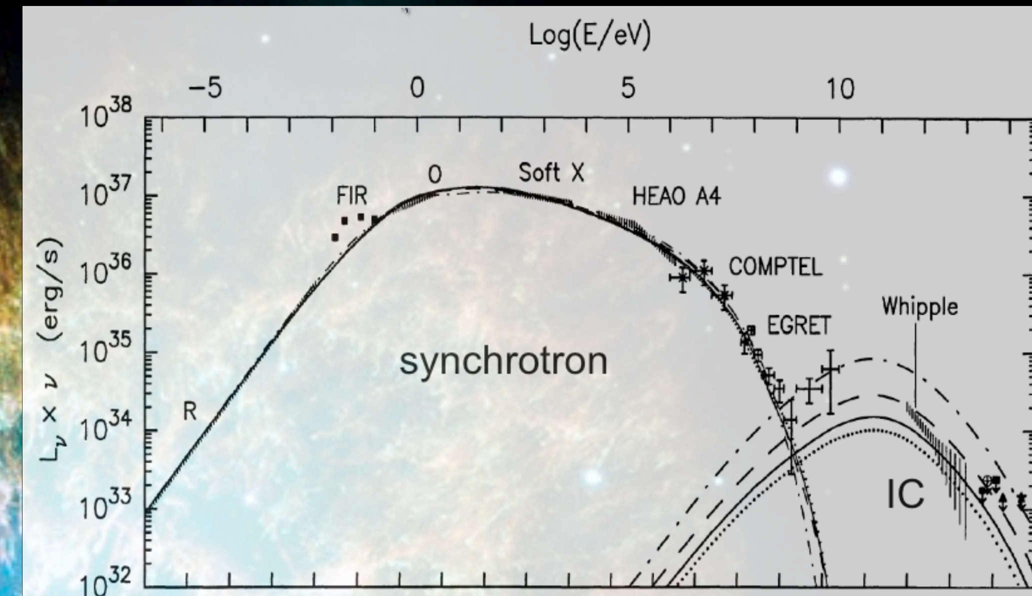
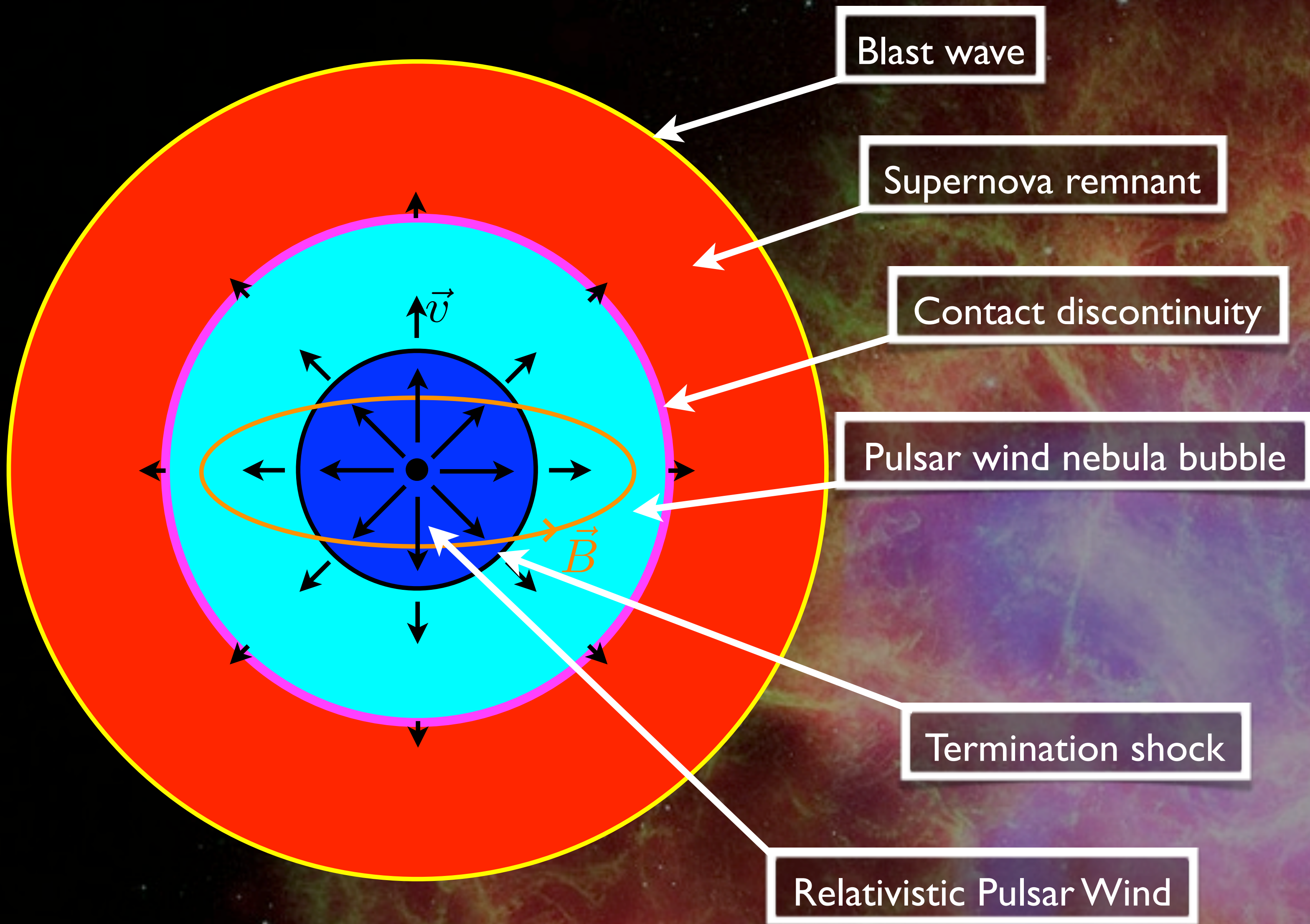
Credits: X-ray: NASA/CXC/ASU/J.Hester et al.; Optical: NASA/HST/ASU/J.Hester et al.



THREE-DIMENSIONAL MHD SIMULATIONS OF THE CRAB NEBULA AND THE SOLUTION TO THE SIGMA-PROBLEM

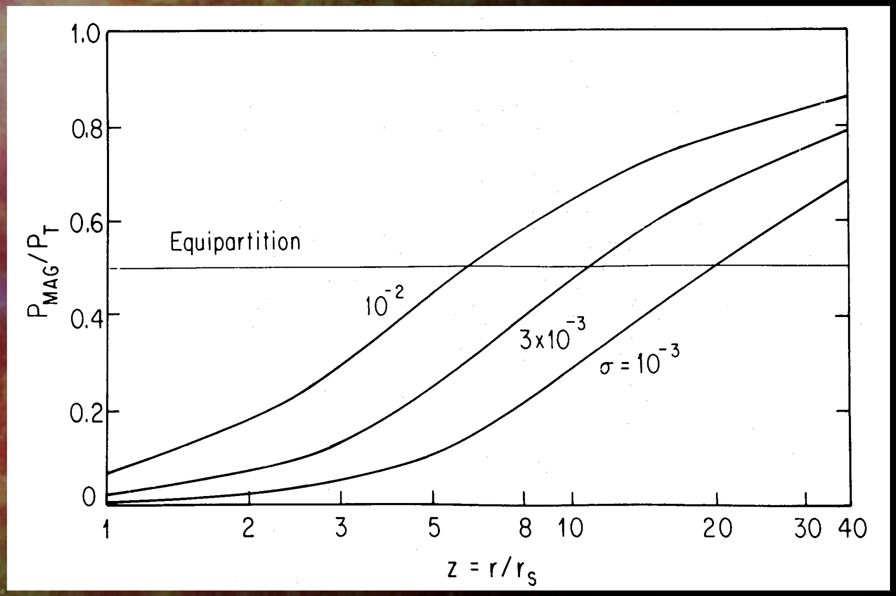
with Serguei S. Komissarov and Rony Keppens

1D Model



Rees & Gunn (1974), Kennel & Coroniti (1984)

- particle dominated relativistic pulsar wind with purely azimuthal magnetic field terminates at shock
- sub-sonic nebula flow velocity decreases to match speed of remnant
- magnetic field increases towards the outer boundary of the nebula



- electrons are accelerated at the termination shock to relativistic energies according to $n \propto E^{-2.2}$
- loose energy due to synchrotron and inverse Compton emission. => Successful to model spectrum from visible to γ-rays

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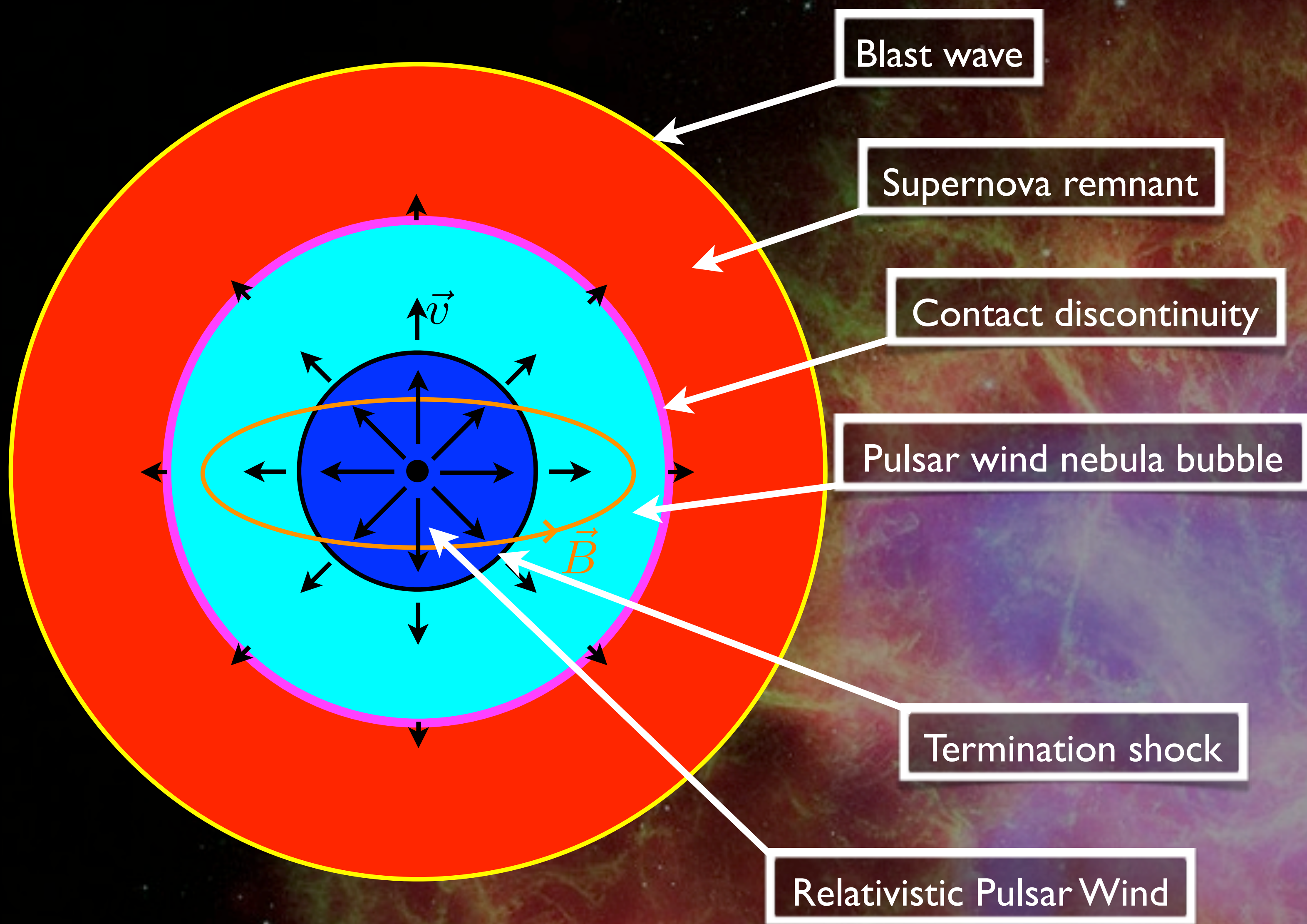

