Stability of Astrophysical Jets – Steps Towards Greater Realism

By

Dinshaw S. Balsara and Jinho Kim (Univ. of Notre Dame)

(dbalsara@nd.edu; jkim46@nd.edu)

Maxim Lyutikov (Purdue University)

Serguei Komissarov (University of Leeds)

Plan of the Talk:-

Introduction

Setting up the Problem – The Need for Greater Realism

Stability of Current-Sheet Free Jets with Top Hat Velocity Profile

Stability of Current-Sheet Free Jets with Sheared Velocity Profile

Conclusions

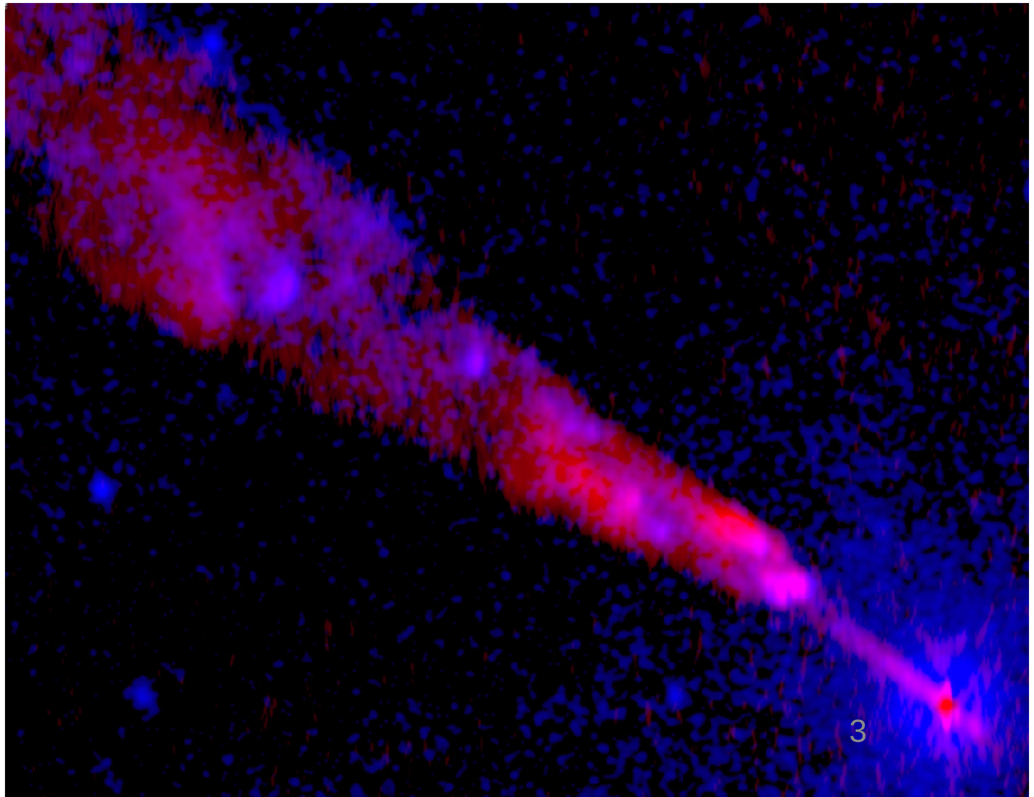
I) Introduction

Extragalactic; Protostellar; X-Ray Binaries; GRB all show evidence of jets

Amazingly stable – Jets propagate thousands to millions of jet radii.

Laboratory jet – destabilizes in ~20 jet radii

Cen A, Astrophysical jet – very stable



Possible Reasons for the Improved Stability of Astrophysical Jets:-

Effect of Environment (Porth & Komissarov 2015)

Magnetic Fields – Current Sheet Free Jets (Gourgouliatos *et al*. 2012)

Sheared Velocity Profile (Mizuno, Lyubarsky *et al*. 2012)

Realization that greater level of realism is needed in Stability Studies.

Older studies of jet stability did not include these effects; Why?

If some of these effects were included, other simplifying assumptions had to be made.
Eg. Zero pressure; Simple magnetic field geometry.

Simplifications were introduced because there was a great desire to use analytical functions, like Bessel functions, to carry out the stability analysis.

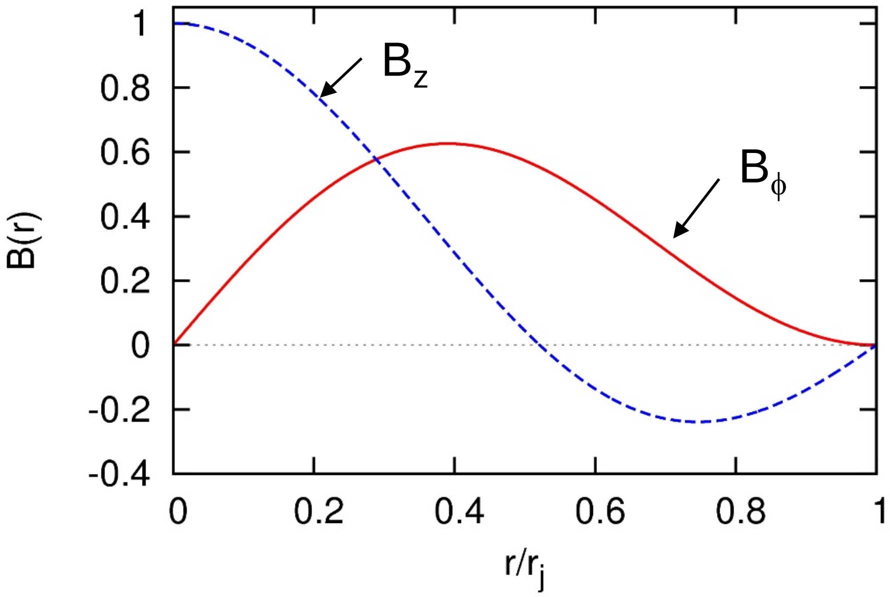
Goal Of This Study:- Include realistic magnetic-fields; realistic Shear; Non-Zero Pressure; Go beyond Analytic functions

II) Setting up the Problem:-

Part 1 of our linear stability analysis:-

Current-Sheet-Free Magnetic Fields

Parametrized by plasma β = gas pressure/mag. pressure



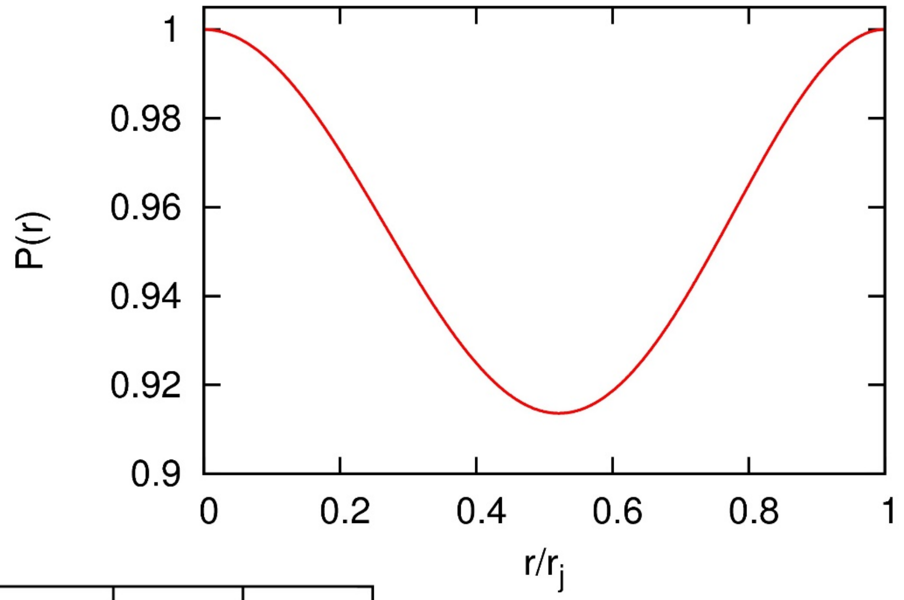
$\Rightarrow$

$\vec{B}$=0!

No current sheet at r_j

Realistic Pressure Profile (solve Grad-Shafranov)

Needed for Equilibrium

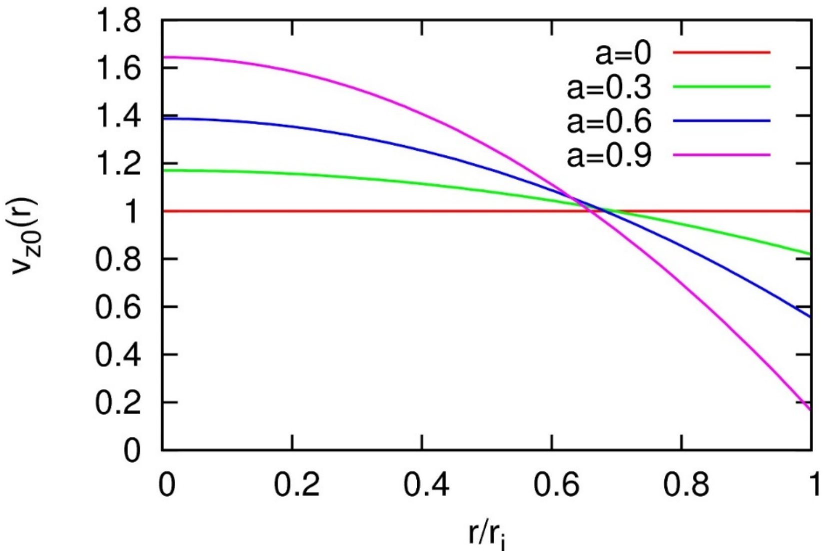


Part 2 of our linear stability analysis:-

Realistic shear profile introduced;

Shear parametrized by “a”

$$v_{z0}(r) = v_{z;\max}\left(1 - a\left(\frac{r}{r_j}\right)^2\right)$$



Linear analysis

1) Start with MHD equation:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0 \qquad \rho \frac{\partial \mathbf{v}}{\partial t} + \rho \mathbf{v} \cdot \nabla \mathbf{v} = (\nabla \times \mathbf{B}) \times \mathbf{B} - \nabla P \qquad \frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B})$$

$$\nabla \cdot B = 0 \qquad P = \kappa \rho^{\Gamma}$$

2) Linearize MHD equations with

$$\delta f(t, r, \phi, z) = \delta f(r)\, exp(i\omega t - im\phi - ikz)$$

3) Obtain two ordinary differential and seven algebraic equations

$\delta v_r, \delta \pi$

$\delta \rho, \delta P, \delta v_\varphi, \delta v_z, \delta B_r, \delta B_\varphi, \delta B_z$

4) For each wavenumber “k”, determine real and imaginary frequency (ω) from boundary condition. ω_R – oscillation; ω_I – growth of instability!

