Broken force-free electrodynamics, current sheets, and your favourite magnetosphere problems

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with

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Force-free electrodynamics

$$\nabla_\mu \left[\cancel{T^{\mu\nu}_{(\mathrm{Matter})}} + T^{\mu\nu}_{(\mathrm{EM})} \right] = 0$$

$$\rho\vec{E} + \vec{J} \times \vec{B} = 0 \qquad \vec{E} \cdot \vec{B} = 0 \qquad B^2 > E^2$$

$$\longrightarrow$$

$$\partial_t \vec{B} = -\nabla \times \vec{E}$$

$$\partial_t \vec{E} = \nabla \times \vec{B} - \vec{J}$$

$$\vec{J} = \left[\vec{B} \cdot (\nabla \times \vec{B}) - \vec{E} \cdot (\nabla \times \vec{E}) \right] \frac{\vec{B}}{B^2} + (\nabla \cdot \vec{E}) \frac{\vec{E} \times \vec{B}}{B^2}$$

Uchida 97, Thompson & Blaes 98, Gruzinov 99, Komissarov 02, Blandford 02,...

Advantages

- Only need BCs for ***E*** & ***B*** — "plasma" maintained as needed
- Infinite magnetization limit — no density floors
- No shocks, since $v_{\mathrm{A}} \rightarrow c$

Problems

- How do you add dissipation? No fluid frame...
- Current sheets — tangential discontinuities in ***B***

“Broken” force-free electrodynamics

Ideal current: $$\vec{J}_{\rm ideal} = \left[\vec{B}\cdot(\nabla\times\vec{B}) - \vec{E}\cdot(\nabla\times\vec{E})\right]\frac{\vec{B}}{B^2} + (\nabla\cdot\vec{E})\frac{\vec{E}\times\vec{B}}{B^2}$$

Parallel current found from: $$\partial_t\left(\vec{E}\cdot\vec{B}\right) = \left(\nabla\times\vec{B}\right)\cdot\vec{B} - \vec{J}\cdot\vec{B} - \vec{E}\cdot\left(\nabla\times\vec{E}\right) = 0$$

Instead, set *target* value: $$\vec{E}\cdot\vec{B}|_{\rm target} = \eta\vec{J}\cdot\vec{B}$$

Now drive to target value: $$\partial_t\left(\vec{E}\cdot\vec{B}\right) = \gamma\left(\vec{E}\cdot\vec{B}|_{\rm target} - \vec{E}\cdot\vec{B}\right)$$

$$\boxed{\vec{J}_{||\rm resist} = \frac{\vec{B}\cdot\nabla\times\vec{B} - \vec{E}\cdot\nabla\times\vec{E} + \gamma\vec{E}\cdot\vec{B}}{(1+\gamma\eta)\,B^2}\vec{B}}$$

$$\vec{E}\cdot\vec{B}\neq 0$$

$$\vec{E}\cdot\vec{J}\neq 0$$

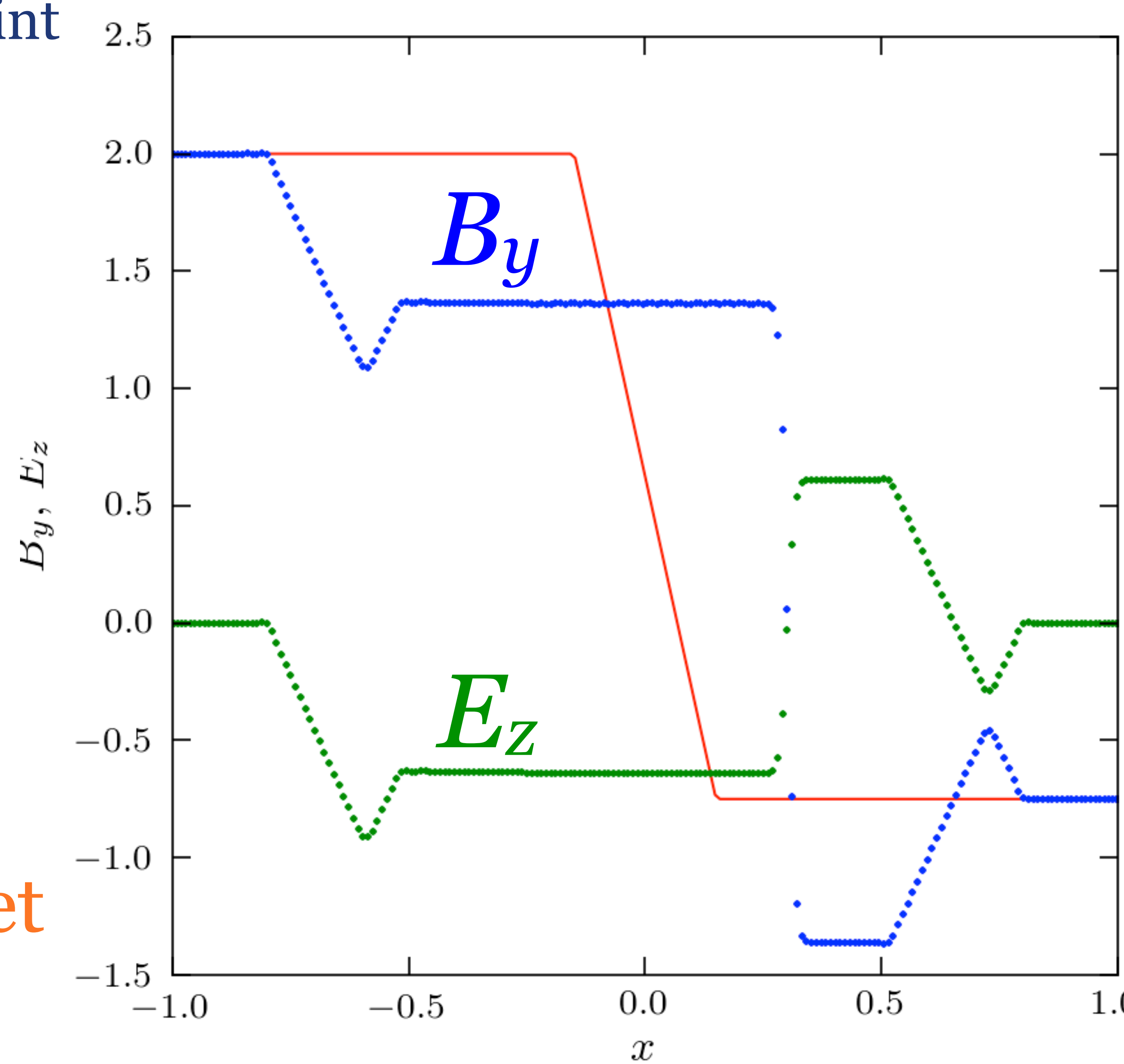
Can have $\eta = \eta\left(\vec{J},\vec{B},\vec{E},\ldots\right)$