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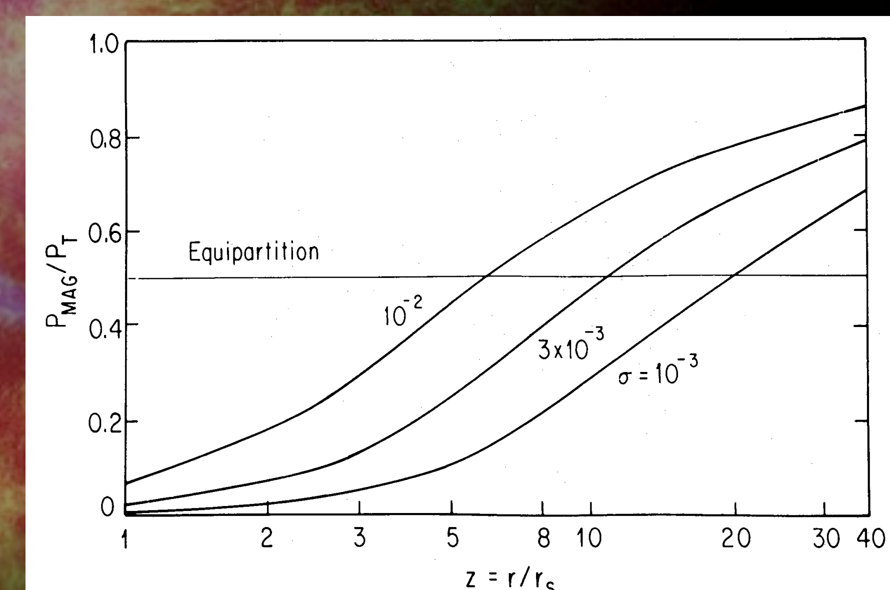

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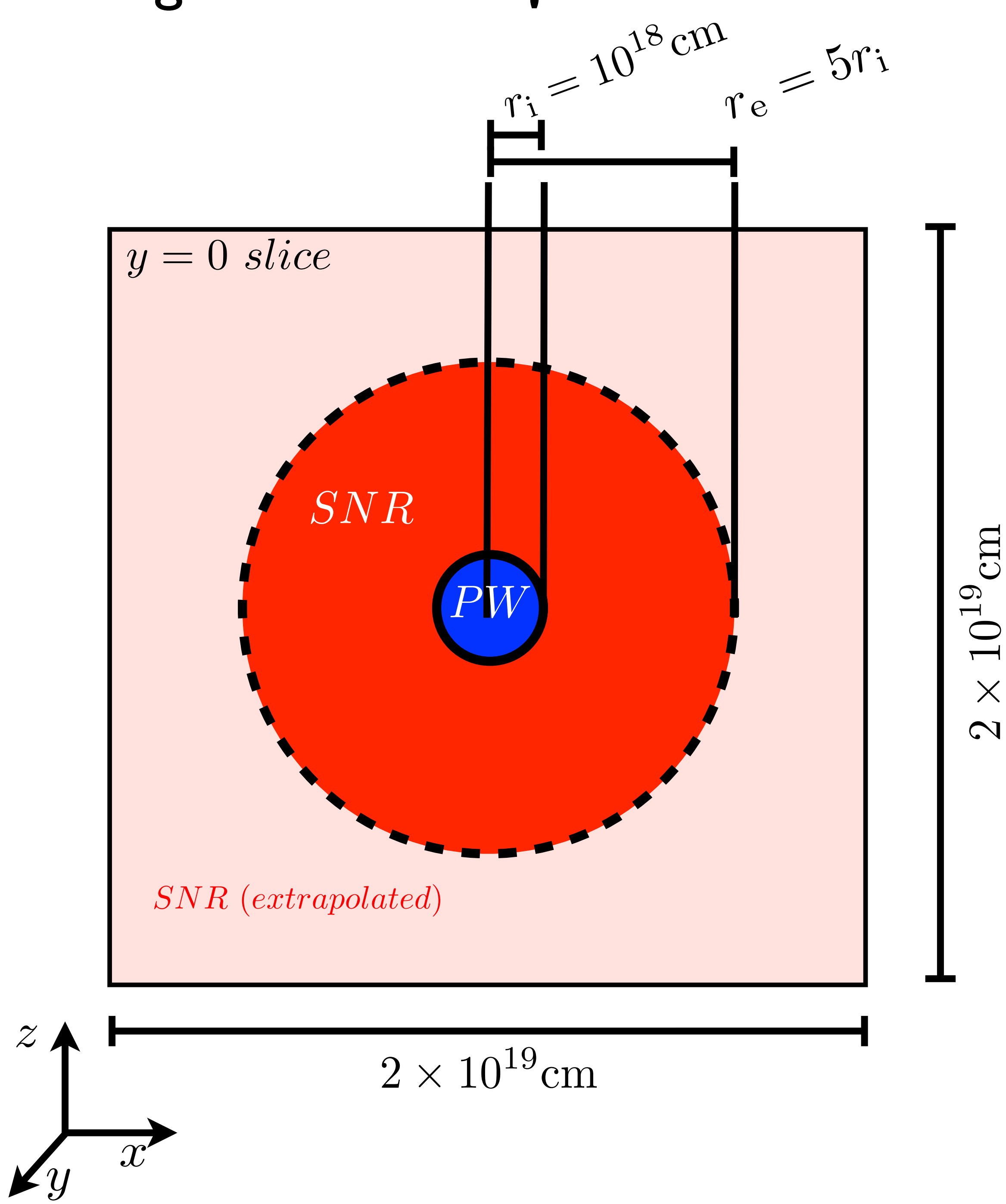
- loose energy due to synchrotron and inverse Compton emission. => Successful to model spectrum from visible to γ-rays

Setup of PWN simulation

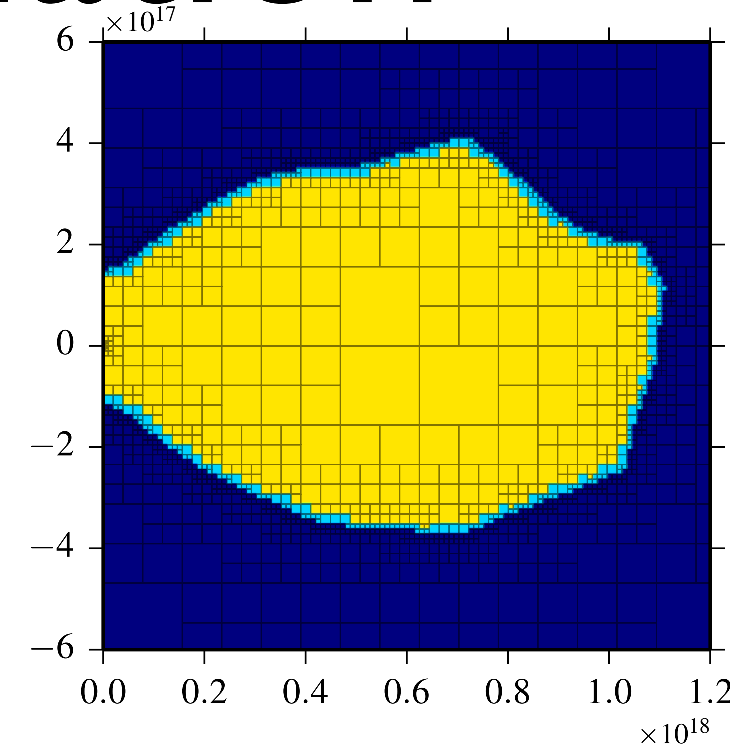
Domain:

3D Cartesian box, 20 lightyears^3
MPI-AMRVAC[1]
ideal SRMHD module,
ideal gas EOS with γ=4/3



AMR:

Base resolution 64^3
PWN on level 5-6; hllc
lim03
Termination shock on
level 8-10; tvdlf minmod



Ejecta (SNR):

$E_\mathrm{e} = 10^{51}\mathrm{erg} \qquad M_\mathrm{e} = 3M_\odot$

contained within r_i and r_e

$$v_r = v_\mathrm{i}\left(\frac{r}{r_\mathrm{i}}\right) \qquad \rho = \rho_\mathrm{e}$$

$$v_\mathrm{i} = \frac{r_\mathrm{i}}{r_\mathrm{e}}\left(\frac{10}{3}\frac{E_\mathrm{e}}{M_\mathrm{e}}\right)^{1/2} = 1495\ \mathrm{km\,s^{-1}}$$

$$\rho_\mathrm{e} = \frac{M_\mathrm{e}}{\int_{r_\mathrm{i}}^{r_\mathrm{e}} 4\pi r^2 dr} = 1.15 \times 10^{-23}\ \mathrm{g\,cm^{-3}}$$

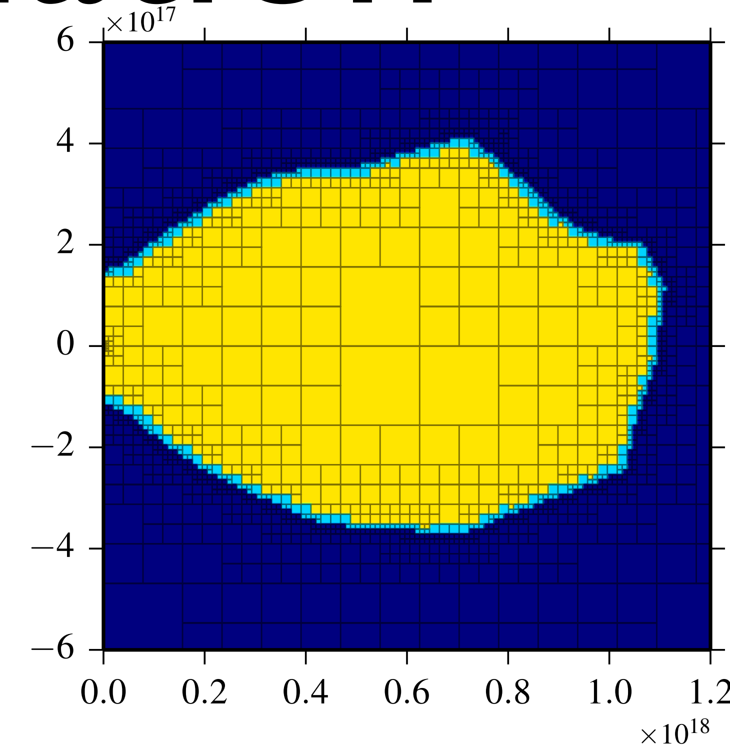
$$r_\mathrm{i} \neq 0 \Rightarrow t_0 = r_\mathrm{i}/v_\mathrm{i} \simeq 210 \text{ years}$$

