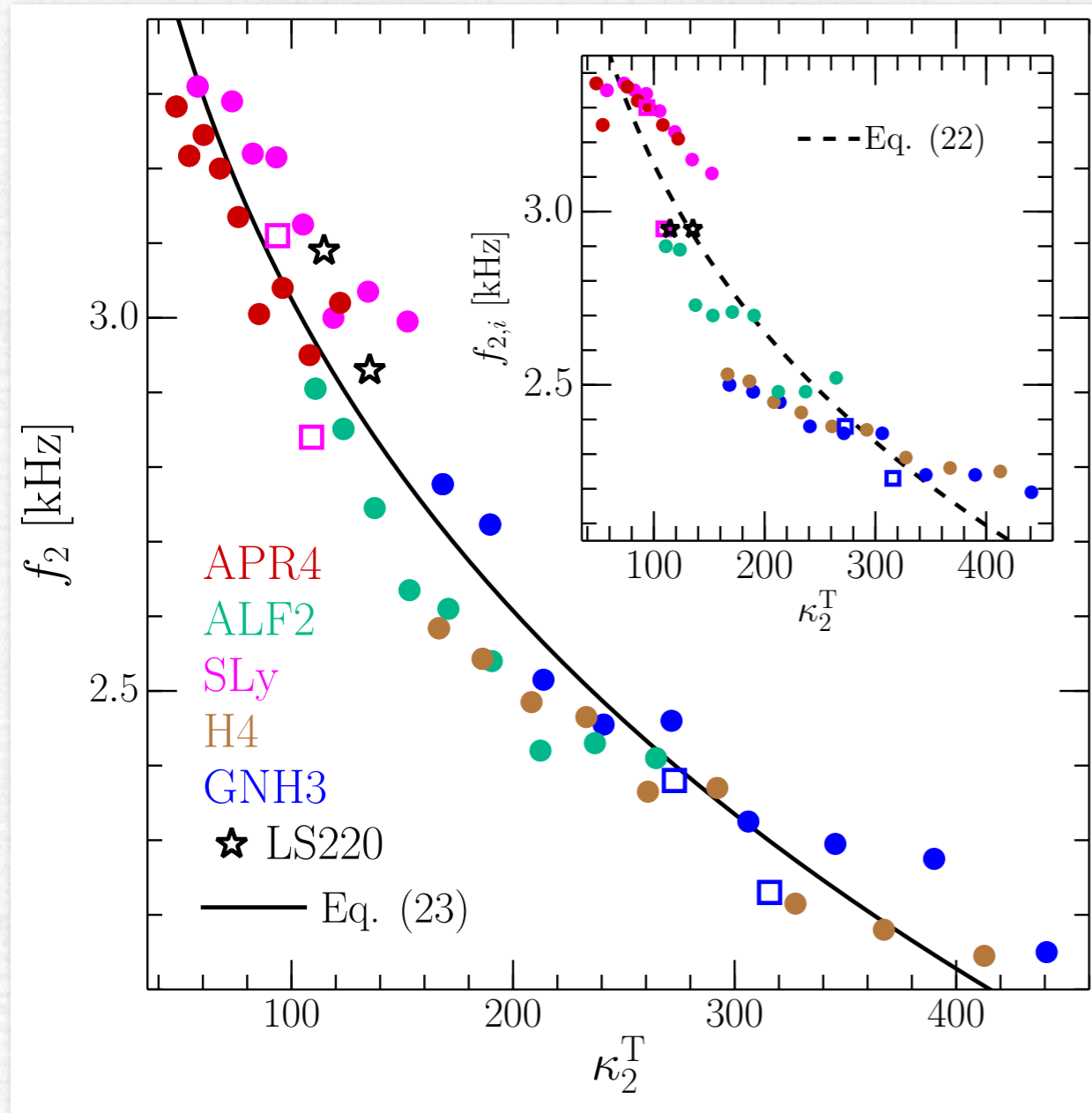


f_1 identification of PSDs is delicate, since created in short time window.

Spectrograms help the identification and results of other groups (Bernuzzi+ 2015, Foucart+ 2015) confirm quasi-universality.

Despite different claims, universality not lost at very low ($1.2 M_{\odot}$), very high ($1.5 M_{\odot}$) masses (LR+ 2016)

Quasi-universal or not? The case for f_2



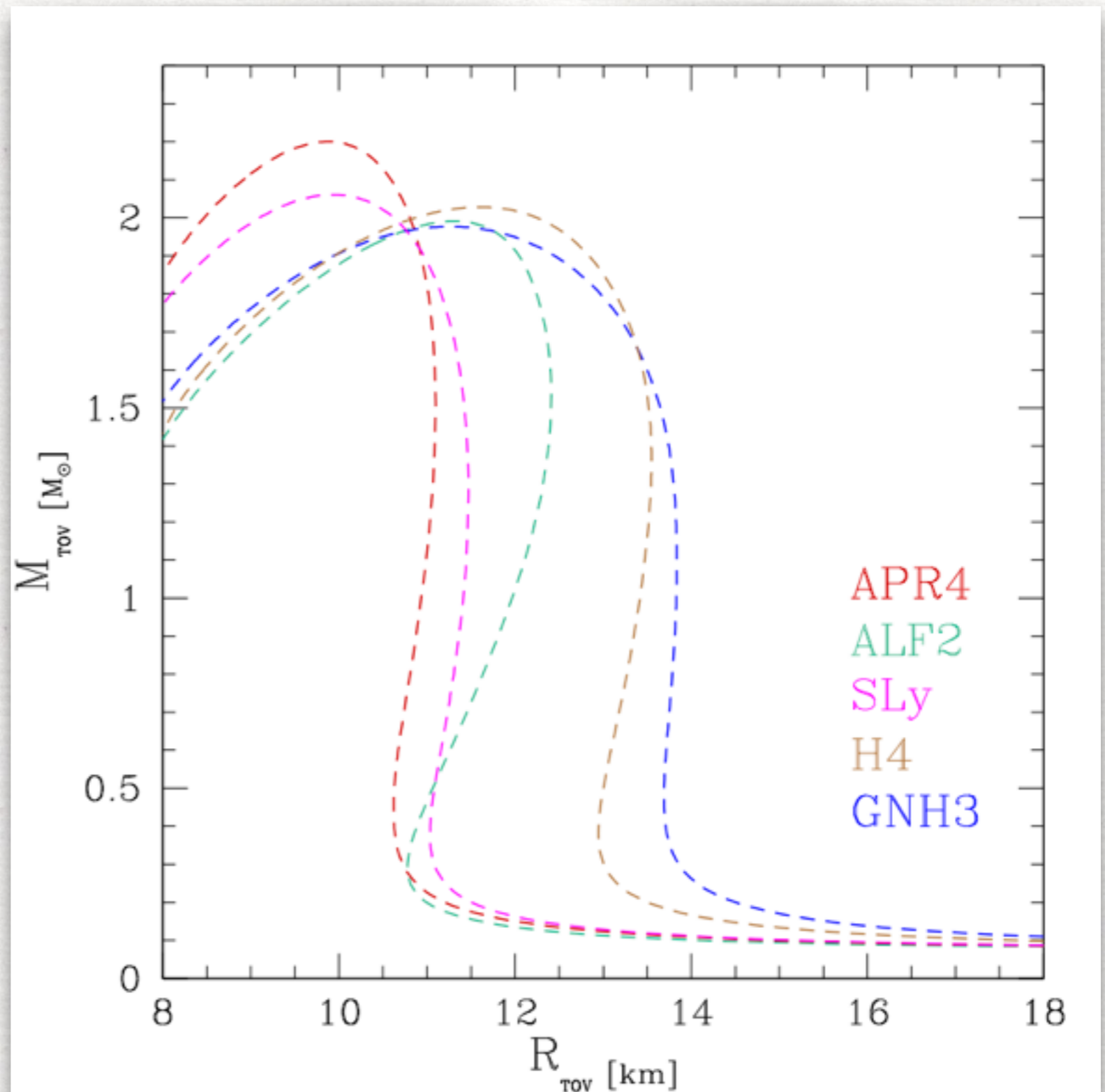
Correlations with stellar properties (Love number) have been found also for f_2 and f_{2-0} peak (Takami+ 2015, Bernuzzi+ 2015, LR+2016)

These correlations are weaker but equally important. Despite its complexity, a complete **analytical** description of pre- and post-merger signal is **possible**.

An example: start from equilibria

Assume that the GW signal from a binary NS is detected and with a SNR high enough that the two peaks are clearly measurable.

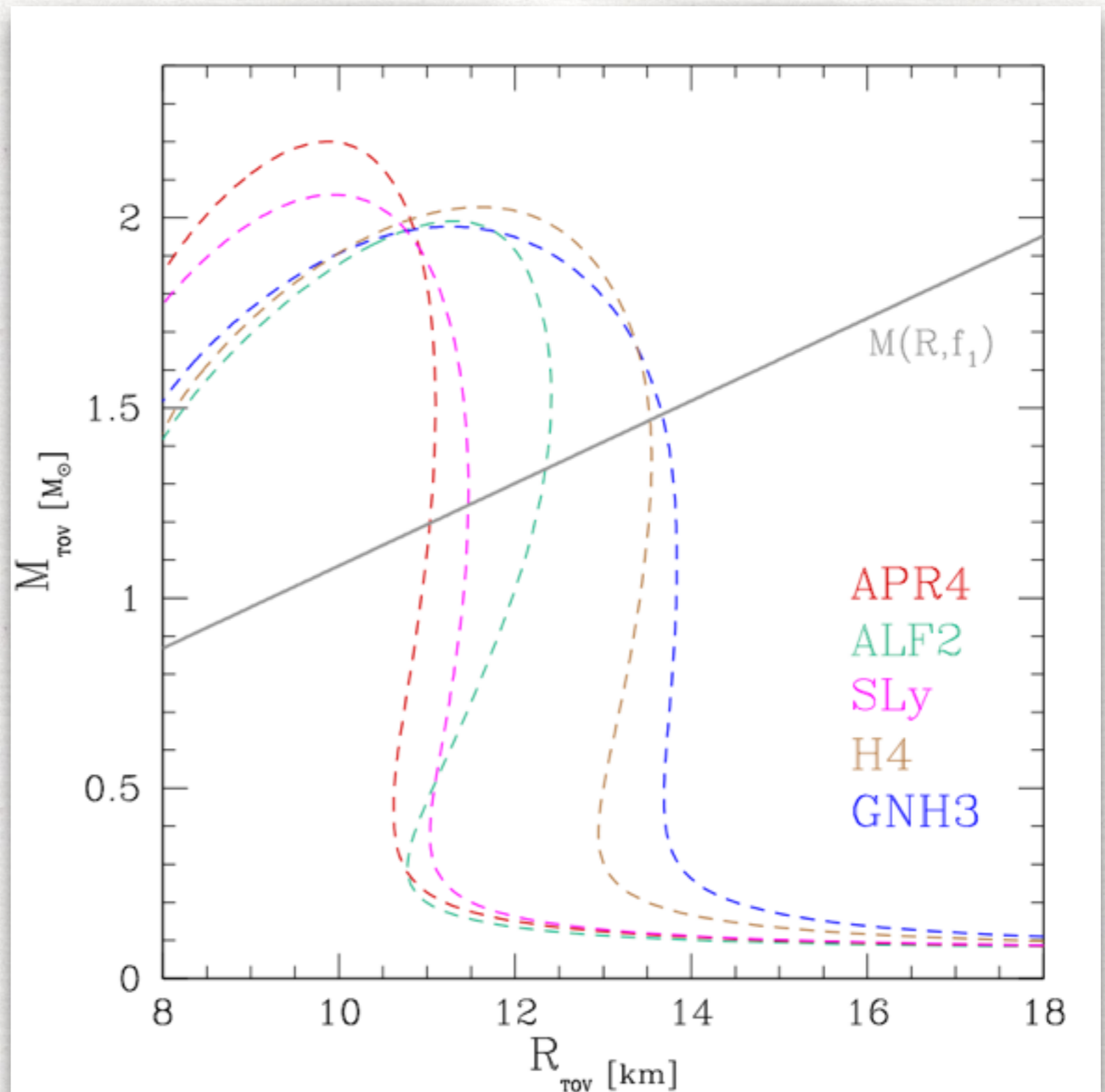
Consider your best choices as candidate EOSs



An example: use the $M(R, f_1)$ relation

The measure of the f_1 peak will fix a $M(R, f_1)$ relation and hence a **single** line in the (M, R) plane.