Current sheet capturing

1. Identify current sheets: B^i changes sign & $B^2 < E^2$, even if *between* grid points
2. Find $\Delta \boldsymbol{E}$ required at that point to set $E^2 \to B^2$ & leave $\boldsymbol{E} \cdot \boldsymbol{B}$ unchanged
3. Reconstruct smooth distribution of $\Delta \boldsymbol{E}$ in nearby region
4. Set $\Delta \boldsymbol{E} \to \Delta \boldsymbol{E}$ - $[\Delta \boldsymbol{E}] \cdot \boldsymbol{B} / B$ at each point
5. $\boldsymbol{E} \to \boldsymbol{E}$ - $\Delta \boldsymbol{E}$ around current sheet

Simulates particle acceleration, pair cascades, and $\boldsymbol{E}$ dissipation

1D moving current sheet

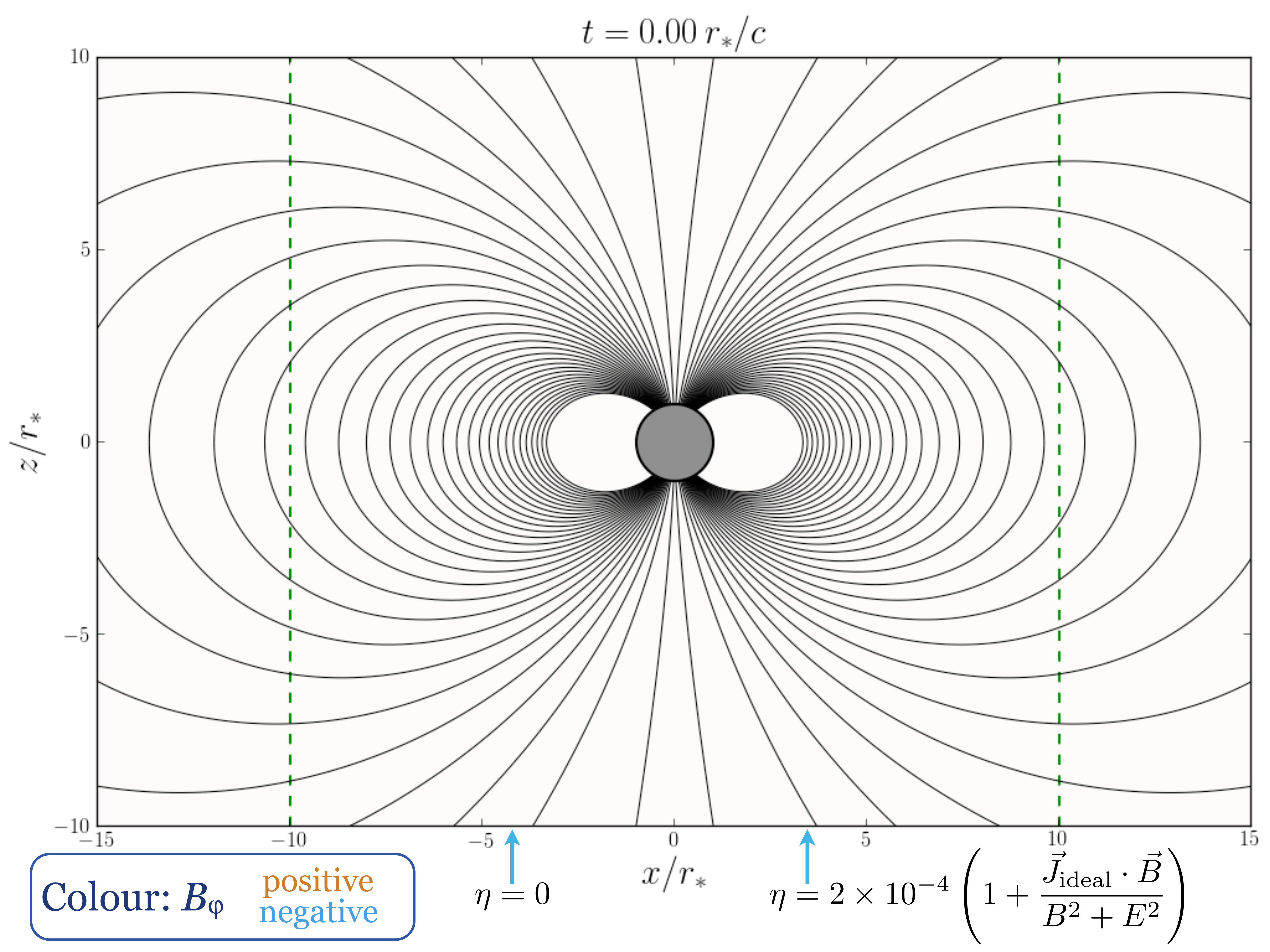


Demonstrations

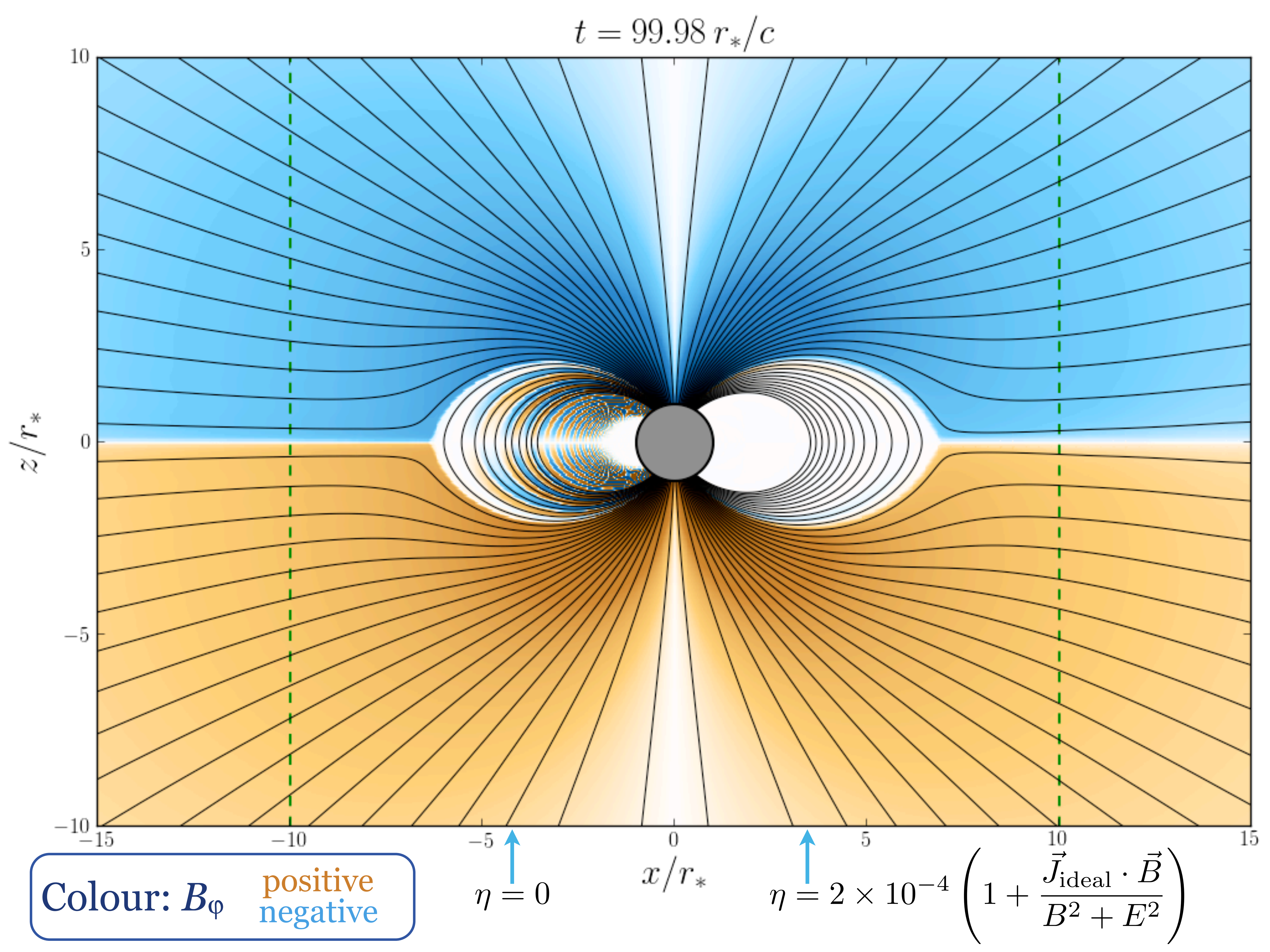
- "Waving" aligned rotator (axisymmetric pulsar)
- Magnetospheric Wald problem