[1] https://gitorious.org/amrvac

Setup of PWN simulation

- Pulsar wind setup

$$L_{\rm tot} = 5 \times 10^{38}\,{\rm erg\,s^{-1}}$$



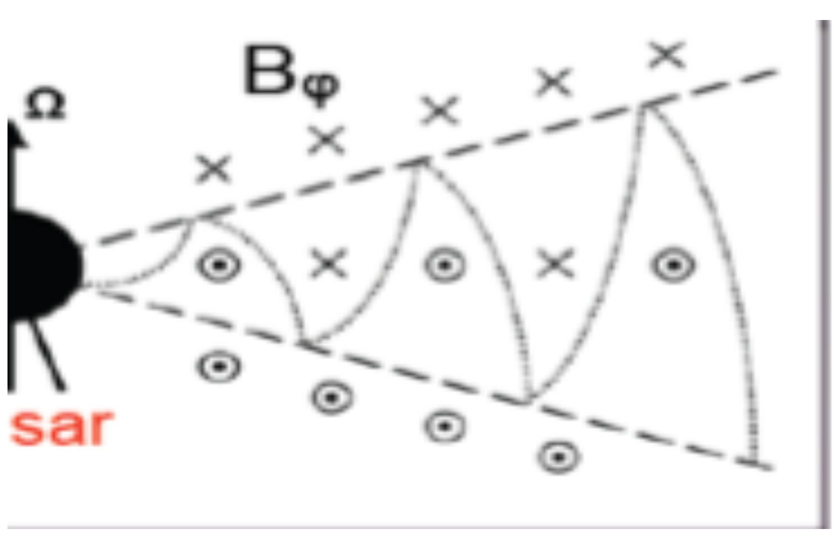
Anisotropic total energy flux

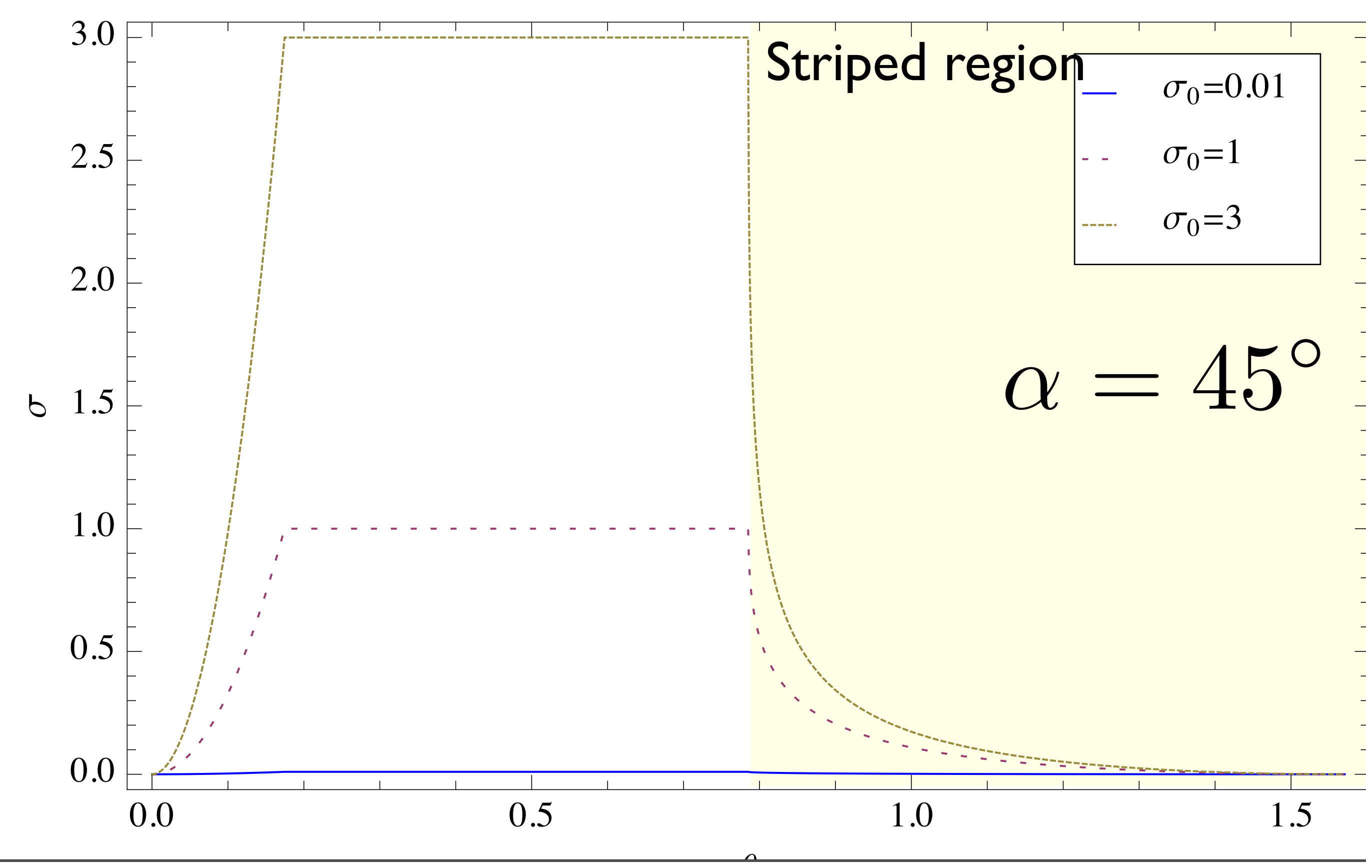
$$f_{\rm tot}(r,\theta) = \frac{1}{r^2}(\sin^2\theta + b) \quad , b = 0.03$$

$$f_{\rm m}(r,\theta) = \sigma(\theta)\frac{f_{\rm tot}(\theta,r)}{1+\sigma(\theta)}$$

$$f_{\rm k}(r,\theta) = \frac{f_{\rm tot}(r,\theta)}{1+\sigma(\theta)}$$

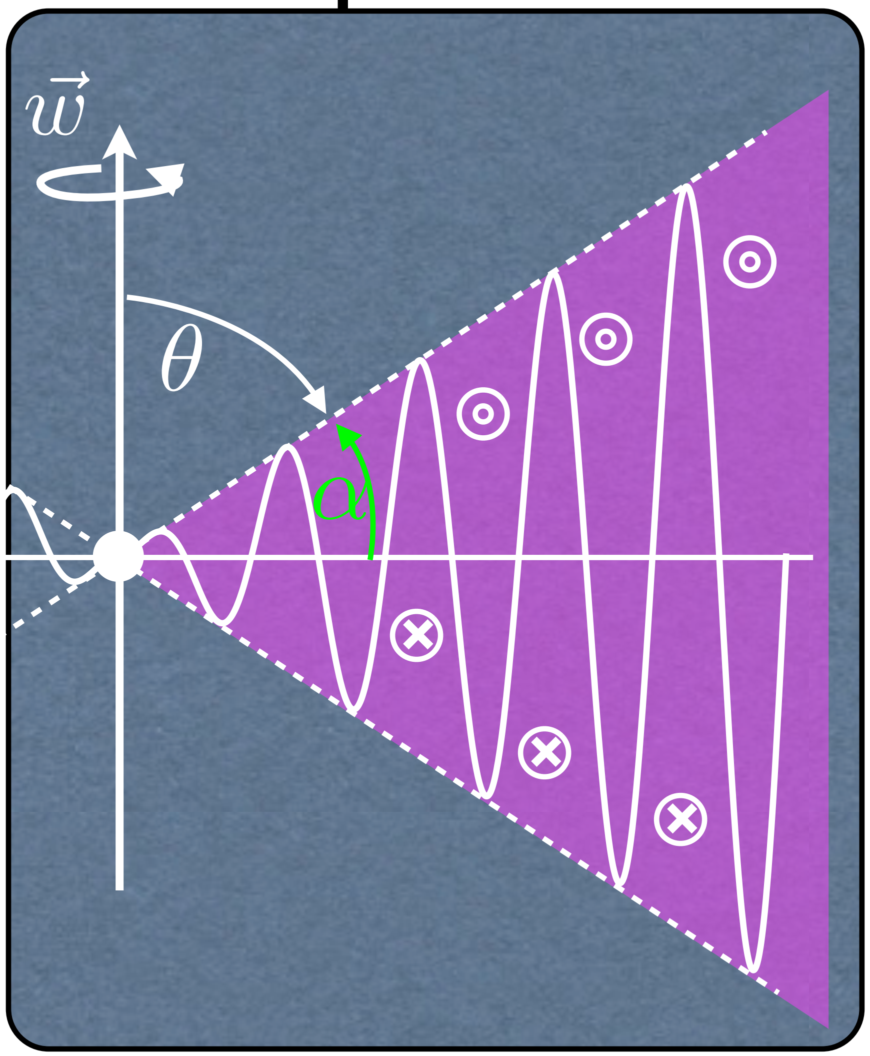
$\Gamma = 10$ Lorentz factor in PW

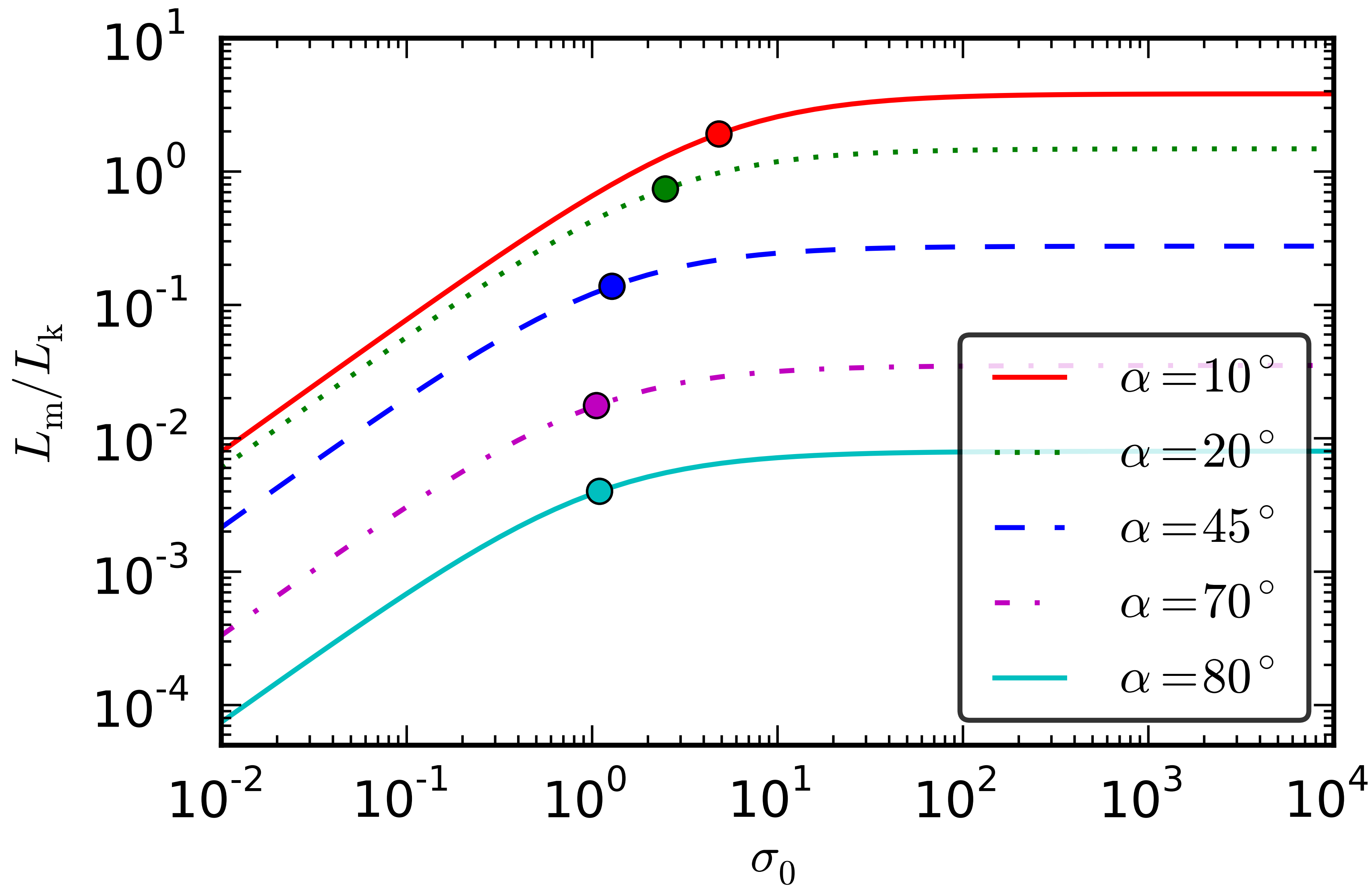




Setup of PWN simulation

- Striped Wind





Magnetization after annihilation:

$$\sigma(\theta) = \frac{\tilde{\sigma}_0(\theta)\chi_\alpha(\theta)}{1 + \tilde{\sigma}_0(\theta)(1 - \chi_\alpha(\theta))}$$

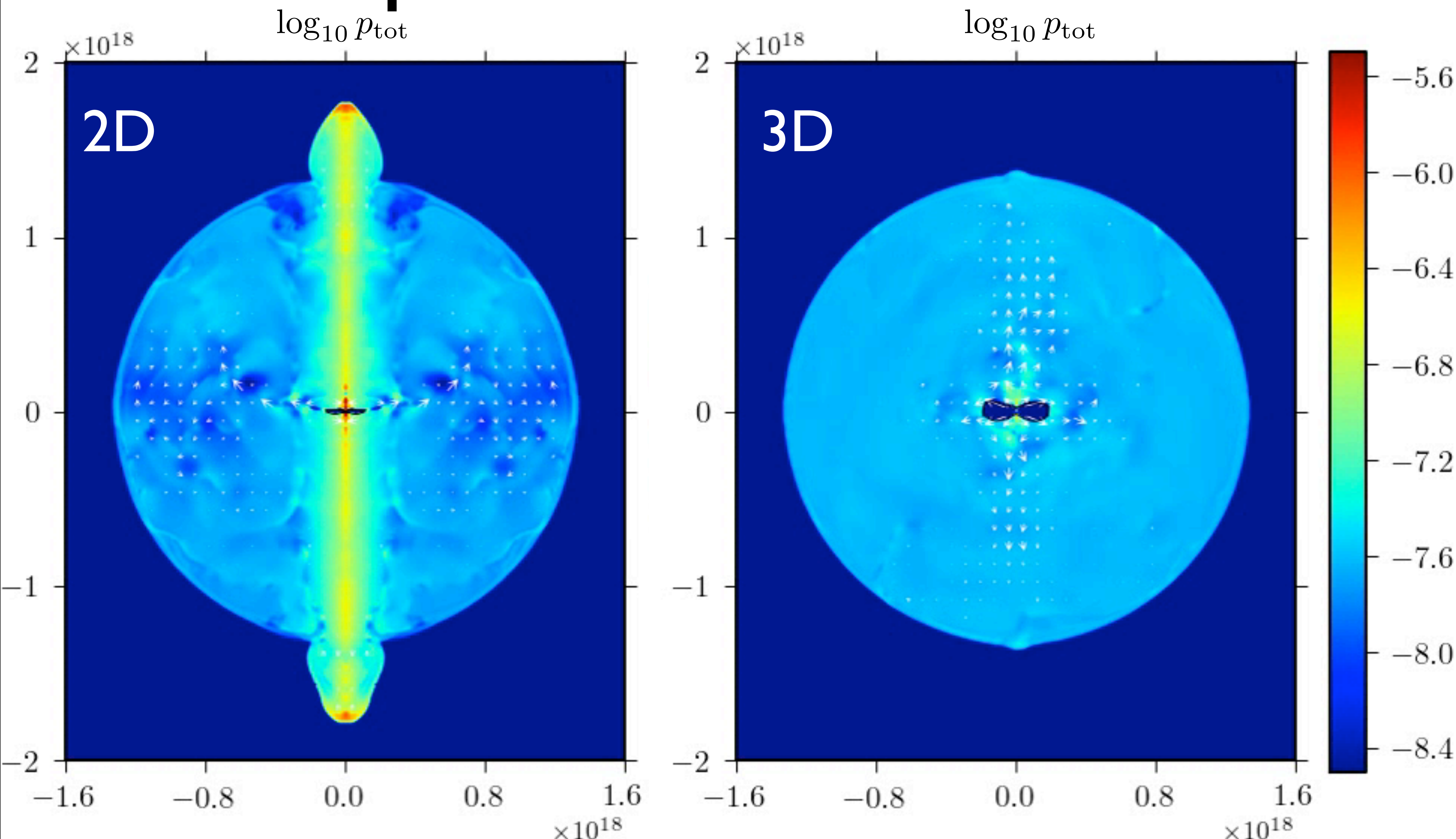
Limits:

$$\sigma_1 \to \chi_\alpha/(1 - \chi_\alpha) \;\; (\tilde{\sigma}_0 \to \infty)$$

$$\sigma_1 \to \tilde{\sigma}_0 \chi_\alpha \;\; (\tilde{\sigma}_0 \to 0)$$

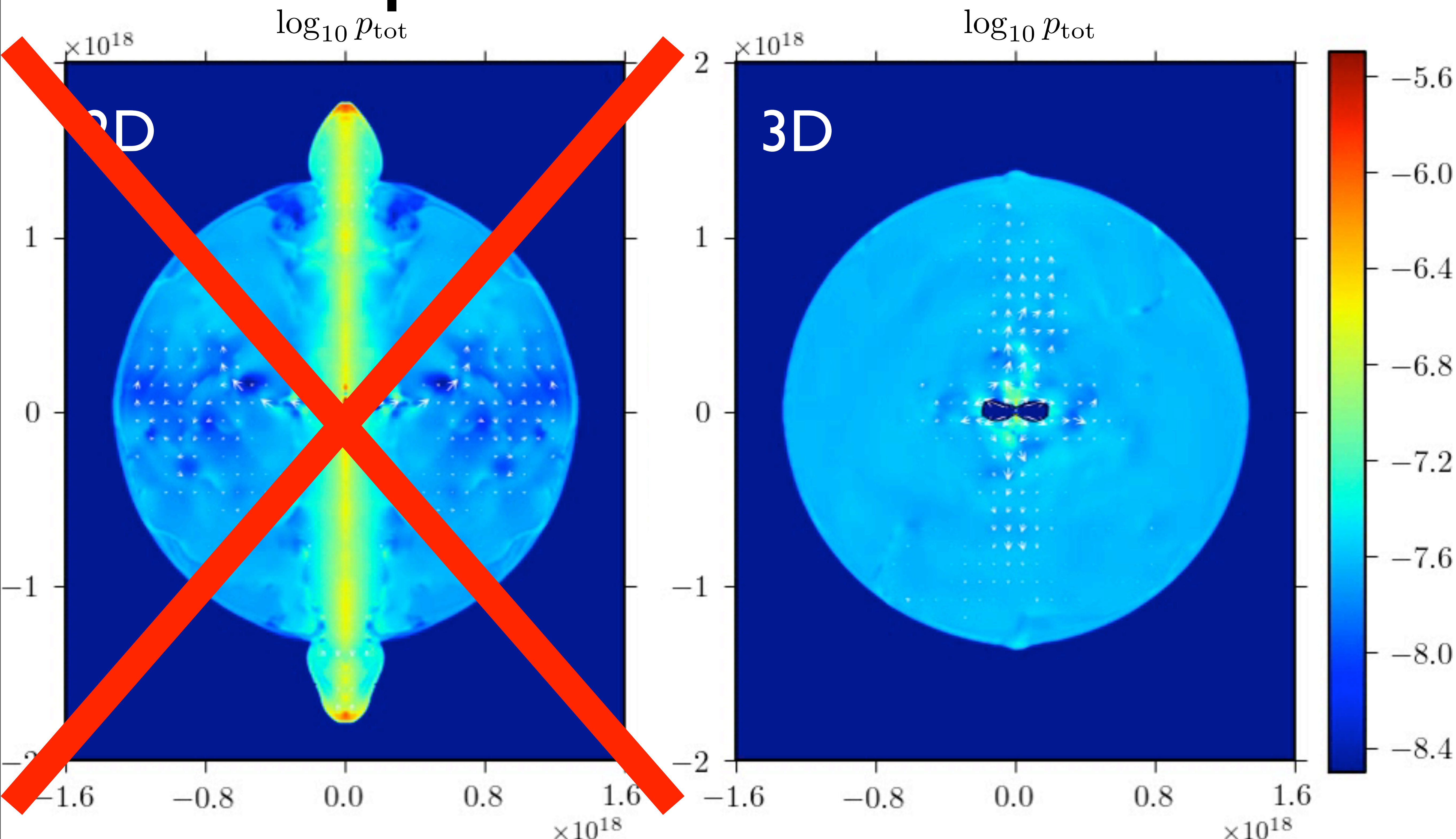
Coroniti 1990, Lyubarski 2003, Sironi & Spitkovsky 2011

Total pressure in 2D and 3D



Total pressure slices for consecutive simulation snapshots 51 years after start of simulation