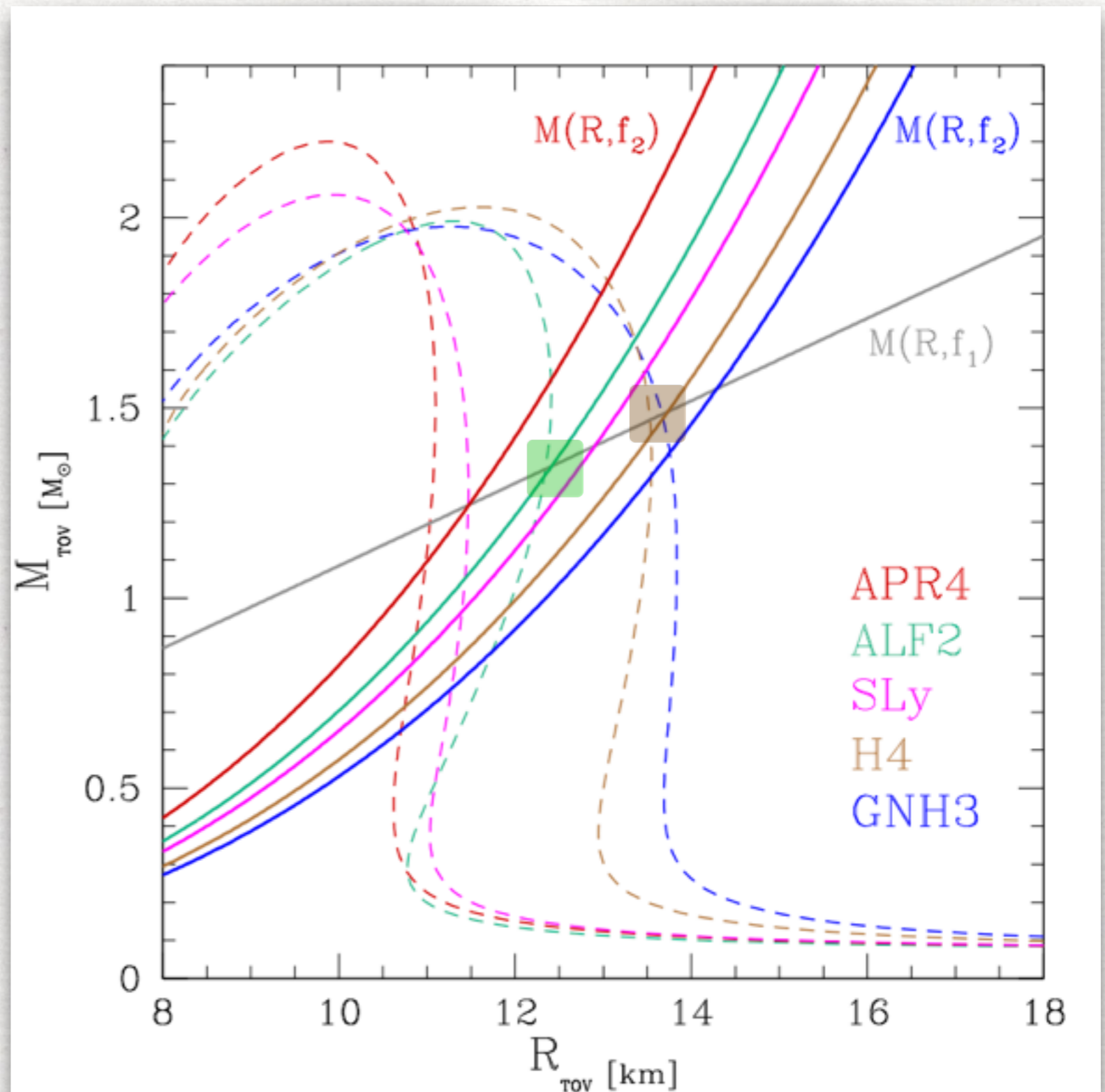
All EOSs will have **one** constraint (crossing)



An example: use the $M(R, f_2)$ relations

The measure of the f_2 peak will fix a relation $M(R, f_2, EOS)$ for each EOS and hence a **number** of lines in the (M, R) plane.

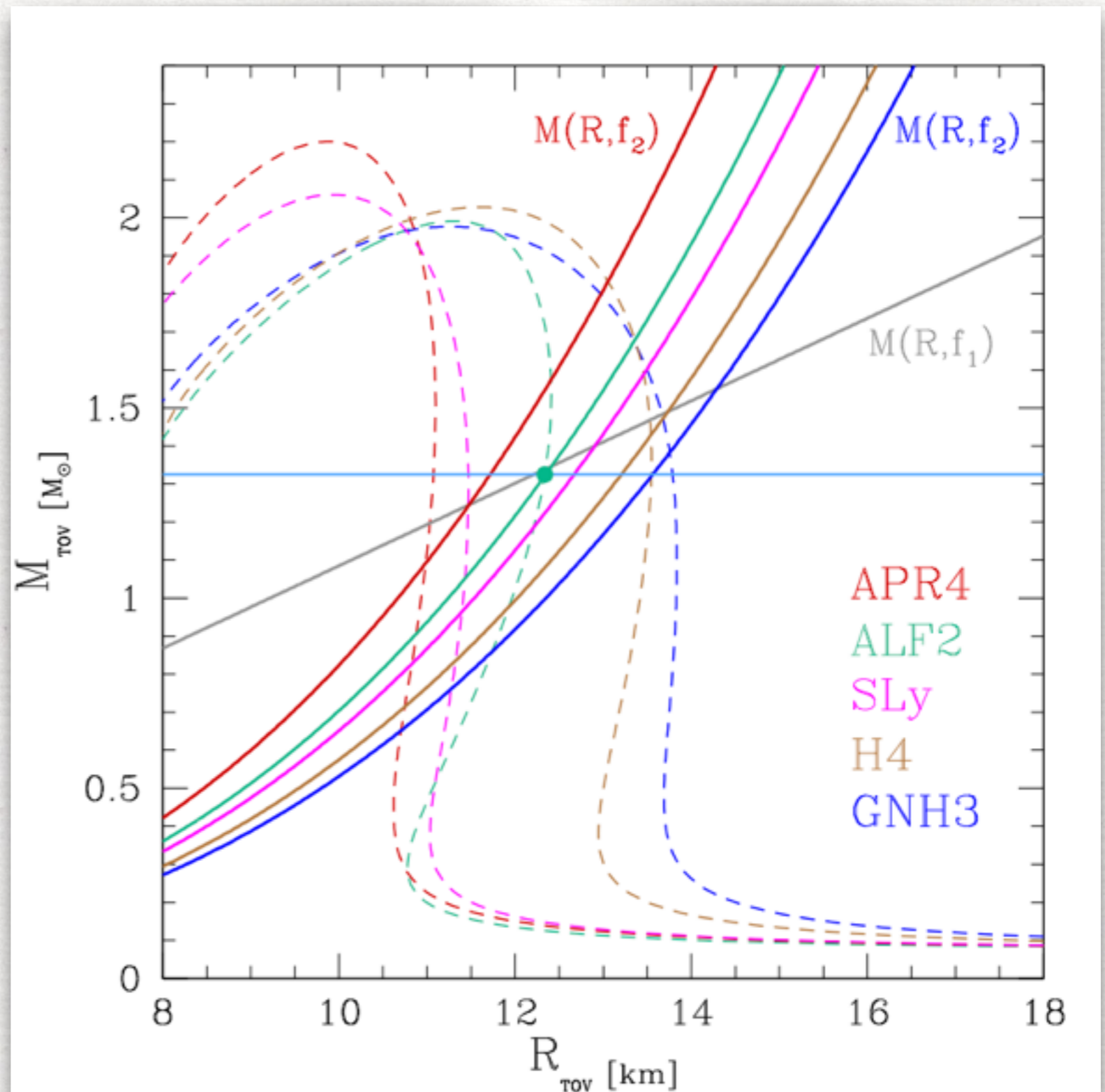
The right EOS will have **three** different constraints (APR, GNH3, SLy excluded)



An example: use measure of the mass

If the mass of the binary is measured from the inspiral, an additional constraint can be imposed.

The right EOS will have **four** different constraints. Ideally, a single detection would be sufficient.