Hydrodynamic Pinch Modes:-

m=0

(Setting up a Baseline)

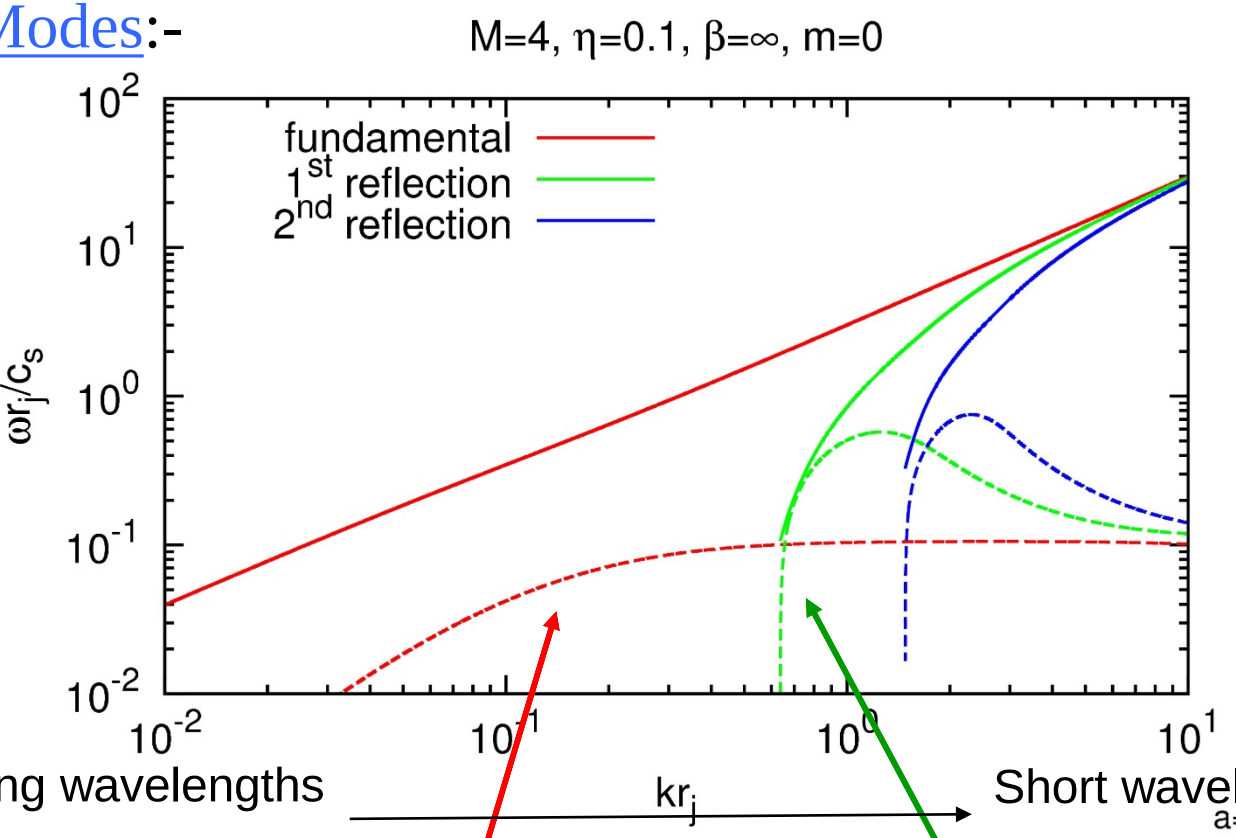
Stability Analysis

+

Pressure Profiles

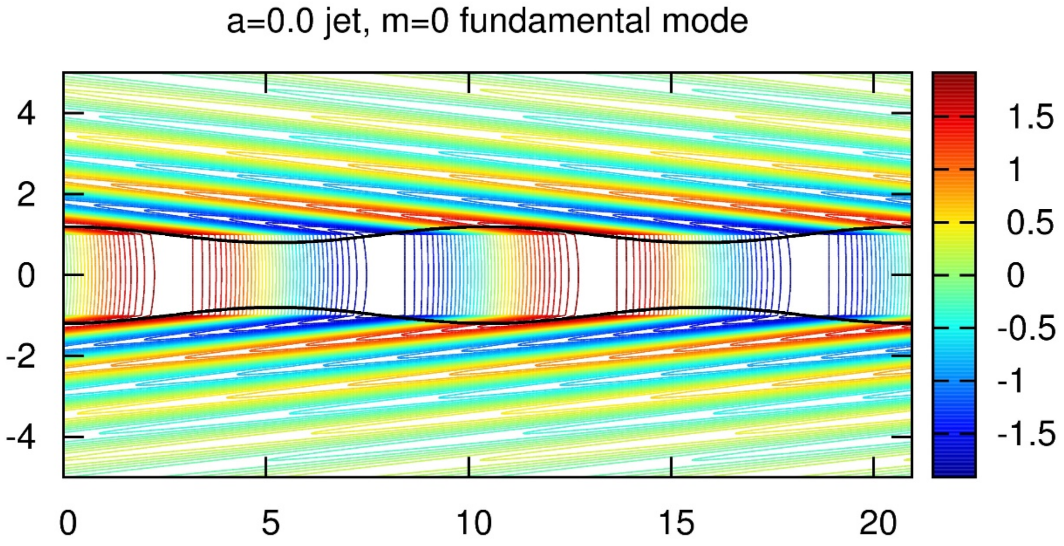
Solid curve – ω_R

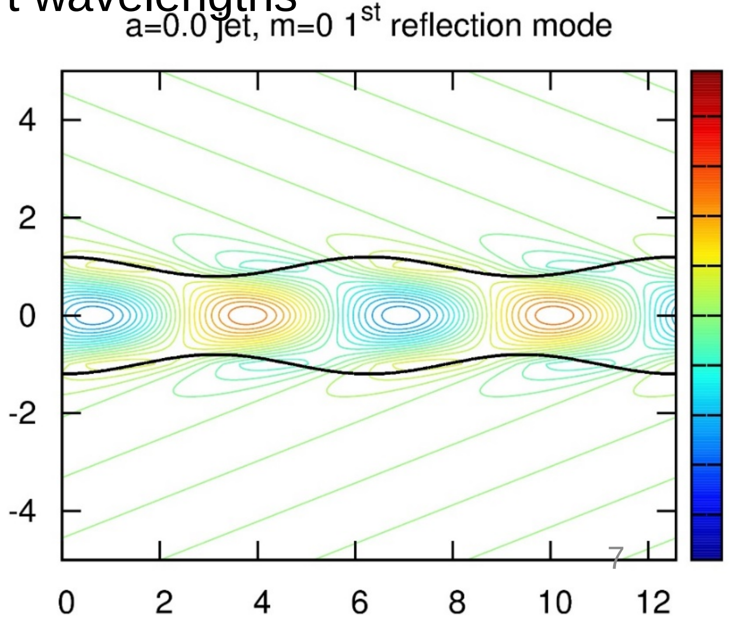
Dashed curve -- ω_I



Long wavelengths

Short wavelengths





Hydrodynamic Kink Modes:-

m=1

(Setting up a Baseline)