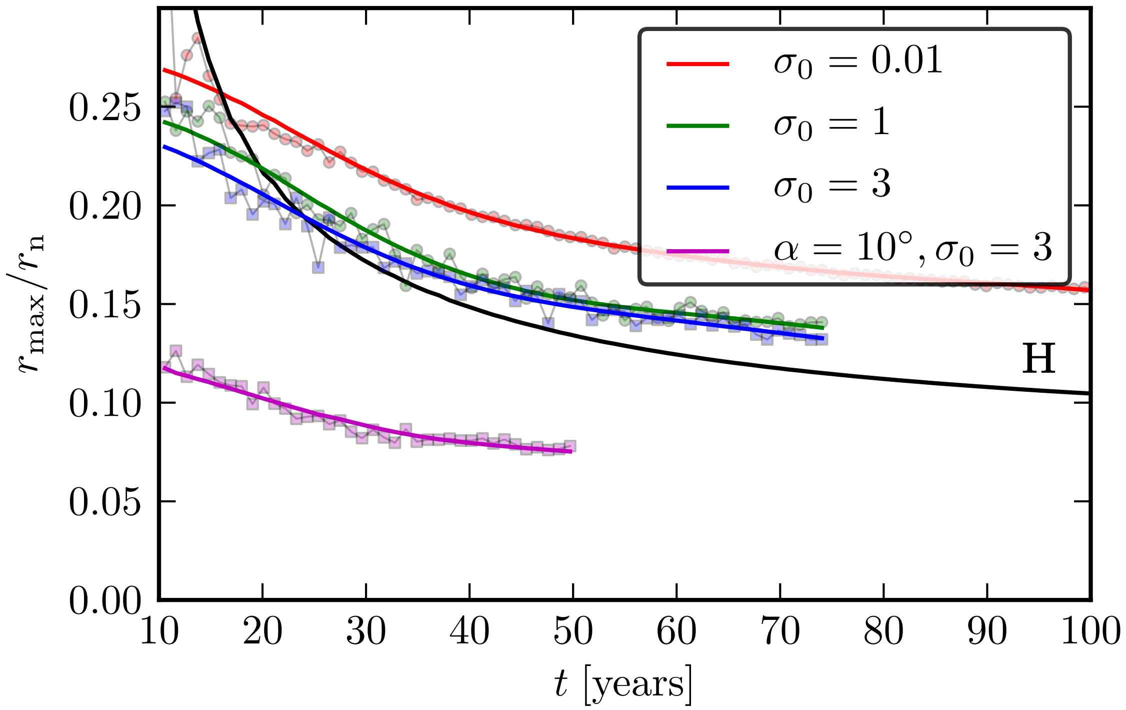
Total pressure in 2D and 3D

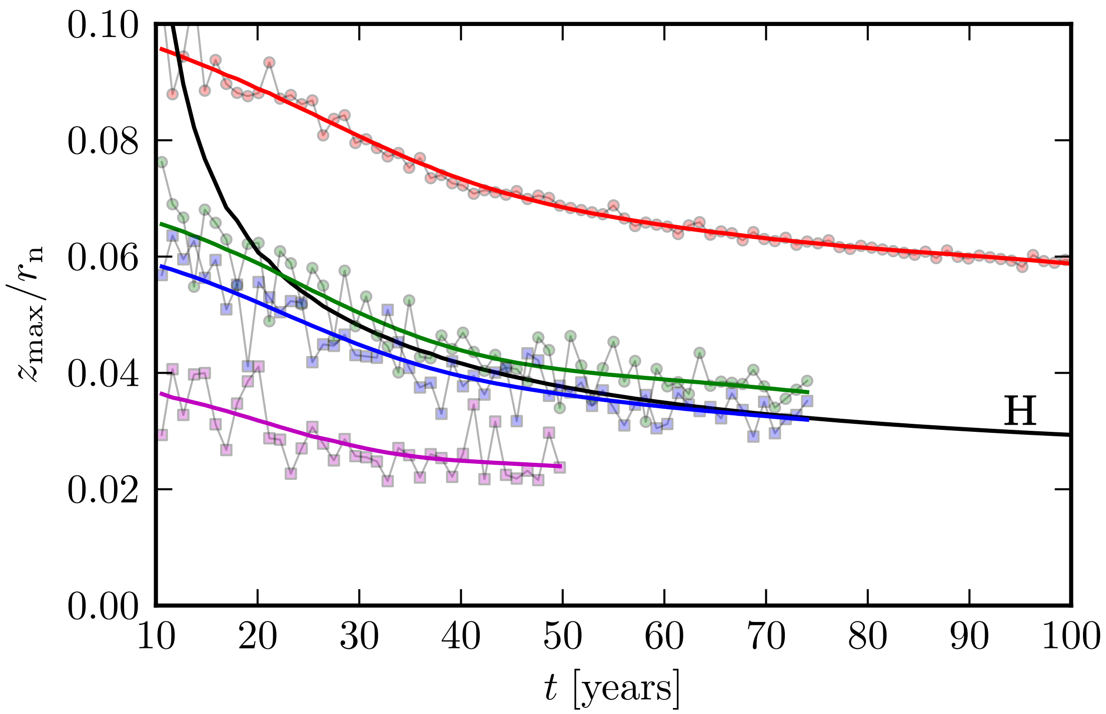


Total pressure slices for consecutive simulation snapshots 51 years after start of simulation

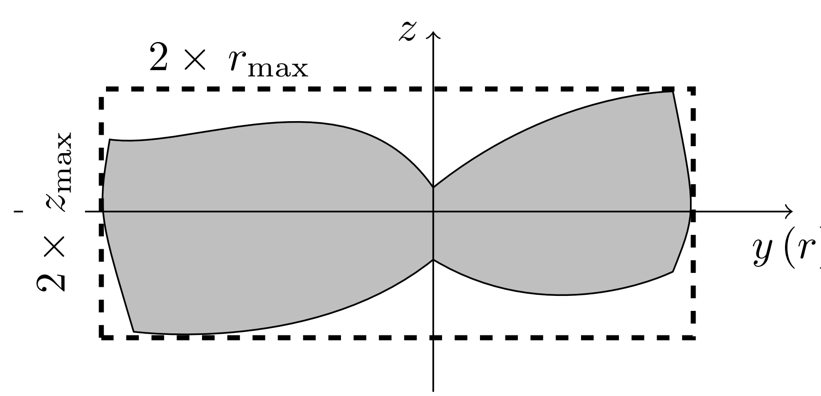
Shock radii

3D

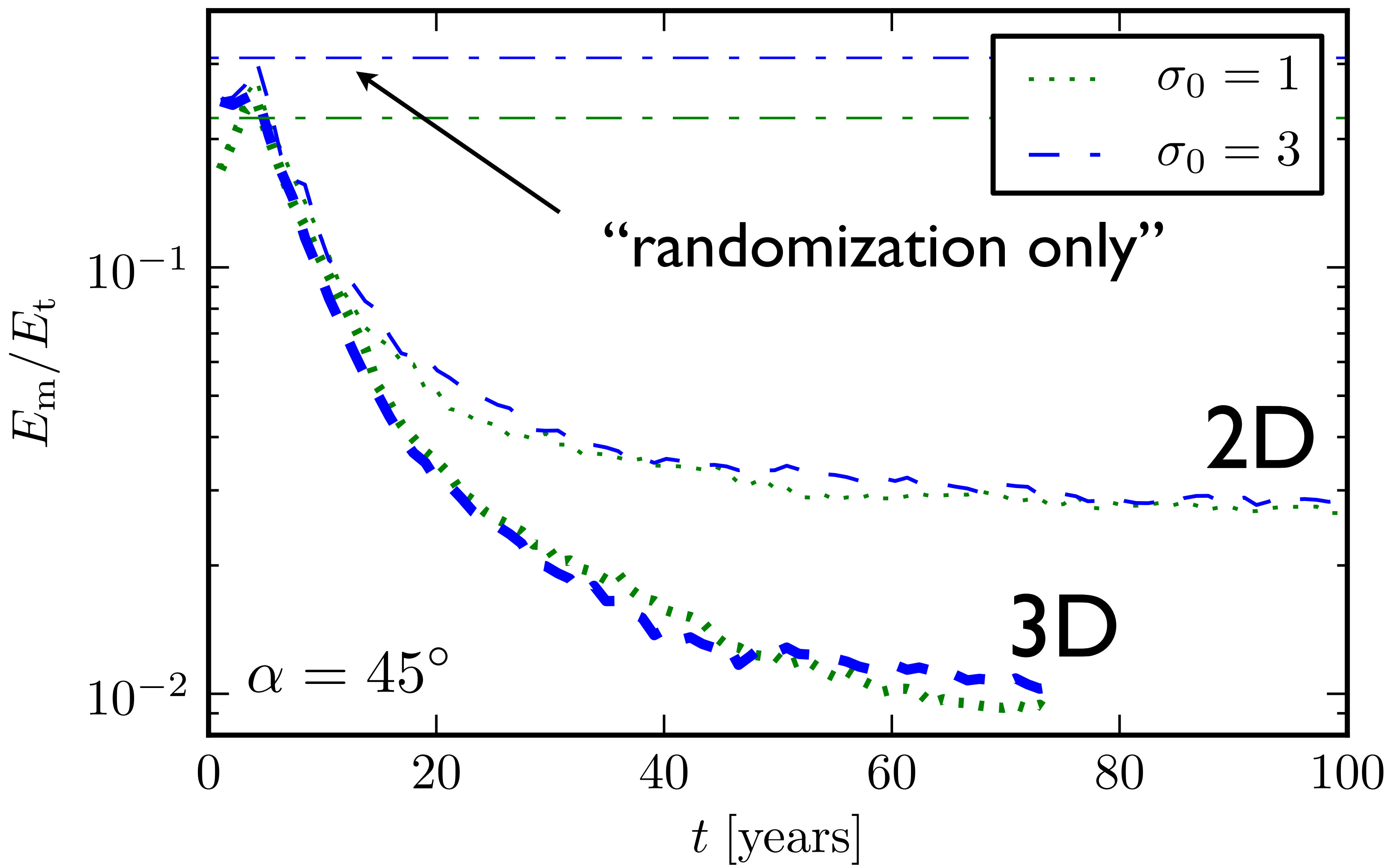




- H is the non-magnetic theory, in self-similar phase: r_{max}/r_n=0.095
- self-similar regime after t~200
- Observations provide r_{max}/r_n=0.085
- Shock sizes in 3D:
  - Don't collapse for high σ_0
  - little dependence on σ_0



What remains of the sigma problem



Dissipation in the nebula

α=45°

2D: thin lines

3D: thick lines

Observed value from fitting Synchrotron and i.Compton emission:

$E_e \cong 30 E_m$

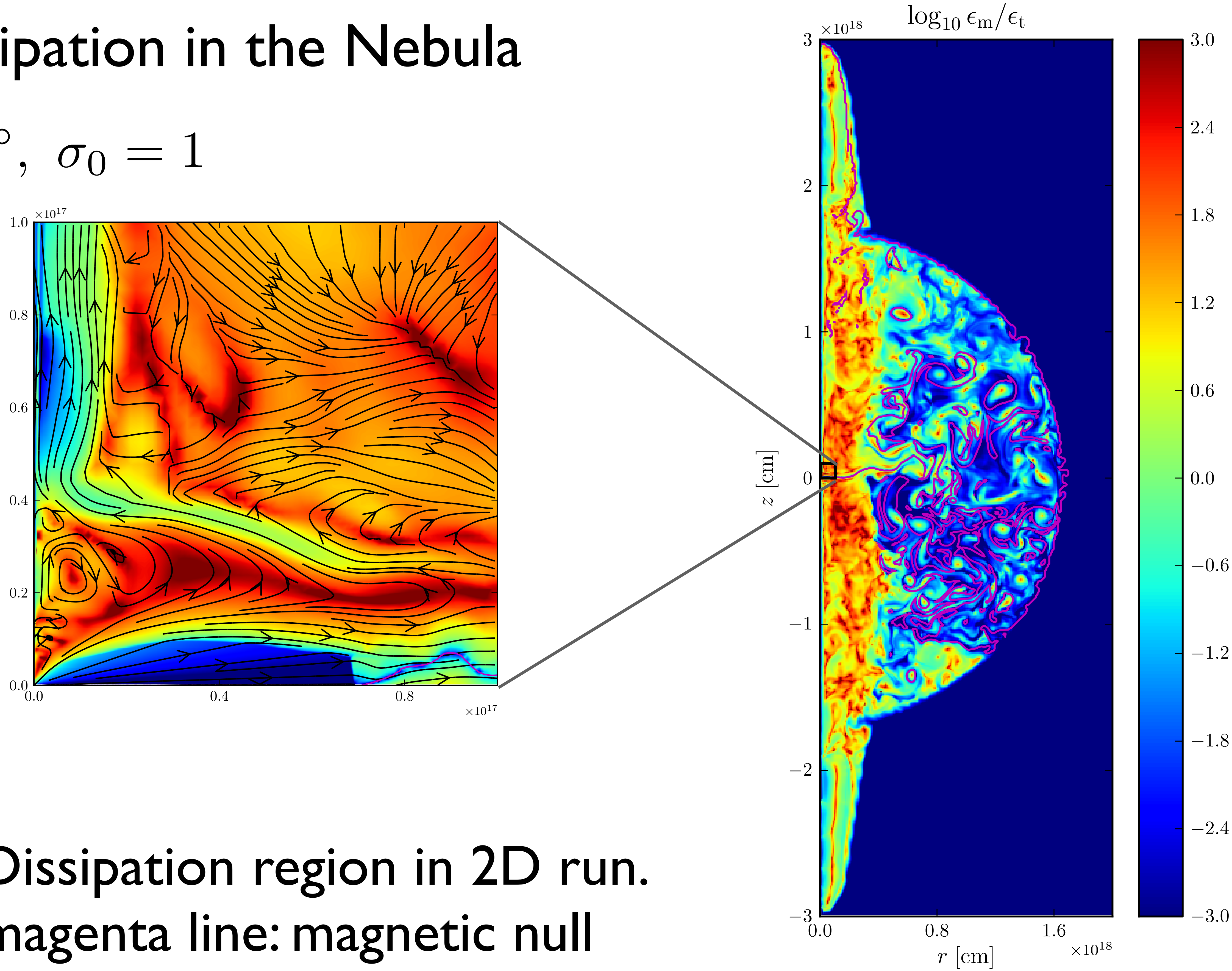
- 2D cases are also fairly dissipative!
- Dynamics dominated by gas pressure

