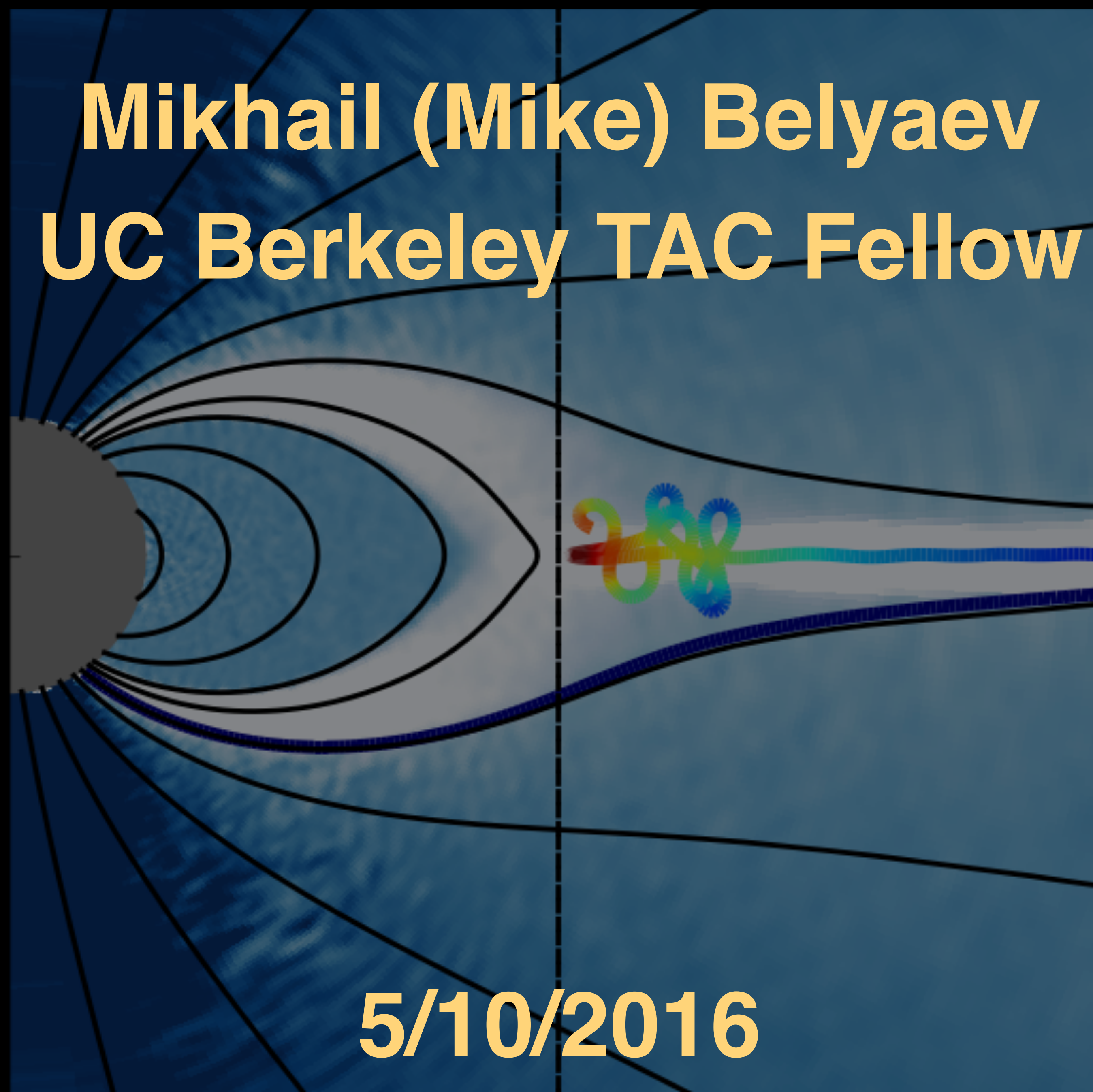
Polar Cap & Y-Point

Theory & PIC Simulation



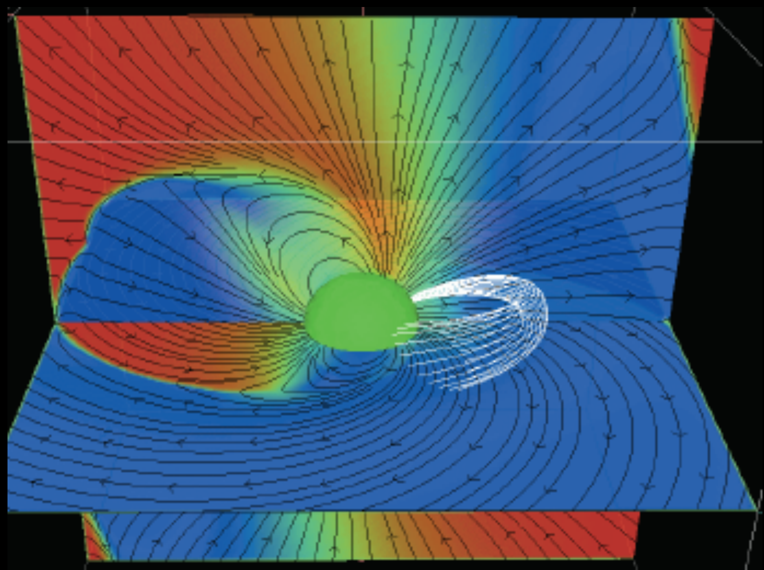
Mikhail (Mike) Belyaev

UC Berkeley TAC Fellow

5/10/2016

Theoretical Backgr

We need to go beyond force-free to understand the emission and connect to observations!

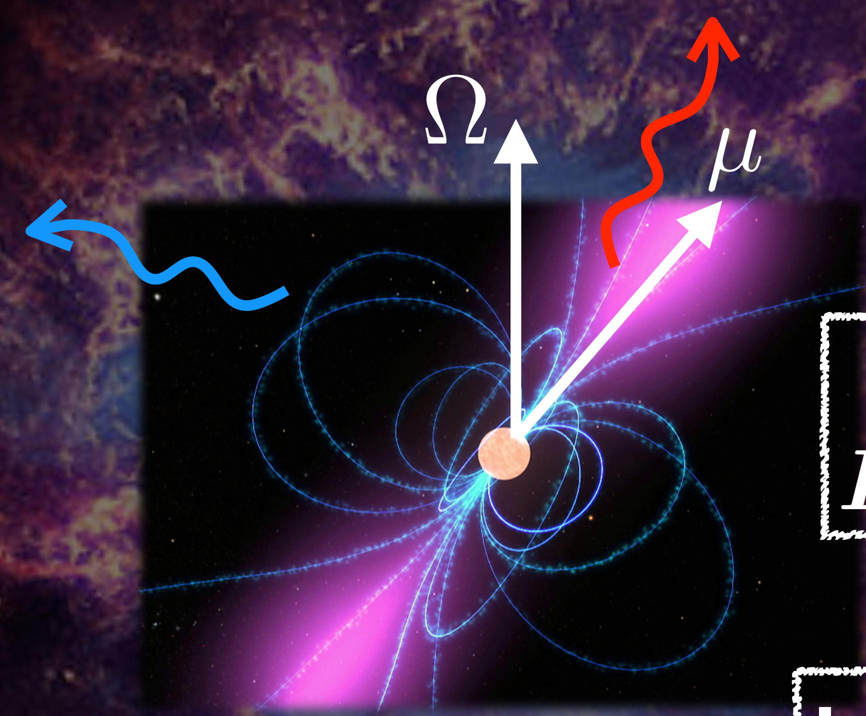


Spitkovsky (2006)

Magnetosphere approximated as force-free due to a high plasma density.

$$\rho \boldsymbol{E} + c^{-1} \boldsymbol{J} \times \boldsymbol{B} = 0 \implies \boldsymbol{E} \cdot \boldsymbol{B} = 0$$

$$B^2 - E^2 > 0 \implies \boldsymbol{V}_d = c\boldsymbol{E} \times \boldsymbol{B} / B^2$$



Gaps
$\boldsymbol{E} \cdot \boldsymbol{B} \neq 0$

Instabilities

Current sheets
$B^2 - E^2 < 0$

Emission in the magnetosphere requires non-force-free effects.

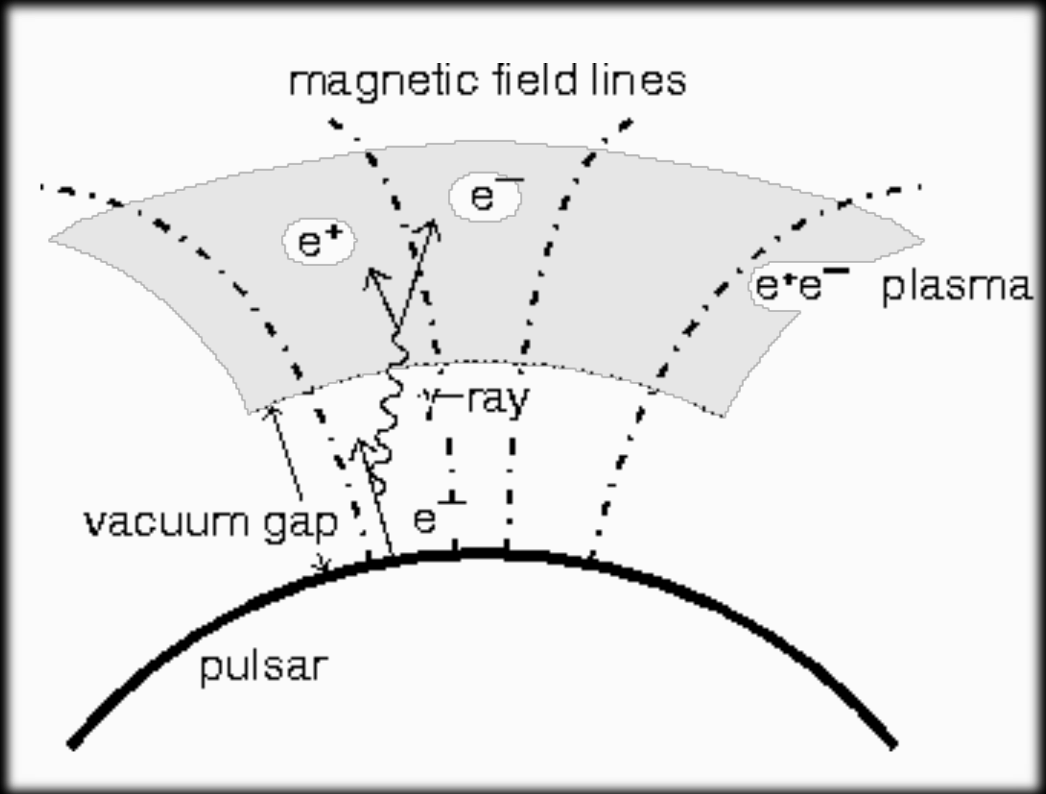
$$\frac{\partial u}{\partial t} + \frac{c}{4\pi} \nabla \cdot (\boldsymbol{E} \times \boldsymbol{B}) = -\boldsymbol{E} \cdot \boldsymbol{J}$$

Polar Cap Pair Production

Where does dense plasma exist in magnetosphere?

$\gamma - B$ **on magnetic field lines**

Pairs made locally — field line by field line



Timokhin & Arons (2013) PP criterion

$$J^\mu J_\mu < 0 \begin{cases} \boldsymbol{J} \cdot \hat{\boldsymbol{r}} / \rho_{GJ} > 0 : \text{no pairs} \\ \boldsymbol{J} \cdot \hat{\boldsymbol{r}} / \rho_{GJ} < 0 : \text{pairs} \end{cases}$$

$$J^\mu J_\mu > 0 : \text{pairs}$$

$$J^\mu J_\mu \equiv -(\rho c)^2 + J^2$$

Axisymmetric Force-Free

Poloidal current flows along magnetic flux surfaces

$$\nabla \cdot (\alpha \boldsymbol{J}) = 0$$

$$\nabla \cdot \boldsymbol{B} = 0$$

$$\rightarrow \quad \alpha \boldsymbol{J}_P \propto \boldsymbol{B}_P$$

Current is set by ***global*** magnetospheric structure

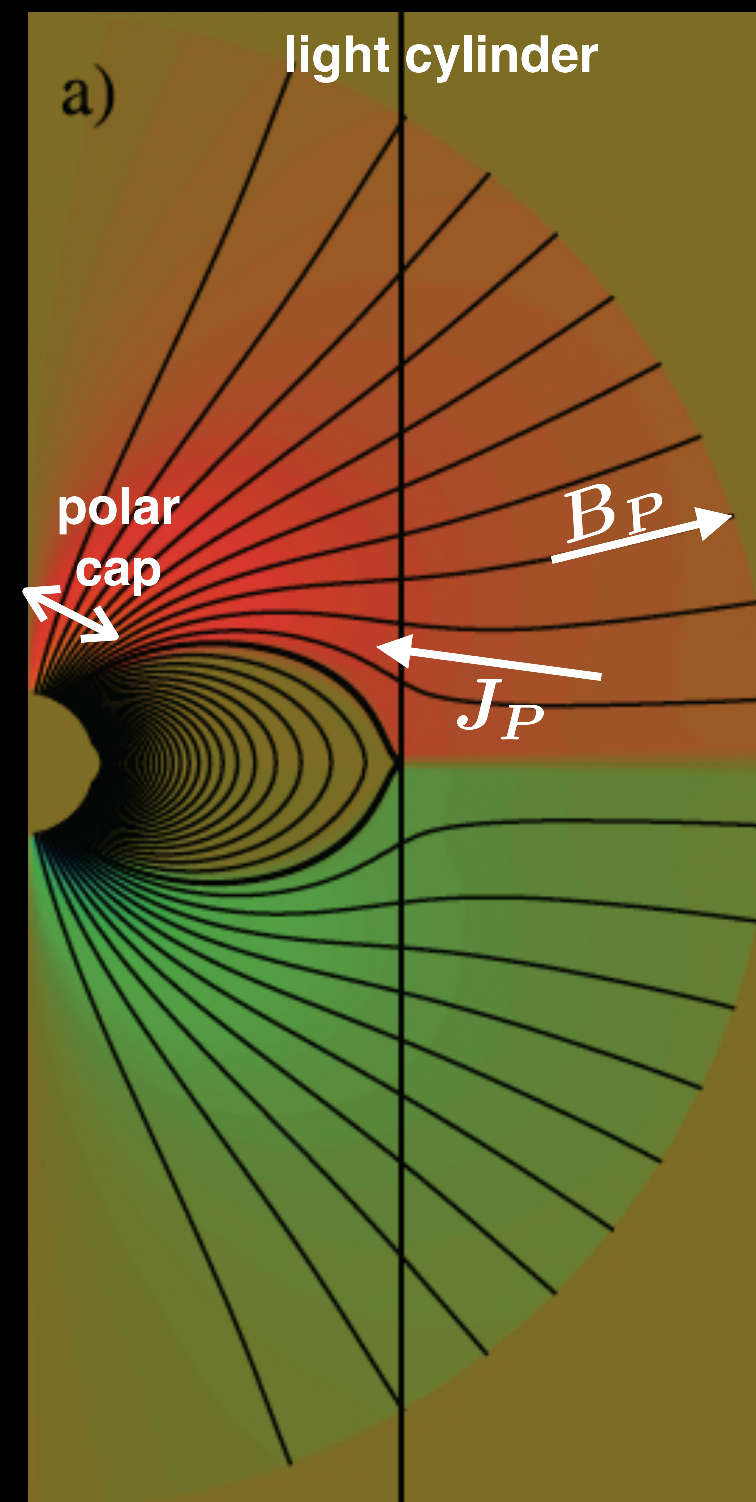
Density is determined ***locally*** as GJ density

$$\boldsymbol{E} = -\boldsymbol{V}_0 \times \boldsymbol{B}/c.$$

$$\boldsymbol{V}_0 \equiv \begin{cases} \boldsymbol{\Omega} \times \boldsymbol{r}, & \text{flat} \\ \alpha^{-1} \left(\boldsymbol{\Omega} - \boldsymbol{\omega}_{LT} \right) \times \boldsymbol{r}, & \text{Kerr} \end{cases}$$

$$\rho_G \equiv -\nabla \cdot \left(\boldsymbol{V}_0 \times \boldsymbol{B} \right) / 4\pi c$$

$$= -\frac{\left(\boldsymbol{\Omega} - \boldsymbol{\omega}_{LT} \right) \cdot \boldsymbol{B}}{2\pi c \alpha} + \frac{\boldsymbol{V}_0 \cdot \left(\nabla \times \boldsymbol{B} \right)}{4\pi c}$$



Spitkovsky (2006)

General Method

We would like to determine $J^\mu J_\mu$ over the entire polar cap.

For a given B field, we know the charge density on the polar cap.

$$\rho_{\rm PC} = -\frac{(\boldsymbol{\Omega} - \boldsymbol{\omega}_{LT})\cdot \boldsymbol{B}}{2\pi c \alpha} + \frac{\boldsymbol{V}_0 \cdot (\boldsymbol{\nabla} \times \boldsymbol{B})}{4\pi c}$$

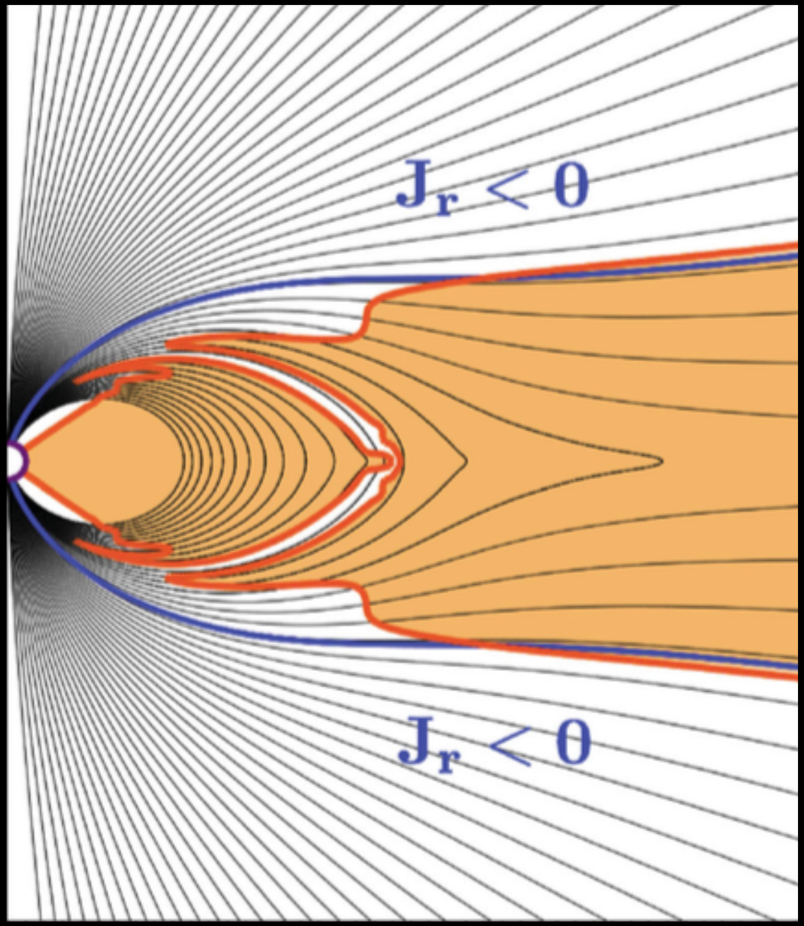
Trace currents back from LC to polar cap.

$$\boldsymbol{J}_\infty = c\rho_\infty \hat{\boldsymbol{r}}, \quad J^\mu J_\mu = 0$$

$$\rho_\infty = -\frac{(\boldsymbol{\Omega} - \boldsymbol{\omega}_{LT})\cdot \boldsymbol{B}}{2\pi c \alpha} + \frac{\boldsymbol{V}_0 \cdot (\boldsymbol{\nabla} \times \boldsymbol{B})}{4\pi c}$$

Last approximation is split-monopole.