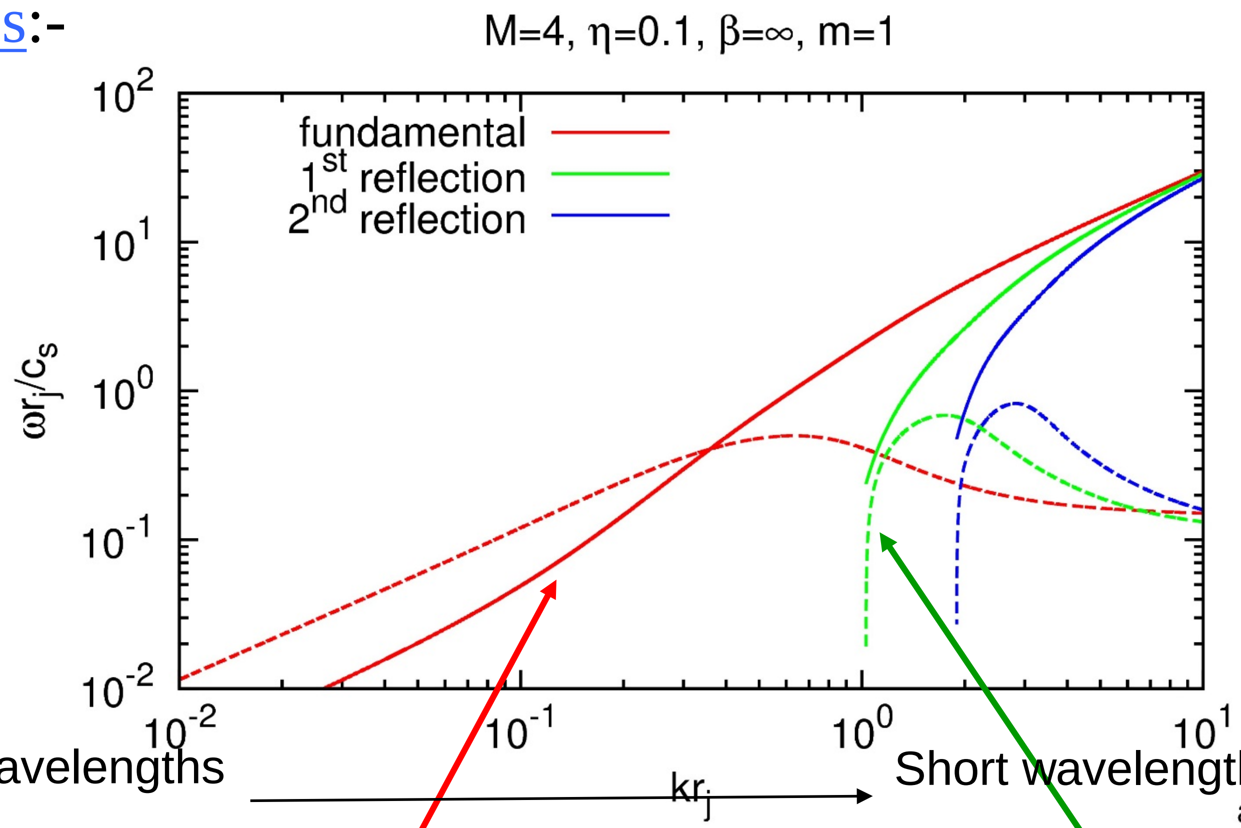
Stability Analysis

+

Pressure Profiles

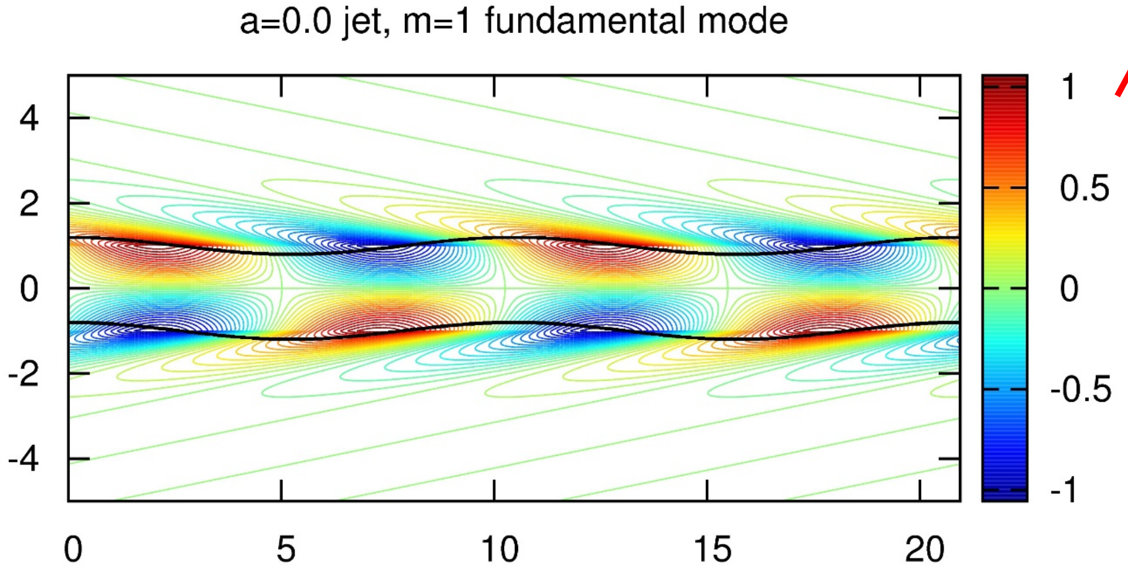
Solid curve – ω_R

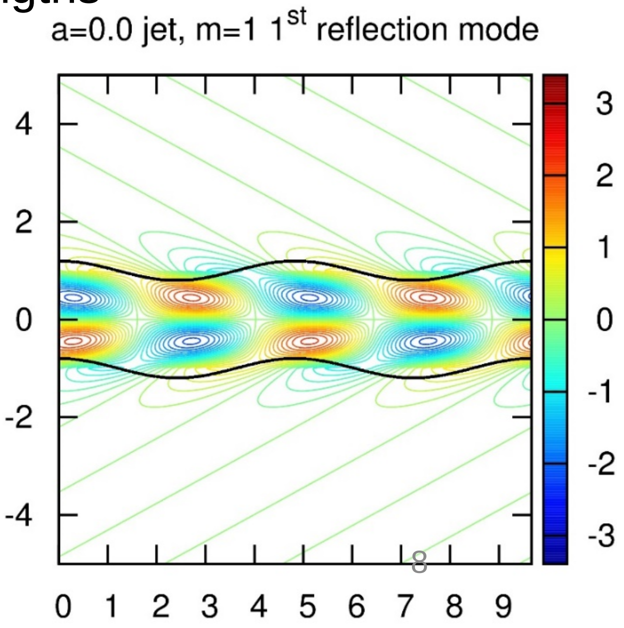
Dashed curve -- ω_I



Long wavelengths

Short wavelengths





Getting rid of all jet instabilities is an improbable task. It is also counterproductive, since some instability is needed for particle acceleration.

We simply ask for the jet to propagate χ jet radii without destabilizing. We want:-

$$\tau > T \quad \text{where} \quad \tau = 1/|\omega_I| \quad \text{and} \quad T = \chi r_j / \mathrm{v}_{z;\,jet} \quad \Rightarrow \quad \frac{\omega_I r_j}{c_s} \le \frac{M}{\chi}$$

We will show this threshold with a dashed line in our plots. Everything below it is "effectively" stable. χ=400 used in this work.

Our fiducial jet has Mach number, M = 4, and jet density ratio, $\rho_{jet} / \rho_{ambient} = 0.1$.

$$\chi = 400 \quad \Rightarrow \quad \frac{\omega_I r_j}{c_s} \le 10^{-2}$$

Parameters to be Varied in this Study:-

β=gas pressure/magnetic pressure. Smaller values of β ➔ larger magnetic fields.

"a" . Larger values of "a" ➔ more shear.

Jet pressure is used as a proxy for jet stability.

III) Magnetized Current-Sheet Free Jets with Top-Hat Velocity Profiles

Fundamental Mode of Pinch Instability (m=0)

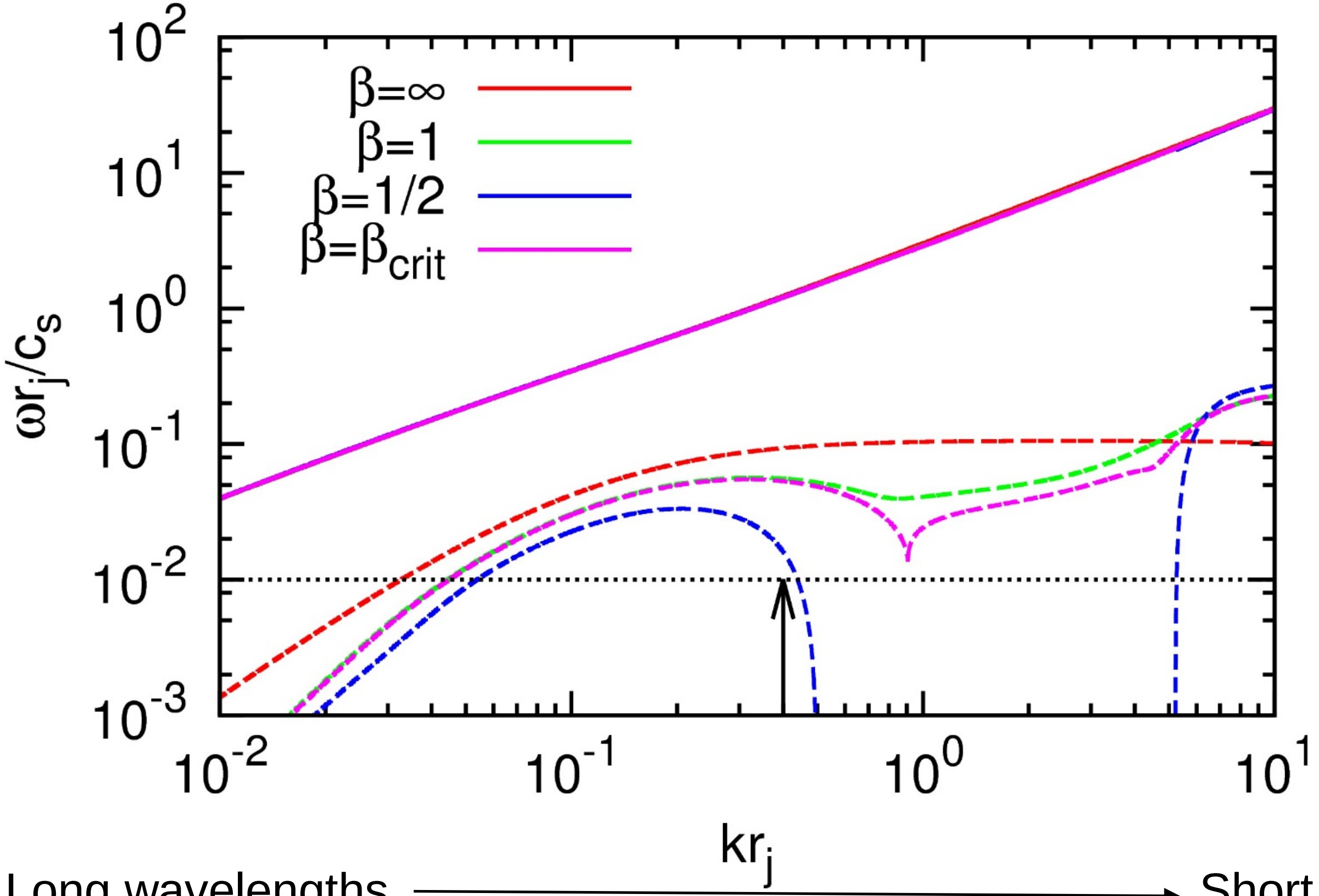
Fiducial Jet *($M=4$, $\eta=0.1$)* various values of magnetic field "β".

($\beta=\infty$ ➔ no field; $\beta=1/2$ ➔ strong field)

Magnetic field helps stabilize the fundamental pinch modes of jets mostly at short wavelengths!

Solid curve – ω_R

Dashed curve -- ω_I



← Threshold for propagating 400 radii with < 1 e-folding.

Long wavelengths ⟶ Short wavelengths

Magnetized Current-Sheet Free Jets with Top-Hat Velocity Profiles

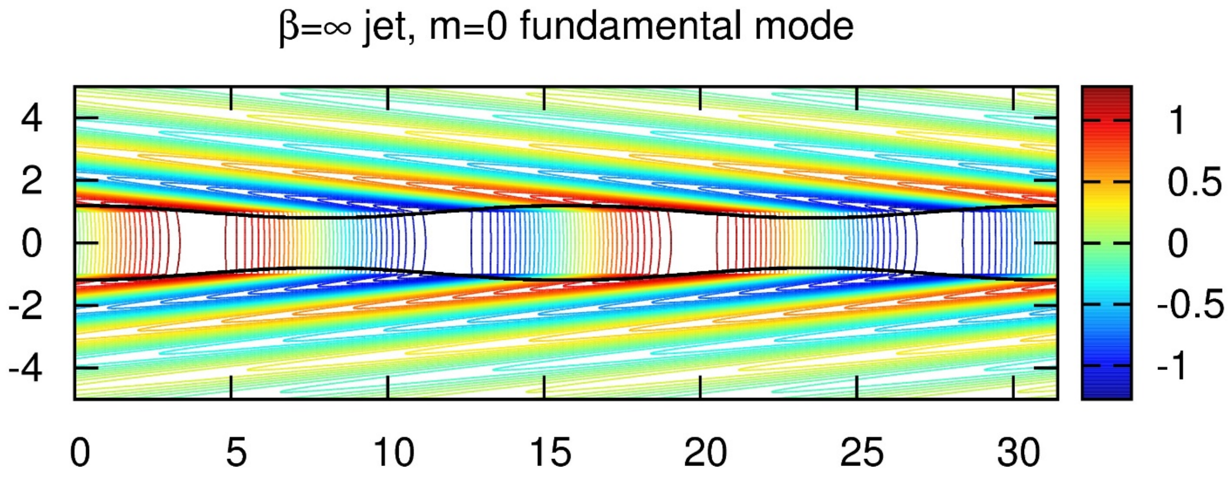
Fundamental Mode of Pinch Instability (m=0)

Fiducial Jet *(M=4, η=0.1)* various values of magnetic field "β".