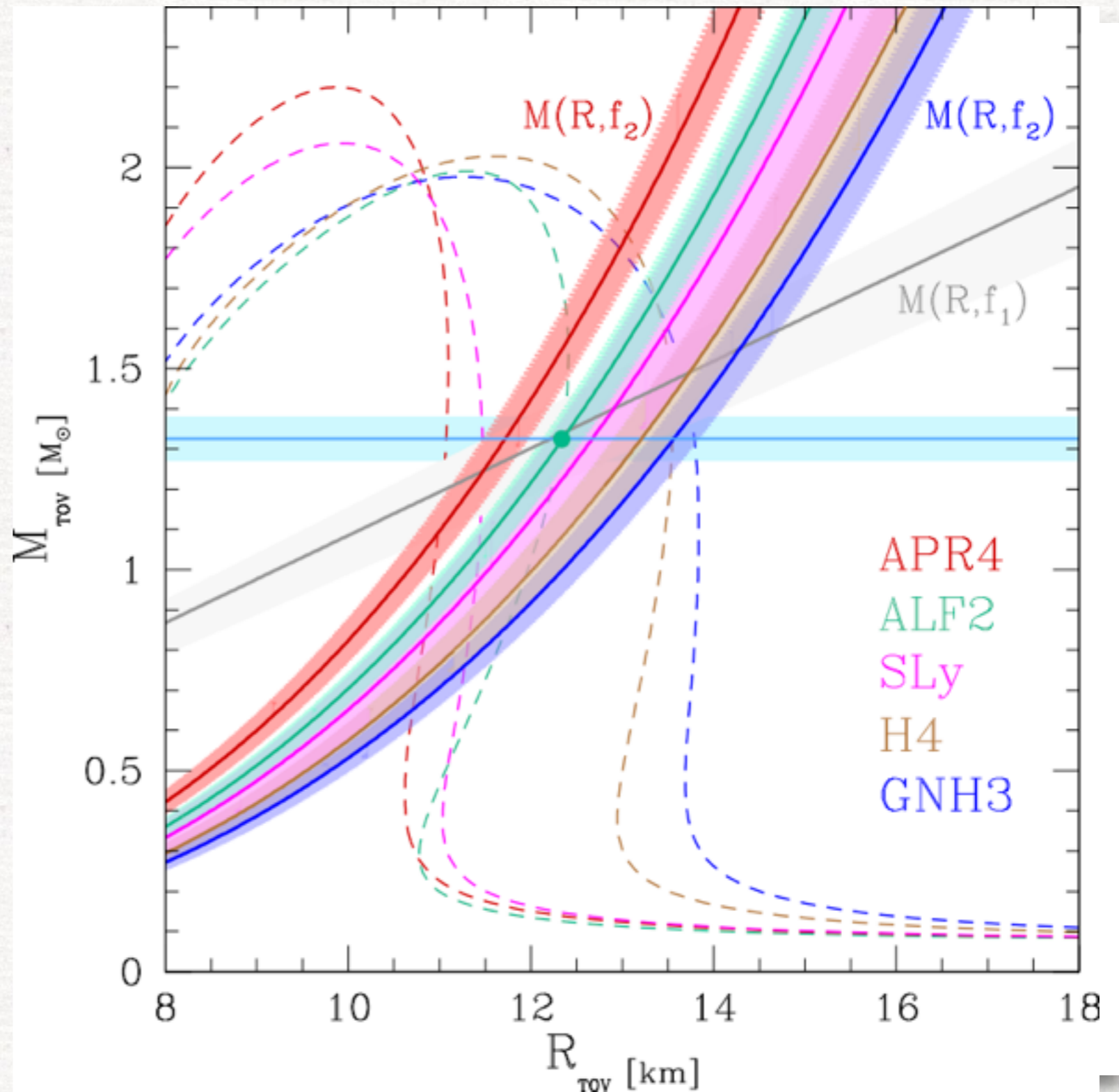


This works for all EOSs considered

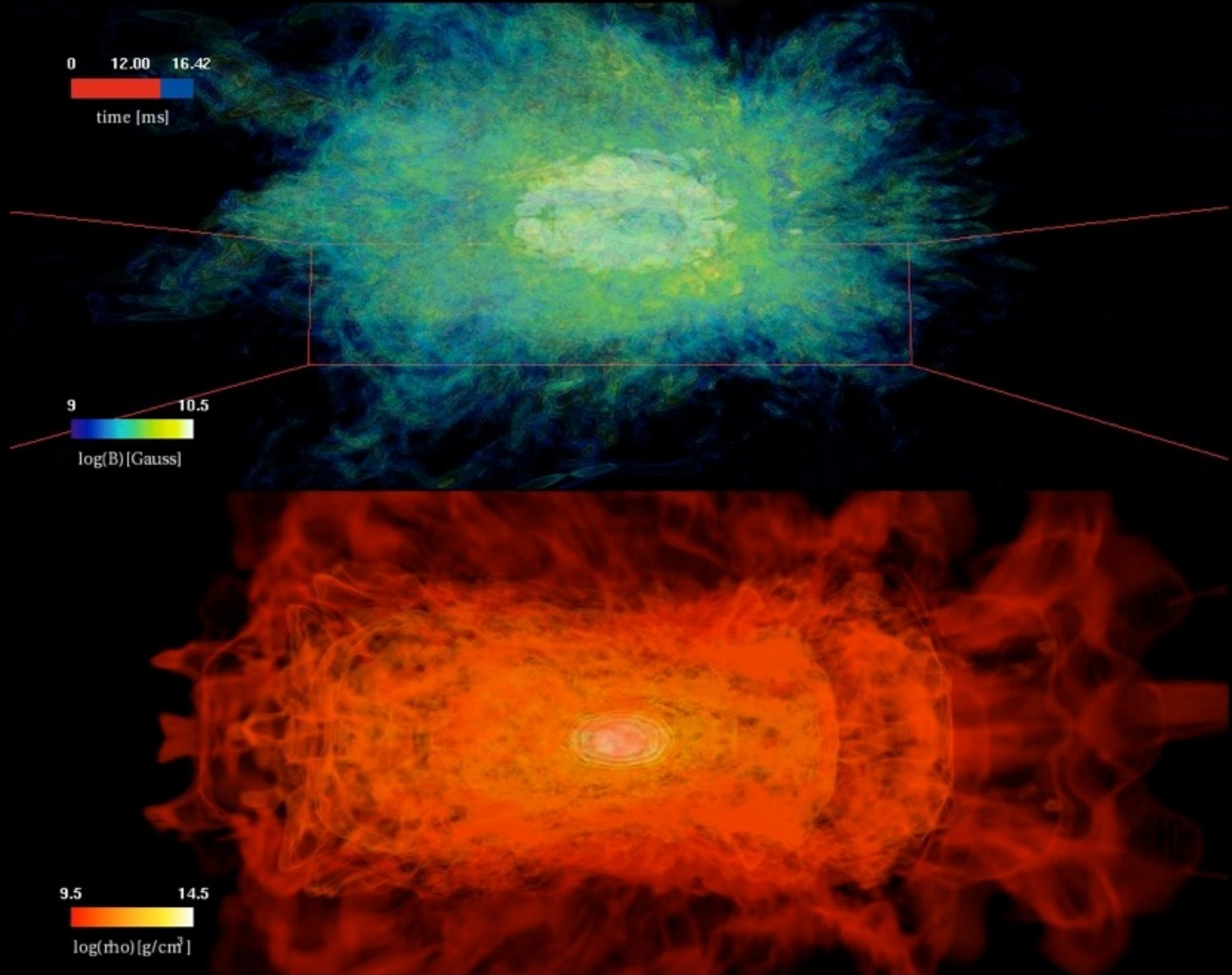
In reality things will be more complicated. The **lines** will be **stripes**; Bayesian probability to get precision on M , R .

Some numbers:

- at 50 Mpc, freq. uncertainty from Fisher matrix is 100 Hz
- at SNR=2, the event rate is 0.2-2 yr⁻¹ for different EOSs.



The role of magnetic fields



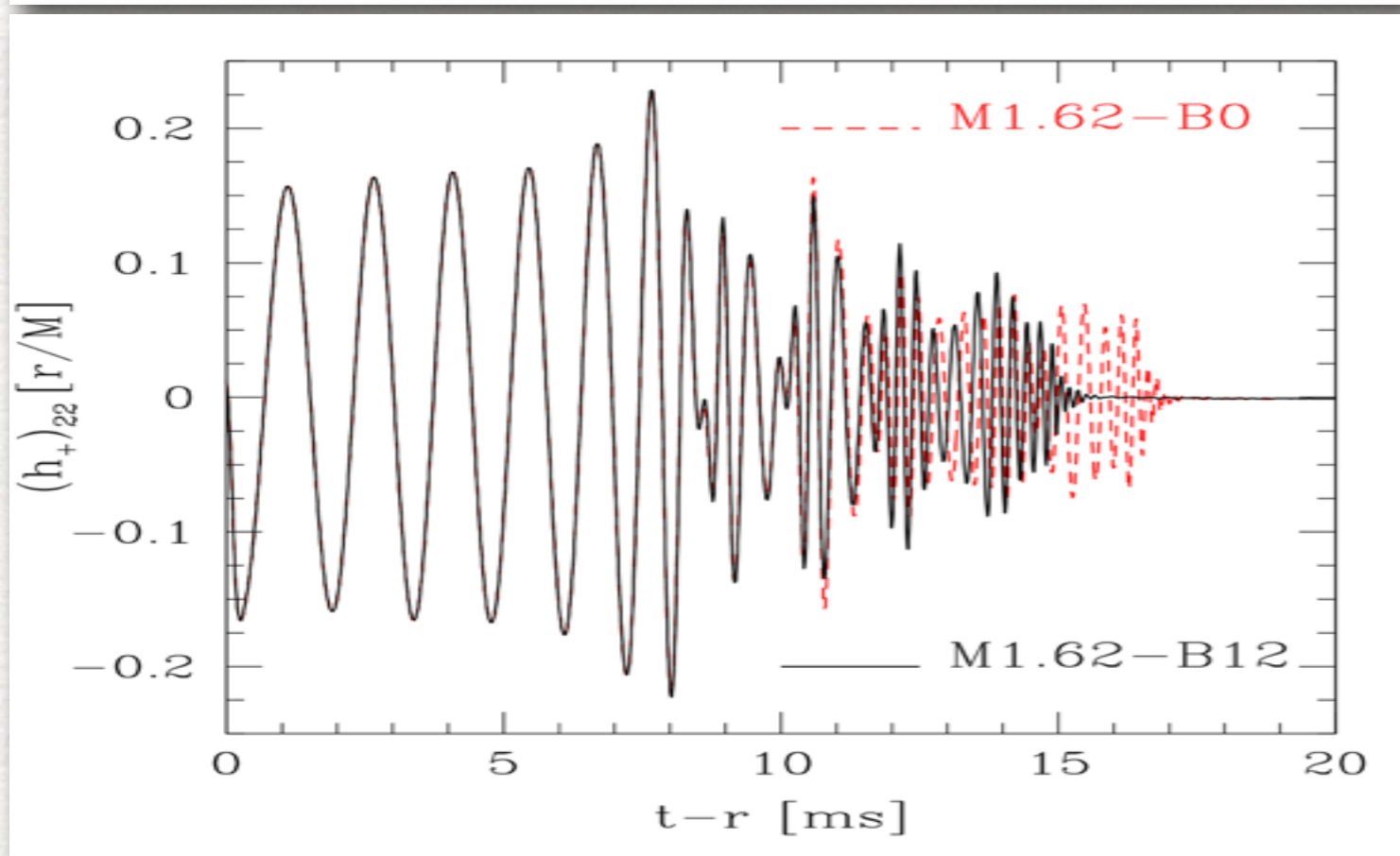
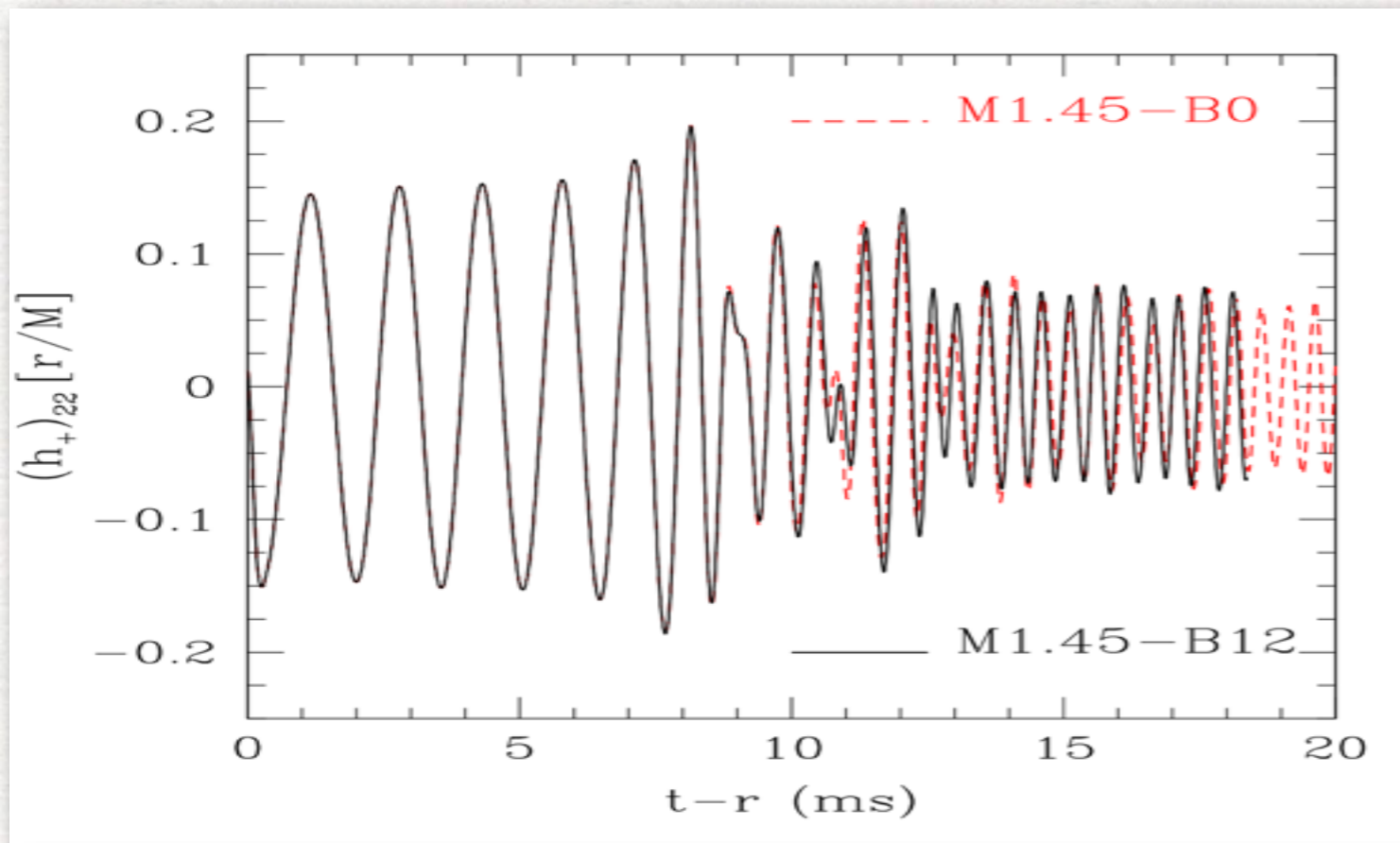
Ideal Magnetohydrodynamics

Most simulations to date make use of **ideal MHD**: conductivity is infinite and magnetic field simply advected.

You can ask some simple questions.

- can B-fields be detected during the inspiral?
- can B-fields be detected in the HMNS?
- can B-fields grow after BH formation?

Waveforms: comparing against magnetic fields



Compare B/no-B field:

- the evolution in the **inspiral** is different but only for ultra large B-fields (i.e. $B \sim 10^{17}$ G). For realistic fields the difference is not significant.

- the **post-merger** evolution is different for all masses; strong B-fields delay the collapse to BH

However, **mismatch** must be computed using detector sensitivity

Can we detect B-fields in the inspiral?