Parfrey et al. (2012)



distributed return current

General Results 1

flat spacetime

$J^\mu J_\mu$ is

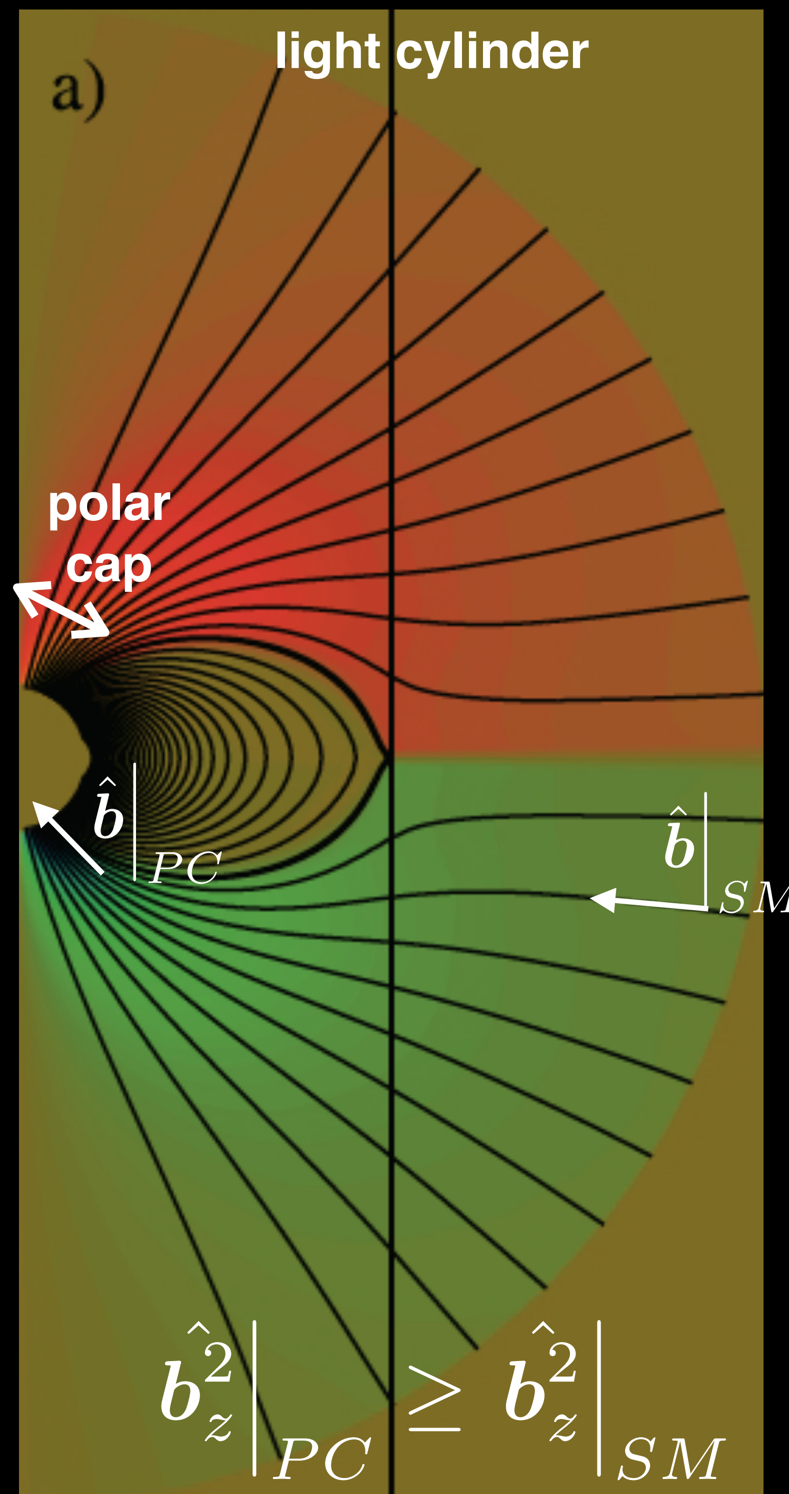
$$\begin{cases} \text{timelike}, & \hat{b}_z^2\Big|_{PC} > \hat{b}_z^2\Big|_{SM} \\ \text{null}, & \hat{b}_z^2\Big|_{PC} = \hat{b}_z^2\Big|_{SM} \\ \text{spacelike}, & \hat{b}_z^2\Big|_{PC} < \hat{b}_z^2\Big|_{SM} \end{cases}$$

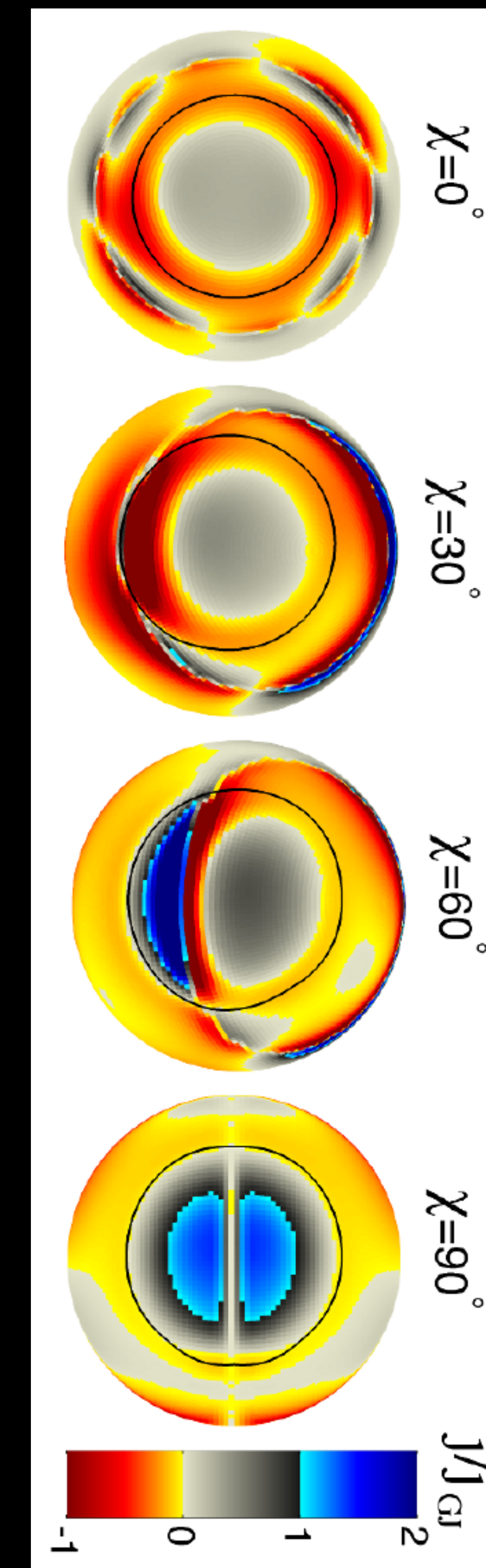
$$\hat{b}_z \equiv \hat{z} \cdot \hat{b}$$

Slowly-rotating Kerr

$J^\mu J_\mu$ is

$$\begin{array}{ll} \text{timelike}, & \left[\left(1 - \frac{\omega_{LT}}{\Omega}\right)\hat{b}_z\right]^2\Big|_{PC} > \hat{b}_z^2\Big|_{SM} \\ \text{null}, & \left[\left(1 - \frac{\omega_{LT}}{\Omega}\right)\hat{b}_z\right]^2\Big|_{PC} = \hat{b}_z^2\Big|_{SM} \\ \text{spacelike}, & \left[\left(1 - \frac{\omega_{LT}}{\Omega}\right)\hat{b}_z\right]^2\Big|_{PC} < \hat{b}_z^2\Big|_{SM} \end{array}$$





Bai & Spitkovsky (2010)

General Results 2

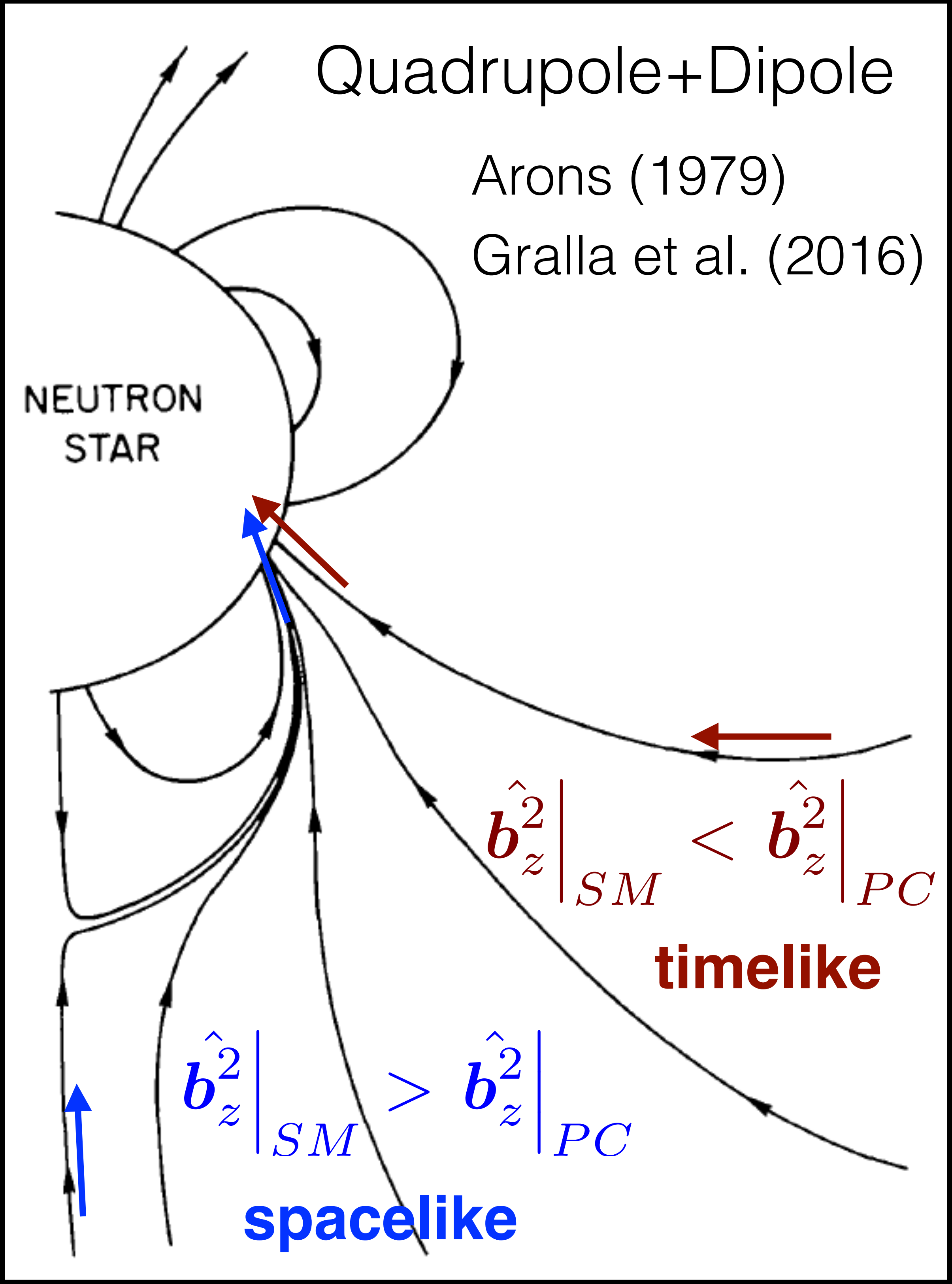
flat spacetime

$$J^\mu J_\mu \text{ is } \begin{cases} \text{timelike}, & \hat{b}_z^2\Big|_{PC} > \hat{b}_z^2\Big|_{SM} \\ \text{null}, & \hat{b}_z^2\Big|_{PC} = \hat{b}_z^2\Big|_{SM} \\ \text{spacelike}, & \hat{b}_z^2\Big|_{PC} < \hat{b}_z^2\Big|_{SM} \end{cases}$$

$$\hat{b}_z \equiv \hat{z} \cdot \hat{\boldsymbol{b}}$$

Slowly-rotating Kerr

$$J^\mu J_\mu \text{ is } \begin{cases} \text{timelike}, & \left[\left(1 - \frac{\omega_{LT}}{\Omega}\right)\hat{b}_z\right]^2\Big|_{PC} > \hat{b}_z^2\Big|_{SM} \\ \text{null}, & \left[\left(1 - \frac{\omega_{LT}}{\Omega}\right)\hat{b}_z\right]^2\Big|_{PC} = \hat{b}_z^2\Big|_{SM} \\ \text{spacelike}, & \left[\left(1 - \frac{\omega_{LT}}{\Omega}\right)\hat{b}_z\right]^2\Big|_{PC} < \hat{b}_z^2\Big|_{SM} \end{cases}$$



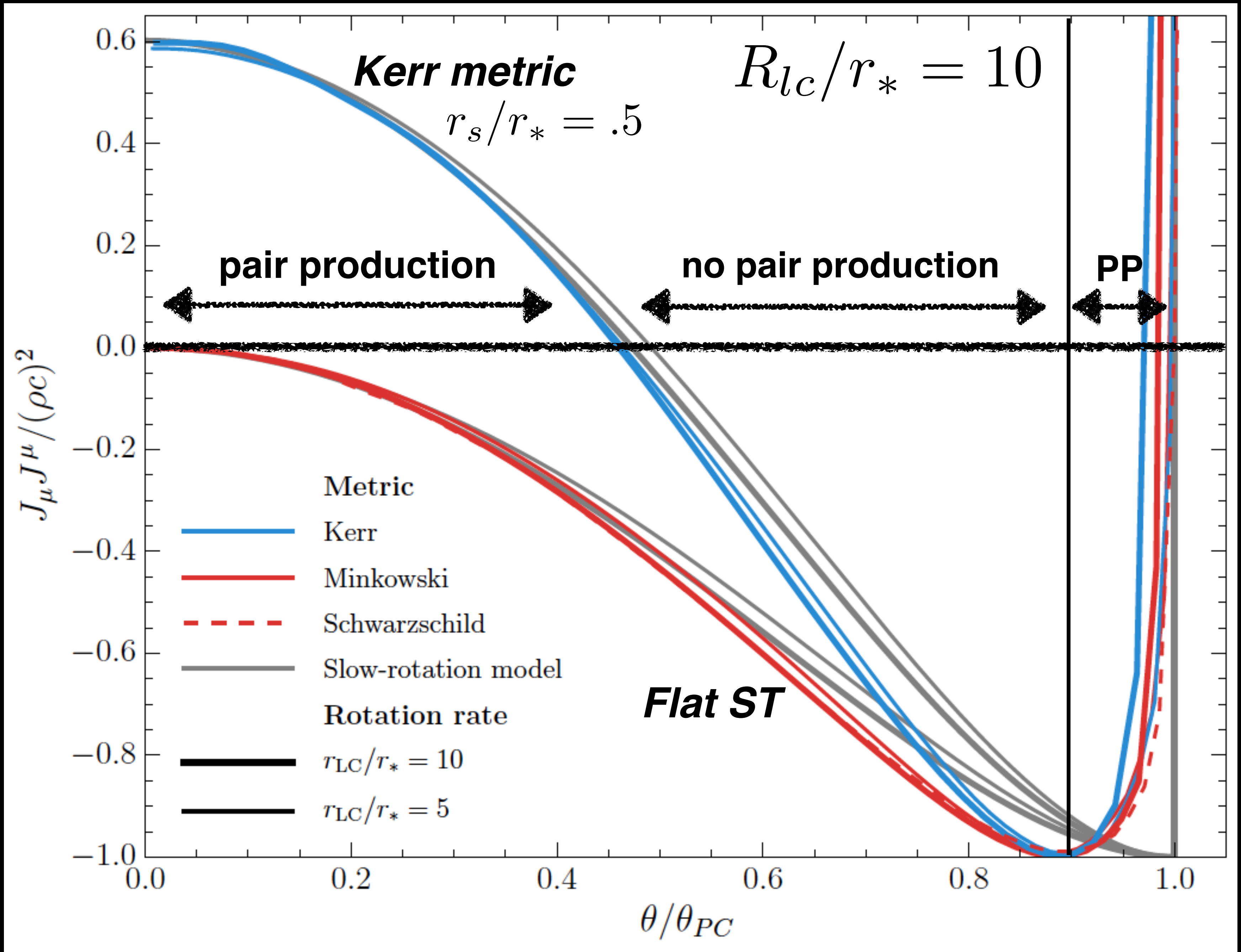
General Results 3

Dipole magnetosphere

Difference between GR and flat ST due exclusively to frame dragging.

With GR, ***two PP regions.*** *Second region due to distributed return current.*

No PP region ***always*** exists, because poloidal current changes sign.