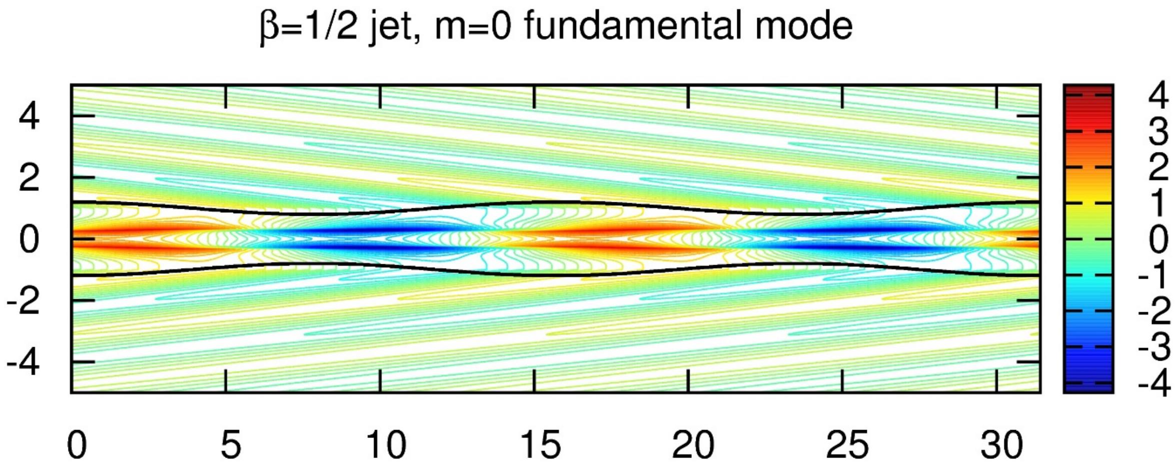
(β=∞ ➔ no field; β=1/2 ➔ strong field) Strong B-field restricts the pressure fluctuations to lie within a narrow channel.

Magnetic field helps stabilize the fundamental pinch modes of jets mostly at short wavelengths!

Notice that toroidal field restricts the perturbations to a narrow channel within the jet.



Unmagnetized



Strongly magnetized

Magnetized Current-Sheet Free Jets with Top-Hat Velocity Profiles

1st reflection Mode of Pinch Instability (m=0)

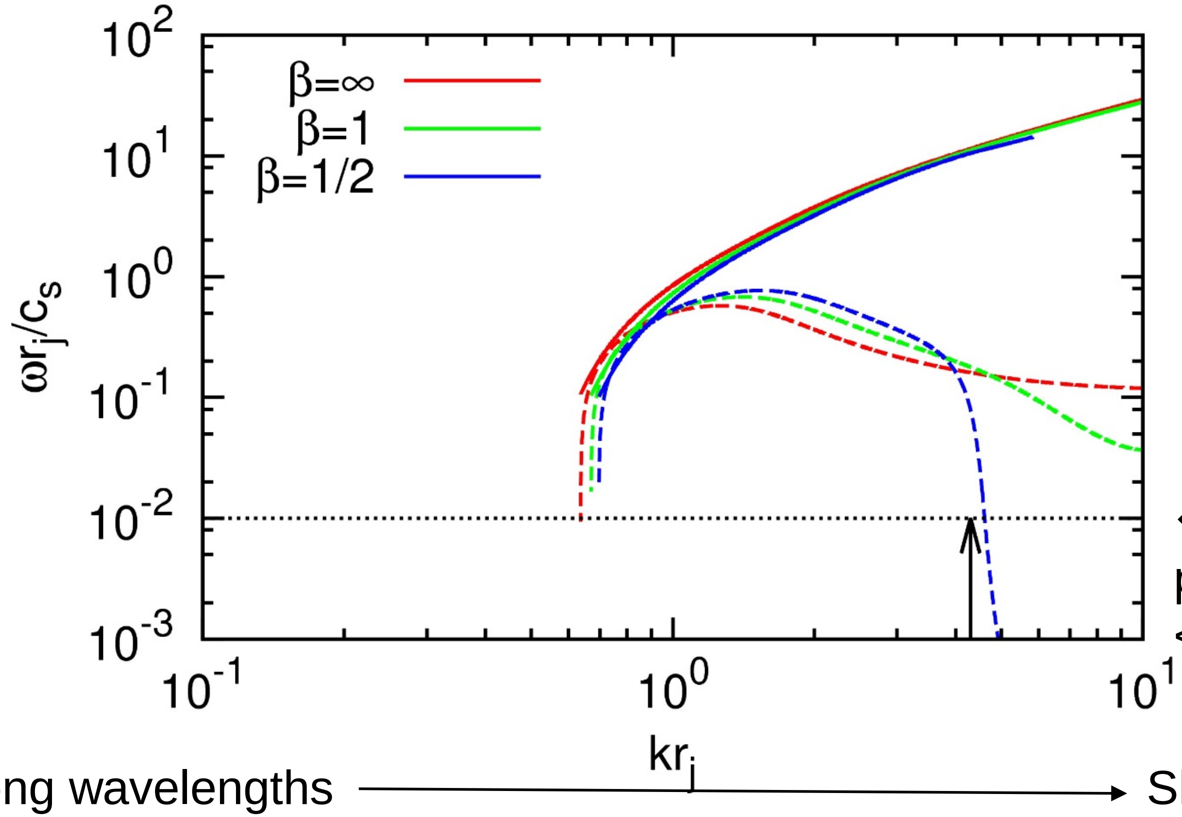
Fiducial Jet *(M=4, η=0.1)* various values of magnetic field "β".

(β=∞ ➔ no field; β =1/2 ➔ strong field)

Magnetic field helps stabilize the 1st reflection pinch modes of jets only at short wavelengths!

Solid curve – ω_R

Dashed curve -- ω_I



← Threshold for propagating 400 radii with < 1 e-folding.

Long wavelengths ——→ Short wavelengths

Magnetized Current-Sheet Free Jets with Top-Hat Velocity Profiles

Fundamental Mode of Kink Instability (m=1)

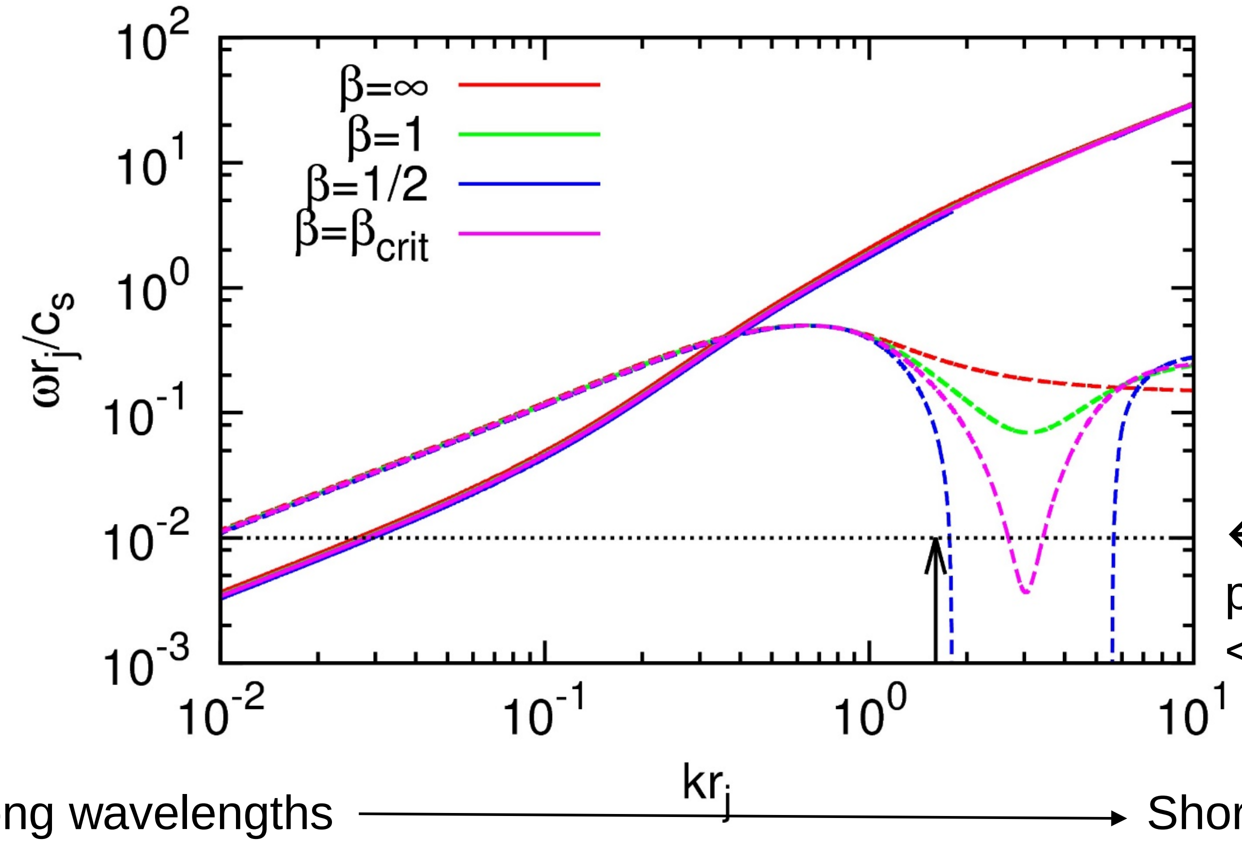
Fiducial Jet *(M=4, η=0.1)* various values of magnetic field "β".

(β=∞ ➔ no field; β =1/2 ➔ strong field)

Magnetic field helps stabilize the fundamental kink modes of jets only at short wavelengths!

Solid curve – ω_R

Dashed curve -- ω_I



← Threshold for propagating 400 radii with < 1 e-folding.