To quantify the differences and determine whether detectors will see a difference in the inspiral, we calculate the **overlap**

$$\mathcal{O}[h_{B1}, h_{B2}] \equiv \frac{\langle h_{B1} | h_{B2} \rangle}{\sqrt{\langle h_{B1} | h_{B1} \rangle \langle h_{B2} | h_{B2} \rangle}}$$

where the scalar product is

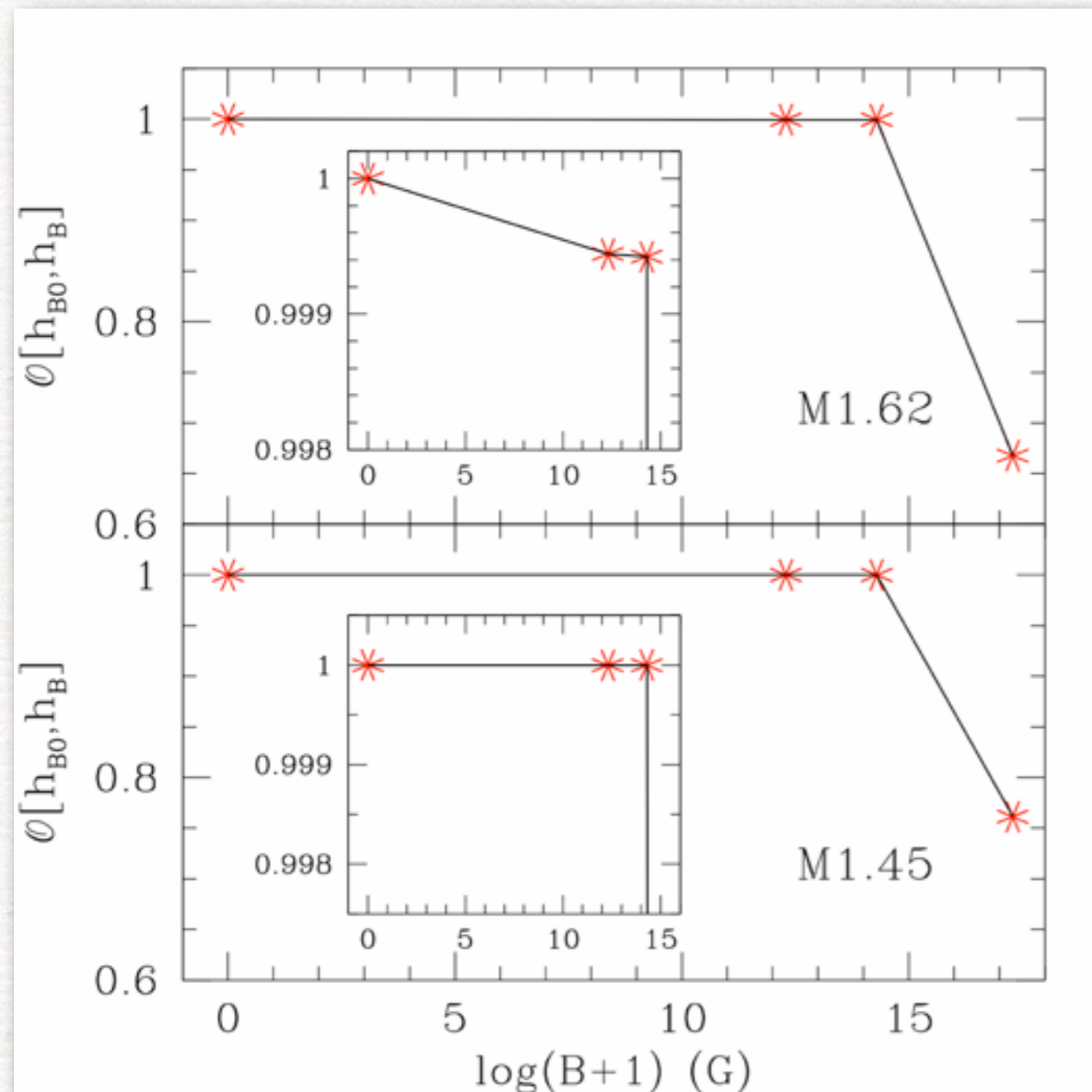
$$\langle h_{B1} | h_{B2} \rangle \equiv 4\Re \int_0^\infty df \frac{\tilde{h}_{B1}(f) \tilde{h}_{B2}^*(f)}{S_h(f)}$$

In essence, at these res:

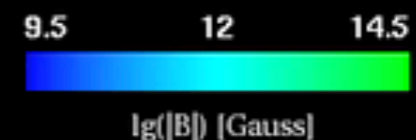
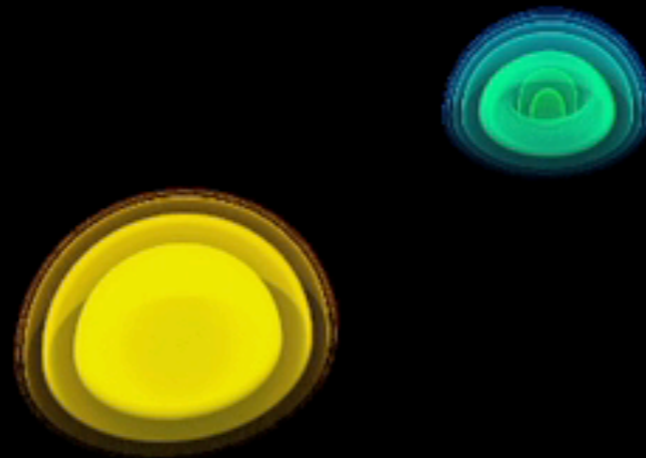
$$\mathcal{O}[h_{B0}, h_B] \gtrsim 0.999$$

$$\text{for } B \lesssim 10^{17} \text{ G}$$

Influence of B-fields on inspiral is **unlikely to be detected**



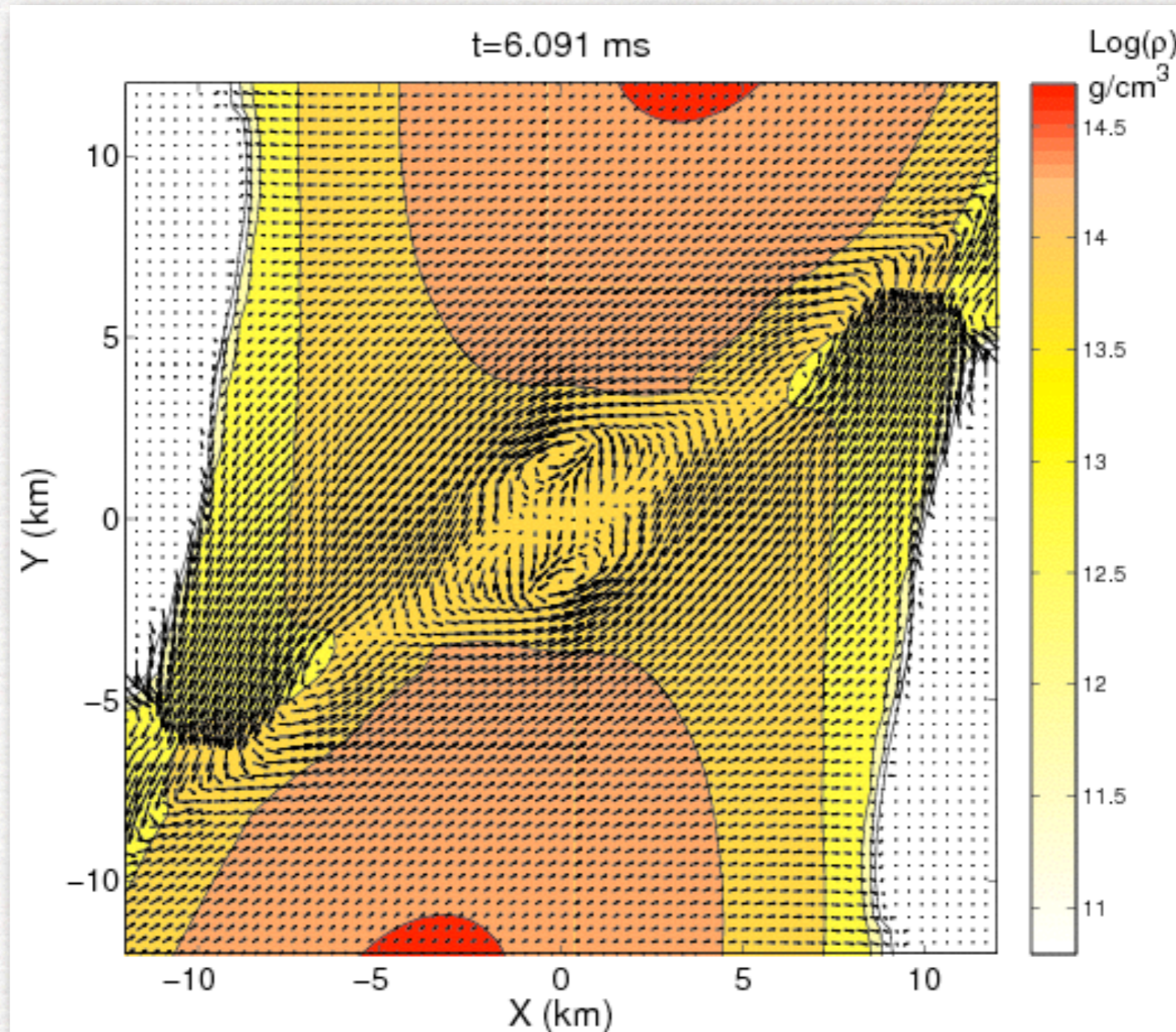
Typical evolution for a magnetized binary
(hot EOS) $M = 1.5 M_{\odot}$, $B_0 = 10^{12}$ G



Animations: LR, Koppitz

MHD instabilities and B-field amplifications

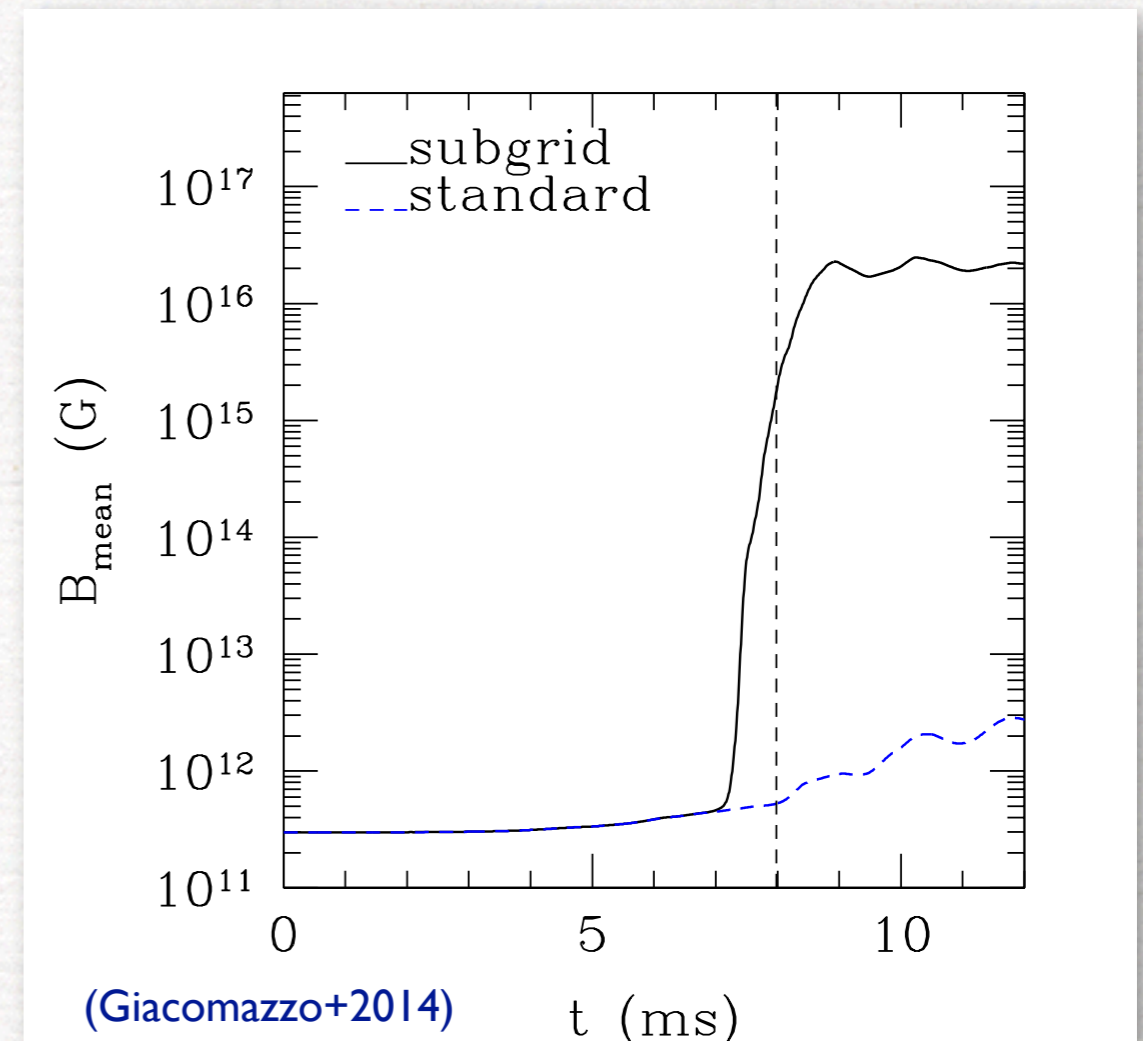
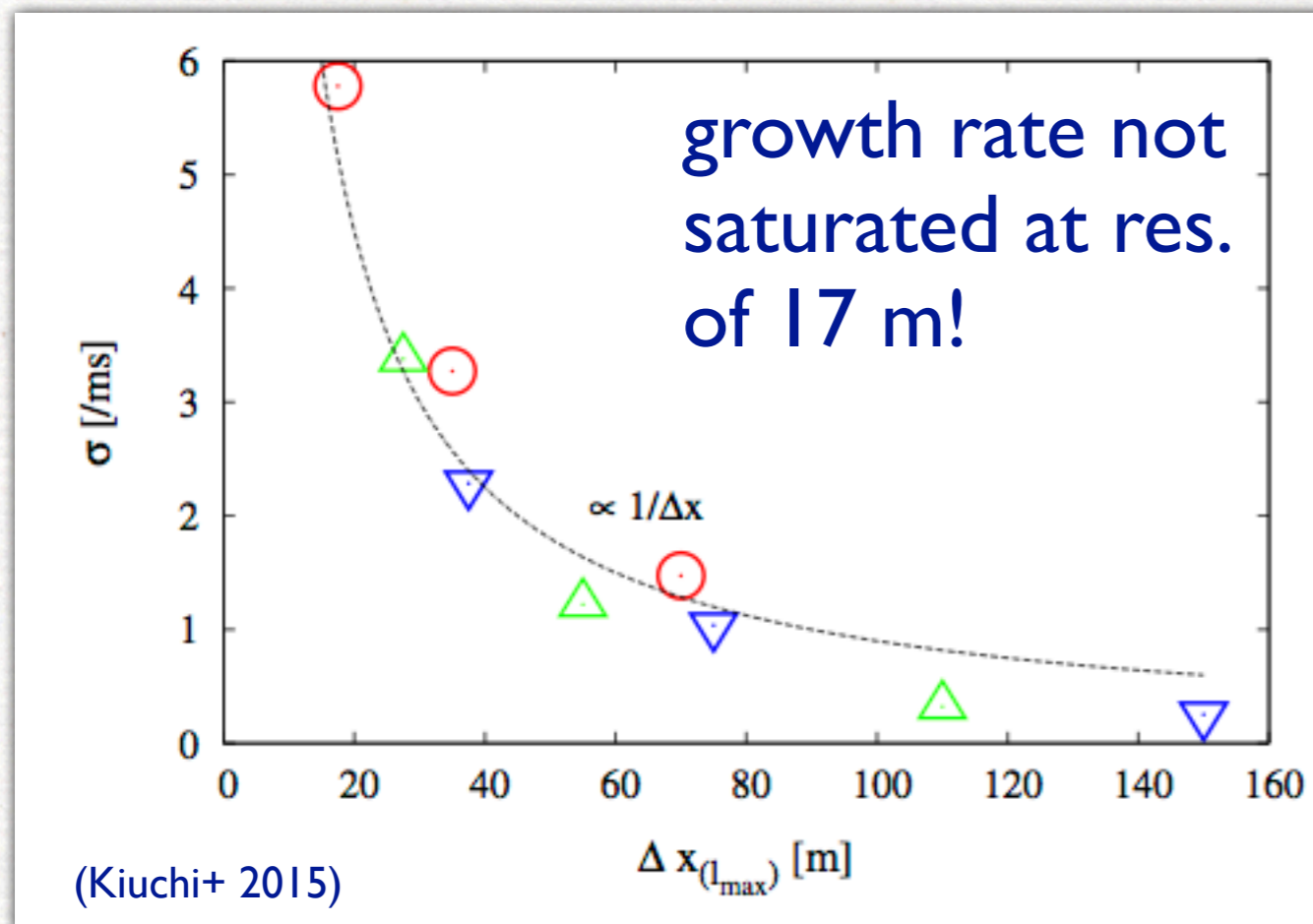
- at the merger, the NS create a strong shear layer which could lead to a **Kelvin-Helmholtz instability**; magnetic field can be amplified