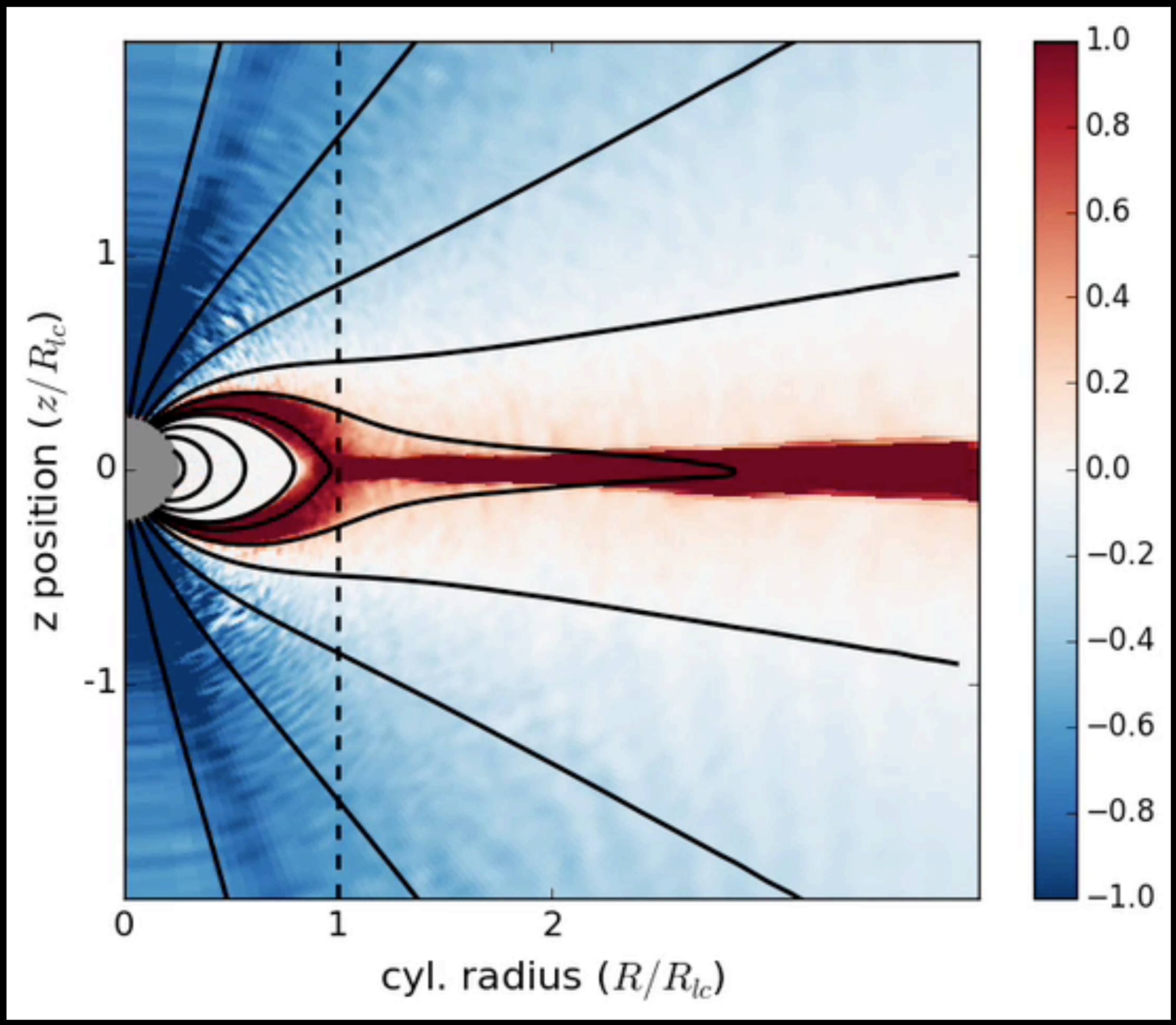


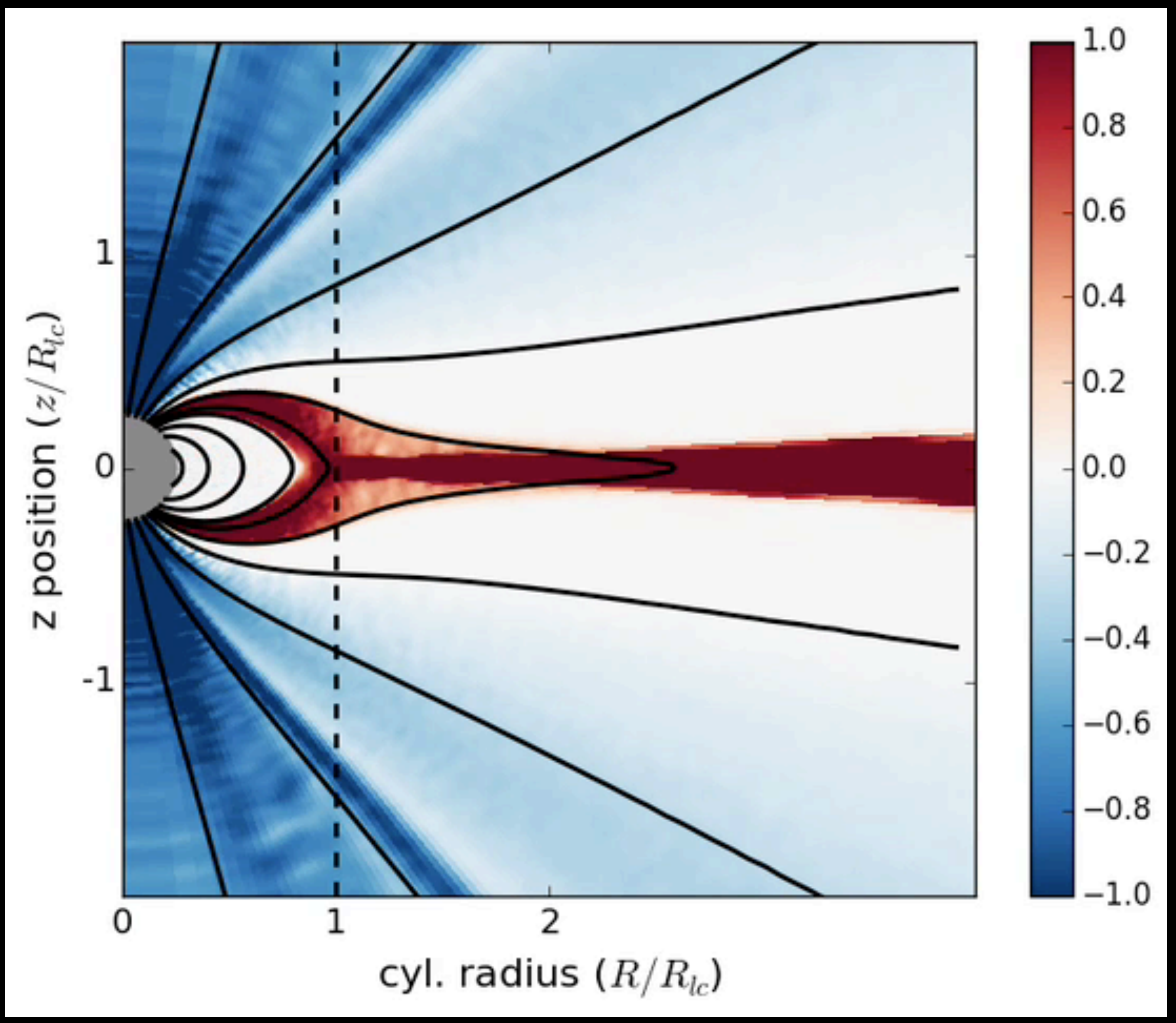
Belyaev & Parfrey (2016)

Magnetospheric Current

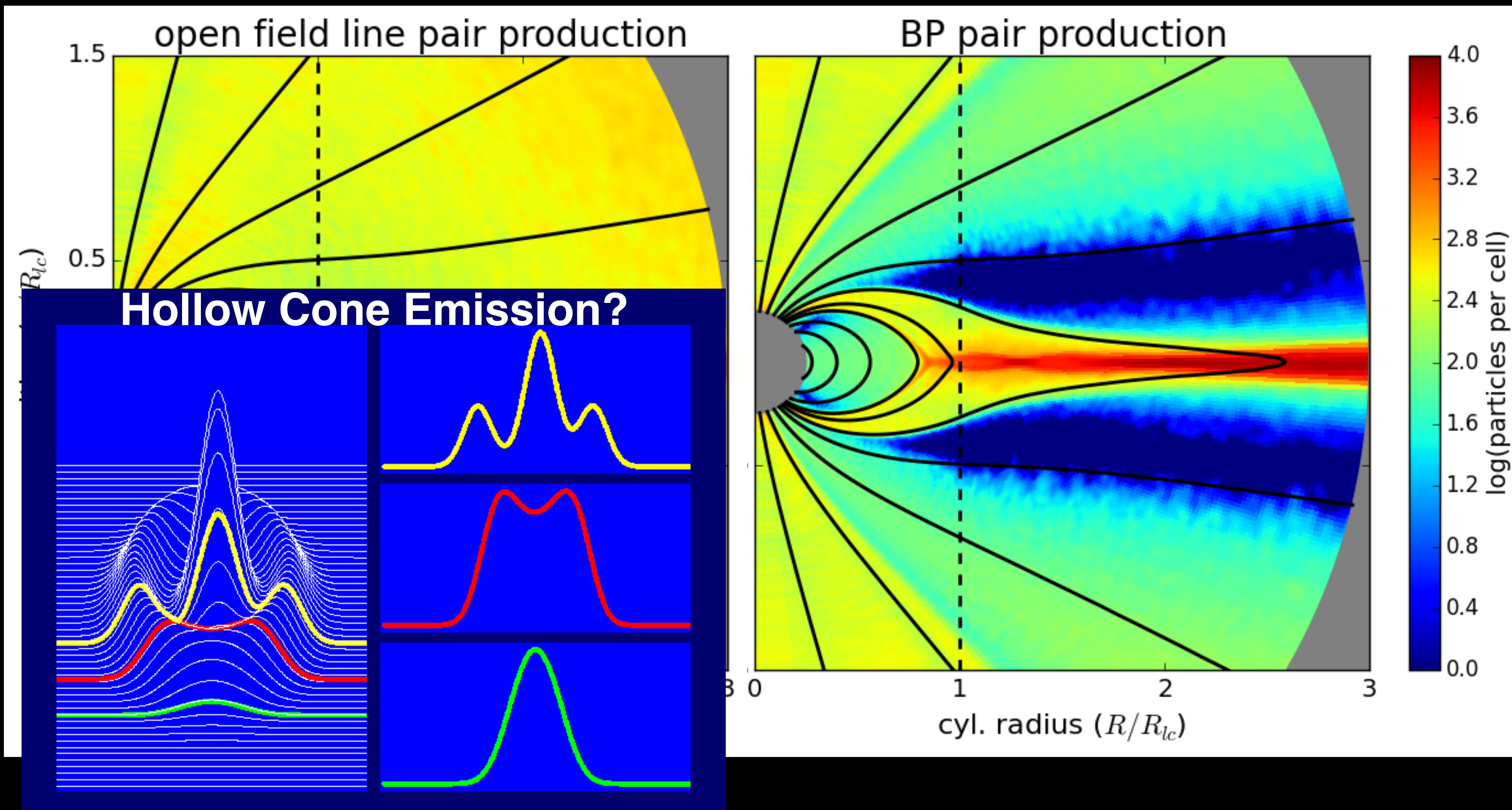
PP Open Field lines

BP Model



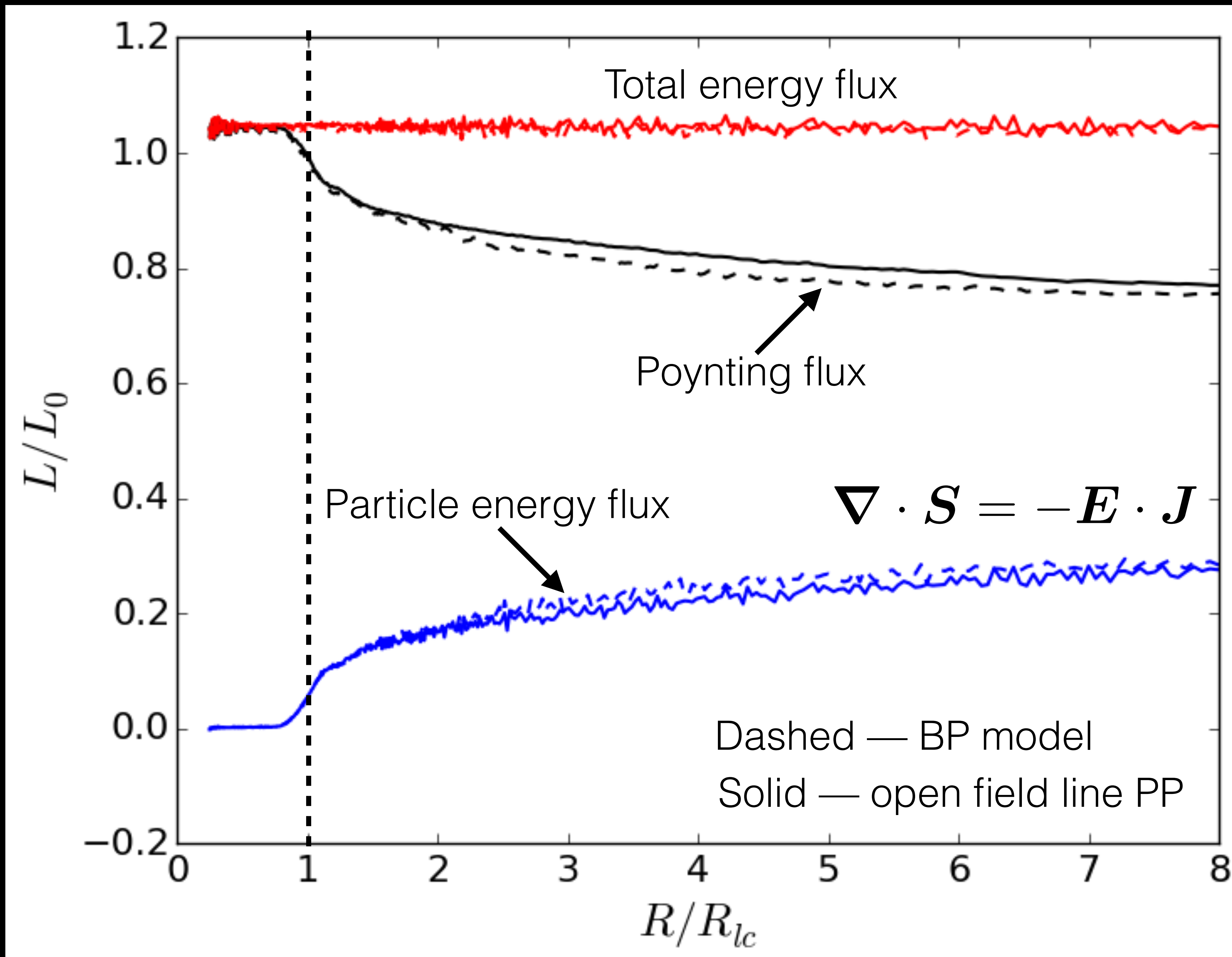


Mind the Gap!

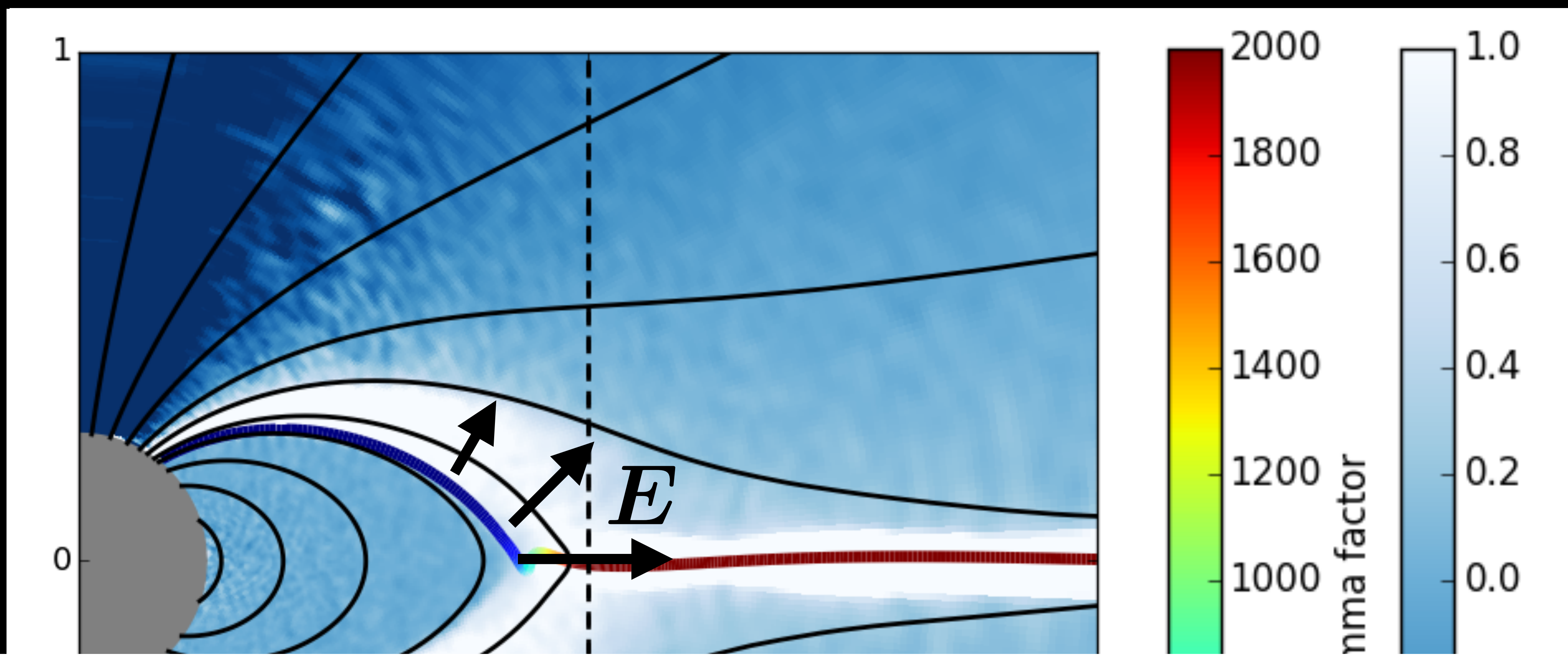


BP PP model has a large outer gap above the current sheet.

Luminosity & Dissipation



Positron Trajectory

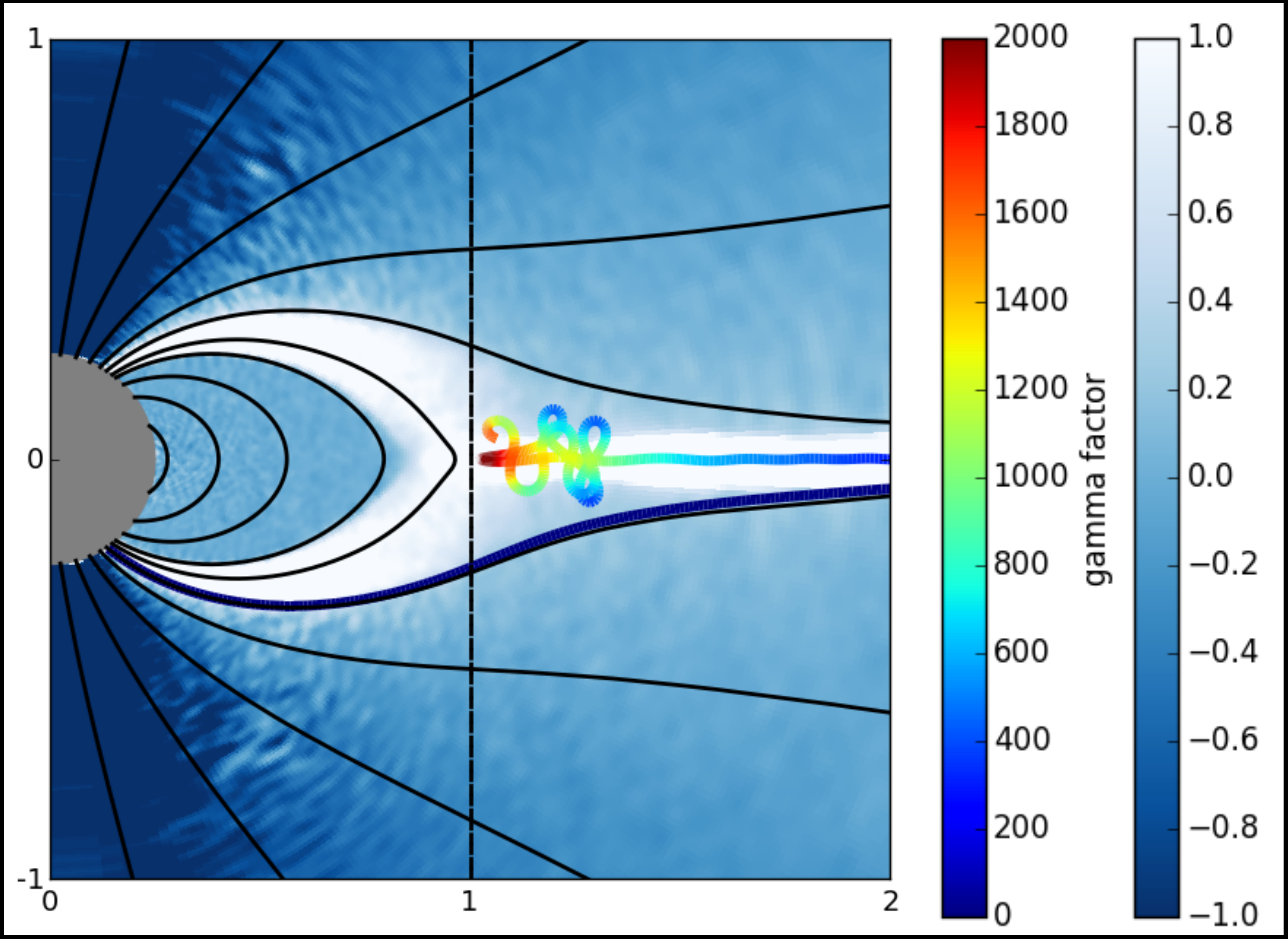


Drift velocity close to speed of light near Y-point:

$$v_{D,\phi} = -E_r/B_z \lesssim c, \quad B_z' = B_z/\gamma_D$$

Particles in closed region accelerate radially across field lines (voltage drop). They cross light cylinder before turning around and escape to infinity in current sheet.