Long wavelengths ⟶ Short wavelengths

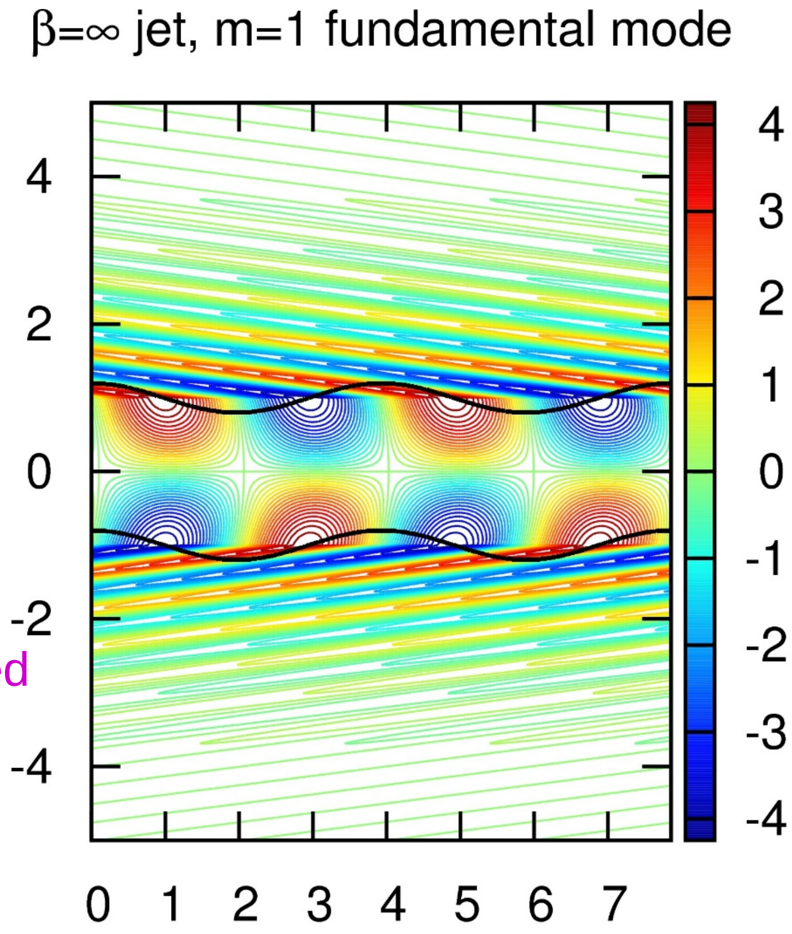
Magnetized Current-Sheet Free Jets with Top-Hat Velocity Profiles

Fundamental Mode of Kink Instability (m=1)

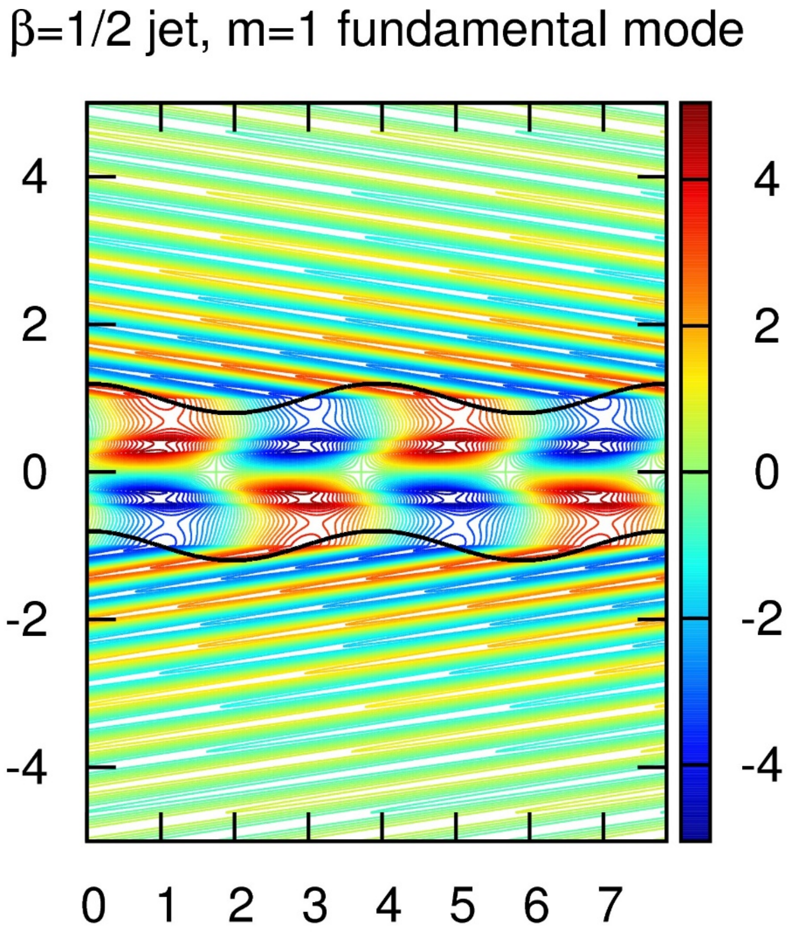
Fiducial Jet *(M=4, η=0.1)* various values of magnetic field "β".

(β=∞ ➔ no field; β=1/2 ➔ strong field) Strong B-field restricts the pressure fluctuations to lie within a narrow channel.

Magnetic field helps stabilize the fundamental kink modes of jets only at short wavelengths!



Unmagnetized



Strongly magnetized

Magnetized Current-Sheet Free Jets with Top-Hat Velocity Profiles

1st reflection Mode of Kink Instability (m=1)

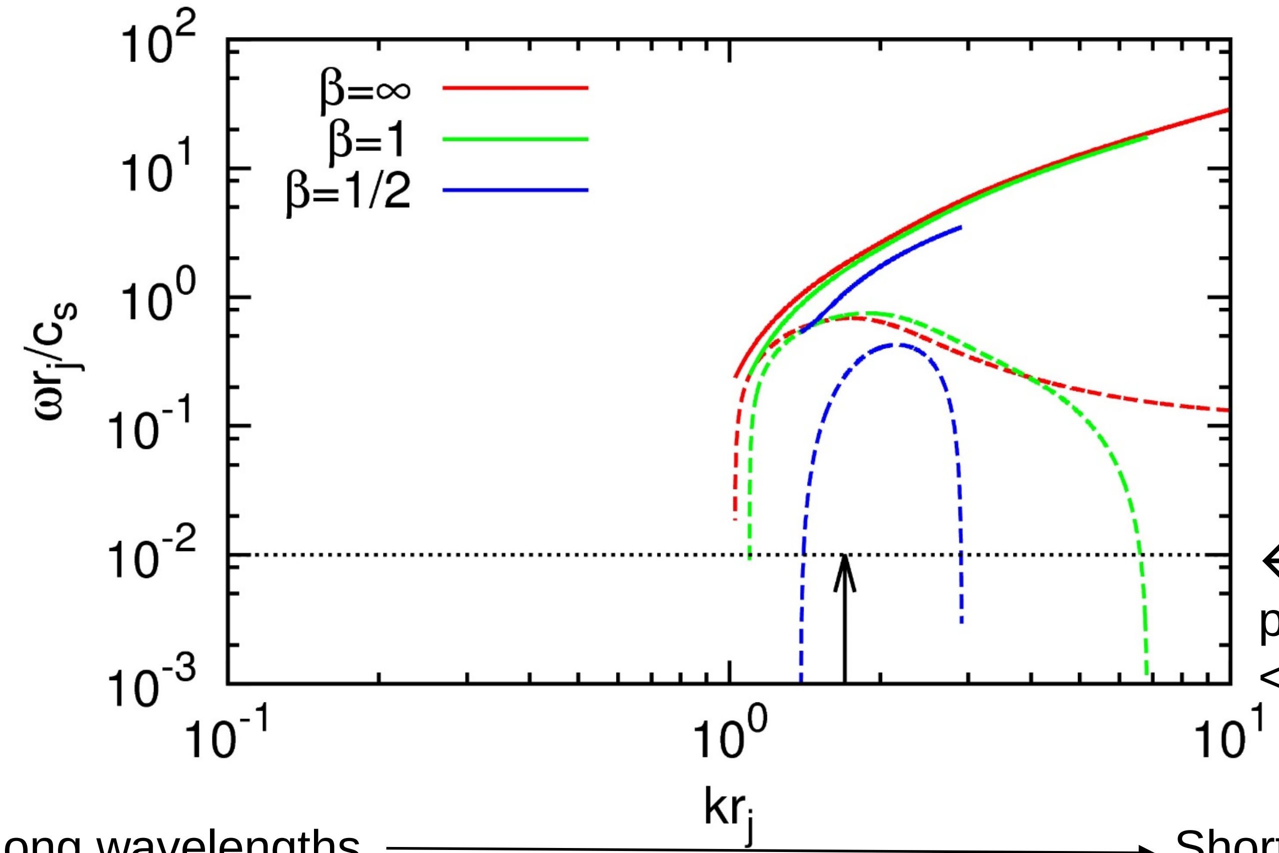
Fiducial Jet *(M=4, η=0.1)* various values of magnetic field "*β*".

(*β*=∞ ➔ no field; *β* =1/2 ➔ strong field)

Magnetic field helps stabilize the 1st reflection kink modes of jets only at short wavelengths!

Solid curve – ω_R

Dashed curve -- ω_I



← Threshold for propagating 400 radii with < 1 e-folding.

Long wavelengths → Short wavelengths

IV) Magnetized Current-Sheet Free Jets with Sheared Velocity Profiles

Fundamental Mode of Pinch Instability (m=0)