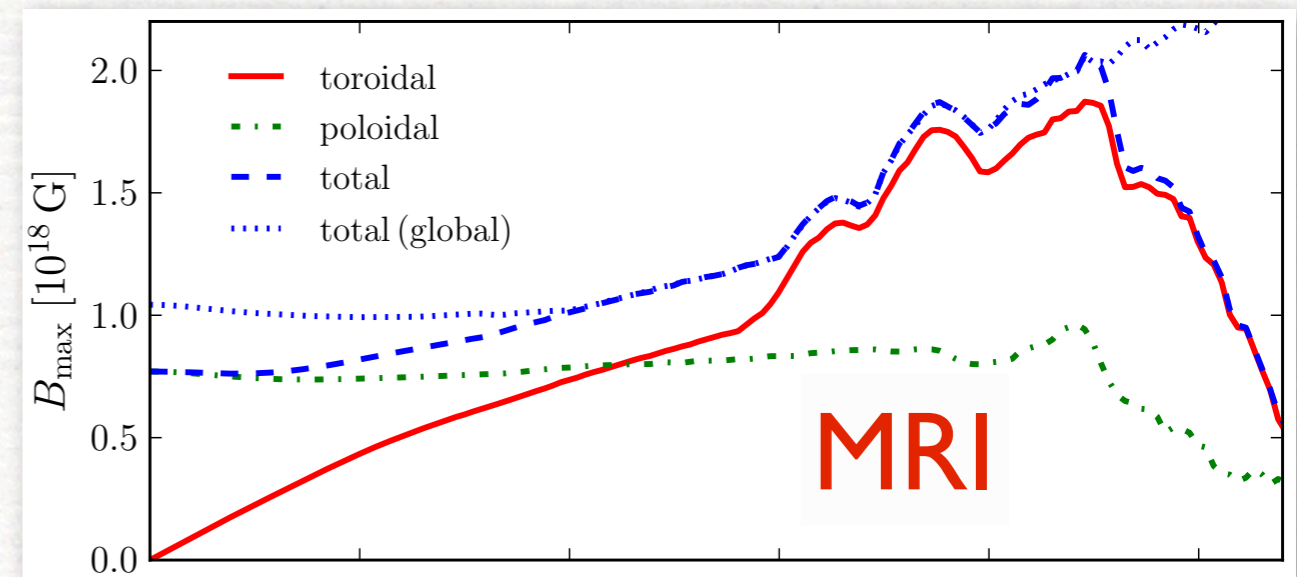
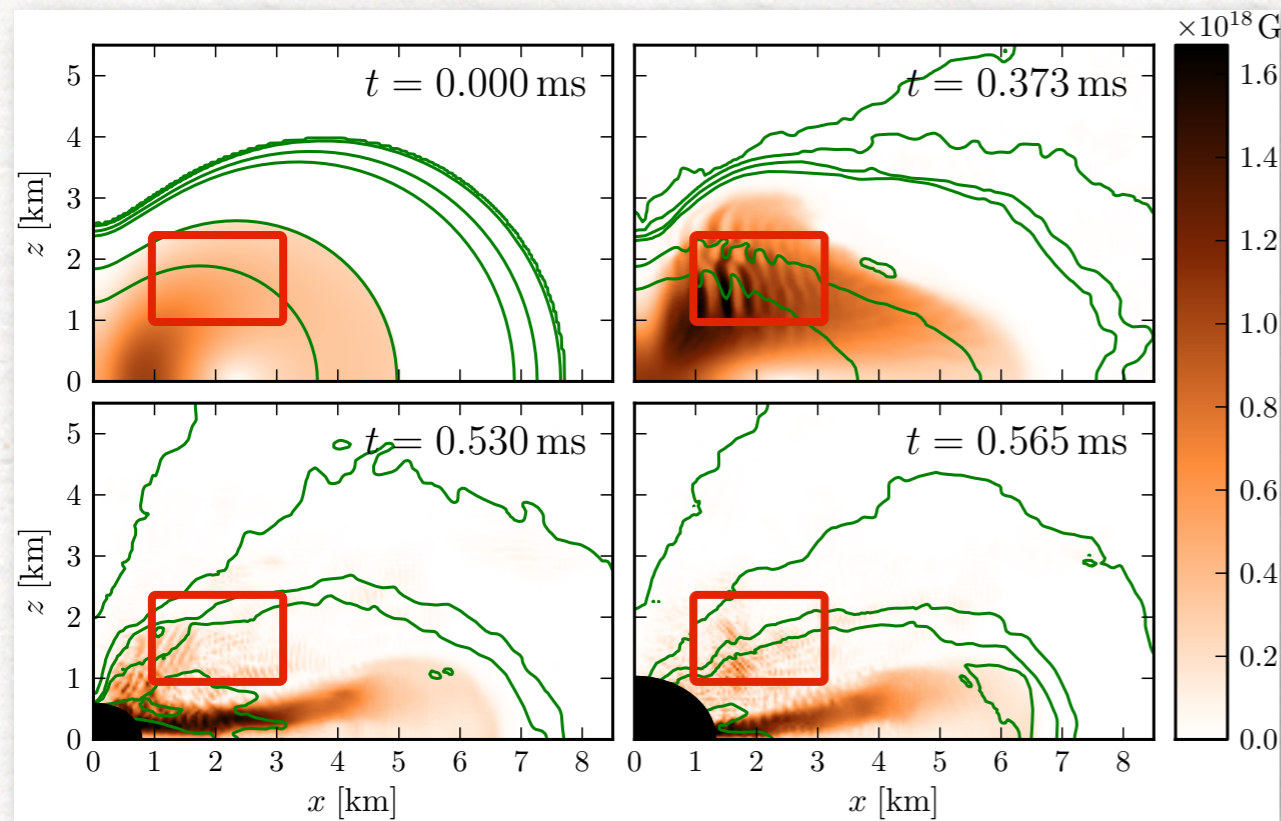
(Baiotti+2008)

MHD instabilities and B-field amplifications

- at the merger, the NS create a strong shear layer which could lead to a **Kelvin-Helmholtz instability**; magnetic field can be amplified
- low-res simulations don't show exponential growth (Giacomazzo+2011)
high-res simulations show increase of ~ 3 orders of mag (Kiuchi+2015)
- sub-grid models suggest B-field grows to 10^{16} G (Giacomazzo+2014)



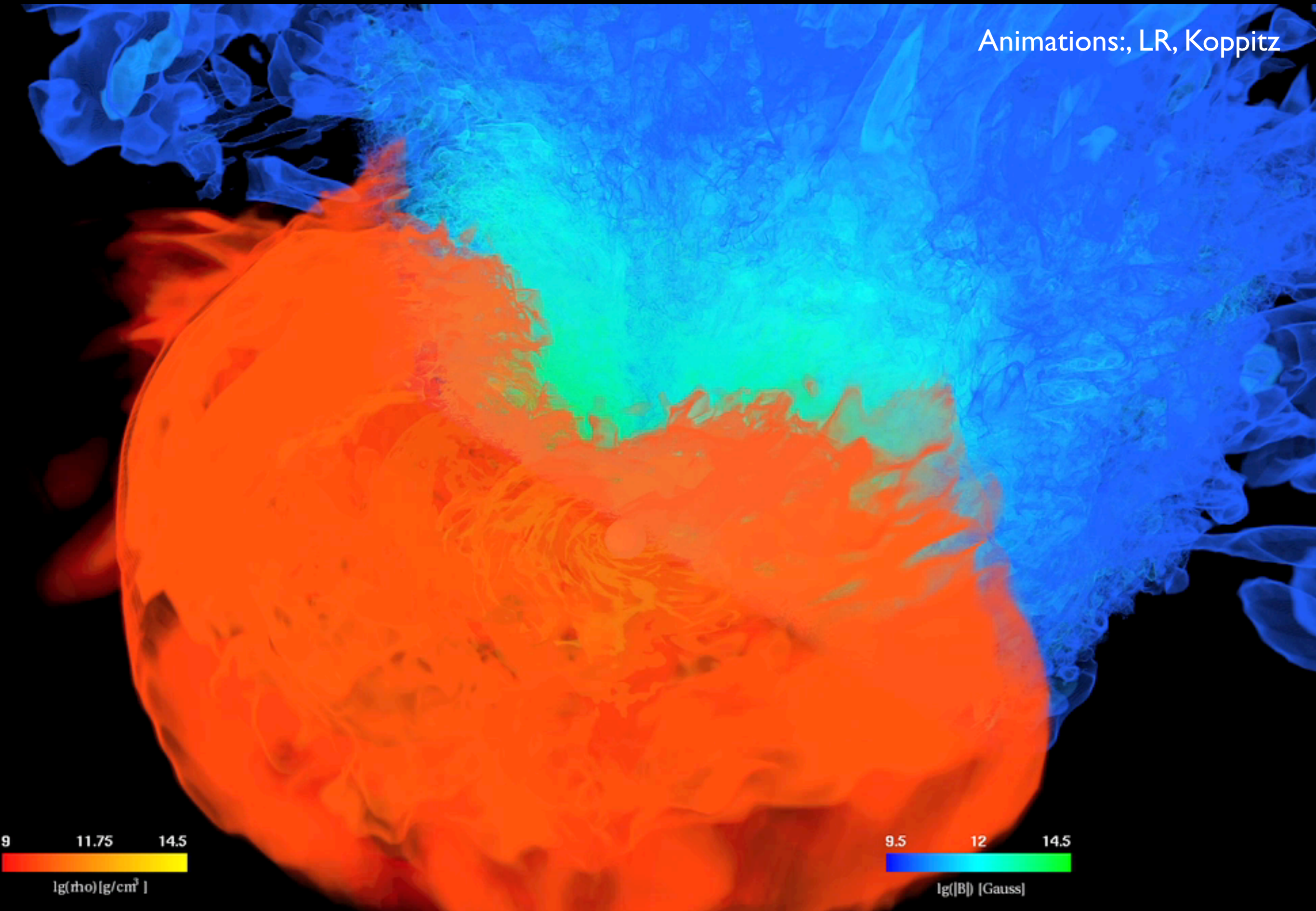
MHD instabilities and B-field amplifications