Electron Trajectory



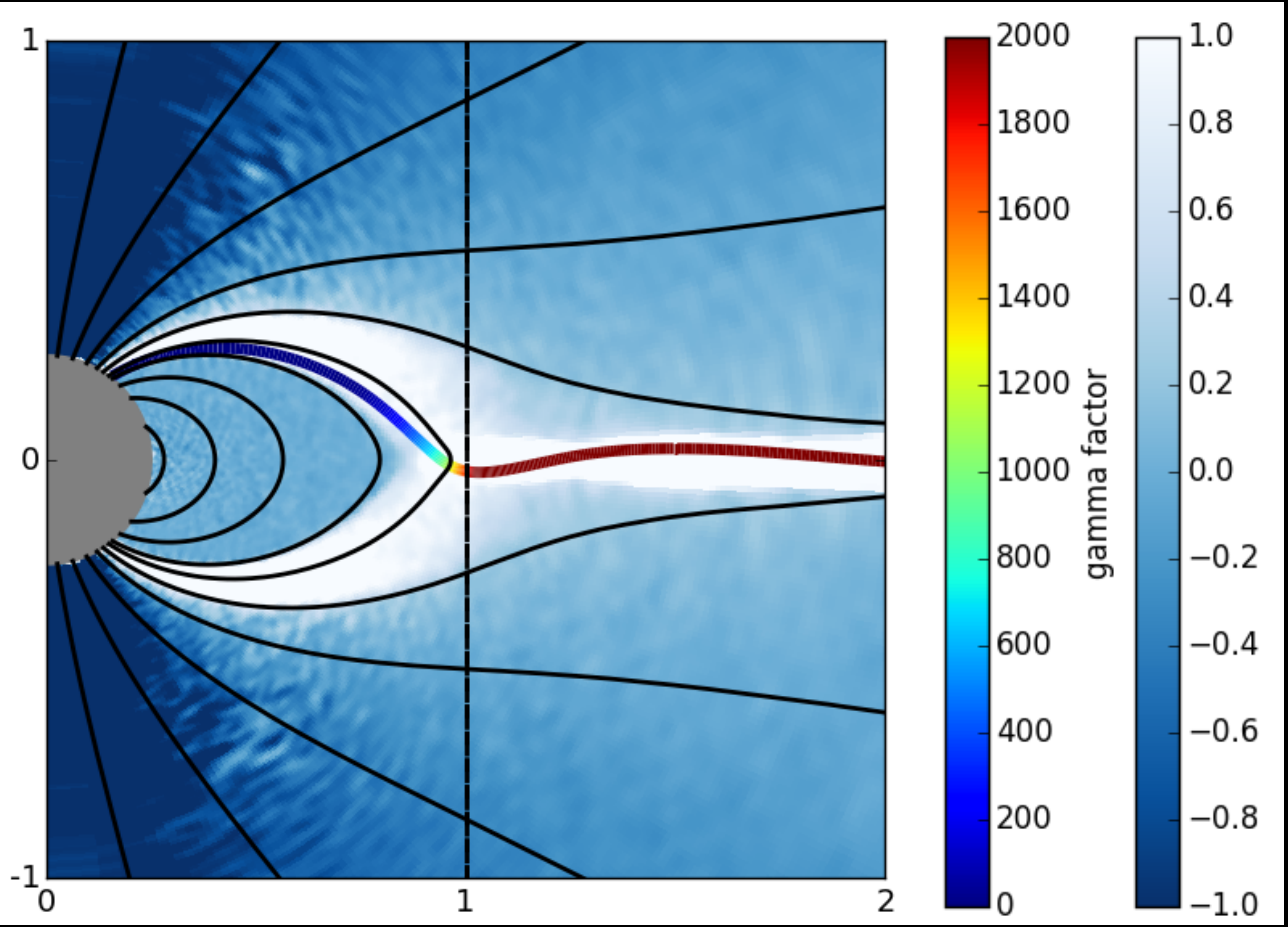
Conclusions

1. **Targeted studies** of polar cap and Y-point beyond force-free limit.

2. **Polar cap** — computed spatial distribution of pair production with implications for radio emission and gaps.

3. **Y-point** — looking into particle trajectories and dissipation. Up to 25% of FF luminosity dissipated in current sheet. Can this be reduced in PIC?

Positron Trajectory 1



First Principles Modeling

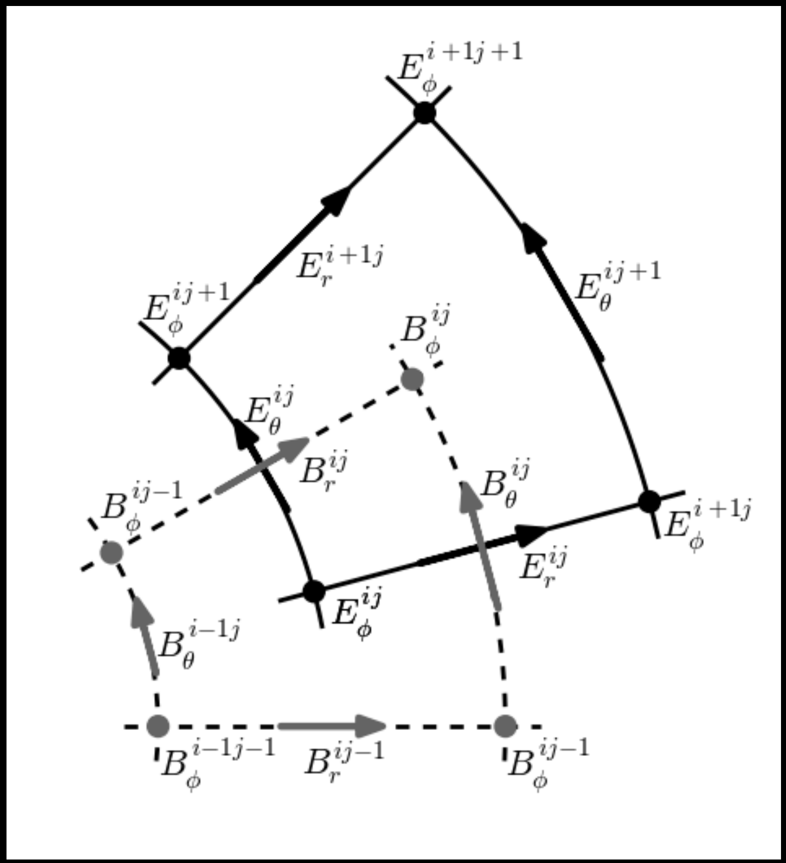
Particle in Cell Method

Computer solves time-dependent Maxwell equations

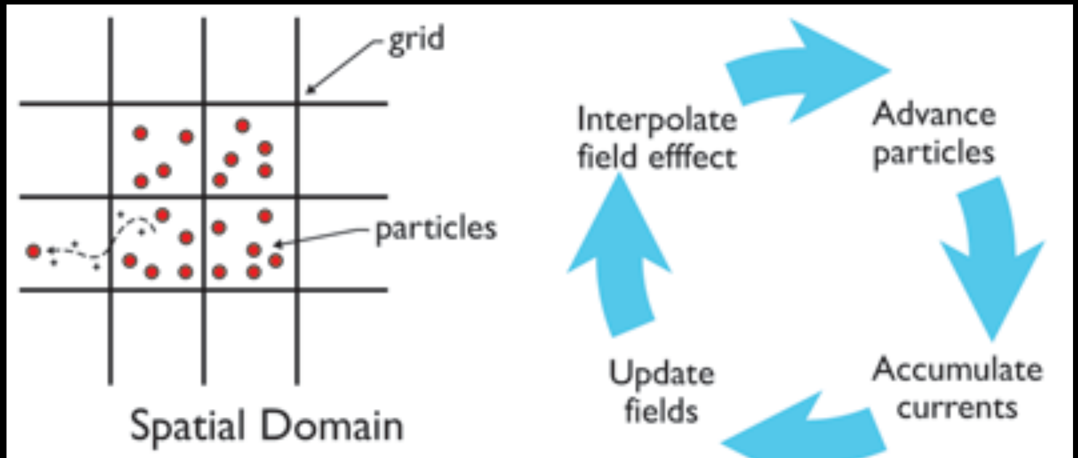
$$\frac{\partial \mathbf{B}}{\partial t} = -c\nabla \times \mathbf{E} \qquad \frac{\partial \mathbf{E}}{\partial t} = c\nabla \times \mathbf{B} + \mathbf{J}$$

Time-independent Maxwell eqns are initial conditions

$$\nabla \cdot \mathbf{E} = \rho \qquad \nabla \cdot \mathbf{B} = 0$$



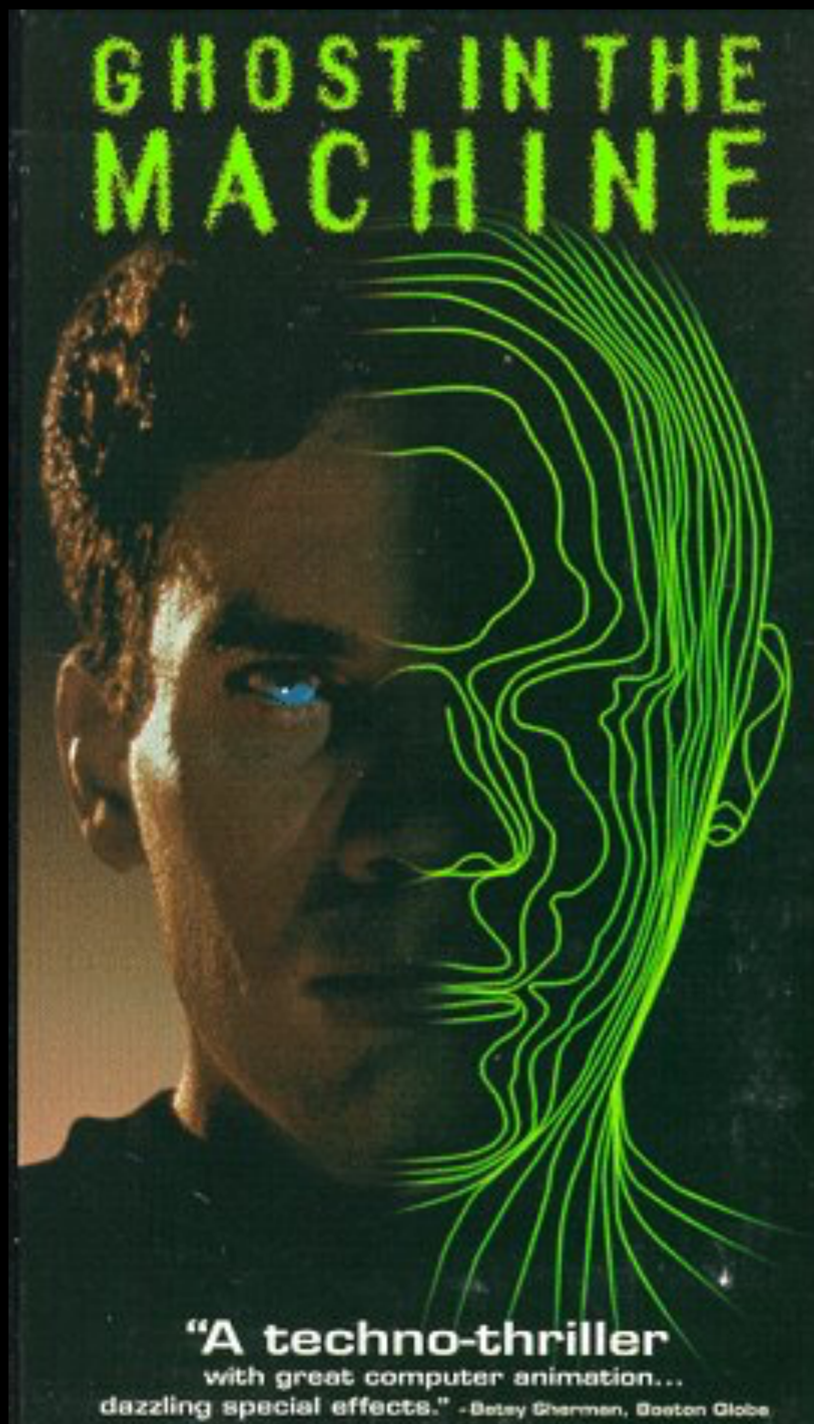
- ⋆ Grid tells particles where to move
- ⋆ Particles tell grid where current flows

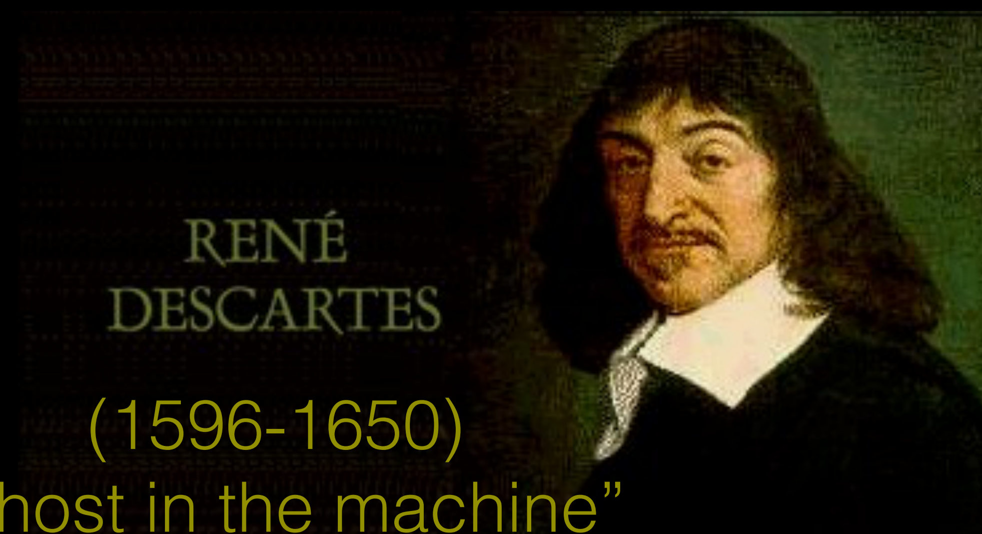


VPIC (LANL)

+ PIC method handles vacuum
+ Reconnection (at high resolution)
+ Relativistic plasma instabilities
+ Particle acceleration & pair physics
- Computationally expensive