Komissarov 04, 05
Nathanail & Contopoulos 14

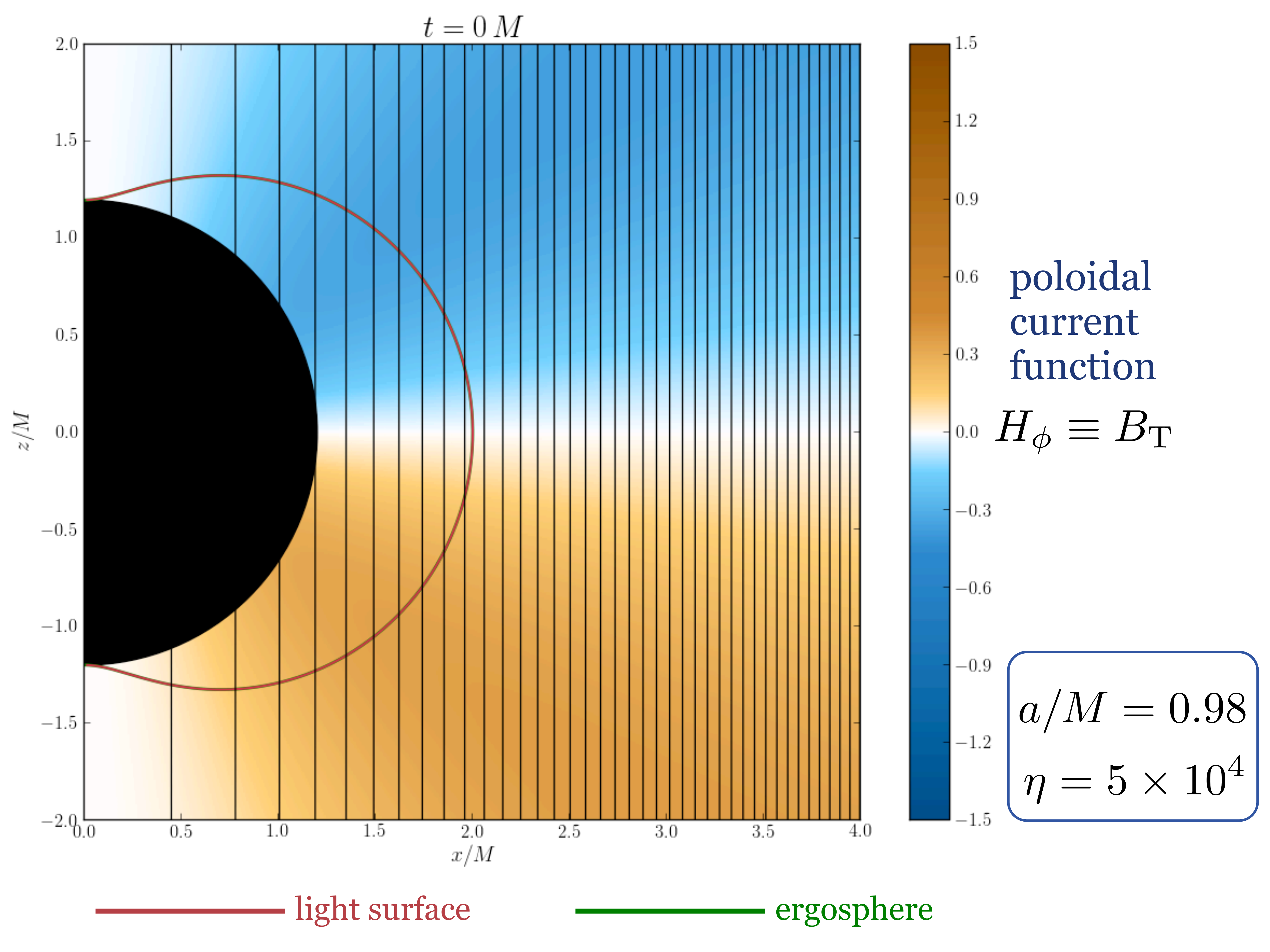
Using pseudospectral GRFFE code *PHAEDRA* (Parfrey+ 12)



$t = 0.00\, r_*/c$
z/r_*
x/r_*
10
5
0
−5
−10
−15
−10
−5
0
5
10
15
Colour: B_φ
positive
negative
$\eta = 0$
$\eta = 2 \times 10^{-4} \left(1 + \frac{\vec{J}_{\mathrm{ideal}} \cdot \vec{B}}{B^2 + E^2}\right)$