Lyutikov (2010), Komissarov (2012)

What remains of the sigma problem

- Dissipation in the Nebula

$$\alpha = 45^\circ,\ \sigma_0 = 1$$



Dissipation region in 2D run.
magenta line: magnetic null

What remains of the sigma problem

- Dissipation in the Nebula

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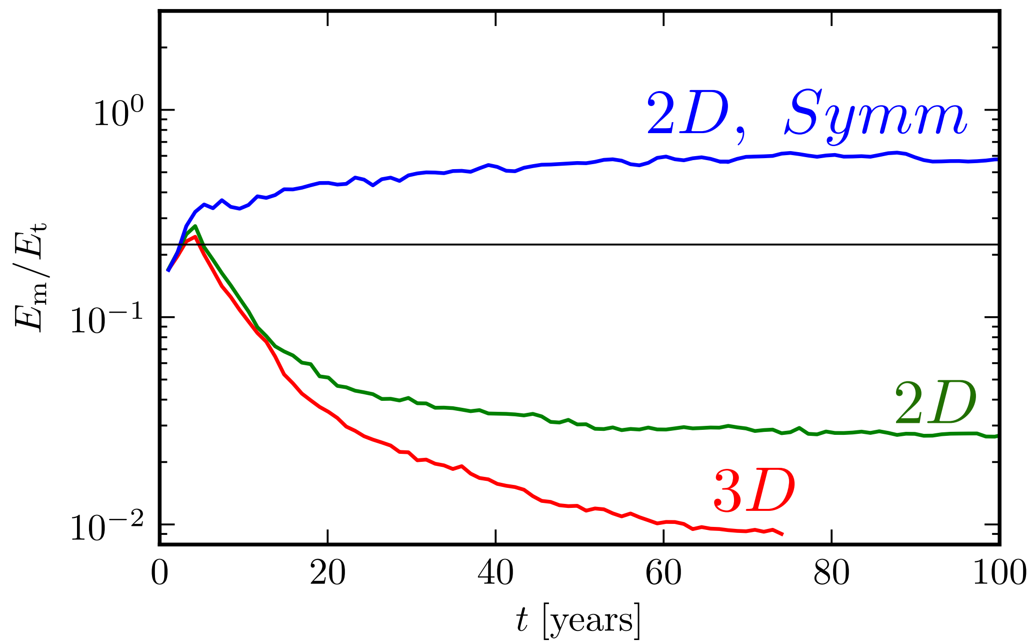
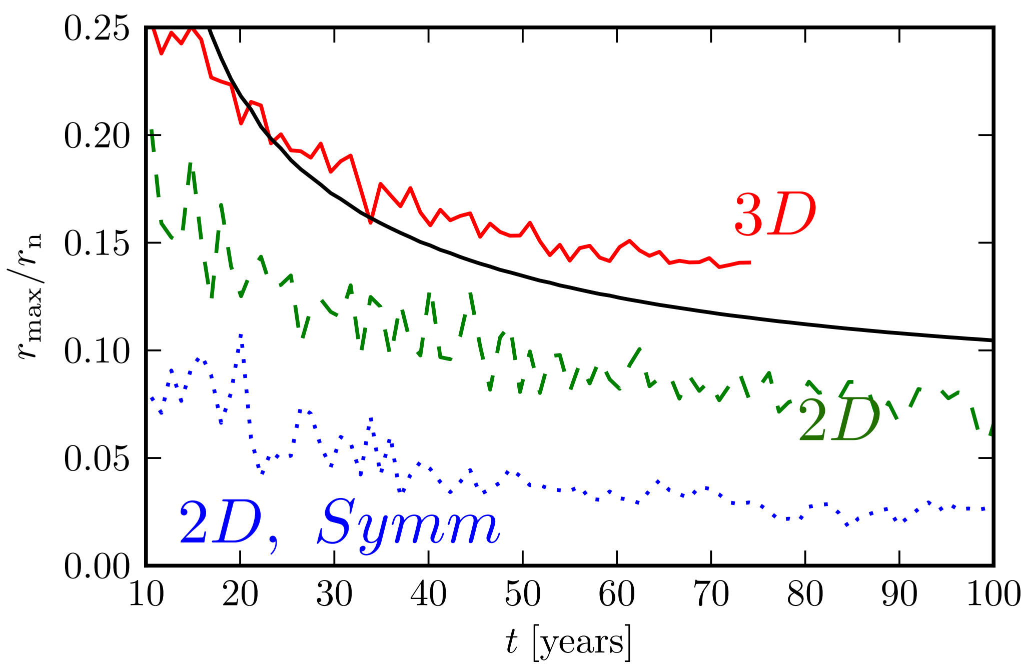
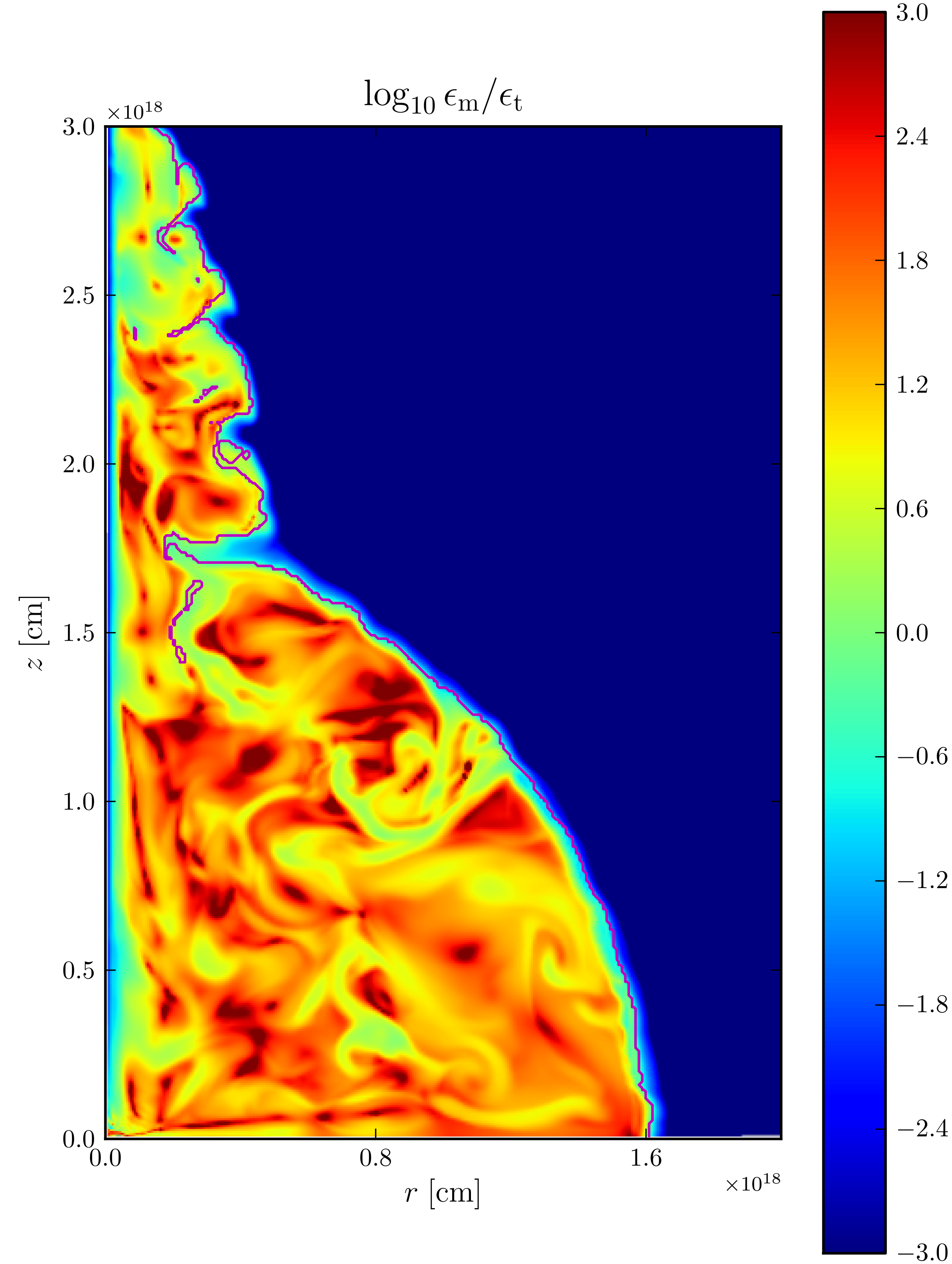
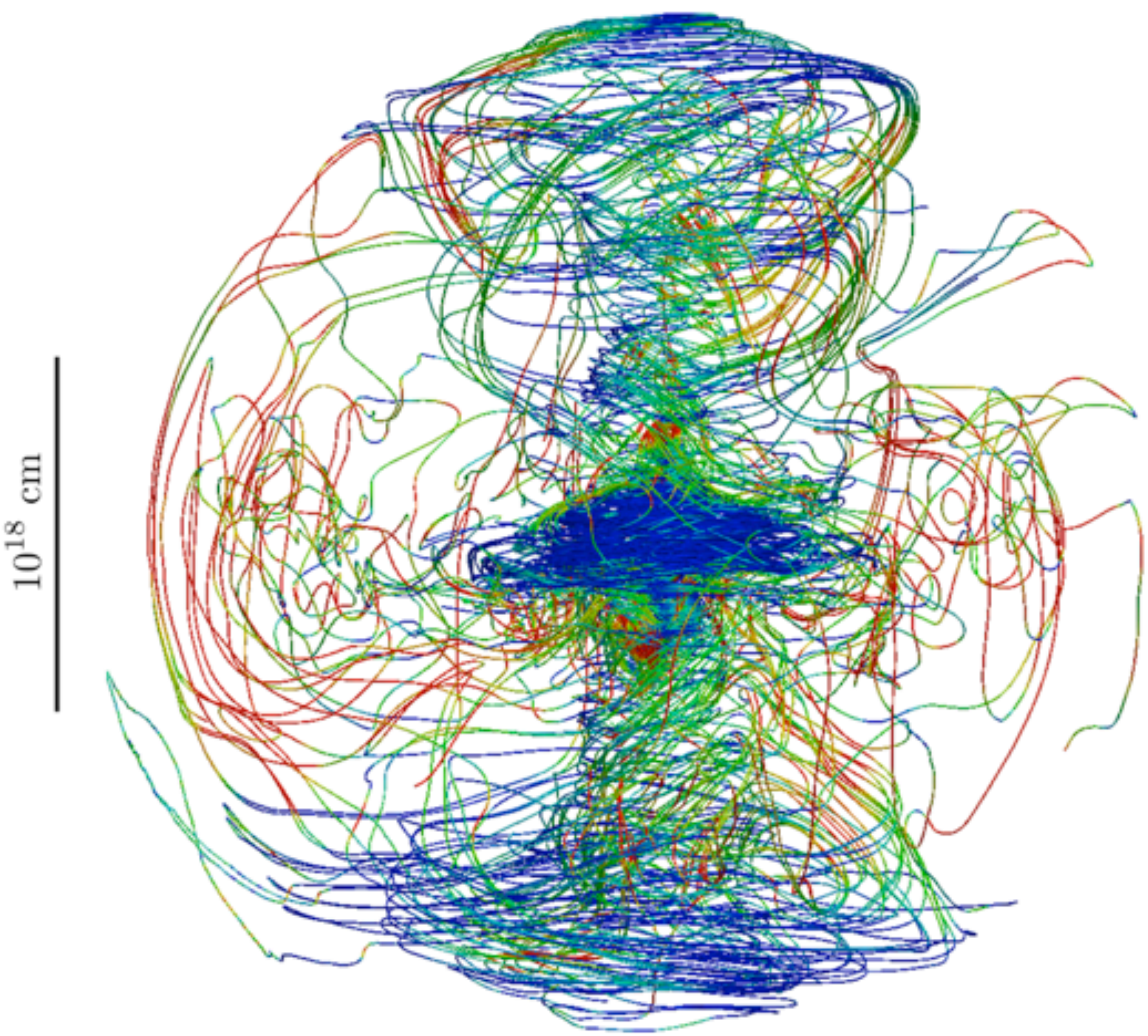