PICsar Studios Presents



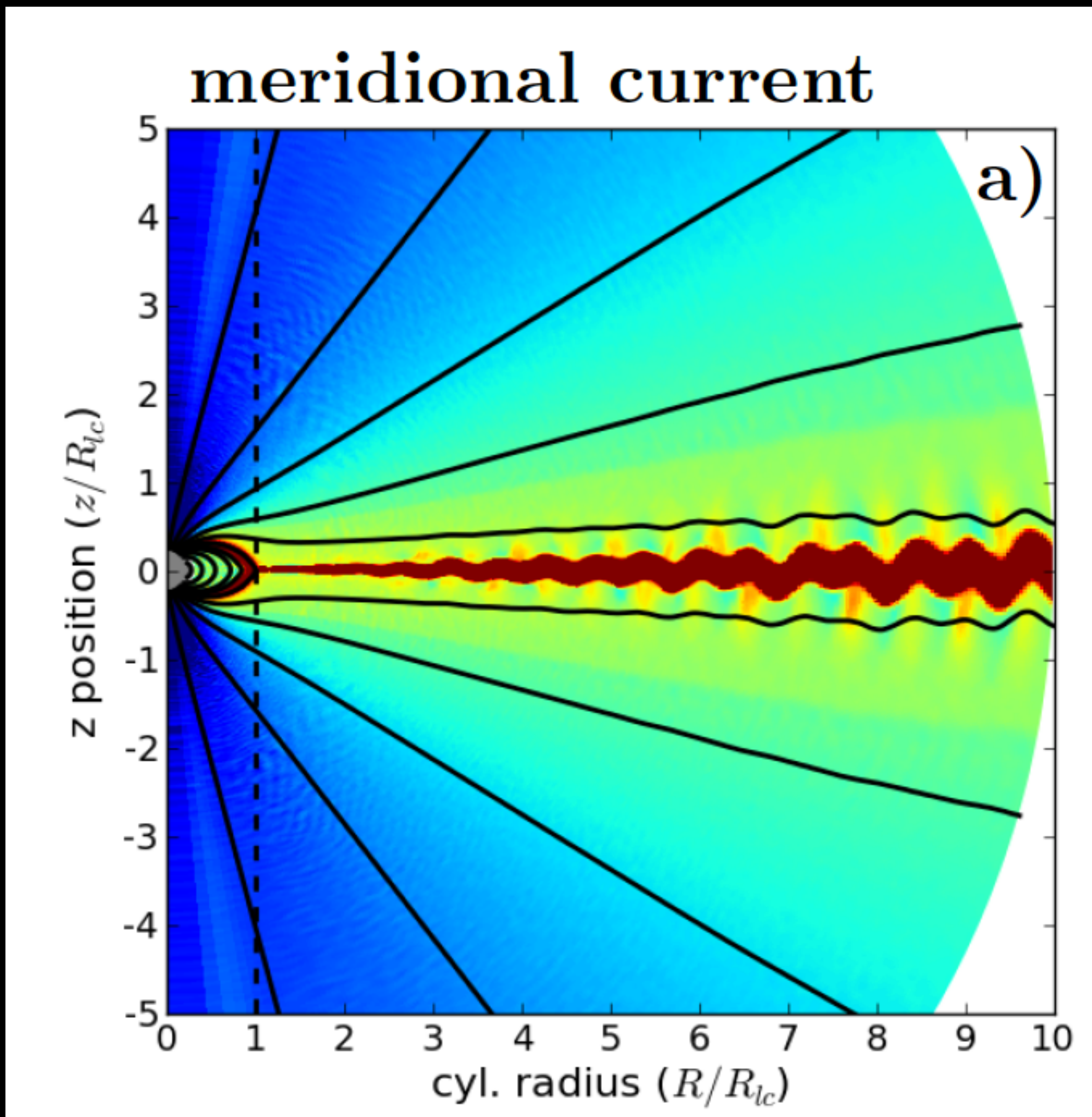


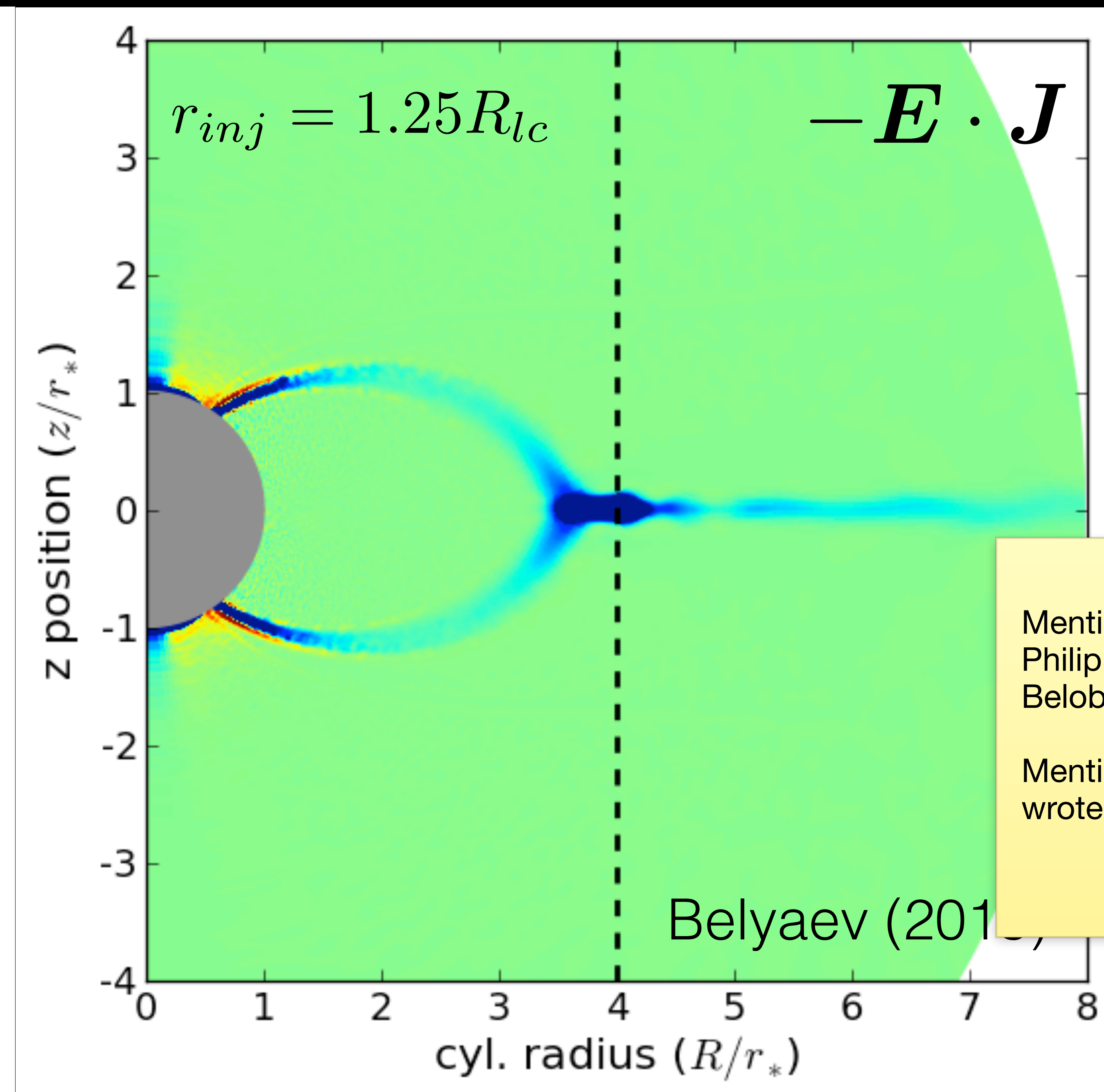
(1596-1650)
"ghost in the machine"

Predicted the development of PIC codes

1. Invented gridded mesh coordinate system (used for the field variables).
2. Presaged introduction of particles aka "little ghosts" into PIC codes.

(Almost) Force-Free Axisymmetric PIC





Mention
Philippov
Beloboro

Mention
wrote.

— Kink instability of current sheet beyond light cylinder.

— ~10% of Poynting flux dissipated near Y-point.

Understanding Pair Production

$$J^\mu J_\mu \equiv -(\rho c)^2 + J^2$$

Timokhin & Arons (2013)

$$J^\mu J_\mu > 0 \text{ pairs}$$
$$J^\mu J_\mu < 0 \text{ no pairs}$$

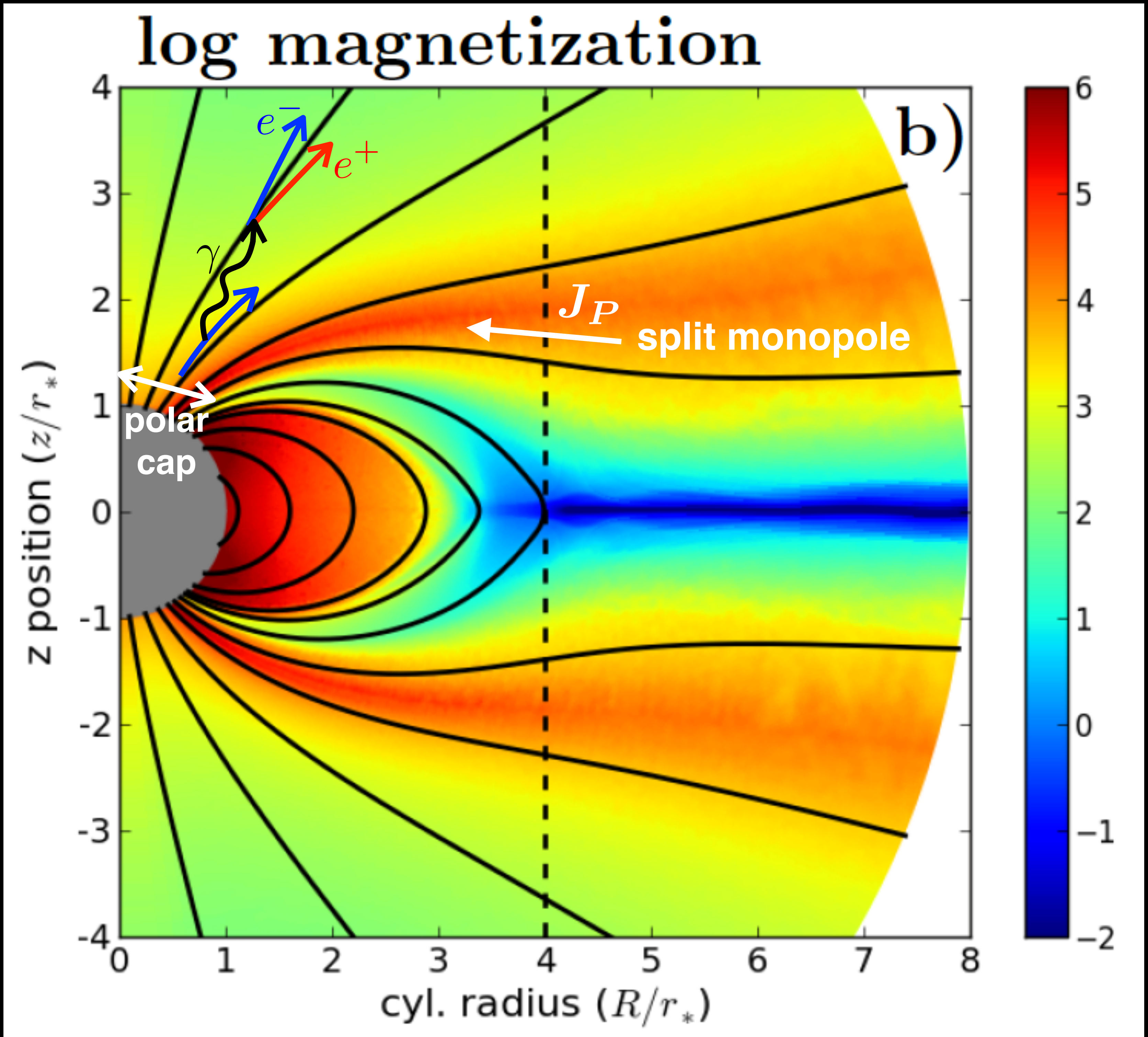
We know charge density

$$\rho_{GJ} \approx -\frac{(\boldsymbol{\Omega} - \boldsymbol{\omega}_{LT}) \cdot \boldsymbol{B}}{2\pi c \alpha}$$

Can extrapolate current

$$\boldsymbol{\nabla} \cdot \boldsymbol{B} = 0, \;\; \boldsymbol{\nabla} \cdot (\alpha \boldsymbol{J}) = 0$$

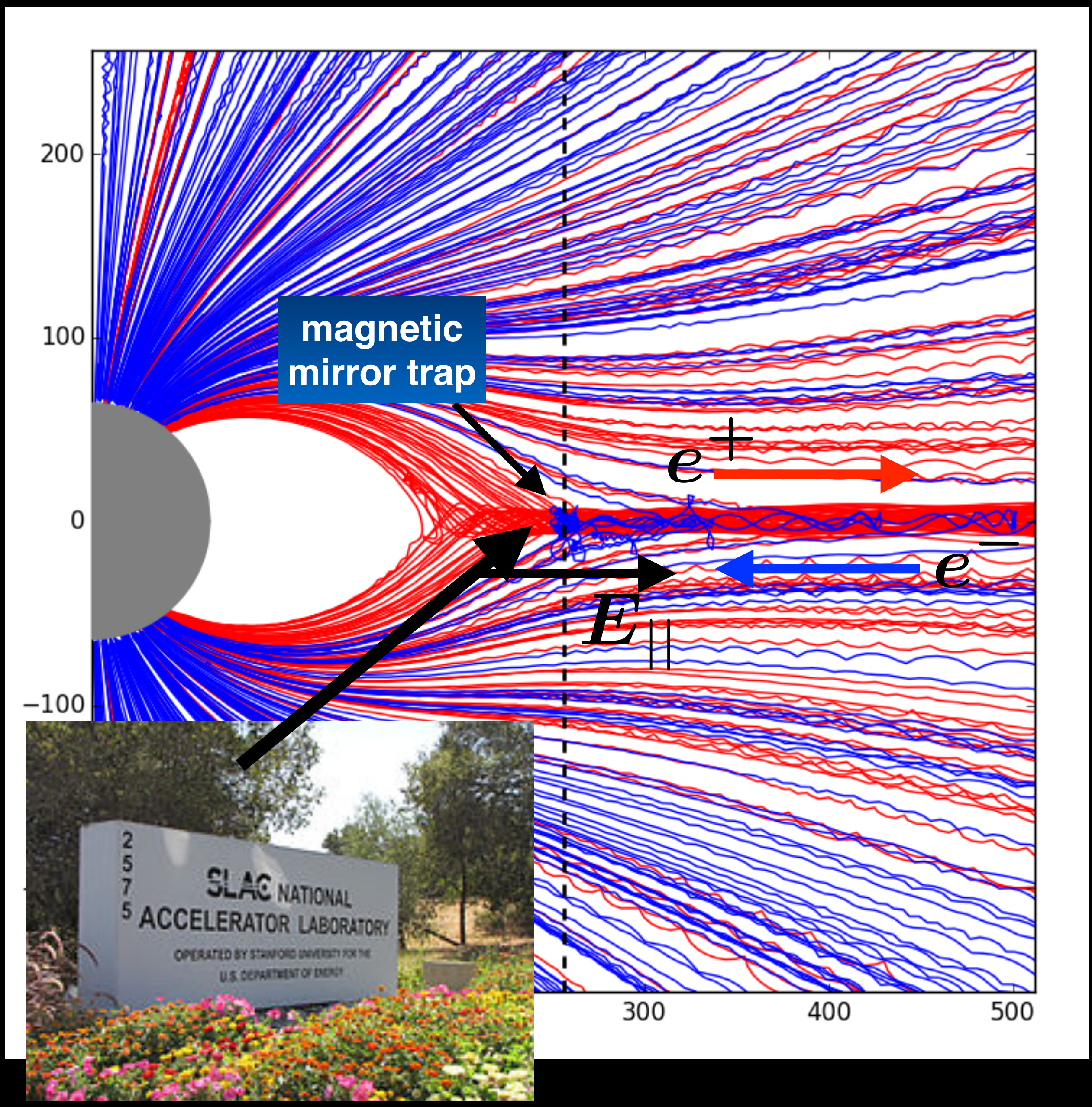
$$\alpha \boldsymbol{J_P} \propto \boldsymbol{B_P}$$



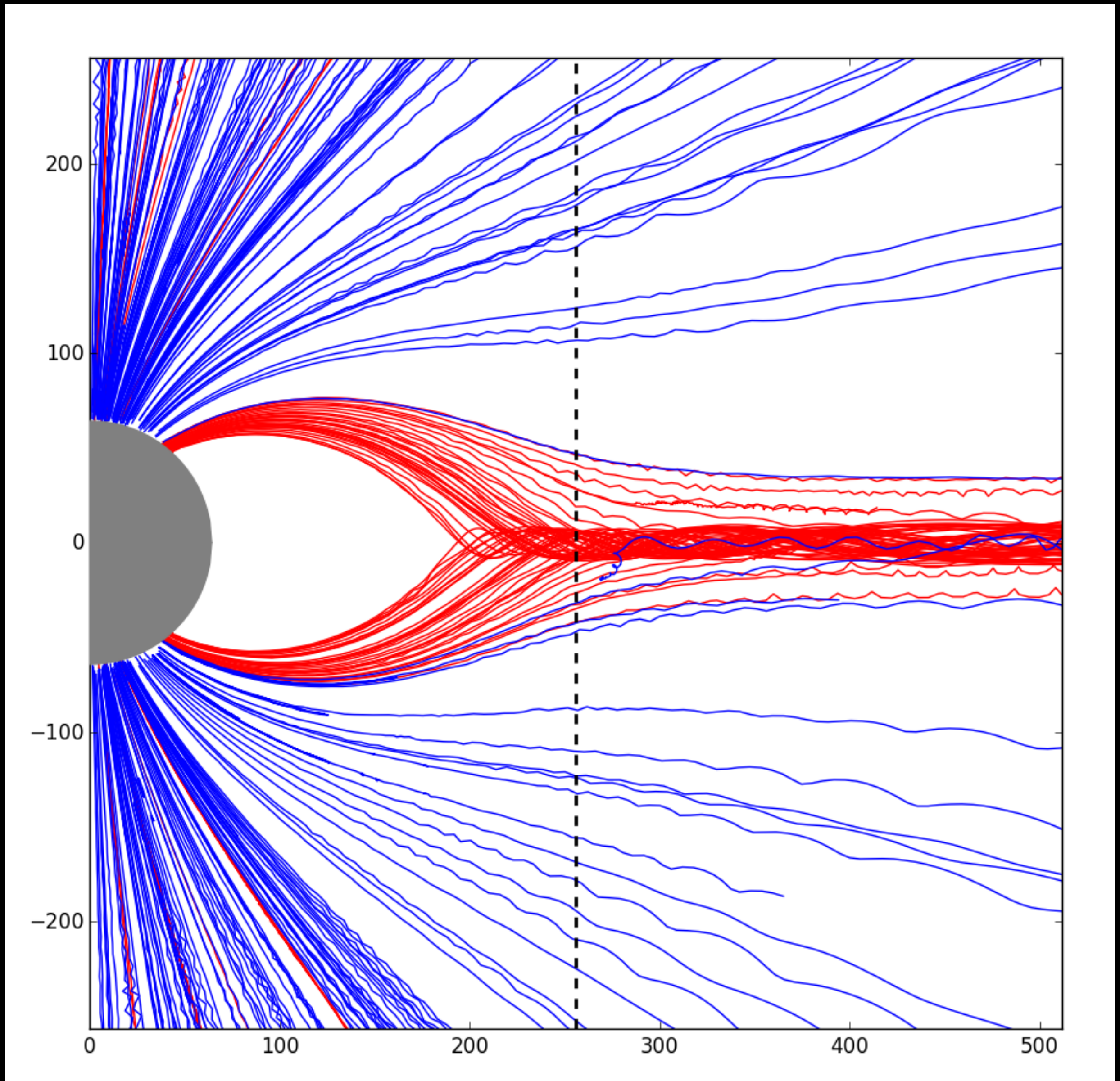
To the Y-point and Beyond!

- Understand how particle acceleration at the Y-point works.
- Atlas of 3D PIC sims to directly model pulsed gamma ray emission.