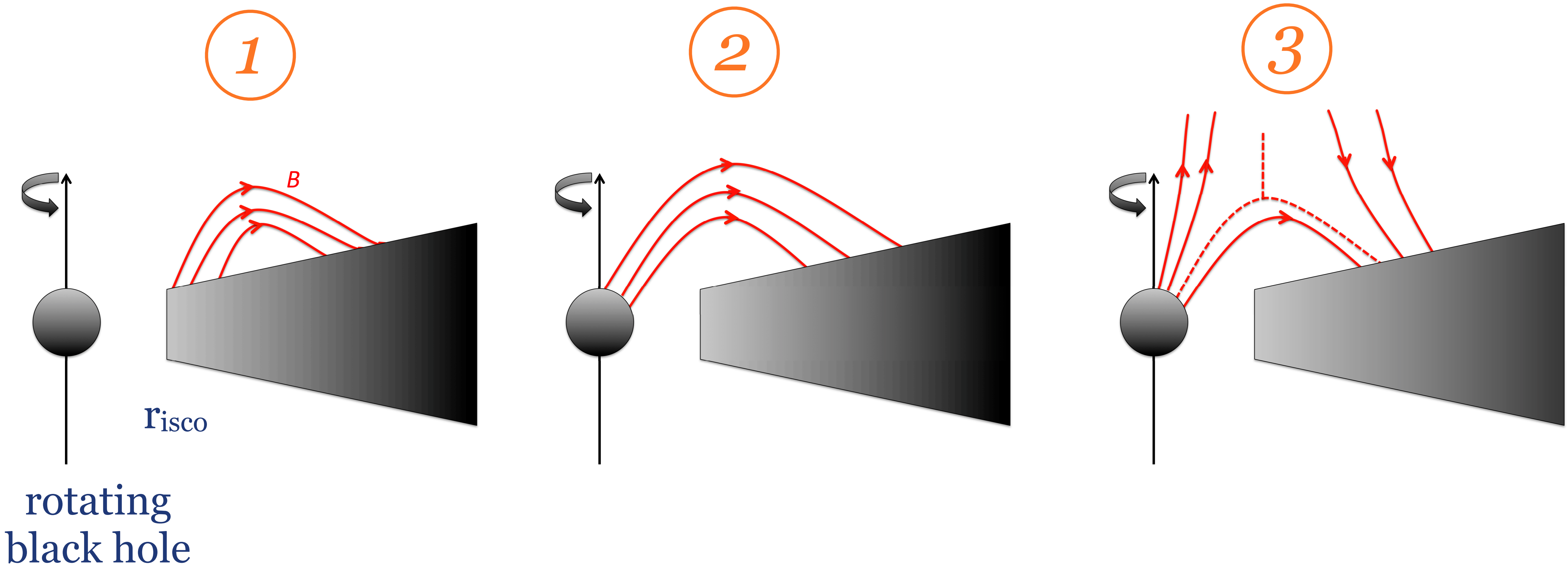


$t = 99.98\, r_*/c$
10
5
0
−5
−10
z/r_*
−15
−10
−5
0
5
10
15
x/r_*
Colour: B_φ
positive
negative
$\eta = 0$
$\eta = 2 \times 10^{-4} \left(1 + \frac{\vec{J}_{\mathrm{ideal}} \cdot \vec{B}}{B^2 + E^2}\right)$



$t = 0\,M$
z/M
x/M
2.0
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
−2.0
0.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
1.5
1.2
0.9
0.6
0.3
0.0
−0.3
−0.6
−0.9
−1.2
−1.5
poloidal
current
function
$H_\phi \equiv B_{\mathrm{T}}$
$a/M = 0.98$
$\eta = 5 \times 10^4$
light surface
ergosphere

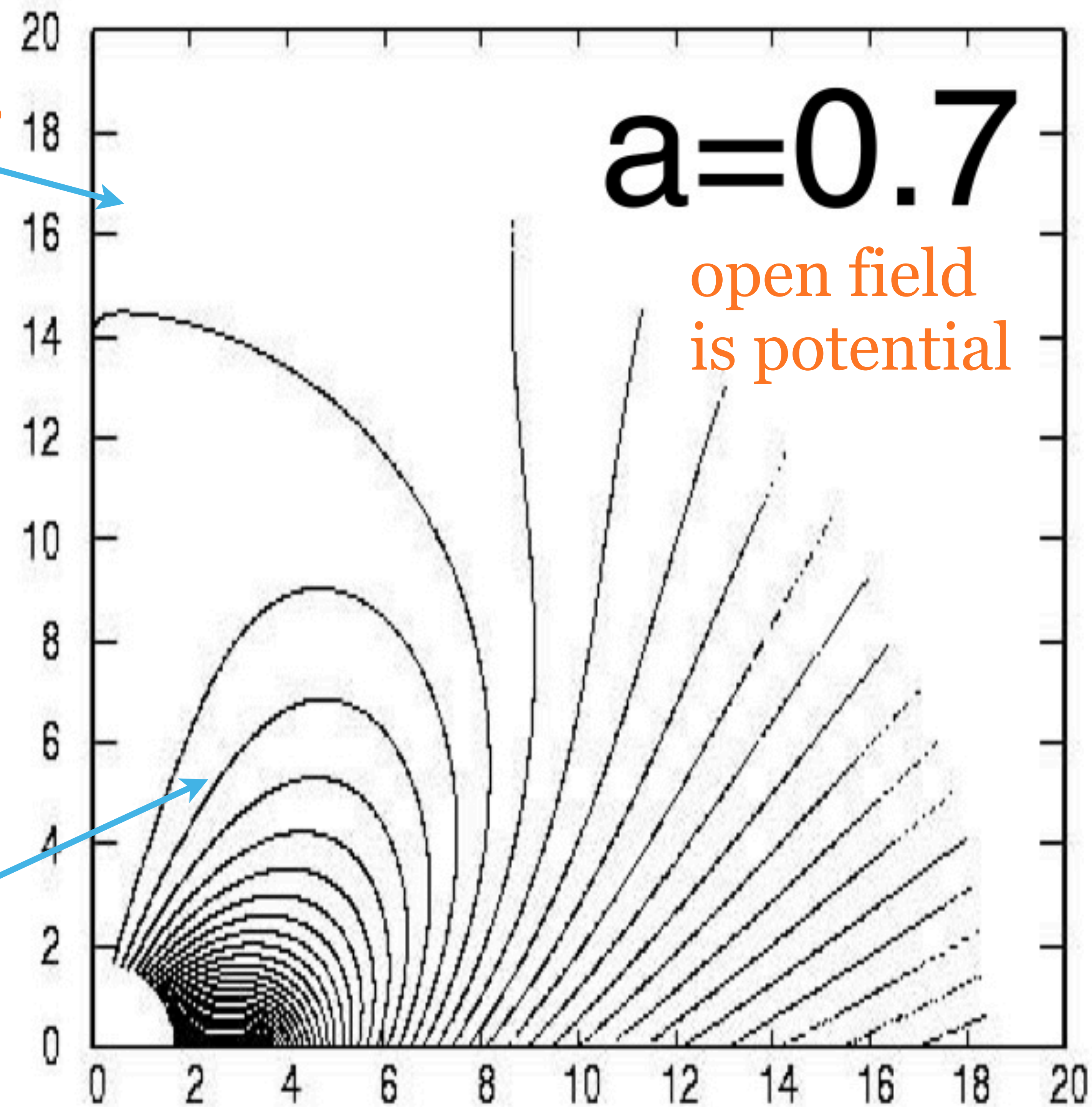
Application: jets from small-scale field