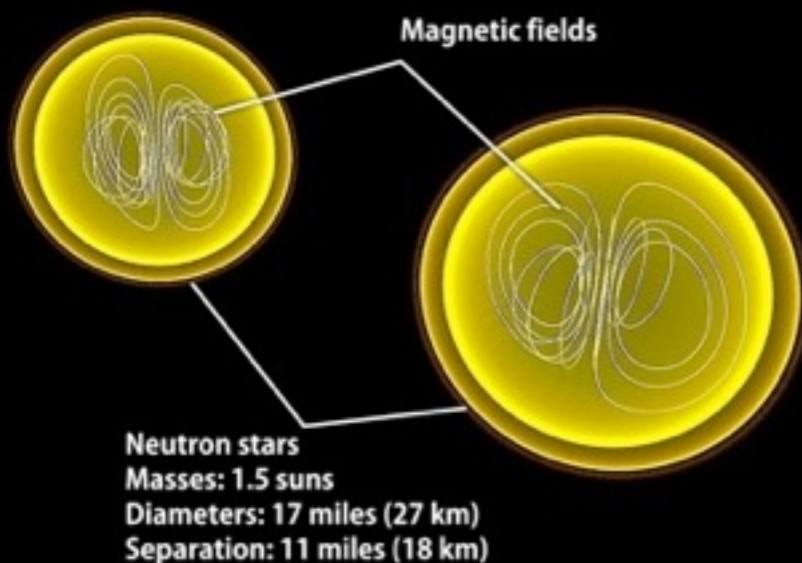


Siegel+2013

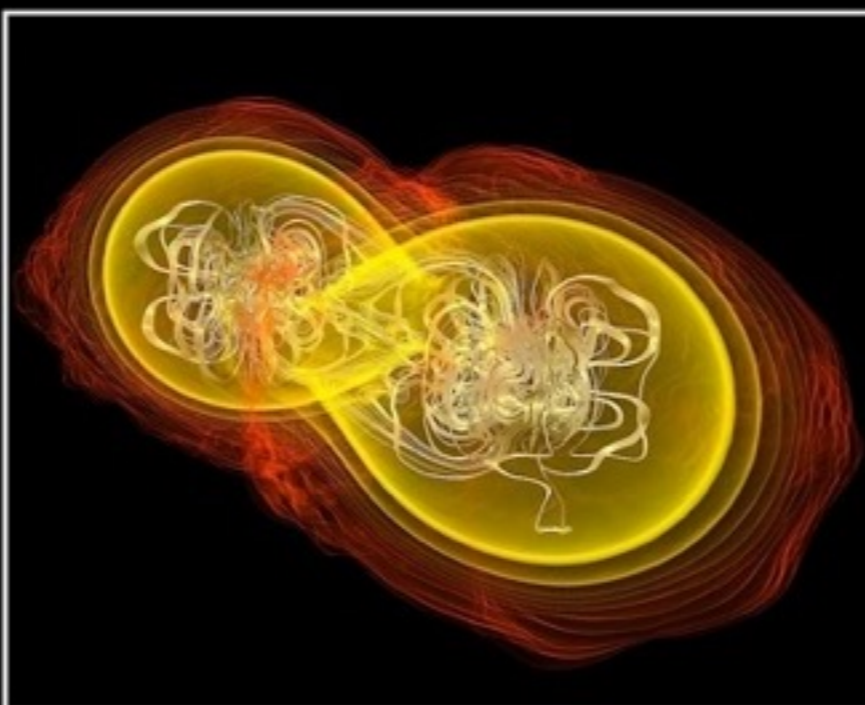
- differentially rotating magnetized fluids develop the **MRI** (**magnetorotational instability**; Velikhov 1959, Chandrasekhar 1960)
- the MRI leads to exponential growth of B-field and to an outward transfer of angular momentum: responsible for accretion in discs
- overall, consensus MRI **can develop** in HMNS (Siegel+2013, Kiuchi+2014)
- degree of amplification is unknown: 2-3 or 5-6 orders of magnitude? What about resistivity? (Kiuchi+2015, Obergaulinger+2015)

Animations:, LR, Koppitz

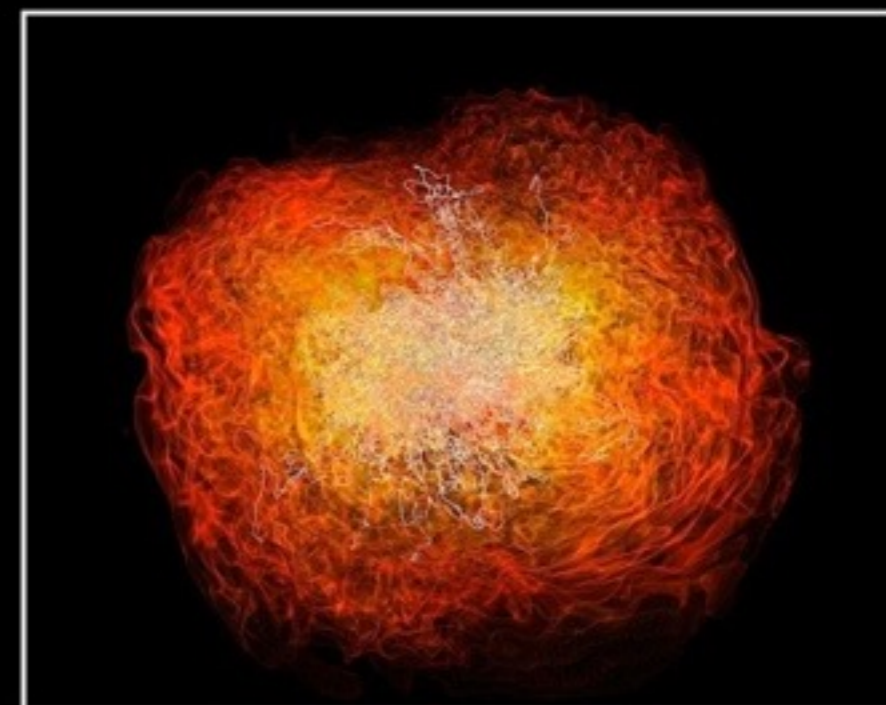




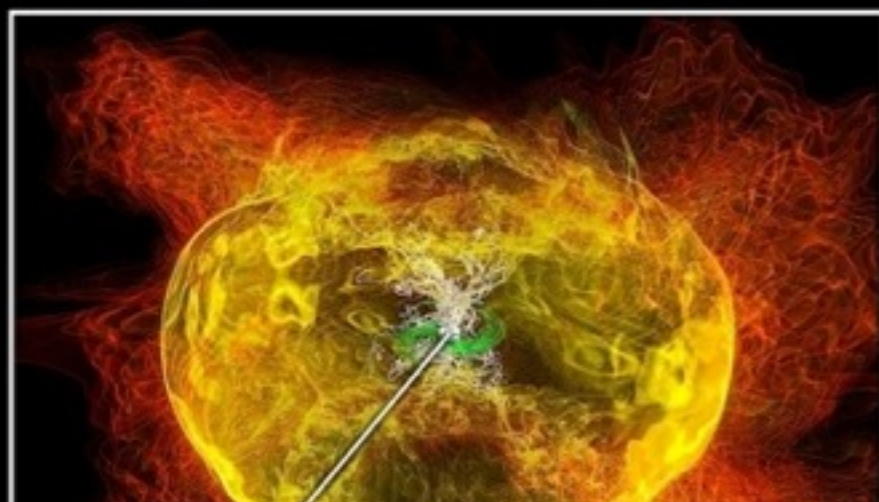
Simulation begins



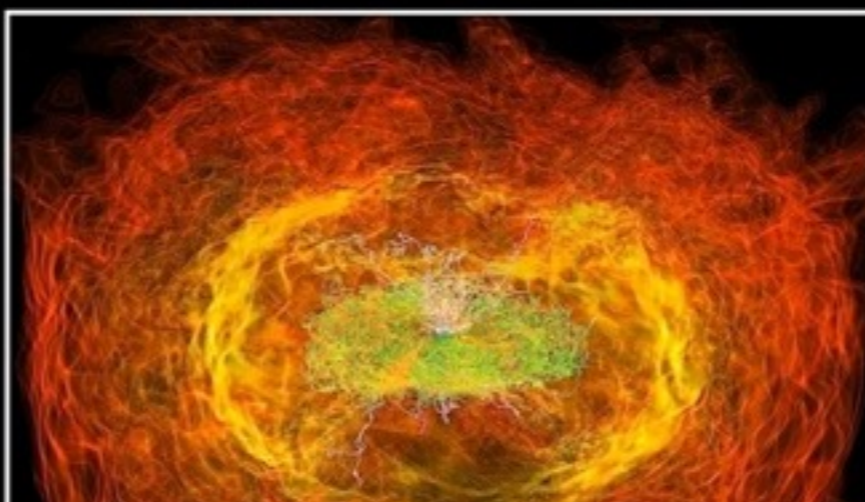
7.4 milliseconds



13.8 milliseconds



Black hole forms
Mass: 2.9 suns
Horizon diameter: 5.6 miles (9 km)



16.2 milliseconds



16.2 milliseconds

These simulations have shown that the merger of a magnetised binary has all the basic features behind SGRBs

$$J/M^2 = 0.83$$

$$M_{\text{tor}} = 0.063 M_{\odot}$$

$$t_{\text{accr}} \simeq M_{\text{tor}}/\dot{M} \simeq 0.3 \text{ s}$$

Resistive Magnetohydrodynamics

Dionysopoulou, Alic, LR (2015)

- Ideal MHD is a good approximation in the inspiral, but not after the merger; match to **electro-vacuum** not possible.
- Main difference in resistive regime is the current, which is dictated by Ohm's law but microphysics is **poorly** known.
- We know conductivity σ is a **tensor** and proportional to density and inversely proportional to temperature.
- A simple prescription with scalar (isotropic) conductivity:

$$J^i = qv^i + W\sigma[E^i + \epsilon^{ijk}v_j B_k - (v_k E^k)v^i],$$

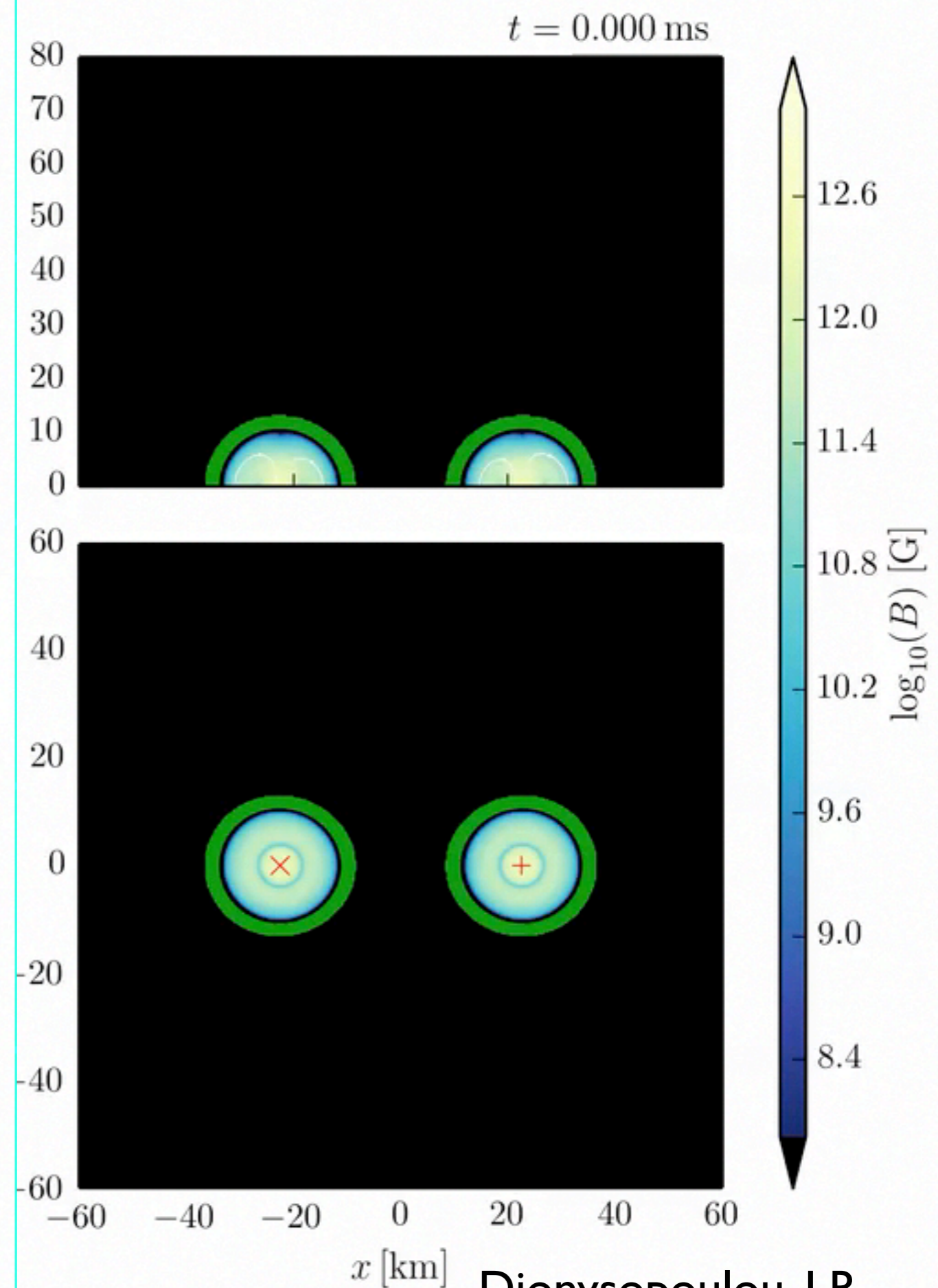
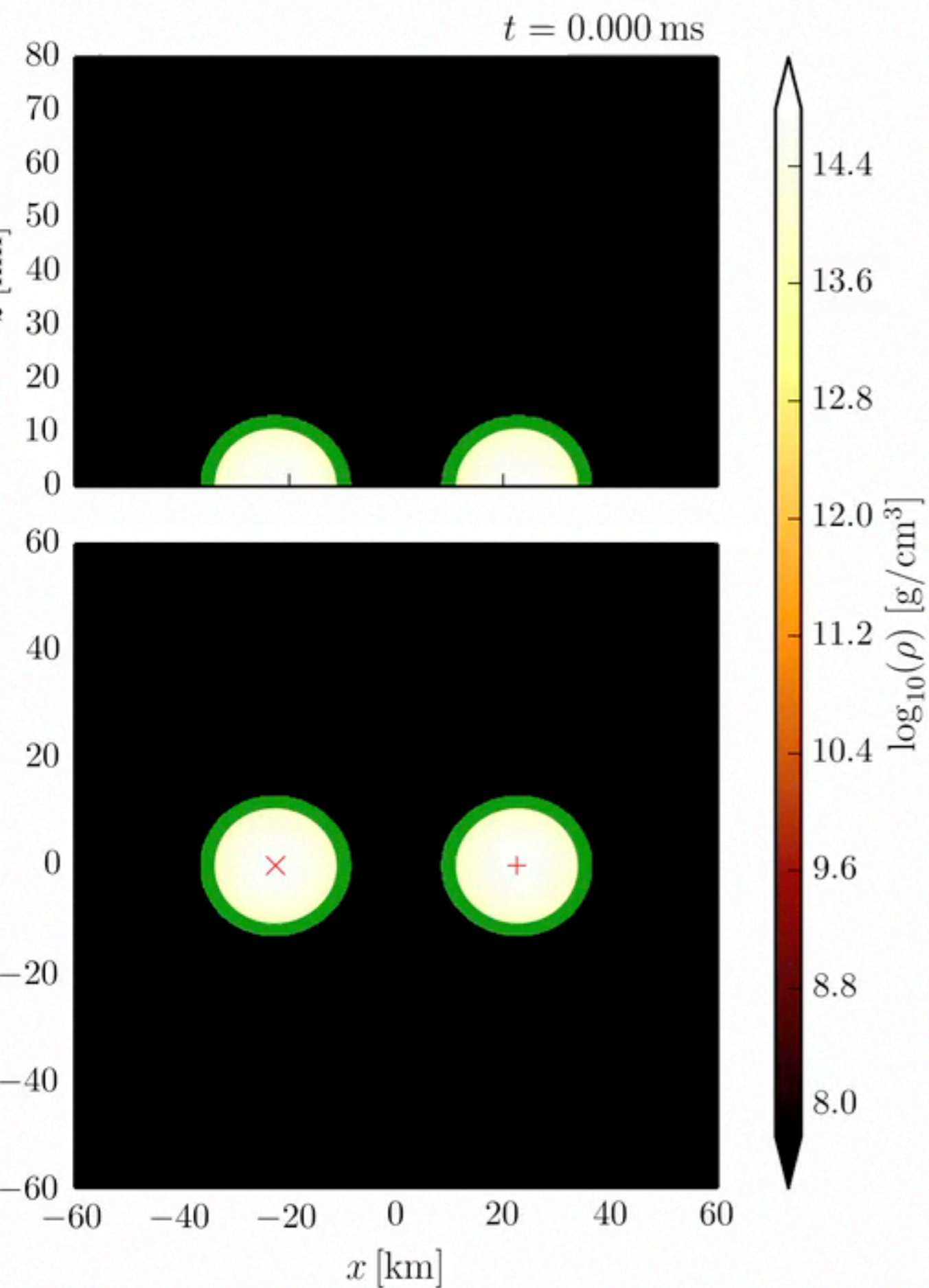
$\sigma \rightarrow \infty$ ideal-MHD (IMHD)

$\sigma \neq 0$ resistive-MHD (RMHD)