Surface Pair Production Everywhere

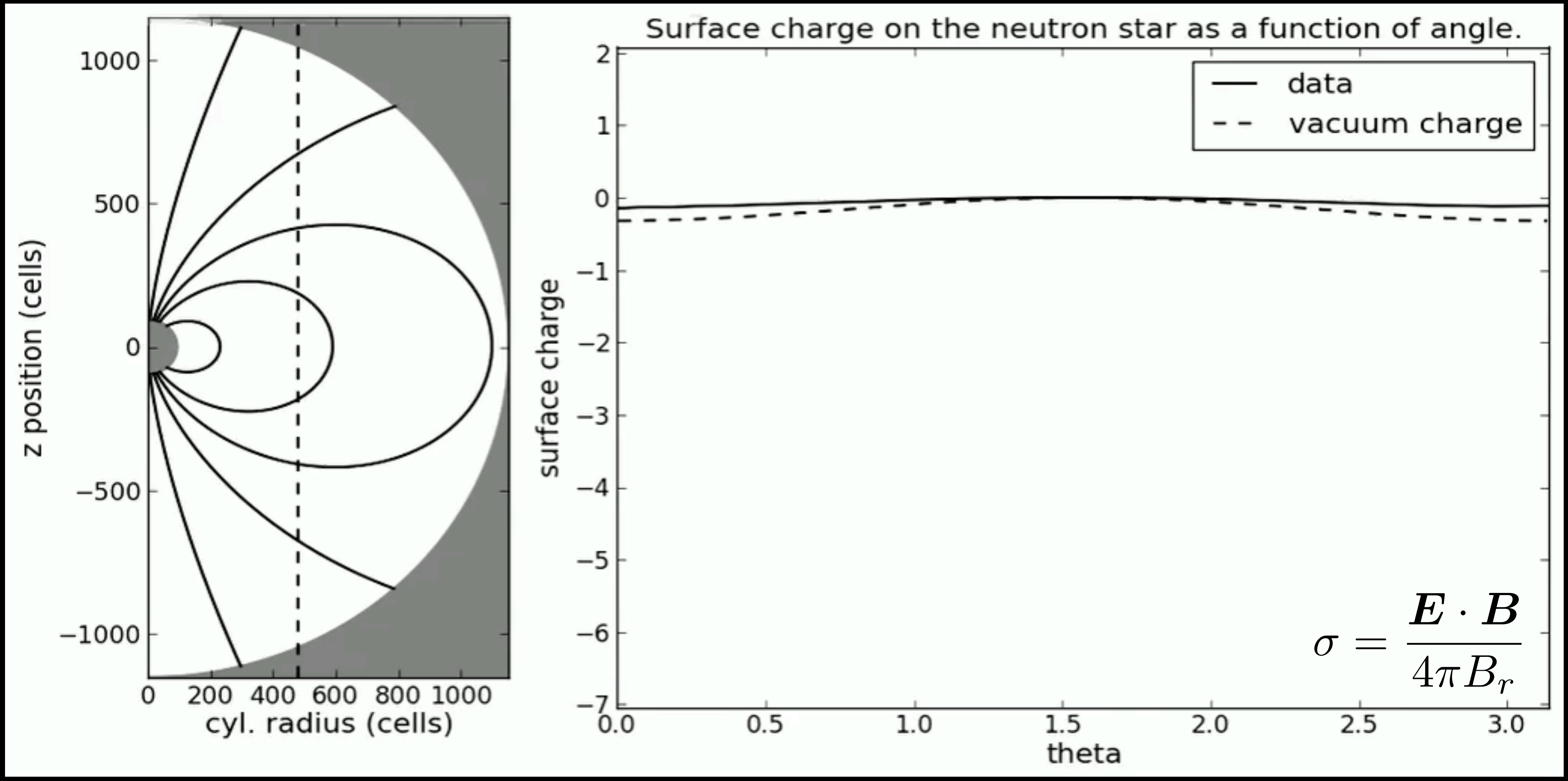


PP as in Belyaev & Parfrey



A Different Injection Scheme

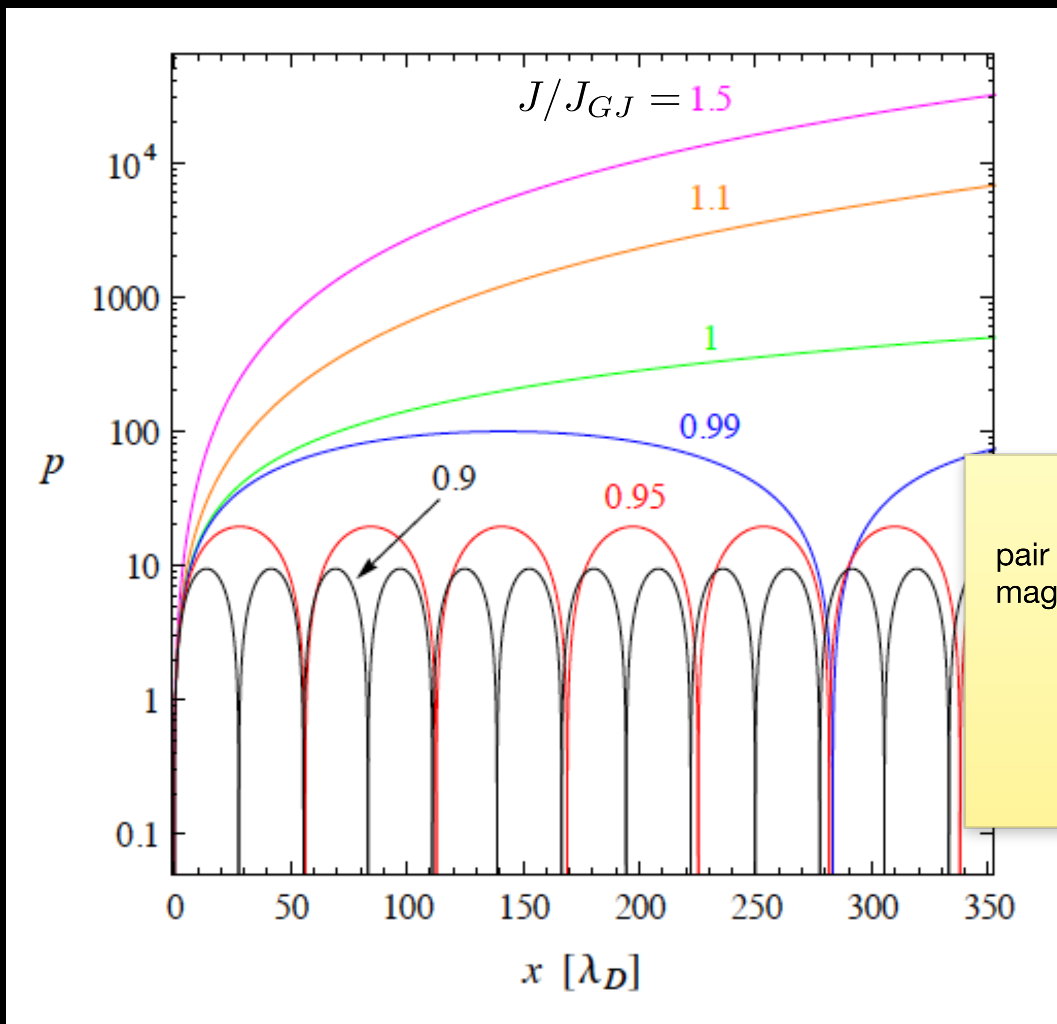
Surface Charge Injection On. Pair Production Off.



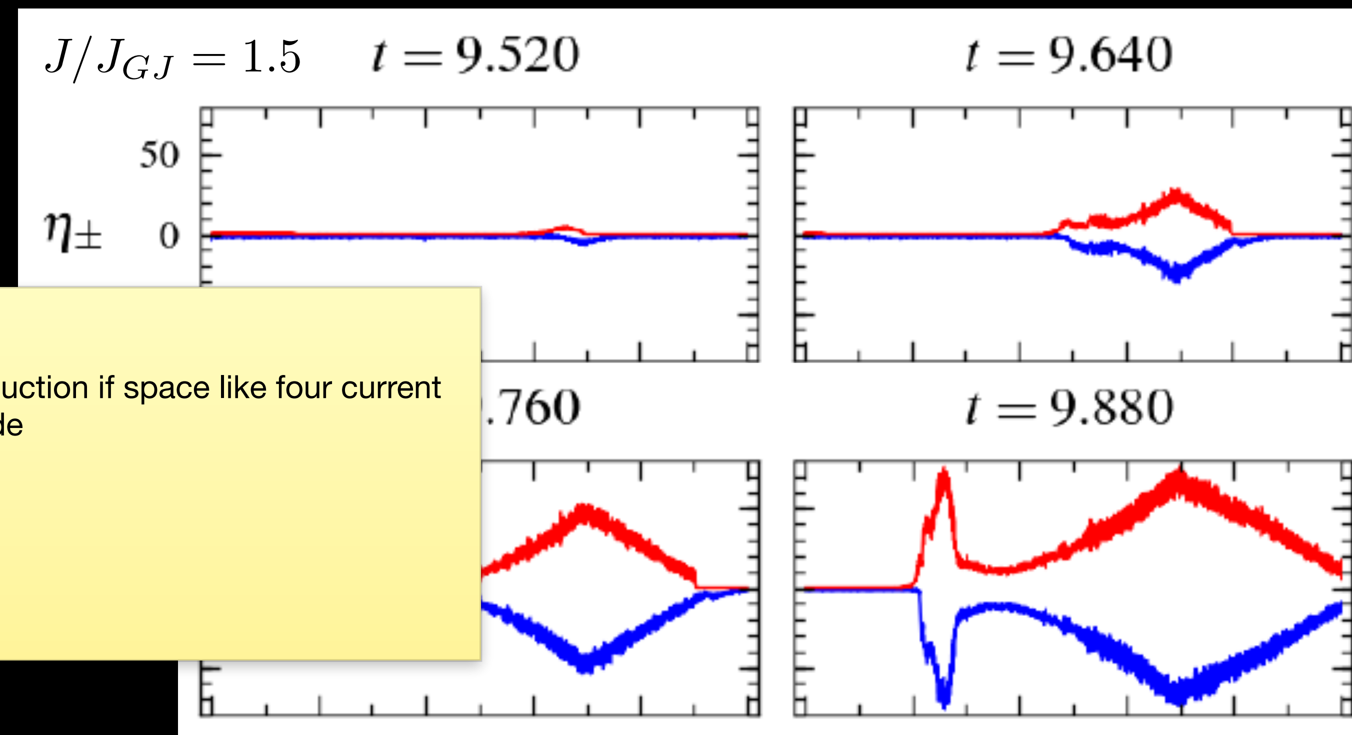
— Without pair production, the electrosphere solution of Krause-Polstorff & Michel (1985) is realized.

— Different charge injection schemes can produce very different solutions!

$\gamma - B$ Pair Production Criteria



$J_{GJ} \equiv \rho_{GJ} c$ $\eta_\pm \equiv \rho_\pm / \rho_{GJ}$



pair production if space like four current magnitude

Timokhin & Arons (2013)

When $J/J_{GJ} > 0$ **PP criteria can be expressed as:**

$J^\mu J_\mu > 0$ pair production $J^\mu J_\mu < 0$ no pair production

$$J^\mu J_\mu \equiv -(\rho c)^2 + J^2 = -J_{GJ}^2 + J^2$$

Understanding the Result

Key Observation:

1) Goldreich-Julian density is determined <u>locally</u>.

$$\boldsymbol{E} = -\boldsymbol{V}_0 \times \boldsymbol{B}/c.$$

$$\boldsymbol{V}_0 \equiv \begin{cases} \boldsymbol{\Omega} \times \boldsymbol{r}, & \text{flat} \\ \alpha^{-1} \left(\boldsymbol{\Omega} - \boldsymbol{\omega}_{LT} \right) \times \boldsymbol{r}, & \text{Kerr} \end{cases}$$

Ferraro's corotation theorem
— No EMFs in steady state —

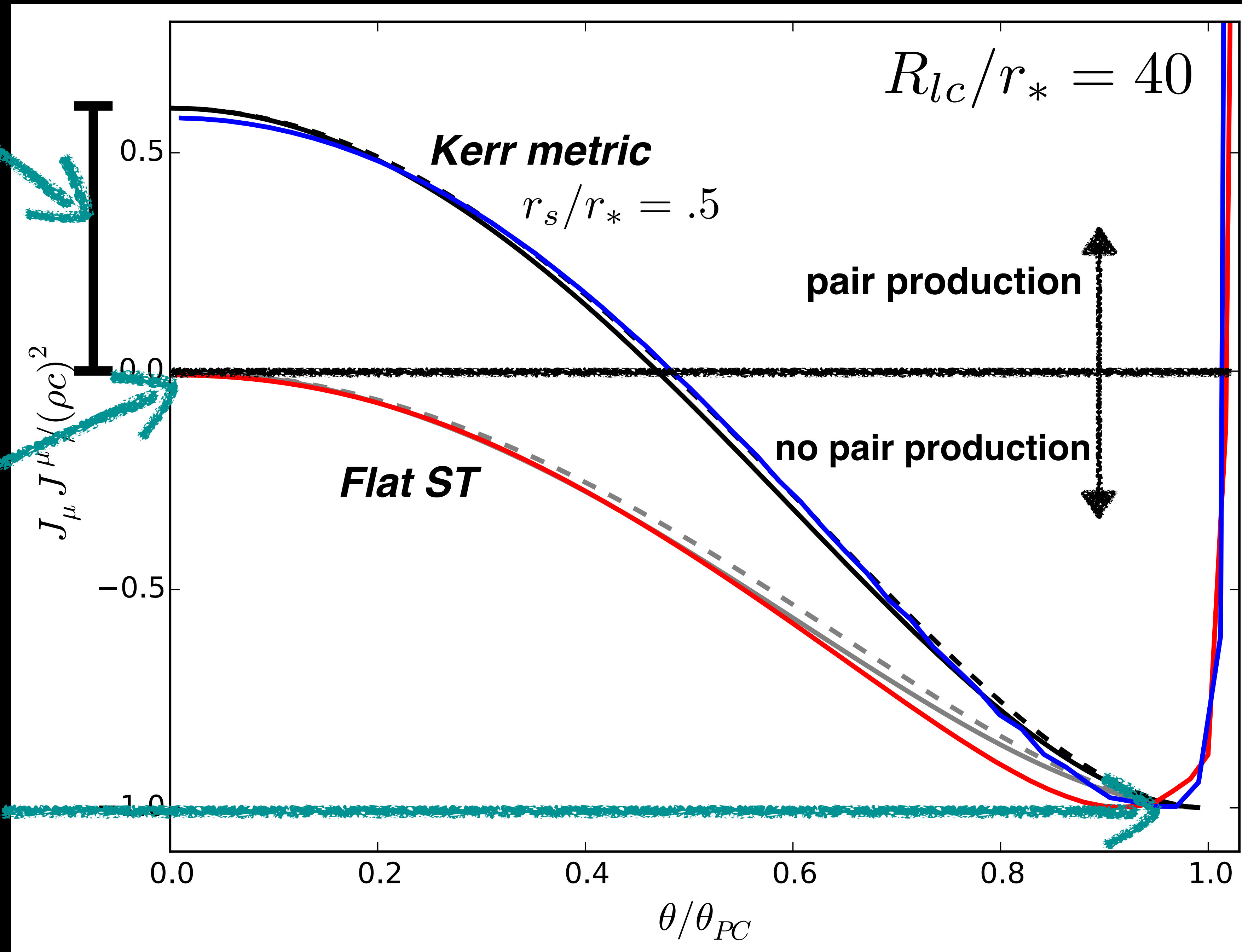
$$\begin{aligned} \rho_G &\equiv -\boldsymbol{\nabla} \cdot \left(\boldsymbol{V}_0 \times \boldsymbol{B} \right) / 4\pi c \\ &= \frac{-\boldsymbol{B} \cdot (\boldsymbol{\nabla} \times \boldsymbol{V}_0) + \boldsymbol{V}_0 \cdot (\boldsymbol{\nabla} \times \boldsymbol{B})}{4\pi c} \\ &= -\frac{(\boldsymbol{\Omega} - \boldsymbol{\omega}_{LT}) \cdot \boldsymbol{B}}{2\pi c \alpha} + \frac{\boldsymbol{V}_0 \cdot (\boldsymbol{\nabla} \times \boldsymbol{B})}{4\pi c} \end{aligned}$$

First term dominates as long as poloidal B field variation larger than scale of polar cap

Difference due exclusively to frame dragging, which lowers the value of the Goldreich-Julian density.

Four current magnitude equals zero on polar axis in flat spacetime.

Last open field line carries no current in analytical model, but simulations show return current has a distributed component.



General Result

$$J^\mu J_\mu \approx -\left(\frac{(\boldsymbol{\Omega}-\boldsymbol{\omega}_{LT})\cdot \boldsymbol{B}}{2\pi\alpha}\right)^2 + \left(\frac{1}{\alpha}\frac{dI}{d\Psi}B_P\right)^2$$

$$= \left(\frac{B_P\Omega}{2\pi\alpha}\right)^2\left[\hat{b}_z^2\Big|_{SM} - \left(\left(1-\frac{\omega_{LT}}{\Omega}\right)\hat{b}_z\right)^2\Big|_{PC}\right]$$

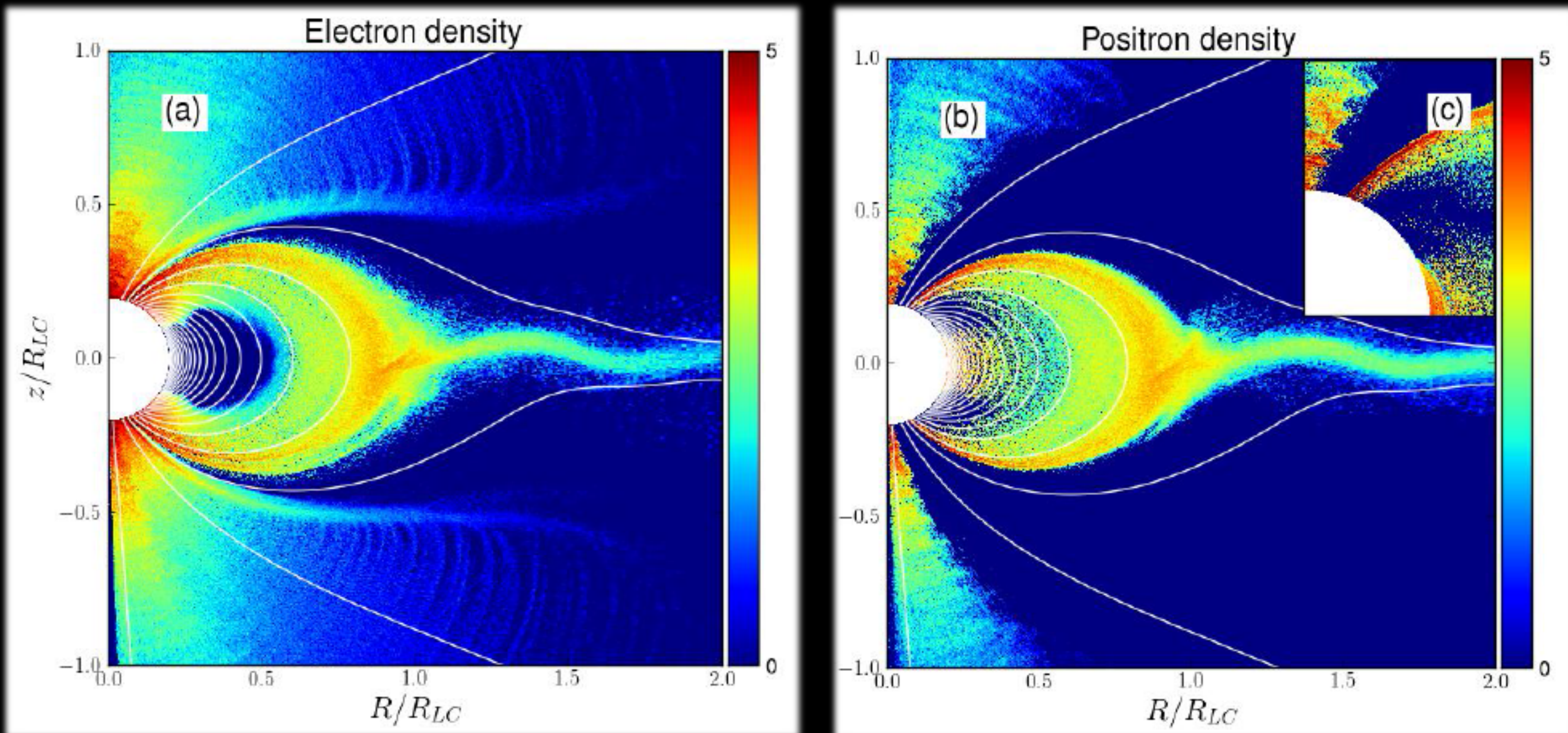
Flat Spacetime

$$J^\mu J_\mu \text{ is } \begin{cases} \text{timelike}, & \hat{b}_z^2\Big|_{PC} > \hat{b}_z^2\Big|_{SM} \\ \text{null}, & \hat{b}_z^2\Big|_{PC} = \hat{b}_z^2\Big|_{SM} \\ \text{spacelike}, & \hat{b}_z^2\Big|_{PC} < \hat{b}_z^2\Big|_{SM} \end{cases}$$

Comparison with PIC results

—Pair production using PIC and GR occurs in same regions as expected analytically.

—No surface pair production in flat spacetime in axisymmetry.