- Field loops created by MRI in disc, on scale ~ *H*
- Rise into corona via Parker instability
- Can open up when first footpoint is accreted
- Swallowed or ejected when second footpoint is accreted

Black hole-disc coupling

no field lines
to infinity



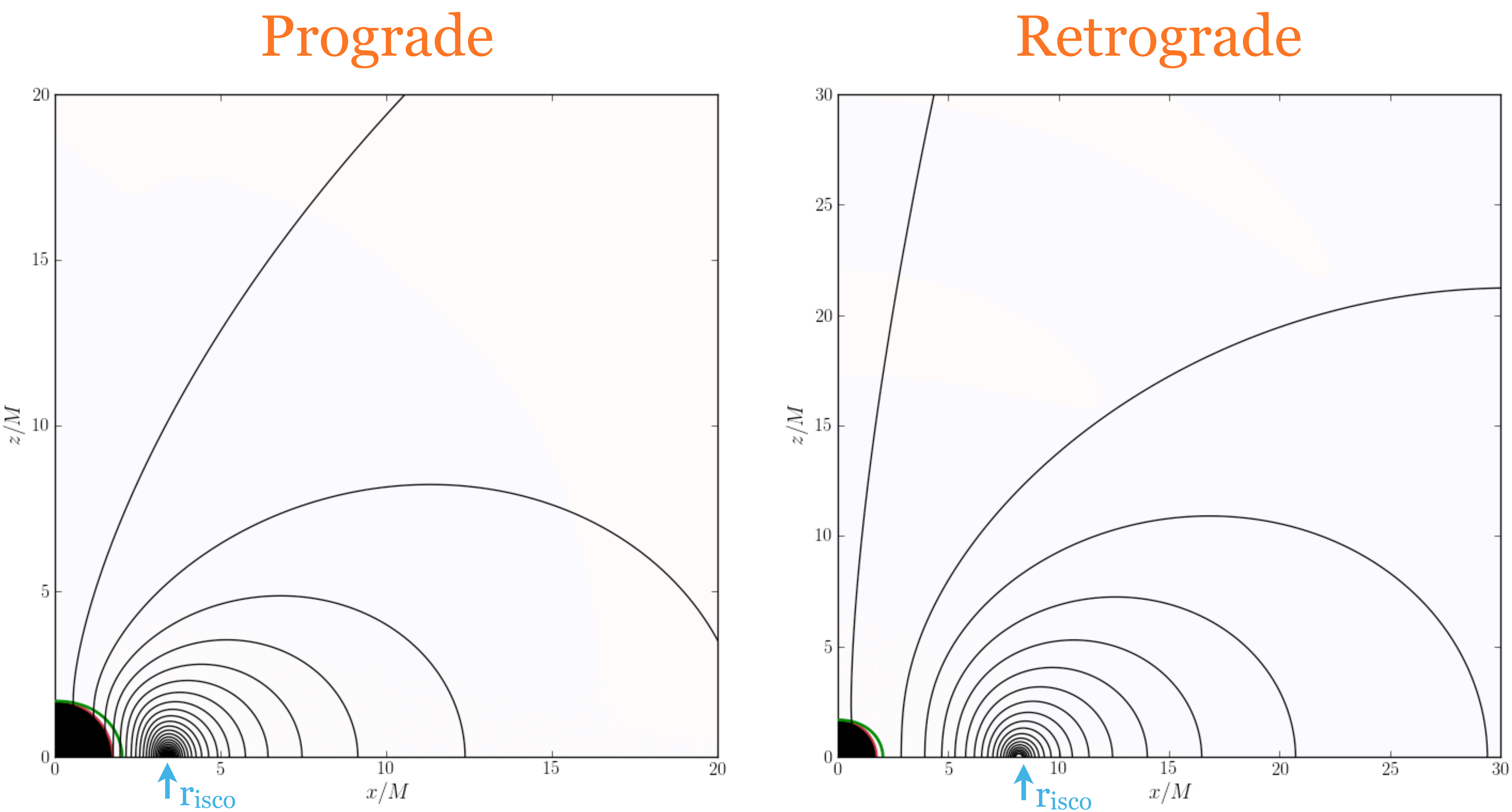
open field
is potential

Steady-state solution of
Grad-Shafranov eqn.