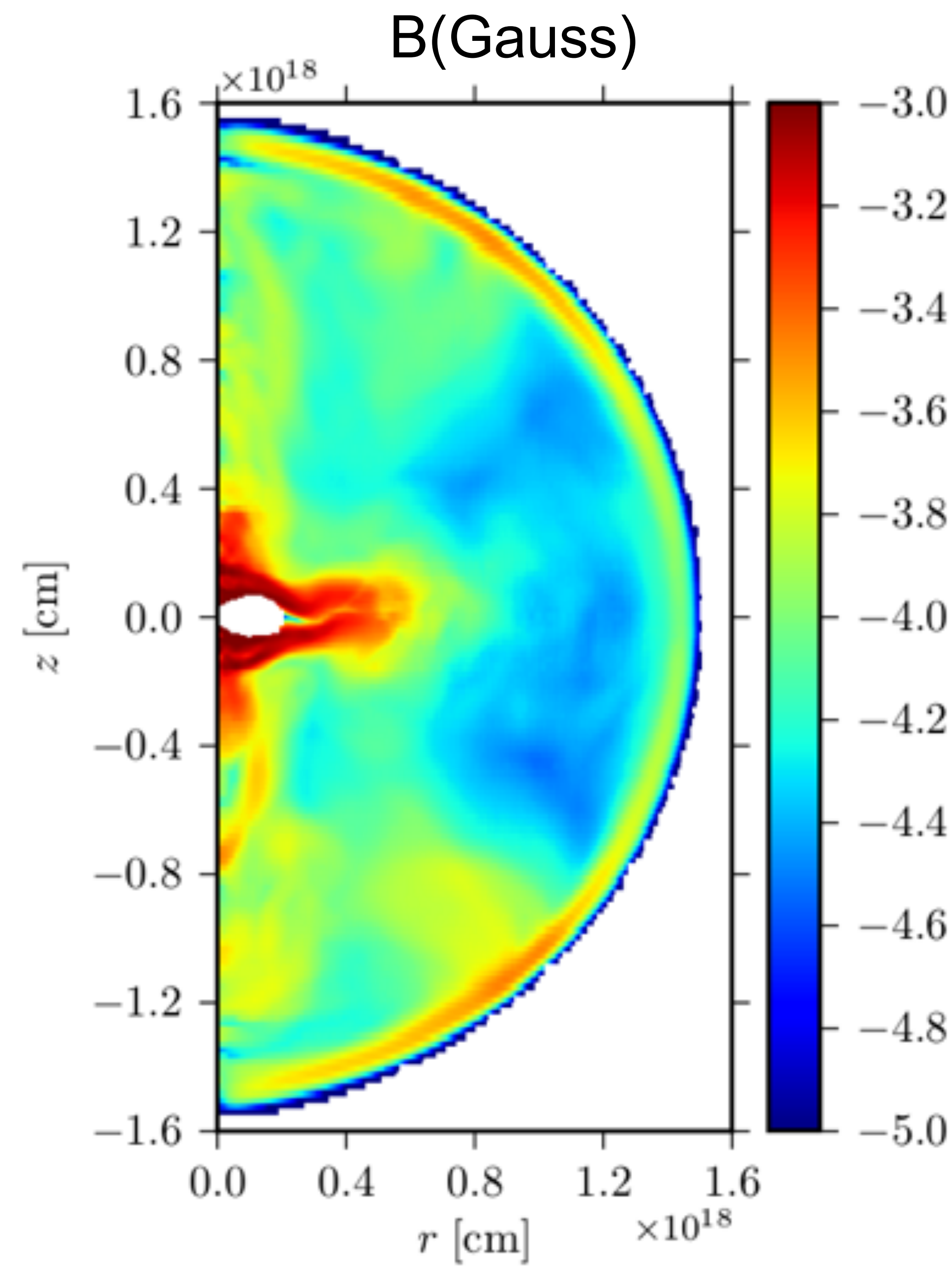




Departure from spherical envelope

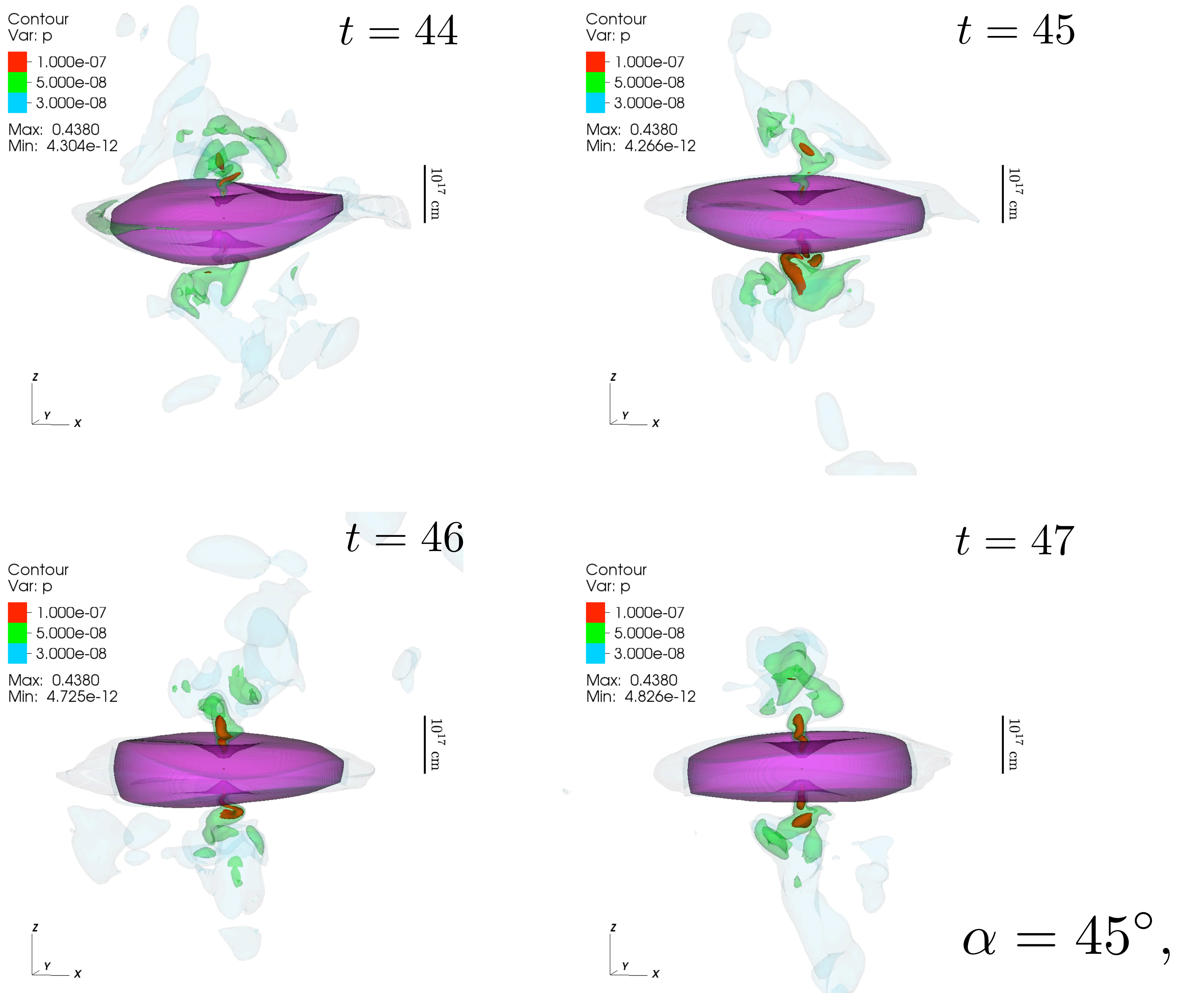
Magnetic field in the nebula





The magnetic field is strongest in the vicinity of the termination shock, where it is still predominantly azimuthal. It is disordered further away from the shock.