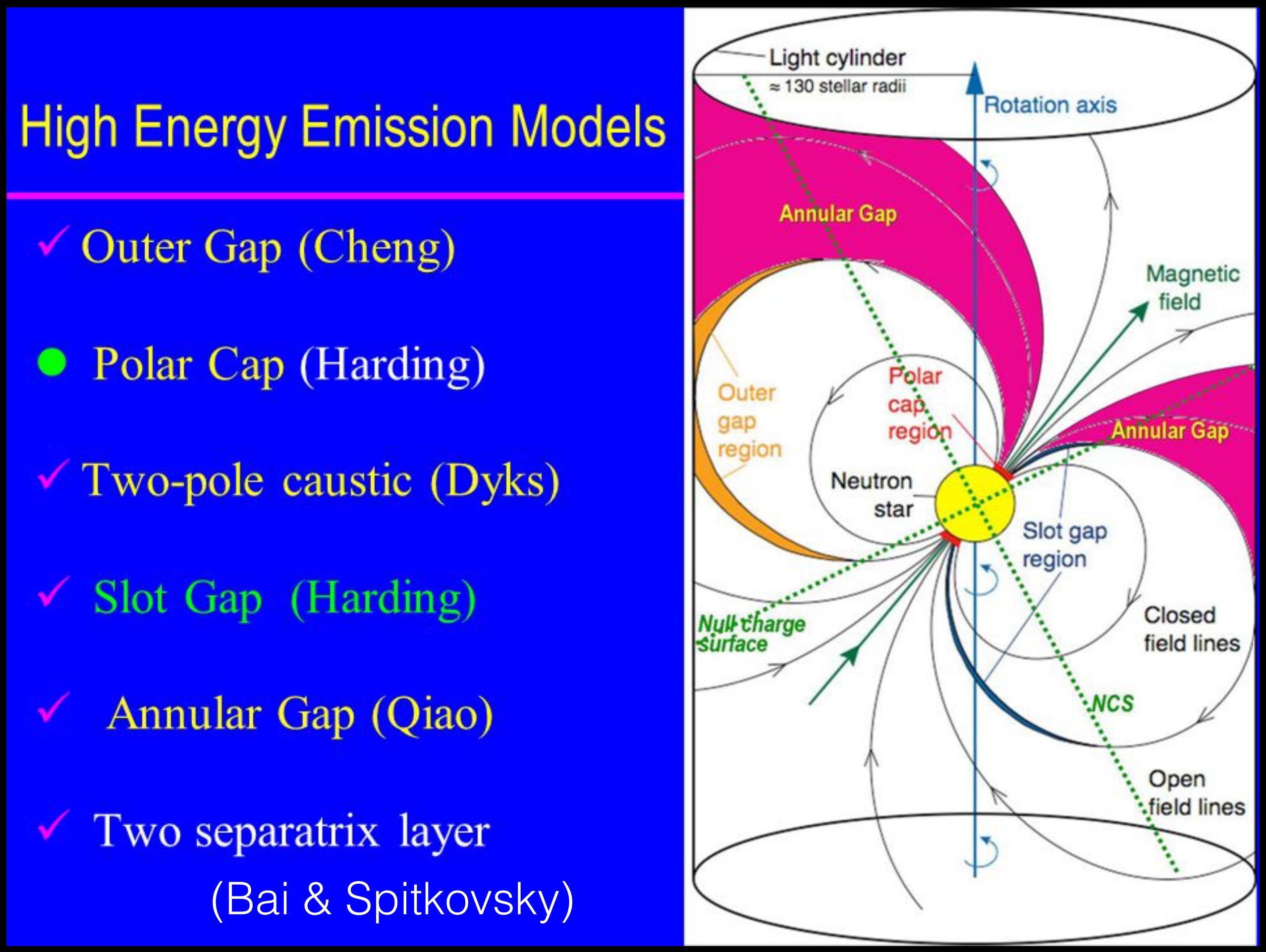


Philippov et al. (2015)

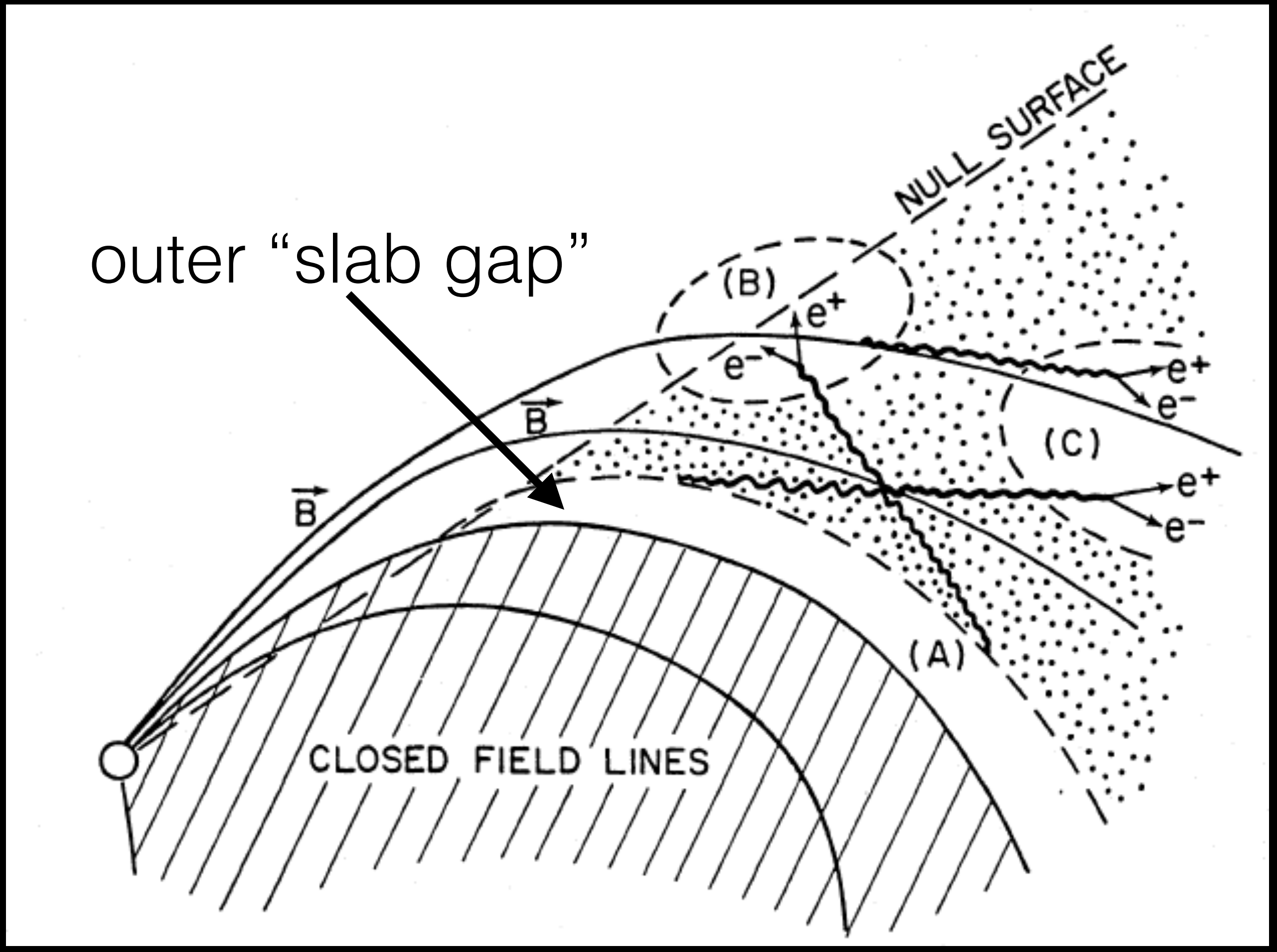
Mind the Gap

$\gamma - B$ **only — wide gap extending from pulsar surface in region of field lines that don't support pair production.**

$\gamma - B$ **&** $\gamma - \gamma$ **— narrow slab gap extending from pulsar surface above the last open field line.**



Slide from presentation by Yuanjie Du



Cheng, Ho & Ruderman (1986)

Summary

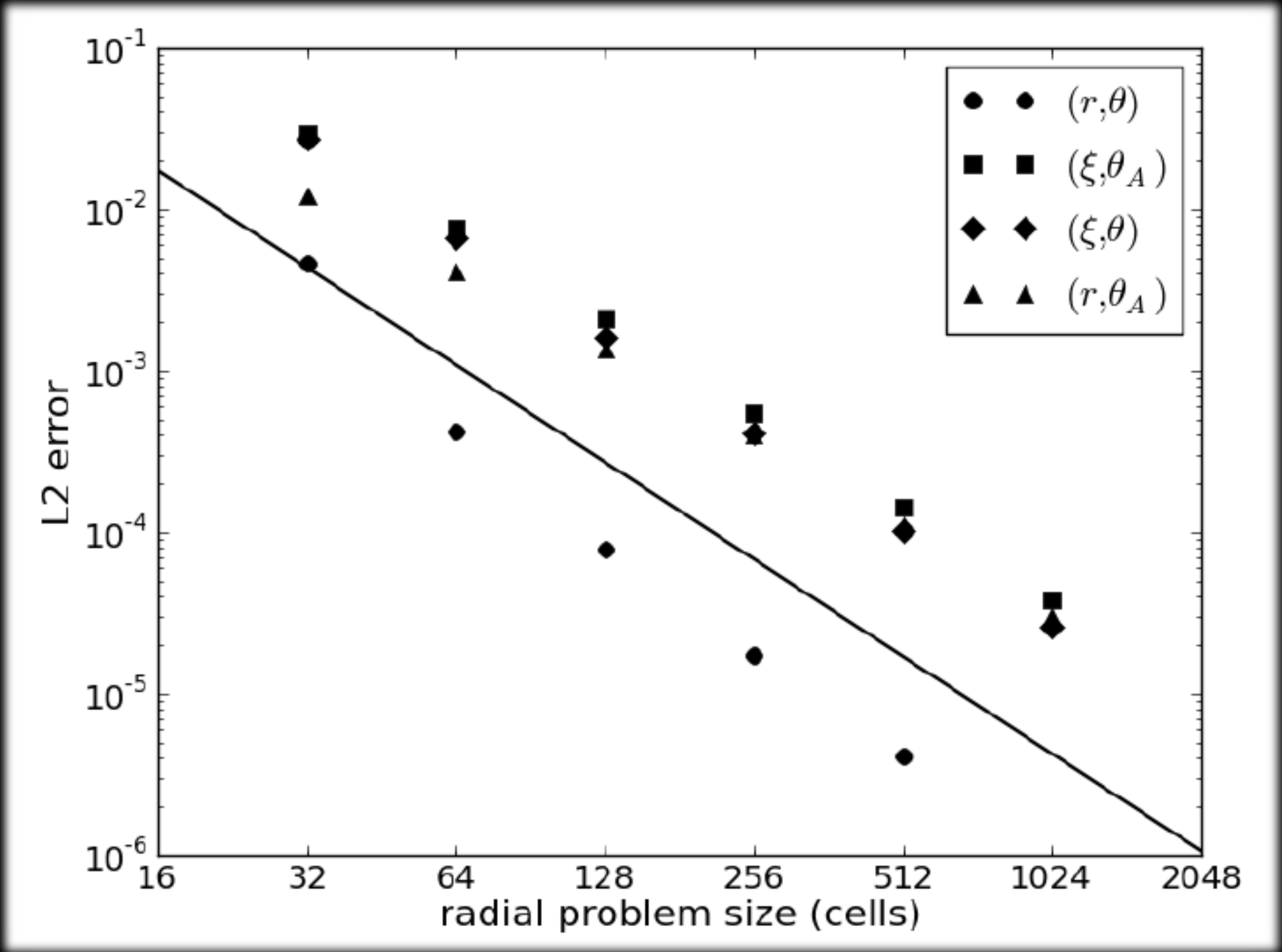
1. We have derived analytically the distribution of pair producing field lines over the polar cap for the aligned rotator for arbitrary magnetic field.

2. In Kerr spacetime, the polar axis is always pair producing, but there is a bundle of open field lines adjoining the last open field line which do not support pair production.

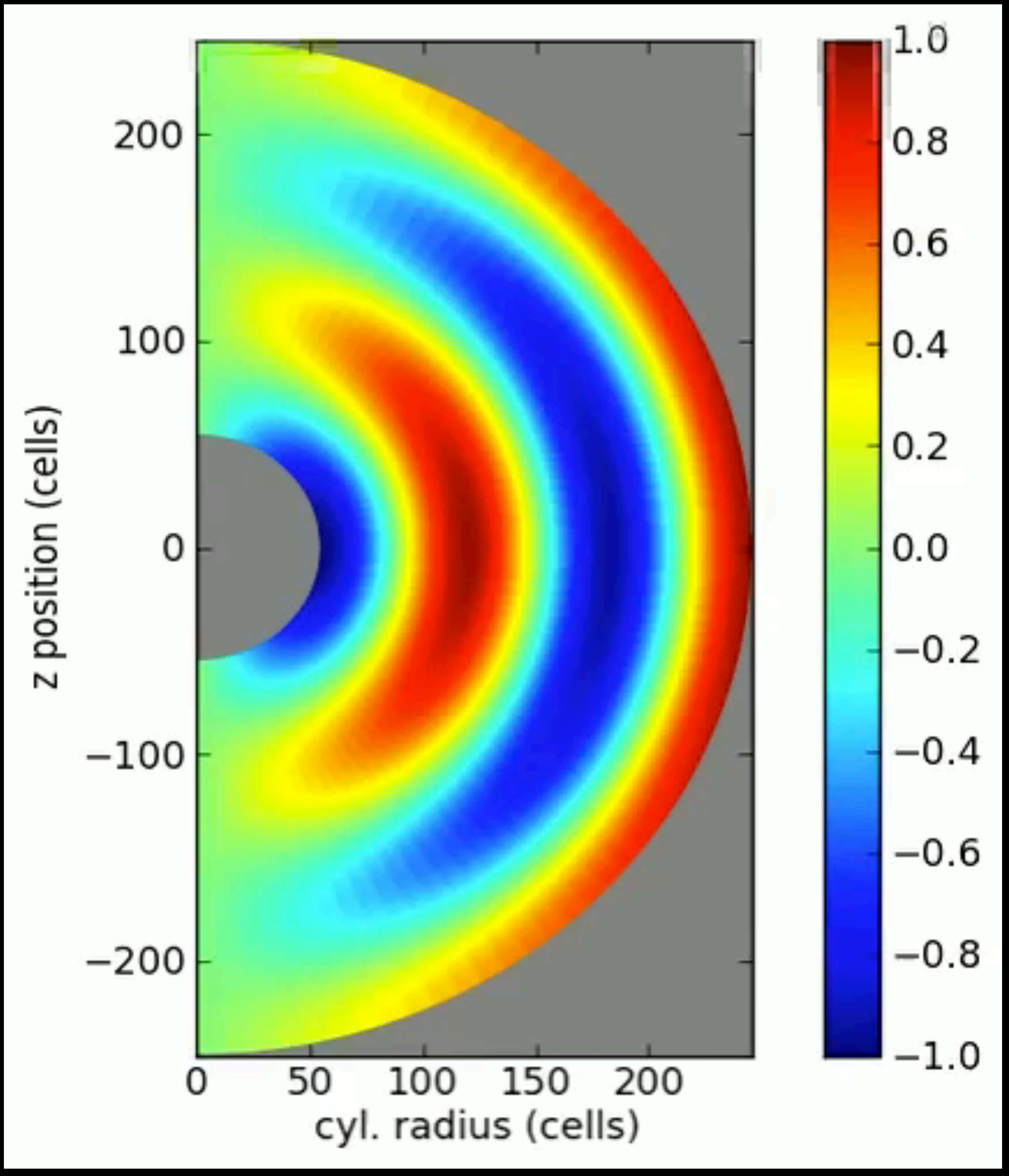
3. In flat spacetime, the current is timelike over the polar cap for the aligned rotator and there is no pair production.

4. Our work has implications for simulating a pair cascade from first principles in PIC simulations, as well as for phenomenological modeling of pulsar emission sites.

Convergence Test

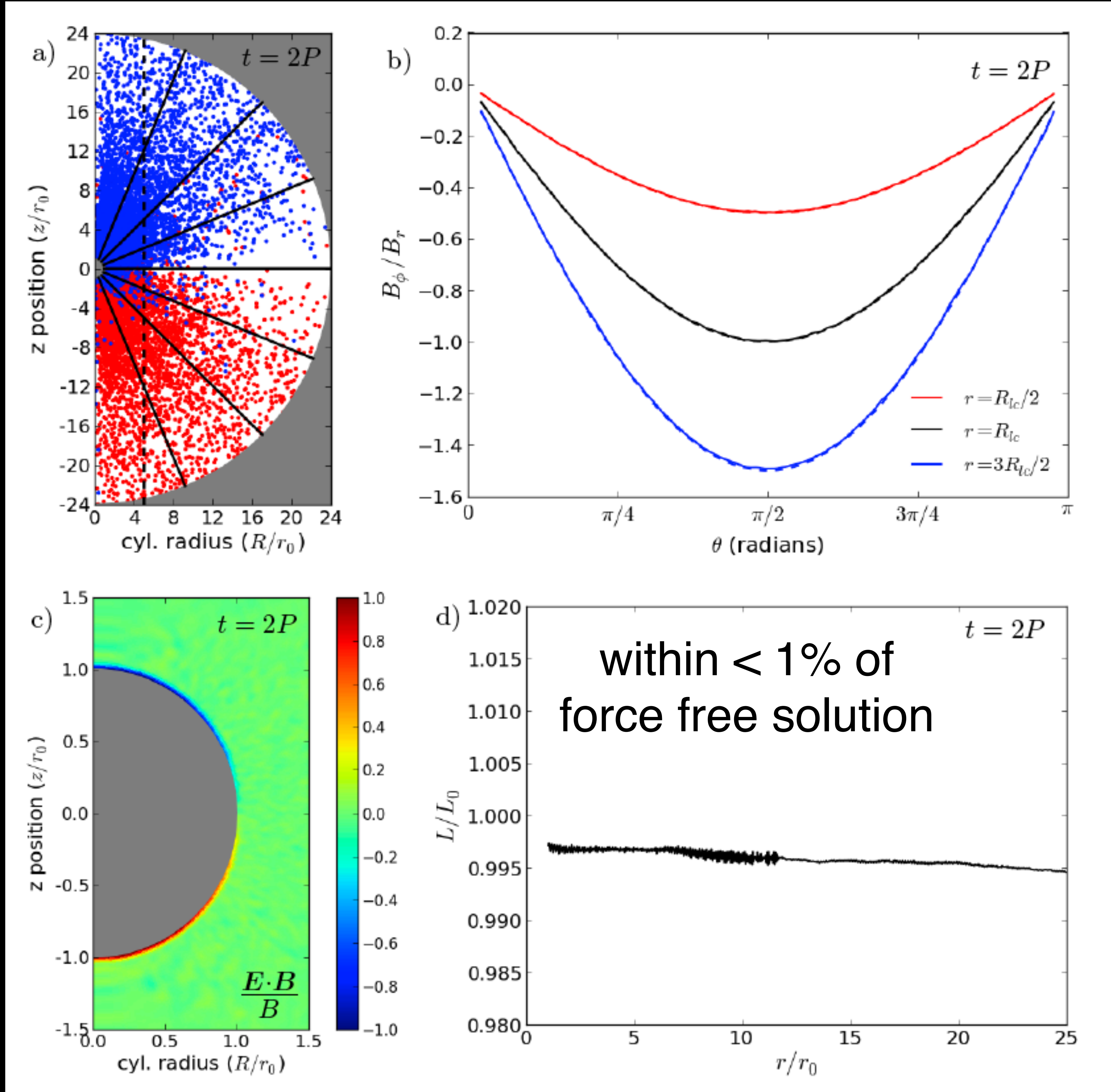


$$B_\phi \propto j_1(kr)\sin(\theta)\cos(kct)$$



Second order convergence in both space and time

Magnetic Monopole



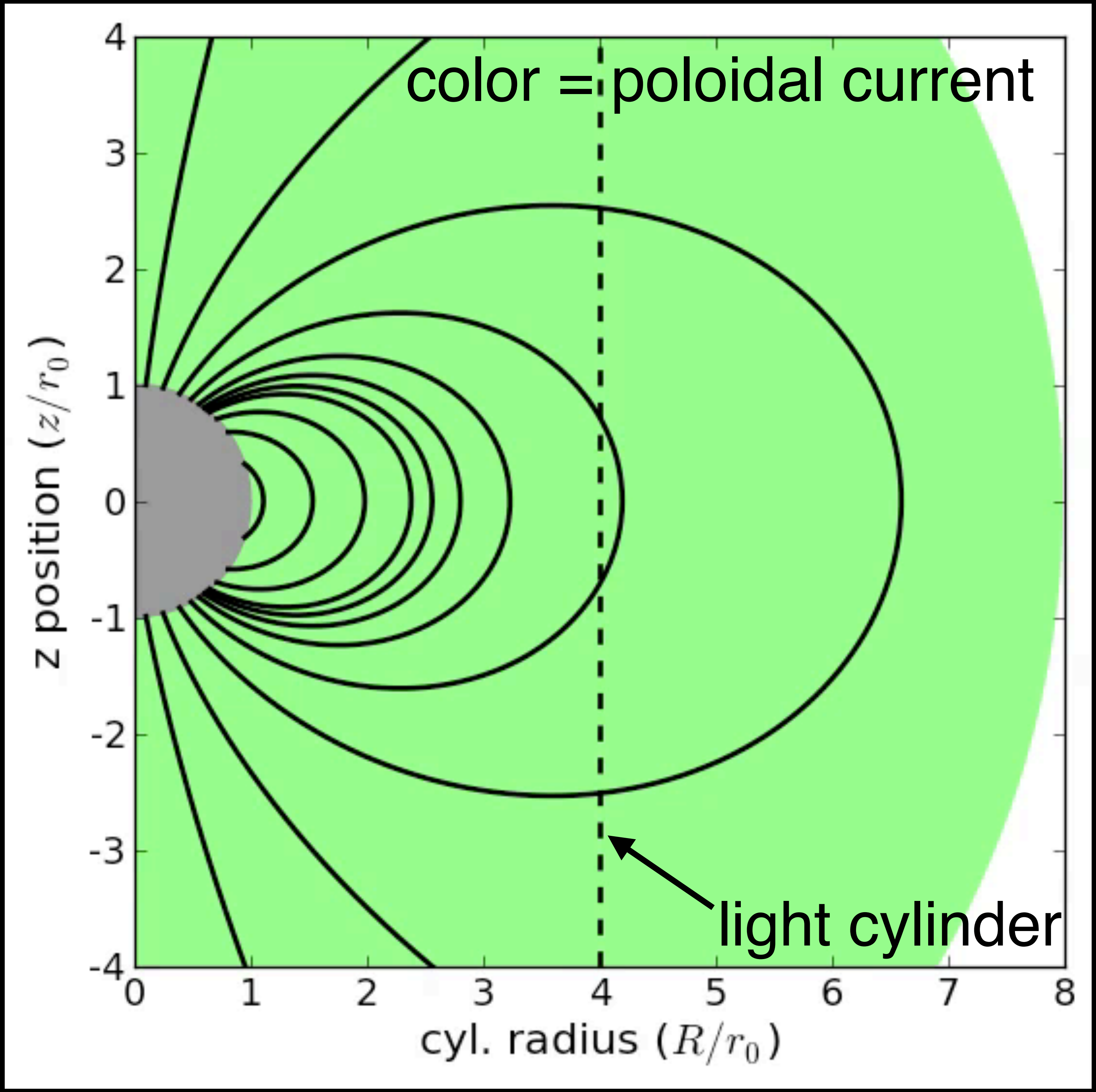
Dipole in Force-Free Limit

Initial conditions:
vacuum dipole fields

Inner boundary:
conducting sphere

Outer boundary:
radiation ($> 100r_0$)