region of closed field
lines connecting the
inner disc to the hole

Uzdensky 05

Quasi-steady BH-disc magnetospheres

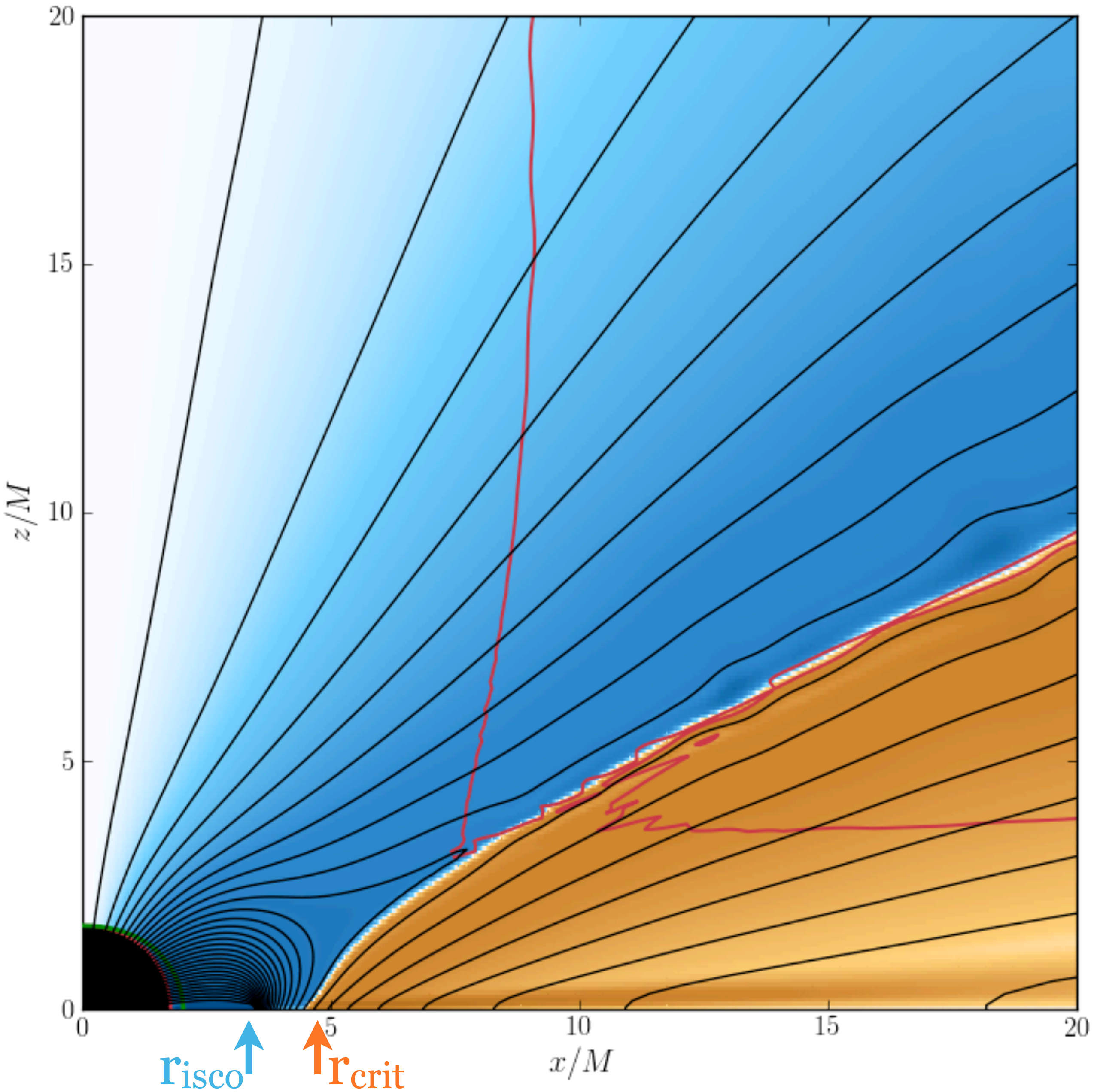


$a/M = 0.7$

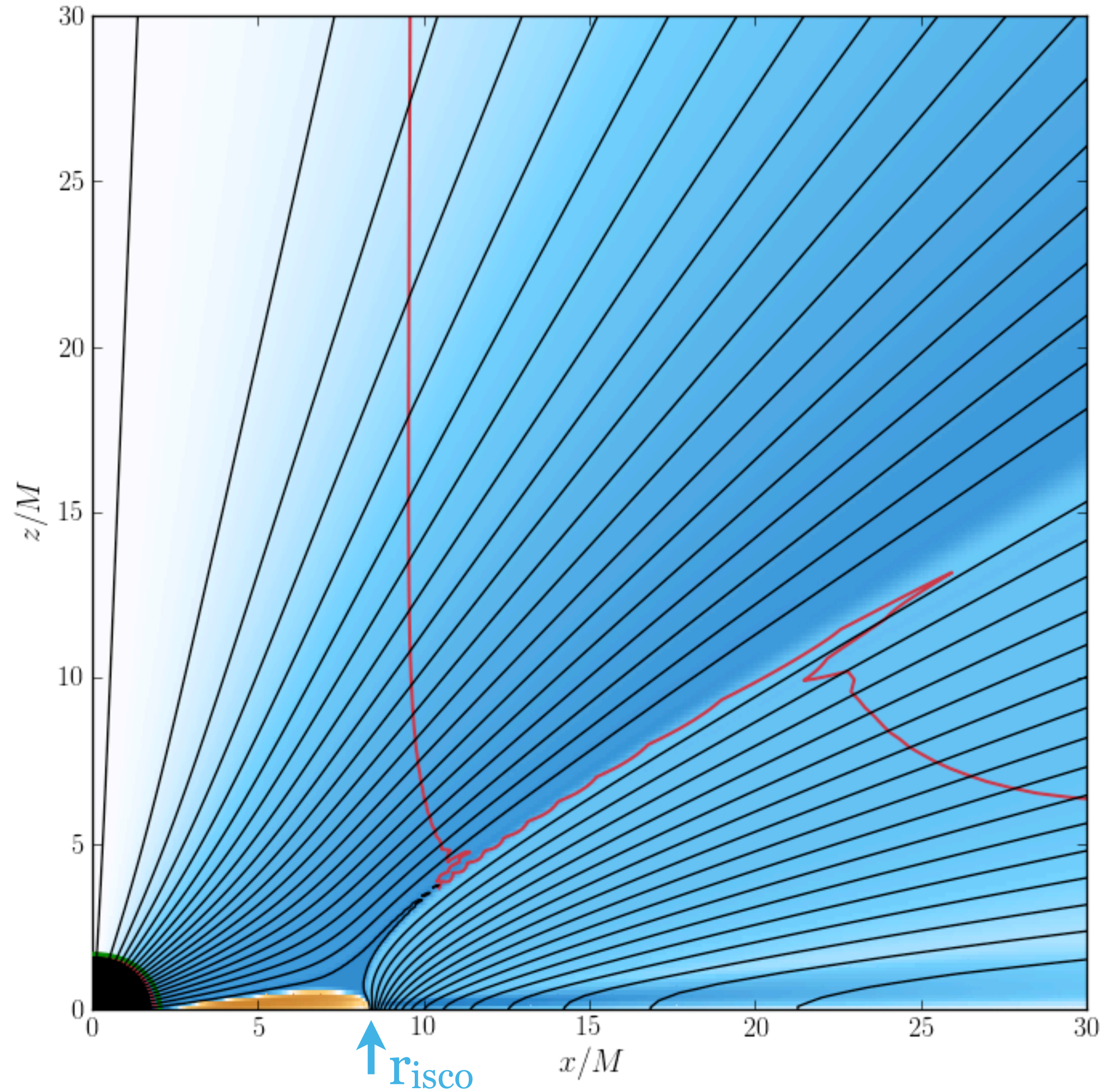
Colour: H_φ positive / negative

Quasi-steady BH-disc magnetospheres

Prograde



Retrograde



closed BH-disc field lines

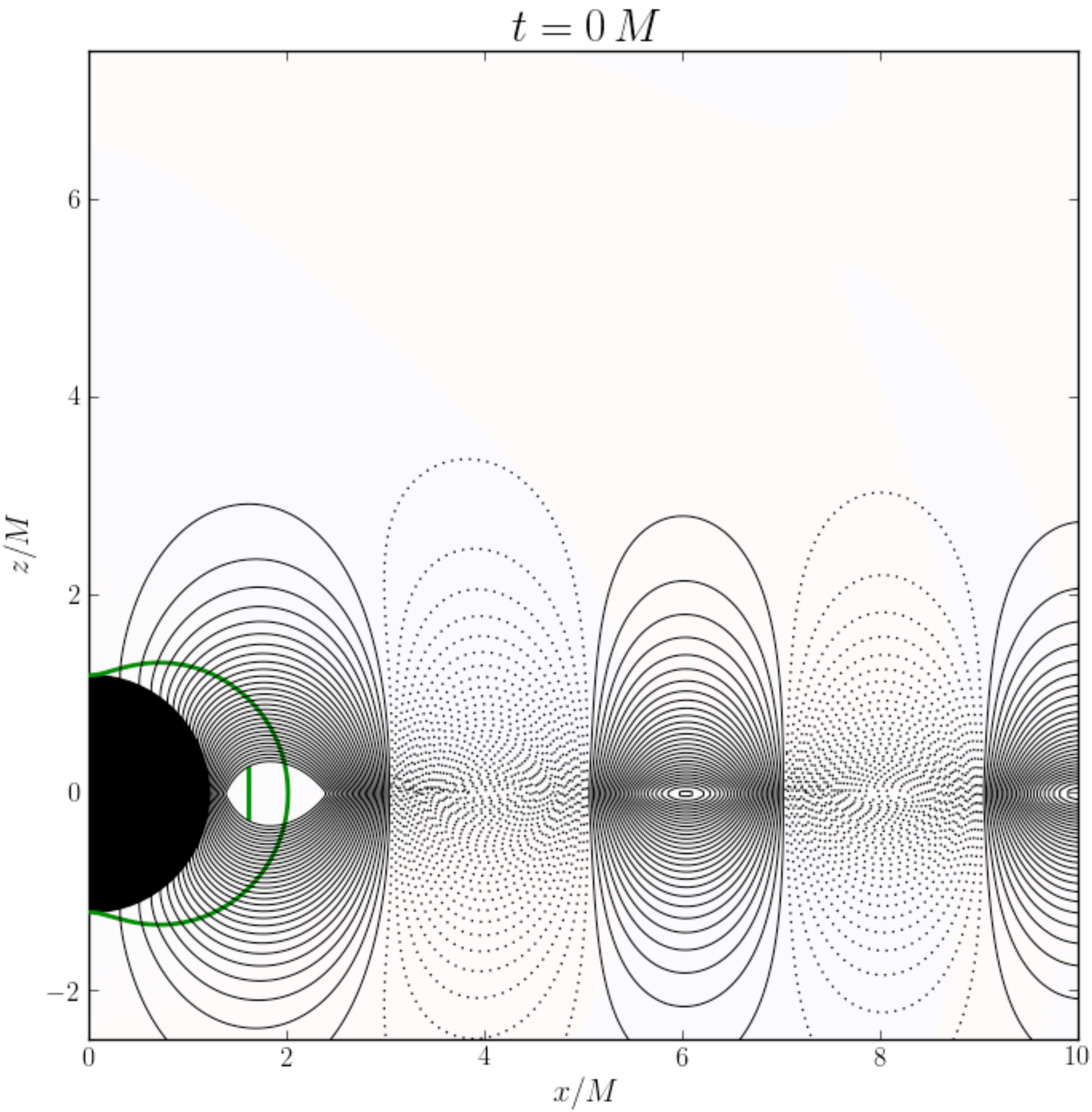
$E^2 > B^2$ inside current sheet

$a/M = 0.7$

Colour: H_φ positive negative

all field lines open

$E^2 < B^2$ inside current sheet



Prograde disc

loop width = 2 *M*

loop direction

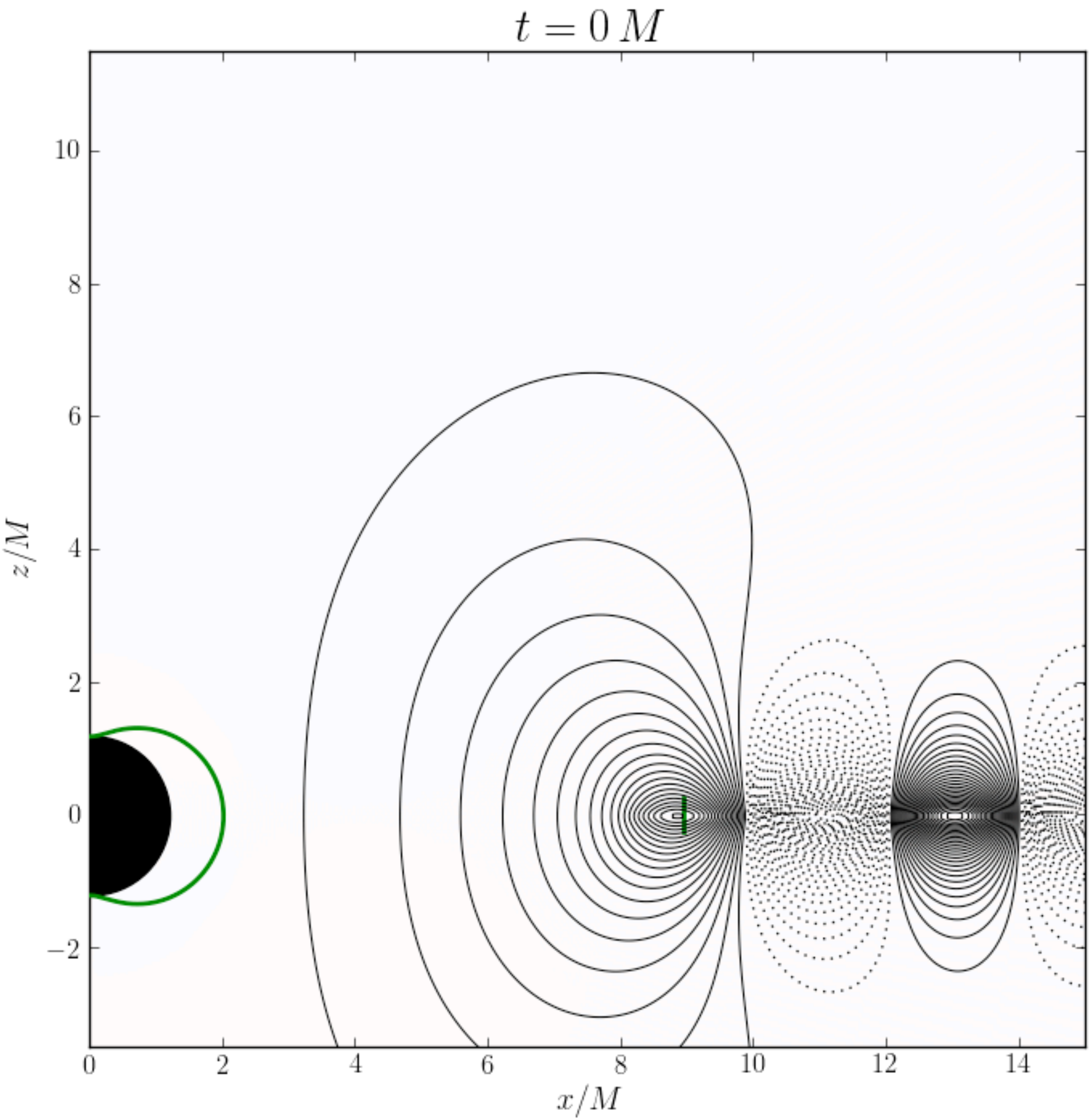
clockwise

anti-clockwise

Colour: H_φ positive negative