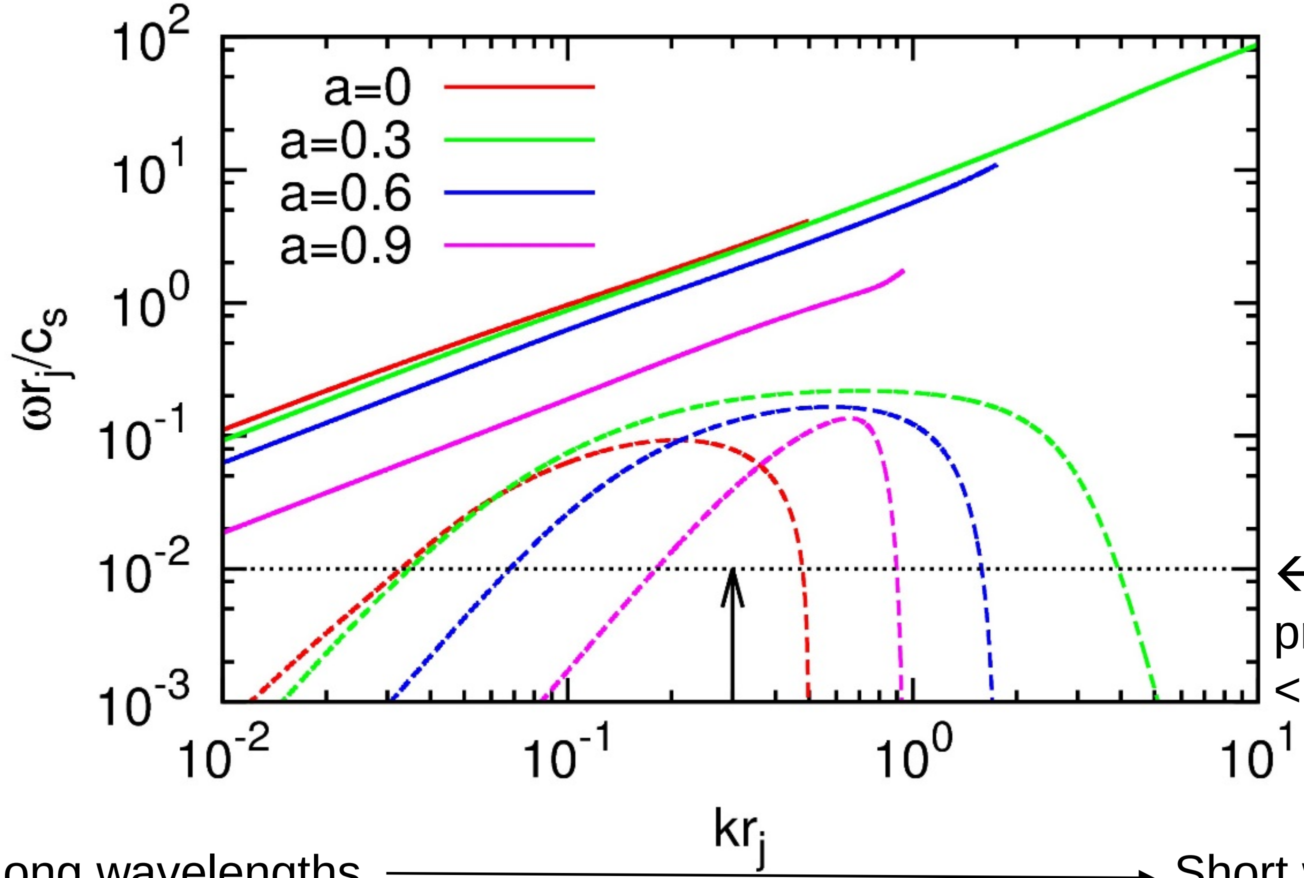
Fiducial Jet *(M=4, η=0.1)* β = 1/2 ; various values of shear "a".

(a=0 ➔ no shear; a=0.9 ➔ high shear)

Shear helps stabilize the fundamental pinch modes of jets at long as well as short wavelengths!

Solid curve – ω_R

Dashed curve -- ω_I



← Threshold for propagating 400 radii with < 1 e-folding.

Long wavelengths → Short wavelengths

Magnetized Current-Sheet Free Jets with Sheared Velocity Profiles

Fundamental Mode of Pinch Instability (m=0)

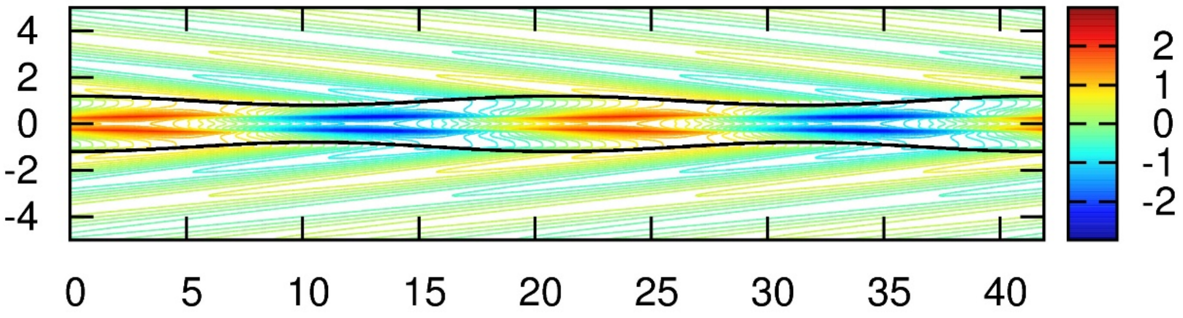
Fiducial Jet *(M=4, η=0.1)* β = 1/2 ; various values of shear "a".

(a=0 ➔ no shear; a=0.9 ➔ high shear) For same boundary amplitude, pressure perturbations decrease with increasing shear.

Shear helps stabilize the fundamental pinch modes of jets at long as well as short wavelengths!

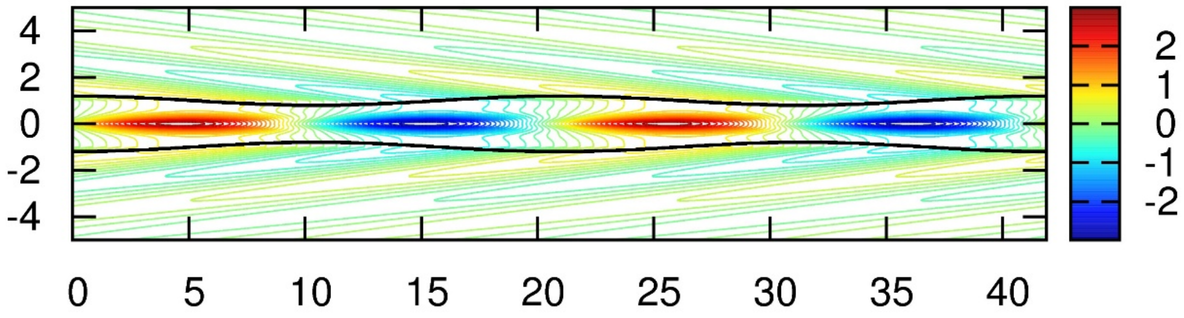
No shear

a=0.0 jet, m=0 fundamental mode



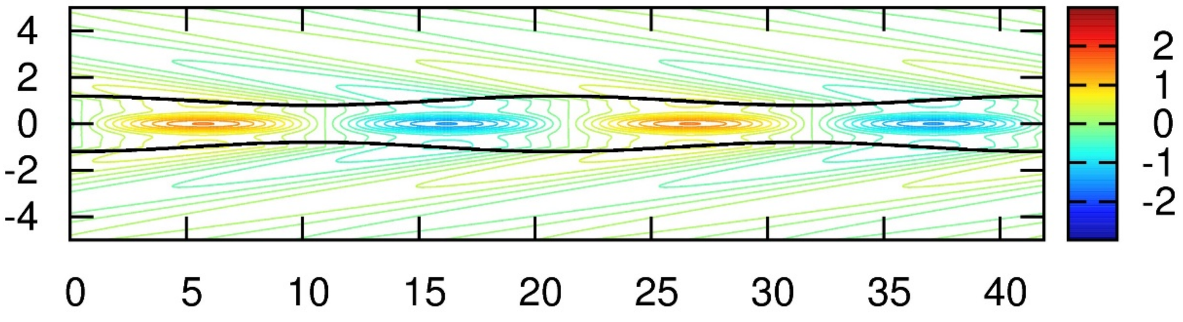
Low shear

a=0.3 jet, m=0 fundamental mode



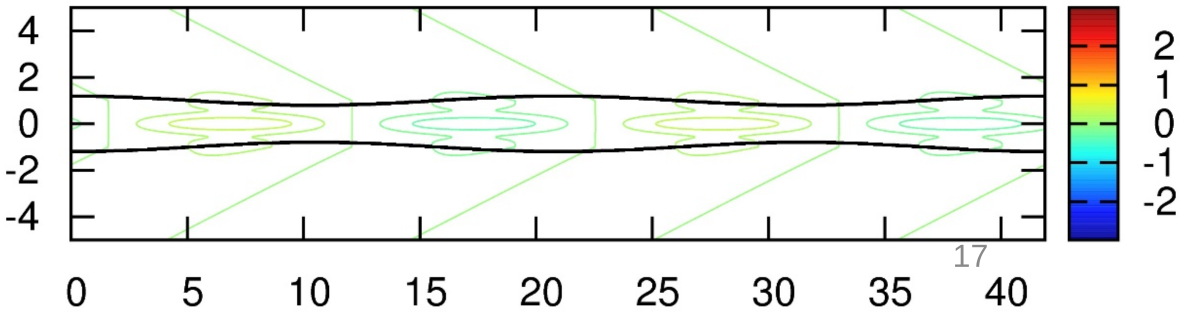
Modest shear

a=0.6 jet, m=0 fundamental mode



High shear

a=0.9 jet, m=0 fundamental mode



Magnetized Current-Sheet Free Jets with Sheared Velocity Profiles

1st Reflection Mode of Pinch Instability (m=0)

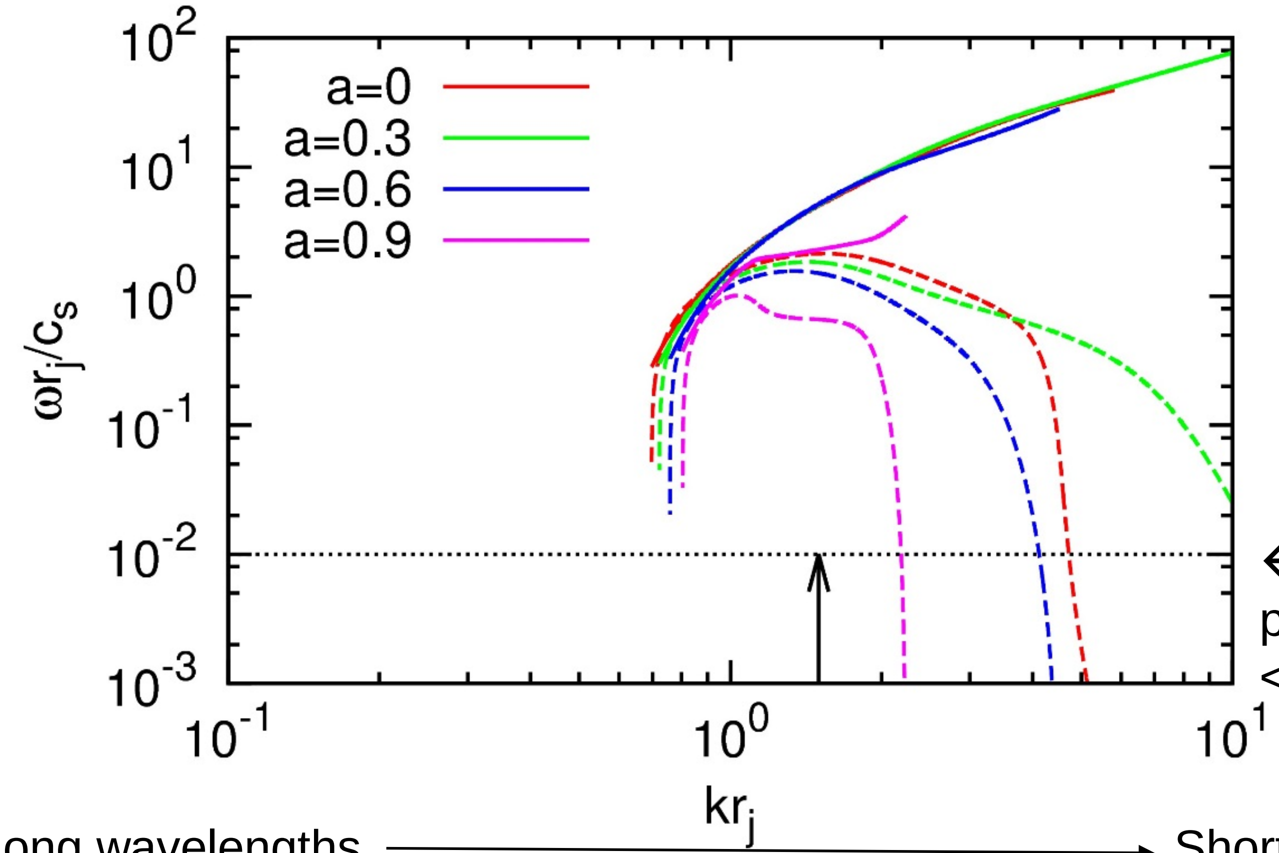
Fiducial Jet *(M=4, η=0.1)* β = 1/2 ; various values of shear "a".