Charge Injection:
surface charge +
volumetric injection



Different Approaches to Charge Injection

Belyaev (2015): Each timestep **inject surface charge** just above neutron star surface. Also, inject charge within a radius $r < r_{inj}$ to **relax to force-free field volumetrically**.

Cerutti et al. (2015): Each timestep **inject a multiple of the Goldreich-Julian density** near the surface. Limit pair multiplicity in injection region to a value of ~10 & give particles a "kick".

Chen & Beloborodov (2014) + Philippov (2015): Directly **simulate pair cascade**. Inject primary particles at surface. Pair production by curvature radiation on B field lines near surface. CB pair production by photon photon collisions in outer gap.

All schemes produce same results in force free limit

Details of Charge Injection

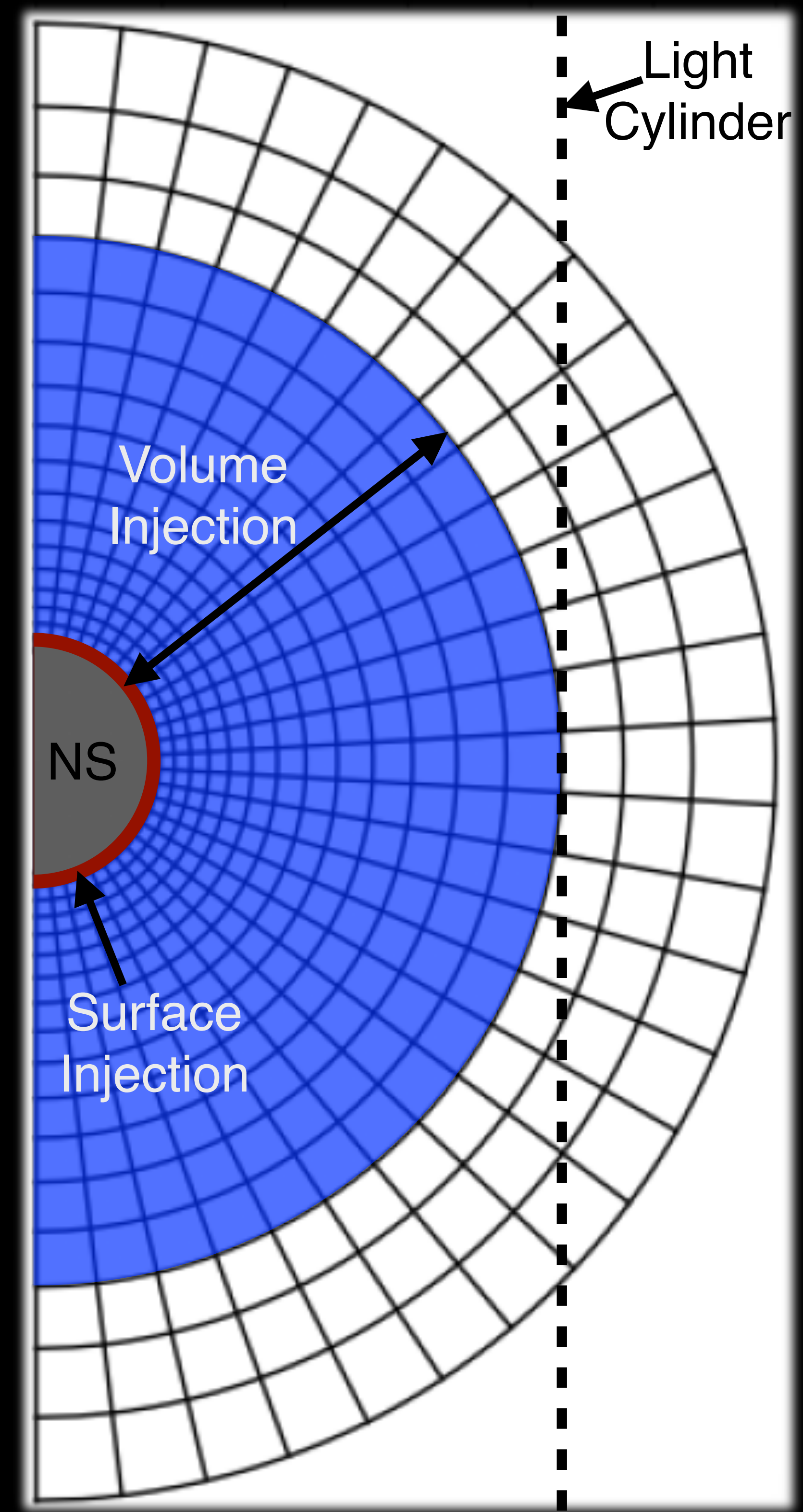
Surface Charge Injection: Inject a fraction of the surface charge each timestep just above NS surface.

Volume Charge Injection: Same basic formula as surface injection but inject throughout a volume. Relaxes $\mathbf{E} \cdot \mathbf{B}$ to zero in time.

$$4\pi q N_{inj}/dA = f_{inj} \frac{\mathbf{E} \cdot \mathbf{B}}{B}$$

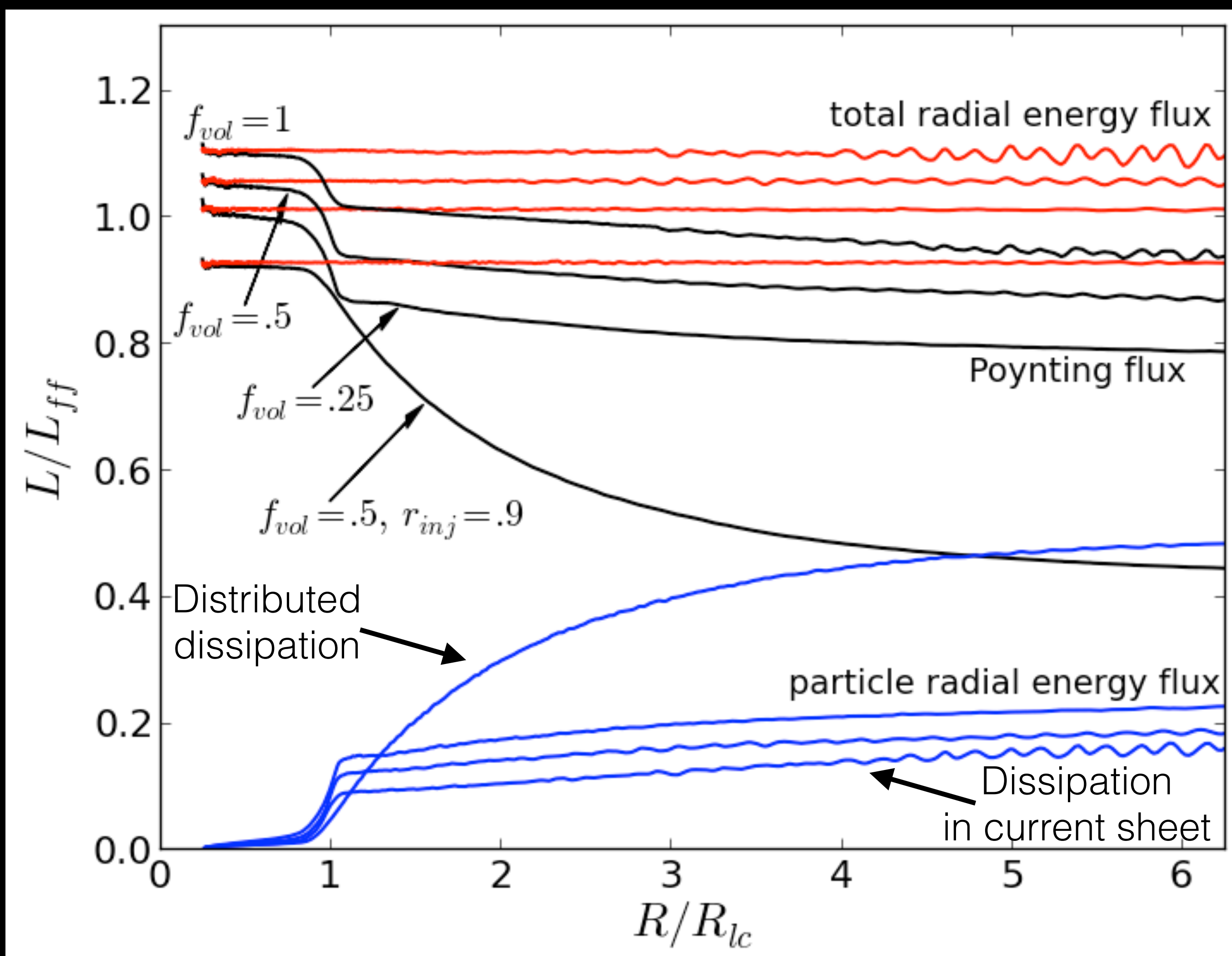
$$f_{inj} \lesssim 1, \quad \text{surface injection}$$

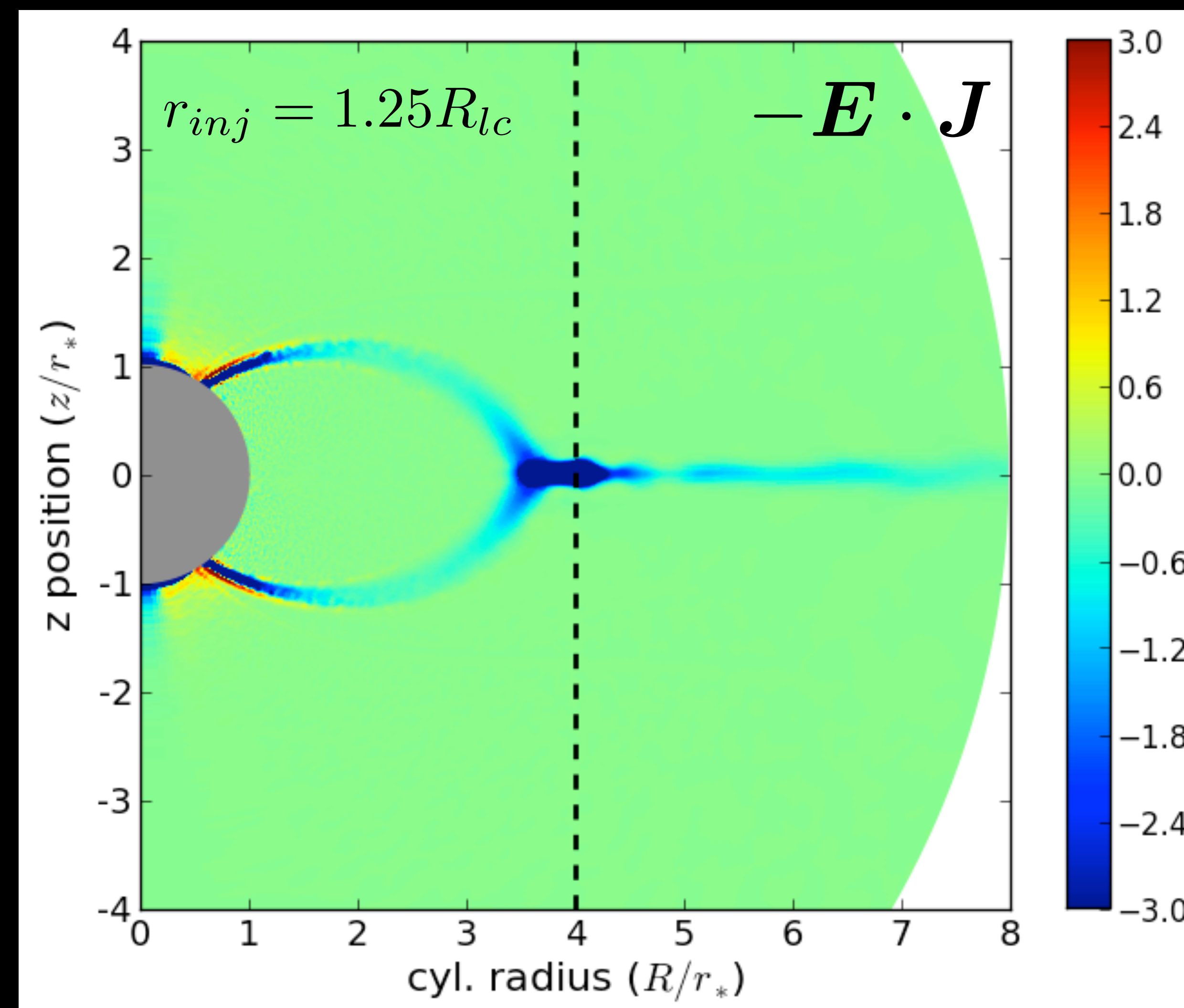
$$f_{inj} \sim c\Delta t/r, \quad \text{vol. injection}$$



Dissipation & Particle Acceleration

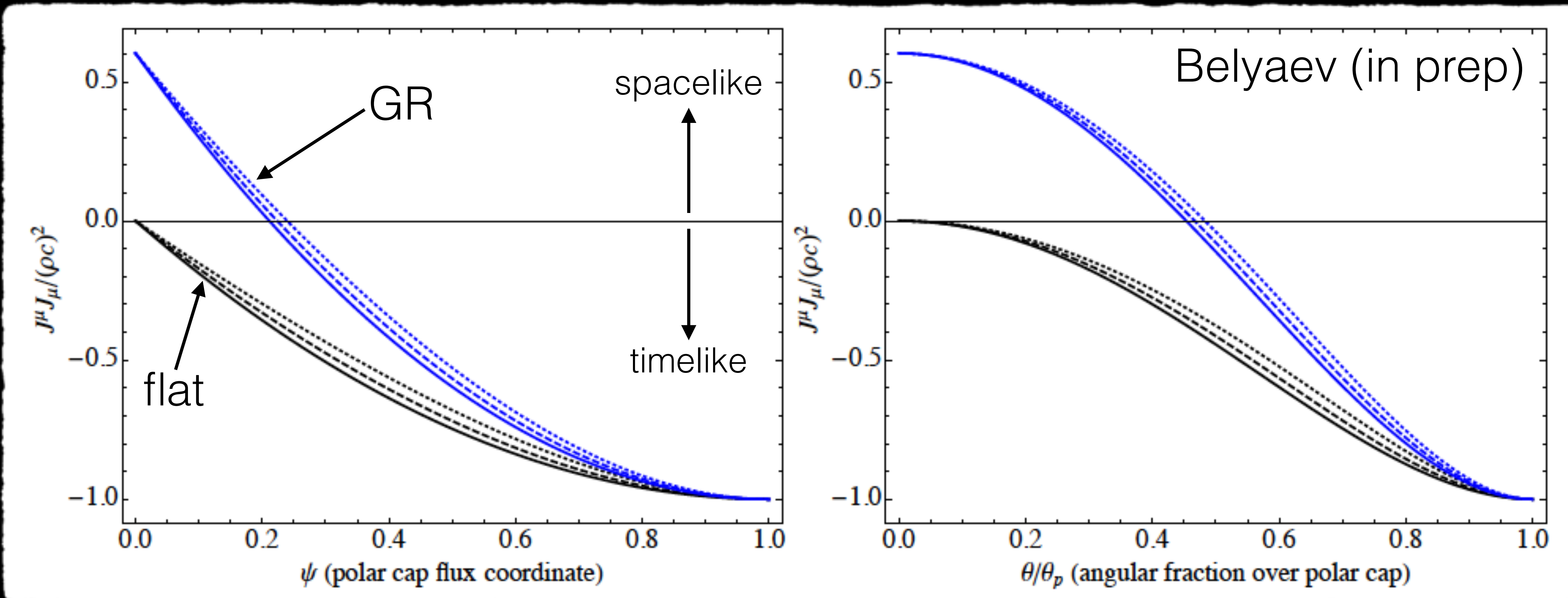
Surface Injection On. Volume Injection On.





Poynting Theorem: $$\frac{\partial u}{\partial t}+\nabla\cdot\boldsymbol{S}=-\boldsymbol{E}\cdot\boldsymbol{J}$$

Polar Cap Currents with GR



Analytic solutions for polar cap currents in axisymmetry.
Results independent of polar cap B field geometry.
Frame dragging creates spacelike region near pole -> pairs
Timelike region near edge of polar cap -> gaps + dissipation?

Conclusions

1. PIC is a useful tool for numerical experiments of pulsar magnetospheres because PIC is accurate in both force-free and vacuum limits.

2. The largest difference between PIC simulations from different groups is the treatment of charge injection. Although, simulations from different groups agree in the force-free limit.

3. Instabilities in the current sheet set in almost immediately beyond the light cylinder. More research is needed to see whether they can dissipate electromagnetic energy and accelerate particles efficiently.

4. PIC results depend sensitively on if the pair cascade at the polar cap is active. Analytic work on polar cap currents with GR (Belyaev in prep).

5. If pair cascade inefficient over even part of the polar cap and volumetric injection due to e.g. photon-photon pair production is inefficient, there are distributed regions of large dissipation where parallel electric field is not completely screened, near but above the current sheet.