$$v_{\text{accrete}} = c/100$$

$$a/M = 0.98$$



Retrograde disc

loop width = 2 M

loop direction

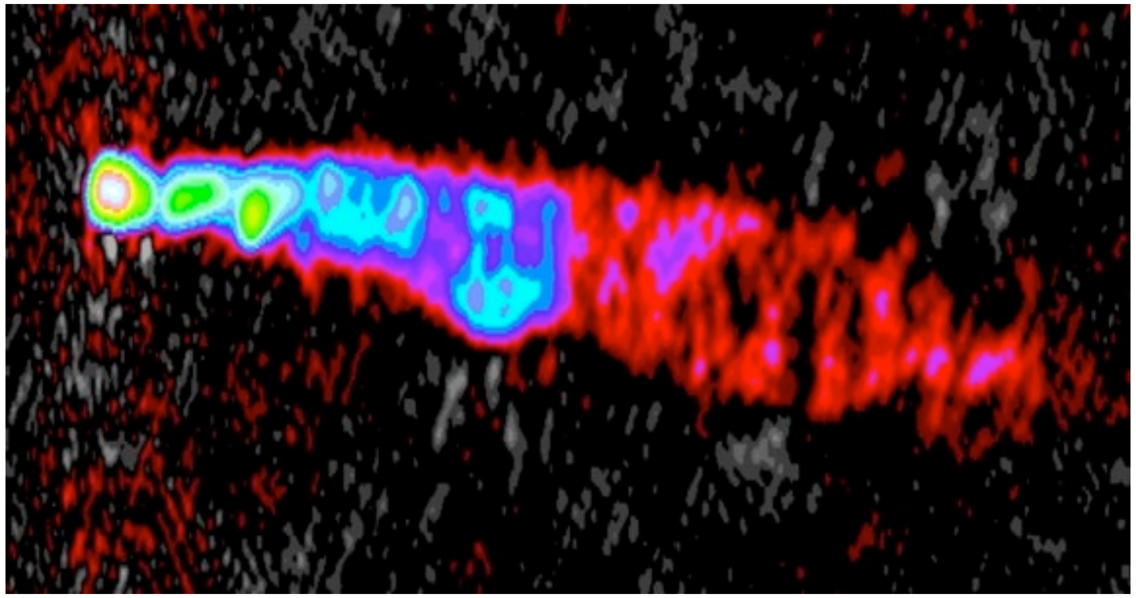
clockwise

anti-clockwise

Colour: H_φ positive negative

$v_{\mathrm{accrete}} = c/100$

$a/M = 0.98$



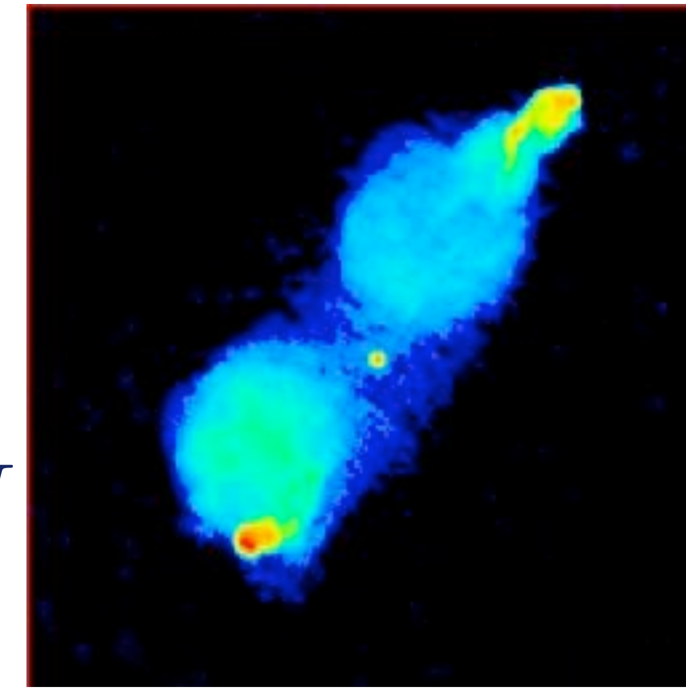
Walker 94

3C 120

$r_{\rm in} \approx 8.6\ M$

Kataoka+ 07

Leahy & Perley 95



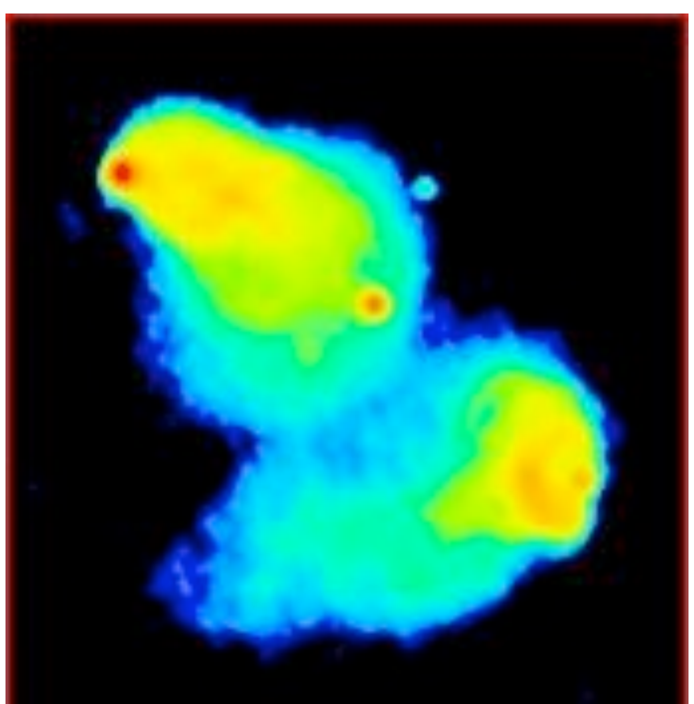
3C 390.3

$r_{\rm in} \gtrsim 20\ M$

Sambruna+ 09

Radio Galaxies

Leahy & Perley 91



3C 382

$r_{\rm in} \approx 10\ M$

Sambruna+ 11

Seyferts $a/M > 0.8$

MCG 6-30-15	Miniutti+ 07
NGC 3783	Brenneman+ 11
NGC 1365	Risaliti+ 13

Summary

- Resistive method for magnetospheric dynamics
  — stiff in the *diffusive* rather than ideal limit
  — suitable for both nearly ideal and very diffusive regimes

- Current sheet capturing method
  — keeps sheets well-behaved and marginally resolved
  — simulates effect of cross-field conductivity when $E^2 > B^2$

- Jets from small-scale loops grown in discs
  — prograde: minimal poloidal scale for jet
  differential-rotation powered magnetic wind
  — retrograde: get BH-powered jet even for small loops
  — possible relevance to (1) radio-loud vs. -quiet AGN?
  (2) jet quenching in XRB soft states?
  (3) retrograde microquasers?