(a=0 ➔ no shear; a=0.9 ➔ high shear)

Shear helps stabilize the 1st reflection pinch modes of jets only at short wavelengths!

Solid curve – ω_R

Dashed curve -- ω_I



← Threshold for propagating 400 radii with < 1 e-folding.

Long wavelengths → Short wavelengths

Magnetized Current-Sheet Free Jets with Sheared Velocity Profiles

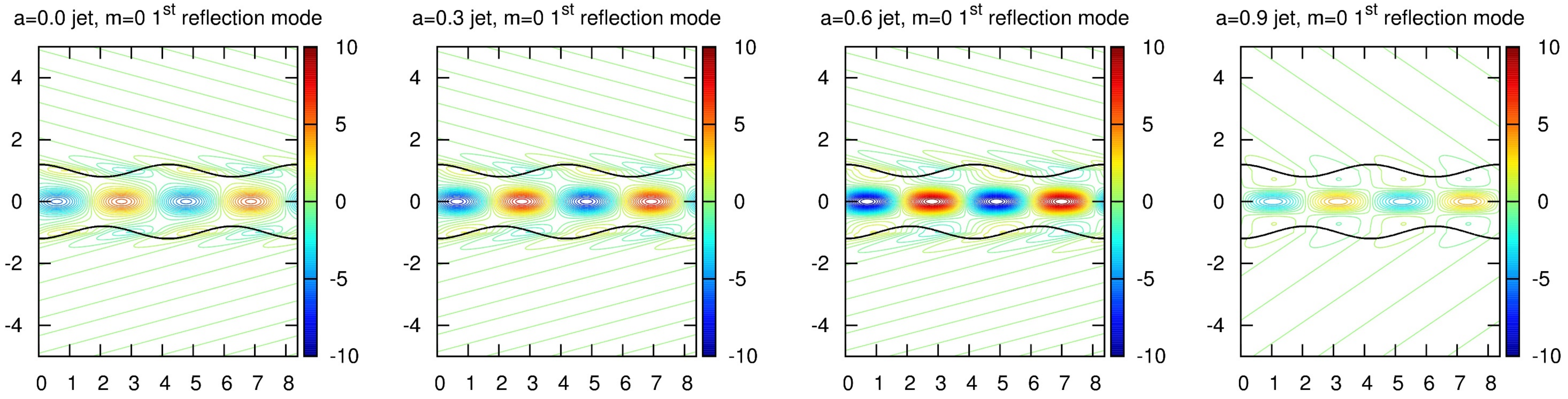
1st Reflection Mode of Pinch Instability (m=0)

Fiducial Jet *(M=4, η=0.1)* β = 1/2 ; various values of shear “a”.

(a=0 ➔ no shear; a=0.9 ➔ high shear) For same boundary amplitude, pressure perturbations decrease with increasing shear.

Shear helps stabilize the 1st reflection pinch modes of jets only at short wavelengths!

No shear | Low shear | Modest shear | High shear



Magnetized Current-Sheet Free Jets with Sheared Velocity Profiles

Fundamental Mode of Kink (m=1)

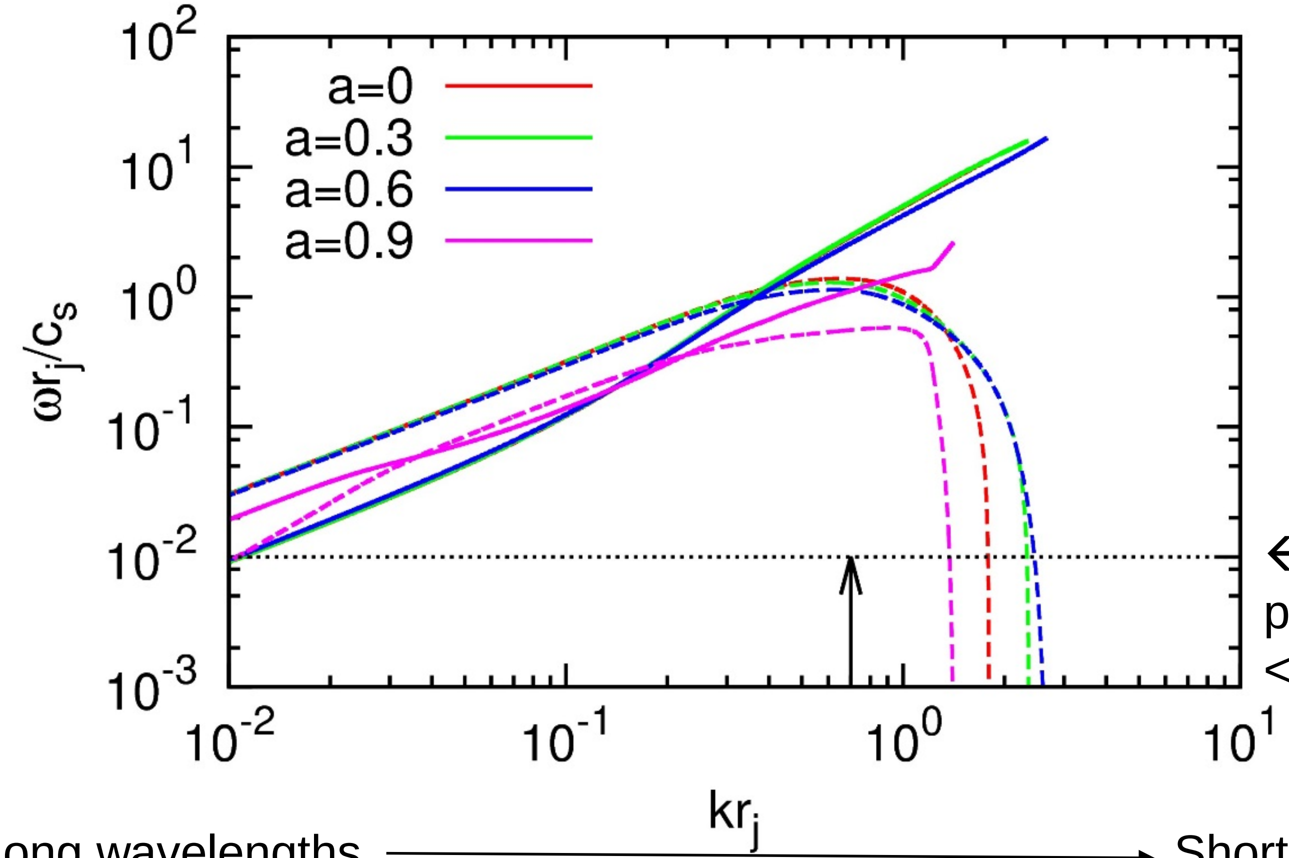
Fiducial Jet *(M=4, η=0.1)* β = 1/2 ; various values of shear “a”.

(a=0 ➔ no shear; a=0.9 ➔ high shear)

Shear helps stabilize the fundamental kink modes of jets at short wavelengths + little at long wavelengths!

Solid curve – ω_R

Dashed curve -- ω_I



← Threshold for propagating 400 radii with < 1 e-folding.

Long wavelengths → Short wavelengths

Magnetized Current-Sheet Free Jets with Sheared Velocity Profiles

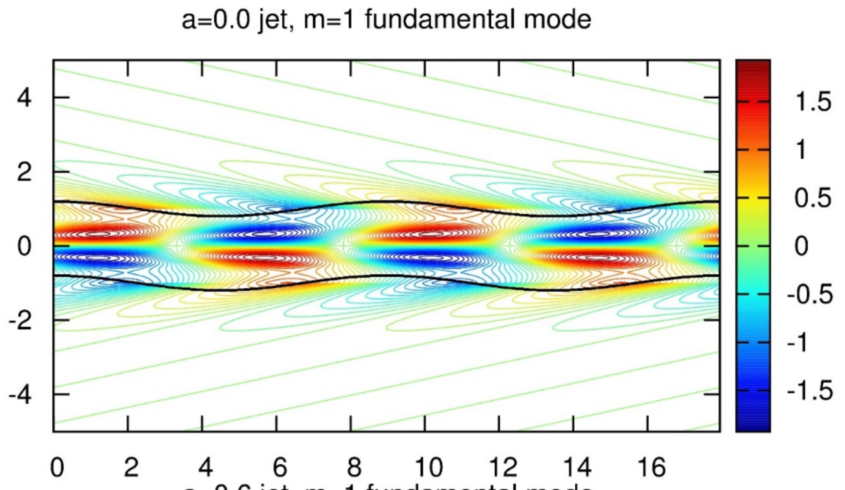
Fundamental Modes of Kink (m=1)

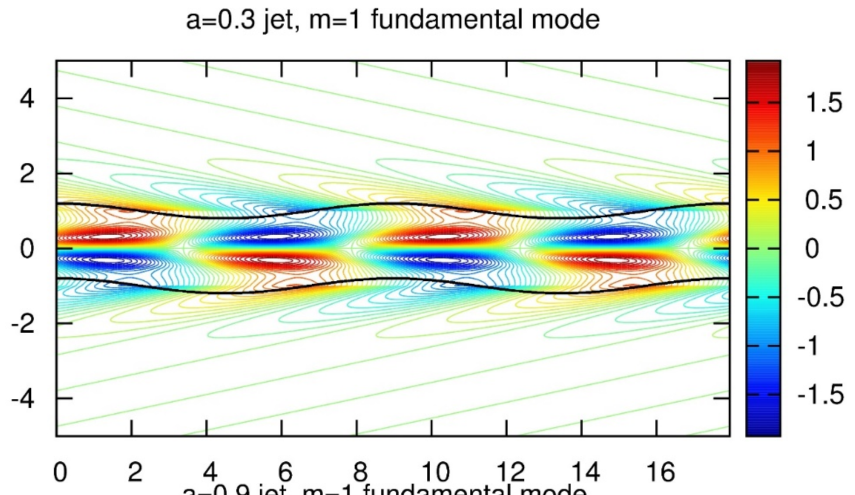
Fiducial Jet *(M=4, η=0.1)* β = 1/2 ; various values of shear “a”.