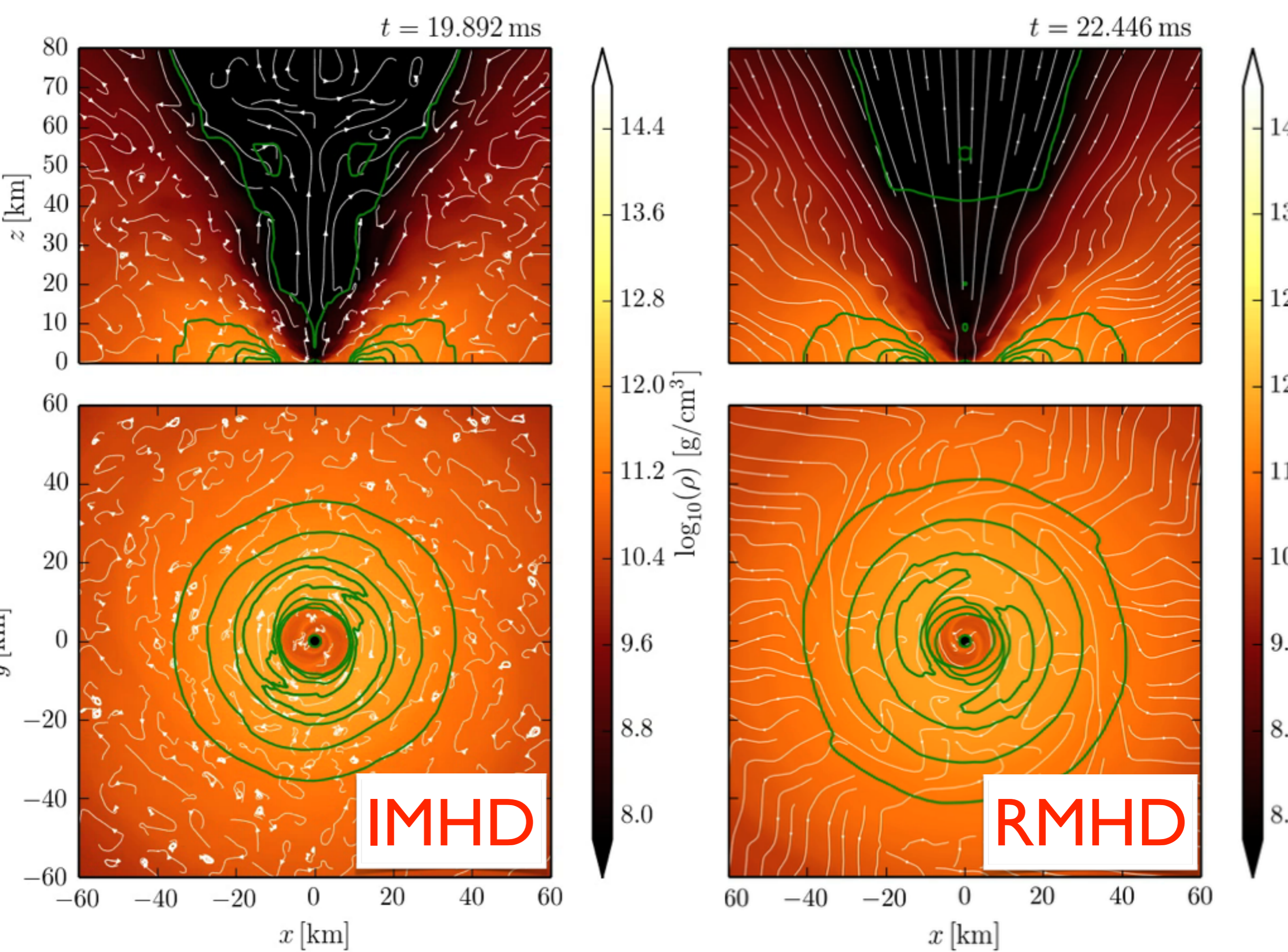
$\sigma \rightarrow 0$ electrovacuum

$$\sigma = f(\rho, \rho_{\min})$$

phenomenological prescription

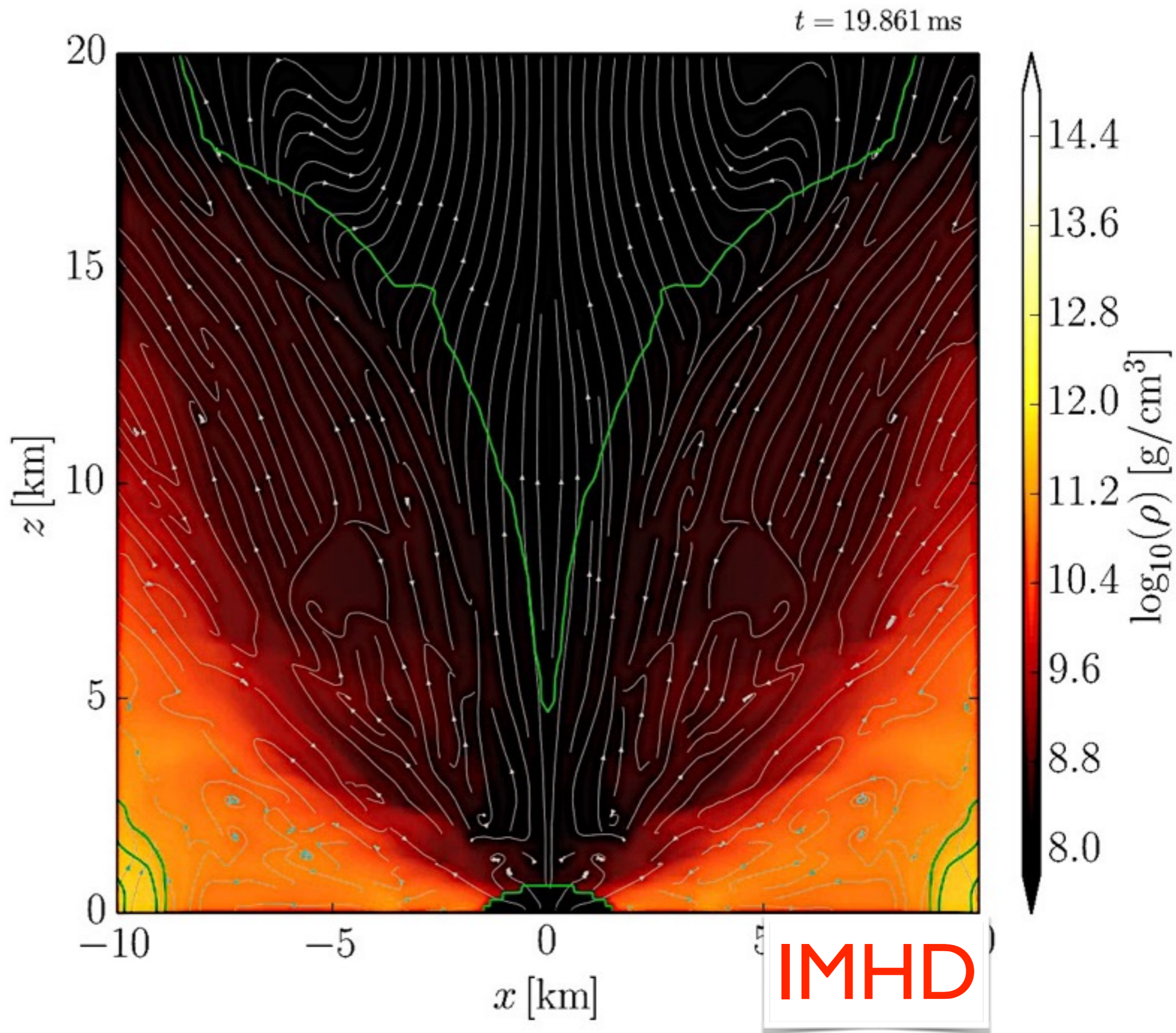


Dionysopoulou, LR



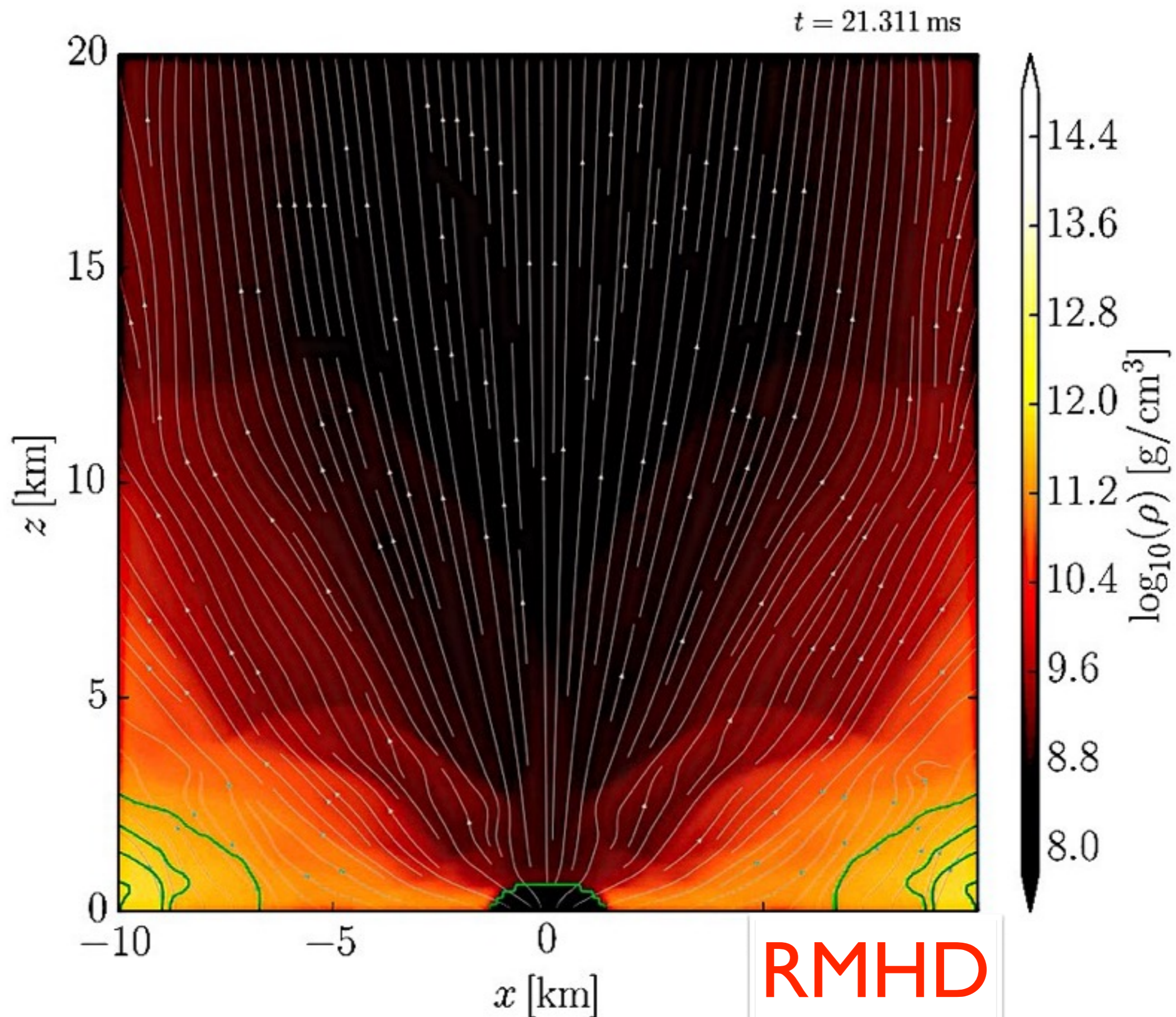
NOTE: the magnetic jet structure is not an outflow. It's a plasma-confining structure.

In IMHD the magnetic jet structure is present but less regular.



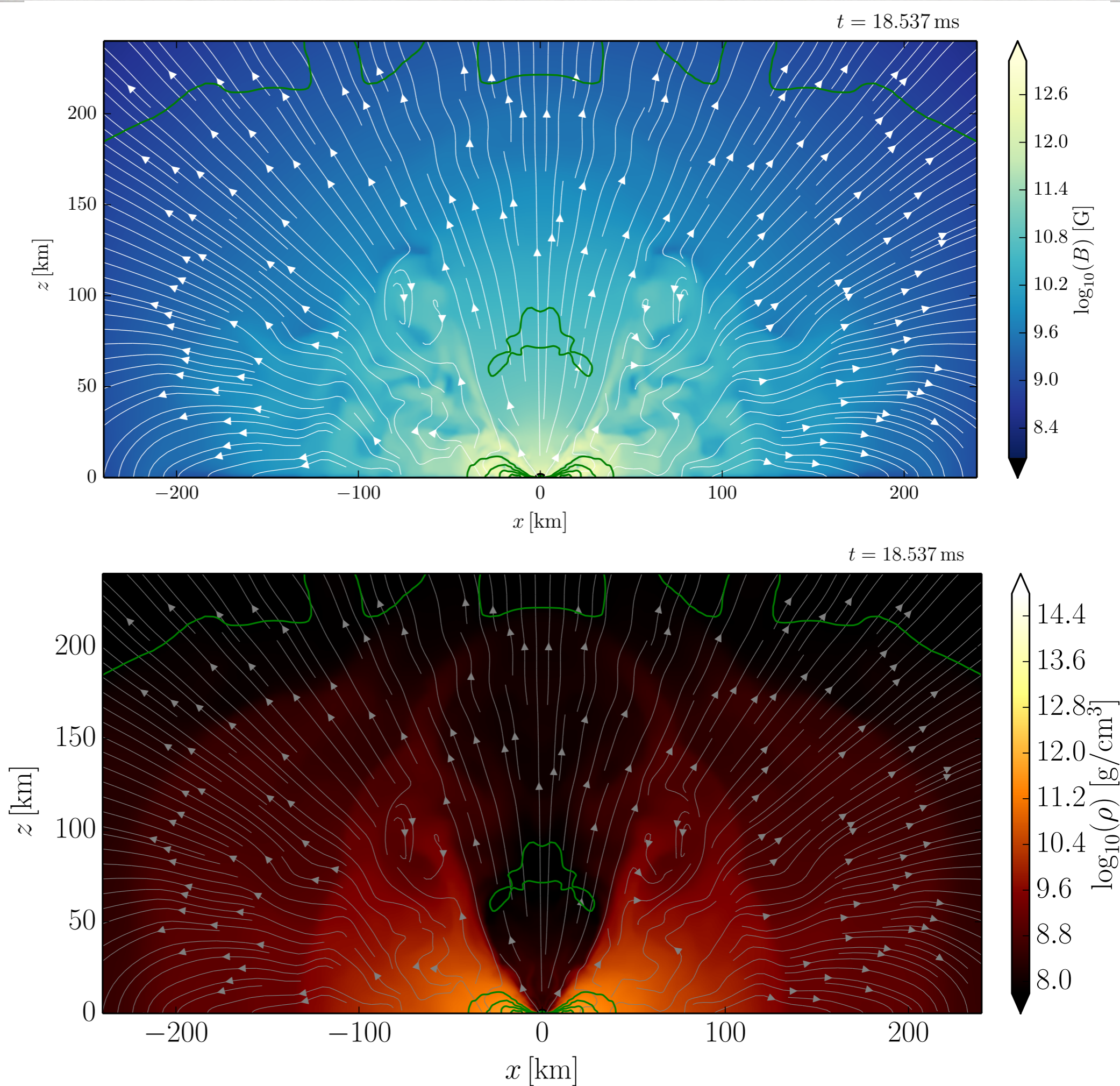
NOTE: the **magnetic jet structure** is **not** an **outflow**. It's a plasma-confining structure.

In **RMHD** the magnetic jet structure is present from the scale of the horizon (res.: $h \sim 150\text{m}$).



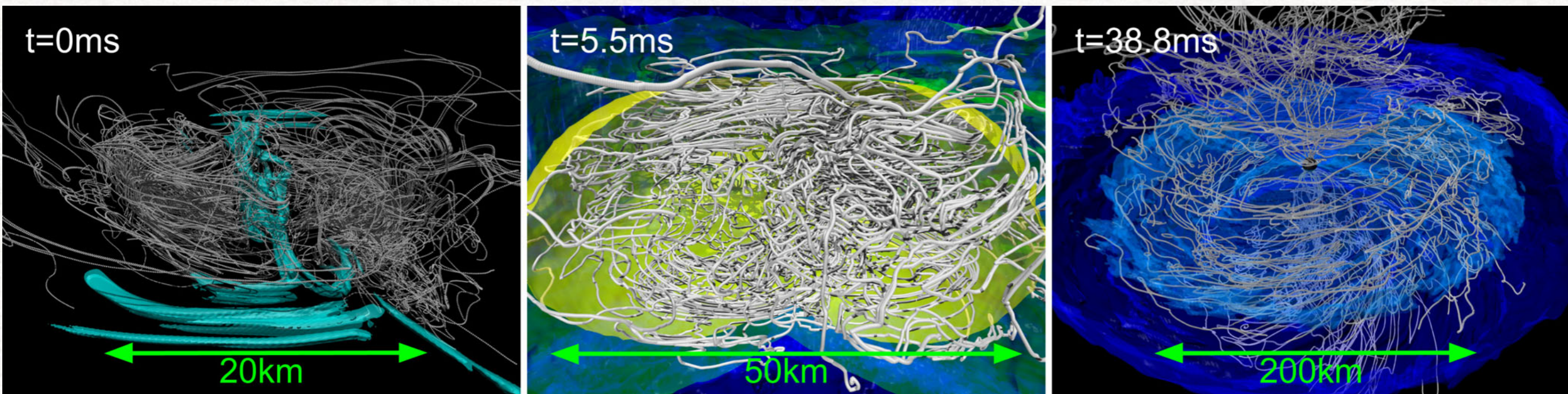
The magnetic jet structure maintains its coherence up to the largest scale of the system.

RMHD

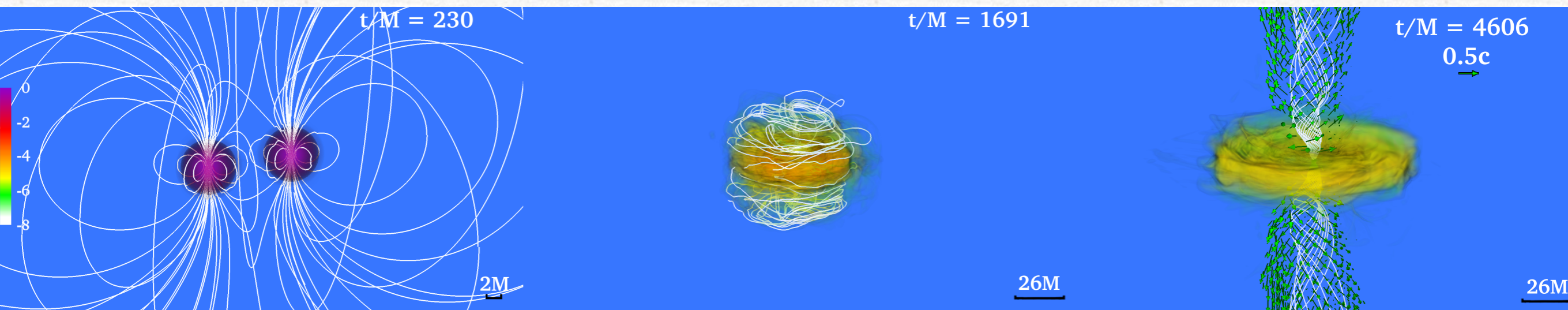


Results from other groups (IMHD only)

With due differences, other groups confirm this picture.



Kiuchi+ 2014



Ruiz+ 2016

Conclusions

- * Modelling of binary NSs in full GR is **mature**: GWs from the inspiral can be computed with precision of binary BHs.
- * GW spectrum shows clear peaks; some are **"quasi-universal"**.
- * If observed, post-merger signal will set tight constraints on EOS.
- * B-fields unlikely to be detected in the inspiral but **important** after the merger: lead to instabilities and EM counterparts.
- * Magnetic jet structure develops, both in IMHD and RMHD; an outflow is possible but not observed in general.

Progress has been huge and general picture is reasonable.

“Details” (turbulence, B-field amplification, dynamo, central engine) much harder.