(a=0 ➔ no shear; a=0.9 ➔ high shear) For same boundary amplitude, pressure perturbations decrease with increasing shear.

Shear helps stabilize the fundamental kink modes of jets at short wavelengths + little for long waves!

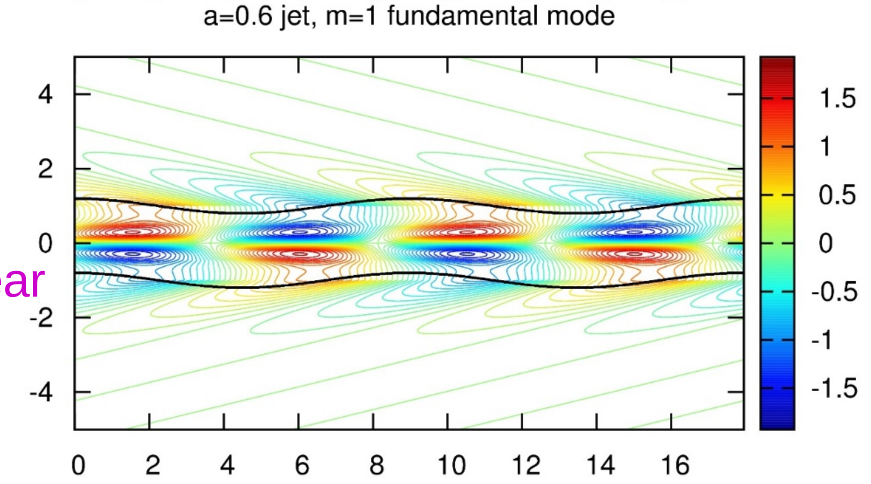
No shear

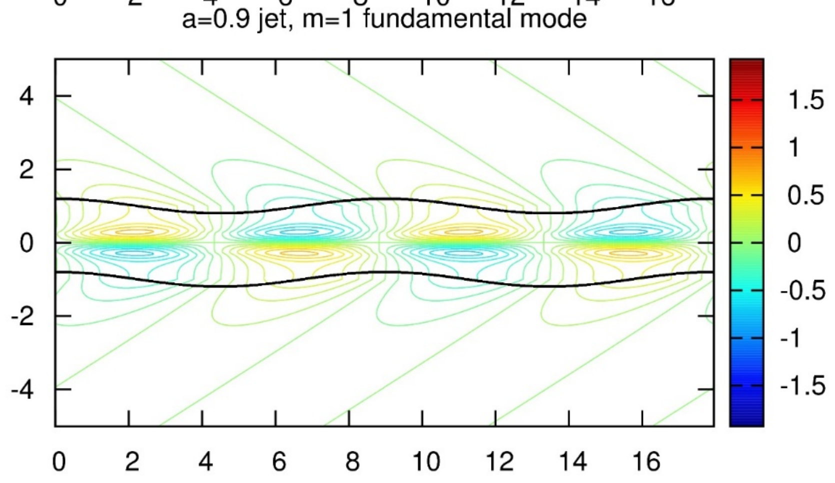




Low shear

Modest shear





High shear

Magnetized Current-Sheet Free Jets with Sheared Velocity Profiles

1st reflection Mode of Kink (m=1)

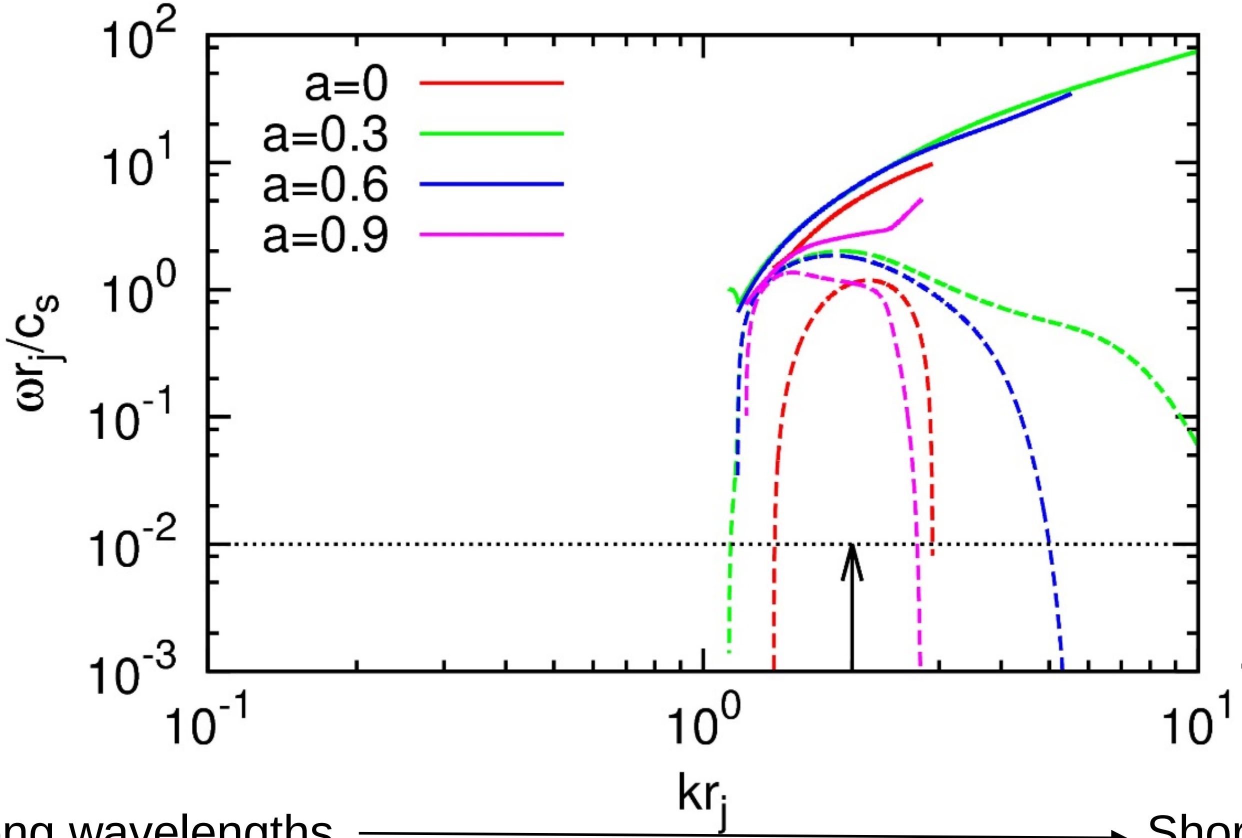
Fiducial Jet *(M=4, η=0.1)* β = 1/2 ; various values of shear "a".

(a=0 ➔ no shear; a=0.9 ➔ high shear)

Shear helps stabilize the 1st reflection kink modes of jets only at short wavelengths!

Solid curve – ω_R

Dashed curve -- ω_I



← Threshold for propagating 400 radii with < 1 e-folding.

Long wavelengths ⟶ Short wavelengths

Magnetized Current-Sheet Free Jets with Sheared Velocity Profiles

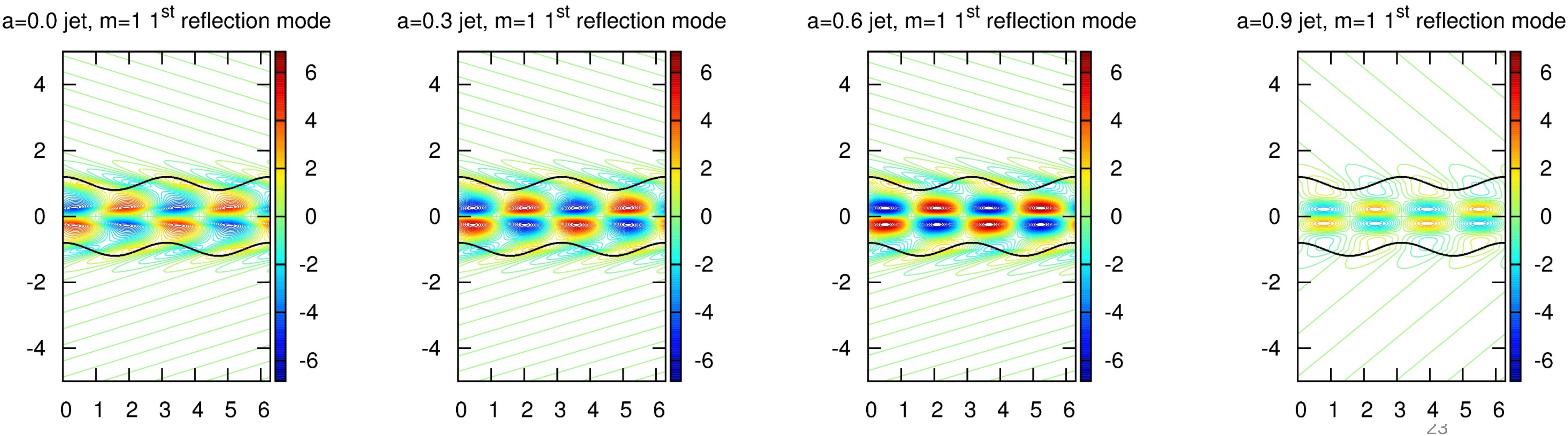
1^{st} reflection Mode of Kink Instability (m=1)

Fiducial Jet *(M=4, η=0.1)* β = 1/2 ; various values of shear “a”.

(a=0 ➔ no shear; a=0.9 ➔ high shear) For same boundary amplitude, pressure perturbations decrease with increasing shear.

Shear helps stabilize the 1^{st} reflection kink modes of jets only at short wavelengths!

No shear | Low shear | Modest shear | High shear



Conclusions

We have carried out the stability analysis of current sheet free jets proposed by Gourgouliatos *et al*. (2012), as well as sheared jets (Mizuno, Lyubarsky *et al*. 2012).

We adopt a different, numerically motivated approach; jet can have any velocity and field profile. No further approximations, like considering only analytic functions, or jets with zero pressure.

We find that current-sheet-free magnetic field can significantly improve the jet stability at short wavelengths for all modes. For fields in excess of equipartition, the stability properties improve dramatically!

Shear, by contrast, improves the stability properties of pinch modes at short and long wavelengths.

Shear also improved the stability properties of kink modes at short wavelengths. Though the improvement at long wavelengths is present, it is not as dramatic.

It is important to get past top hat velocity profiles and simple B-fields in jet stability studies.