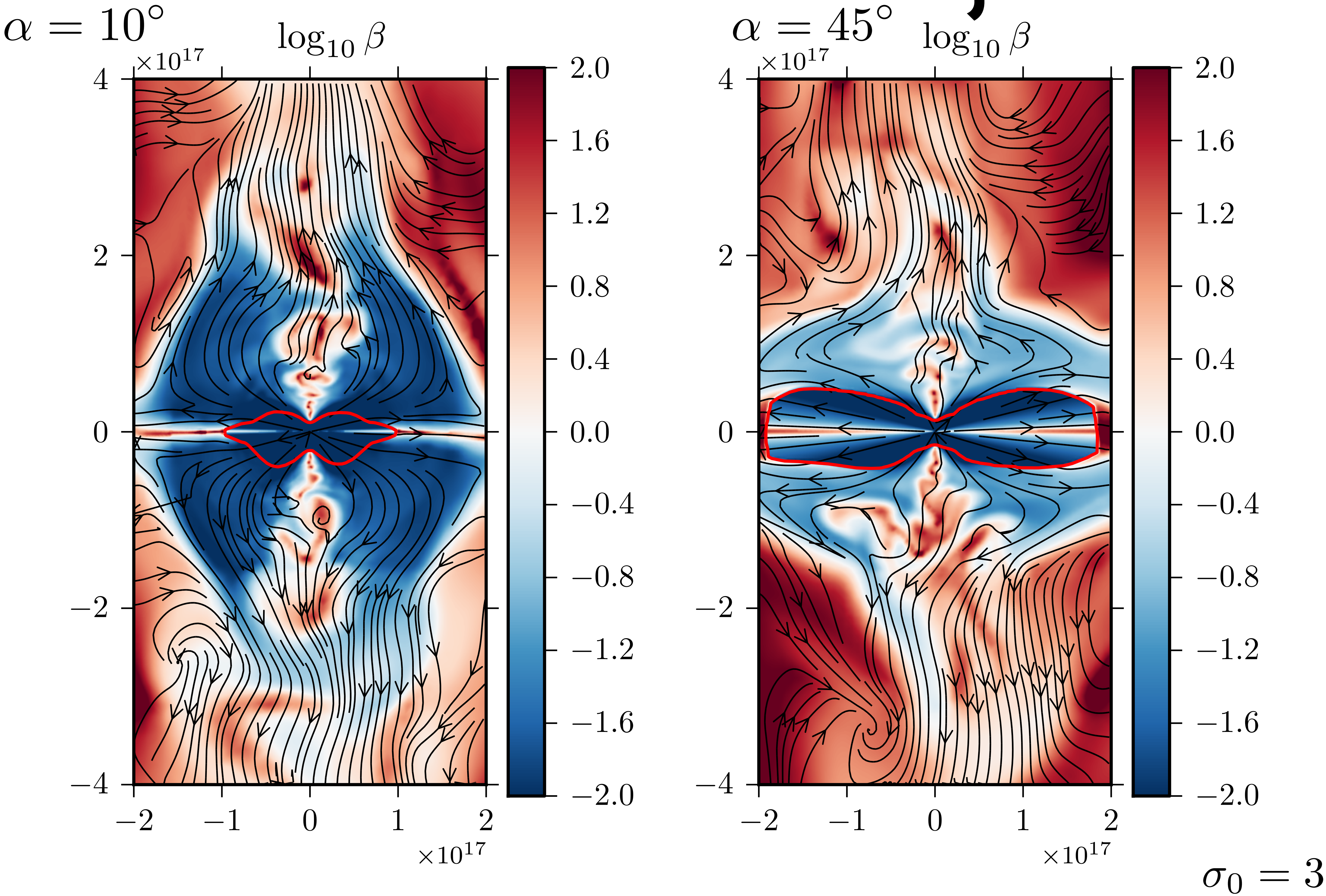
Polar beam and jet



$$\alpha = 45^\circ,\ \sigma_0 = 1$$

Thermal pressure isocontours for consecutive simulation snapshots [years after start of simulation]

Polar beam and jet



Polar beam and jet

Toroidal field jet (non-force free)
visible in 3D as a 'plume' with velocity up to 2/3c

Ok for Crab and Vela jets (e.g. Pavlov+ 2013

Iso contours of the velocity component u_z={1/3c,2/3c} for ten consecutive years

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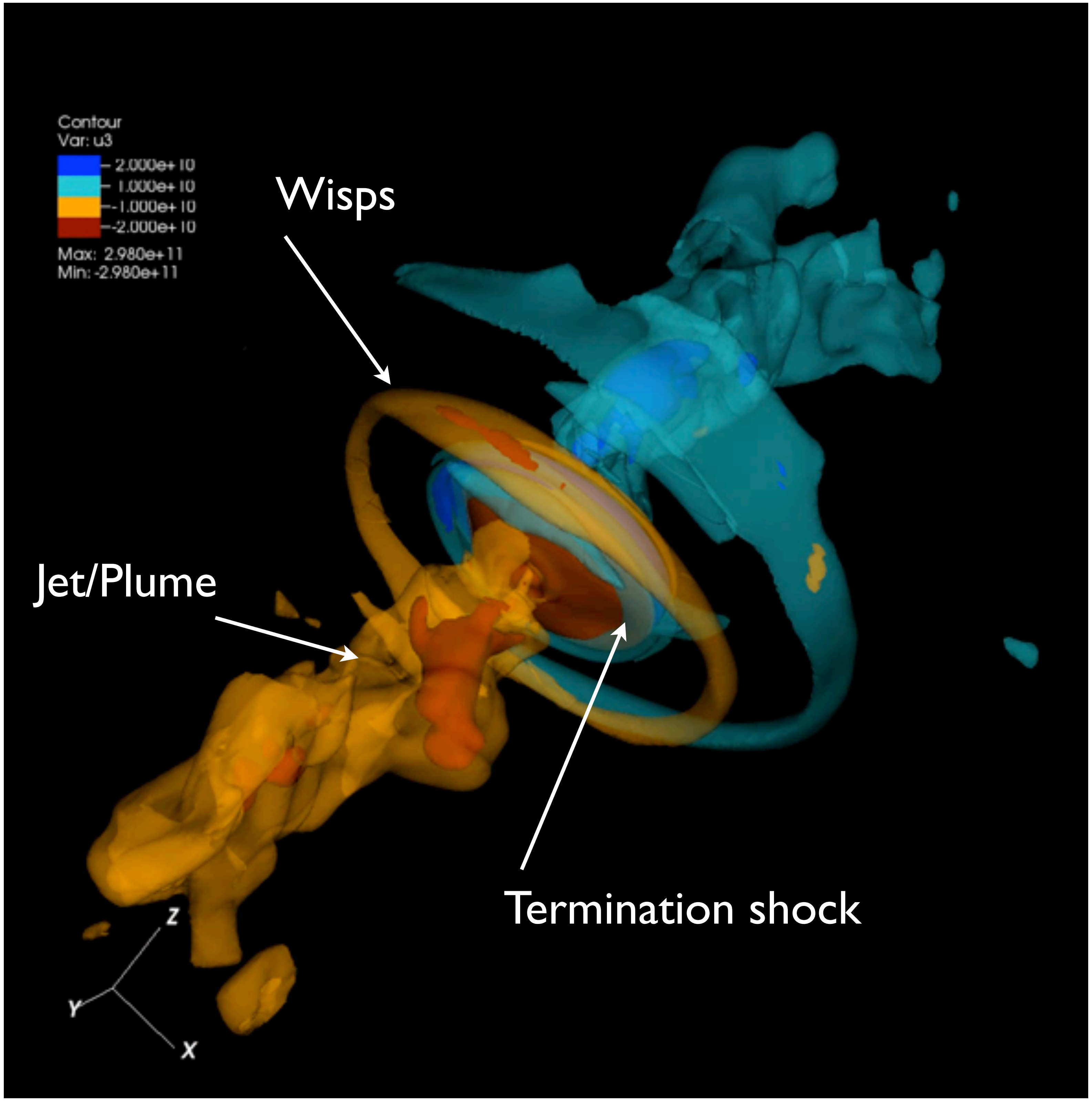

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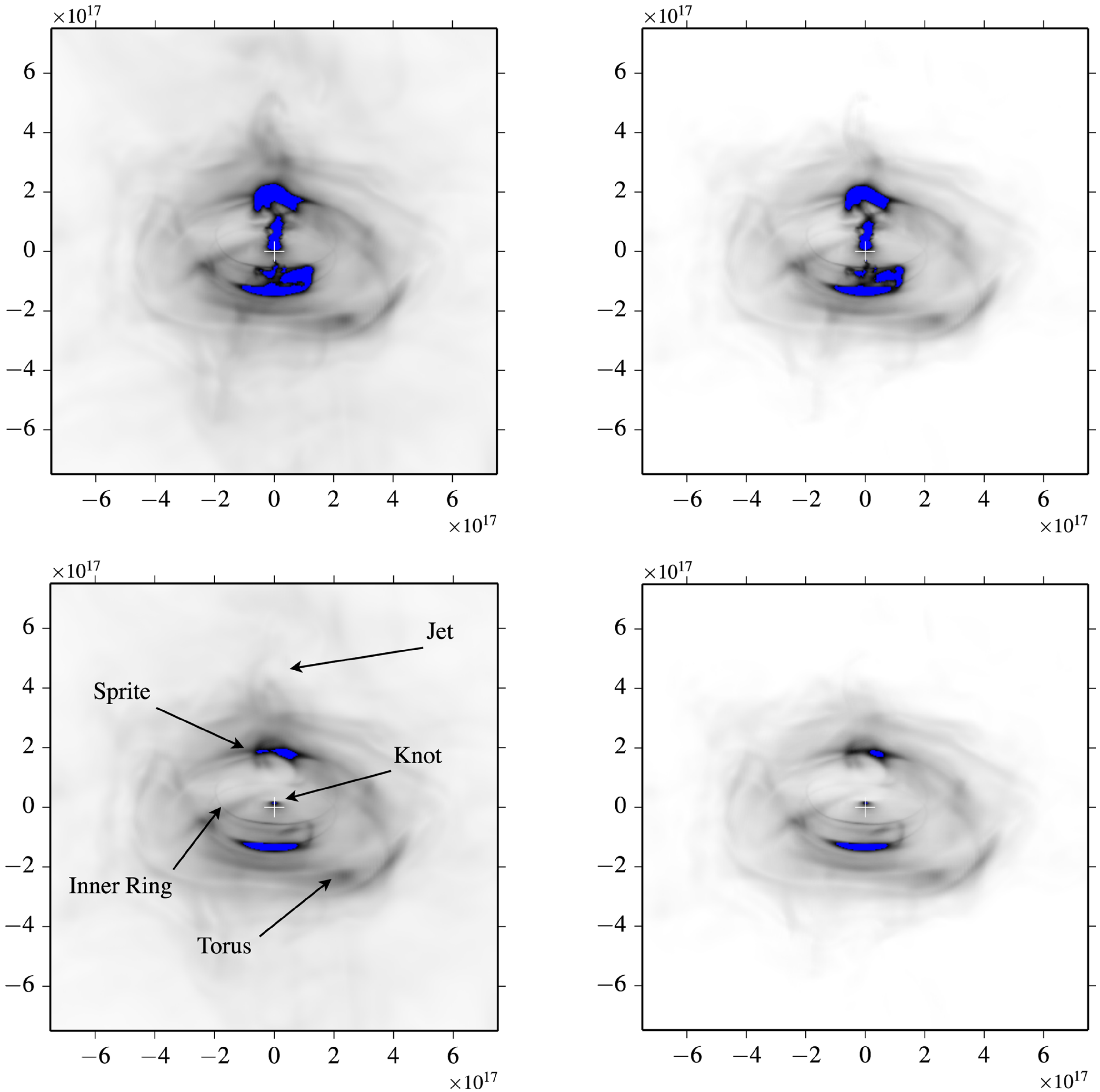
Radiation modeling - Particle injection profiles

as Kennel & Coroniti (1984)

Optical $\nu = 10^{15}$Hz   hard Xray $\nu = 10^{19}$Hz



Recipe A

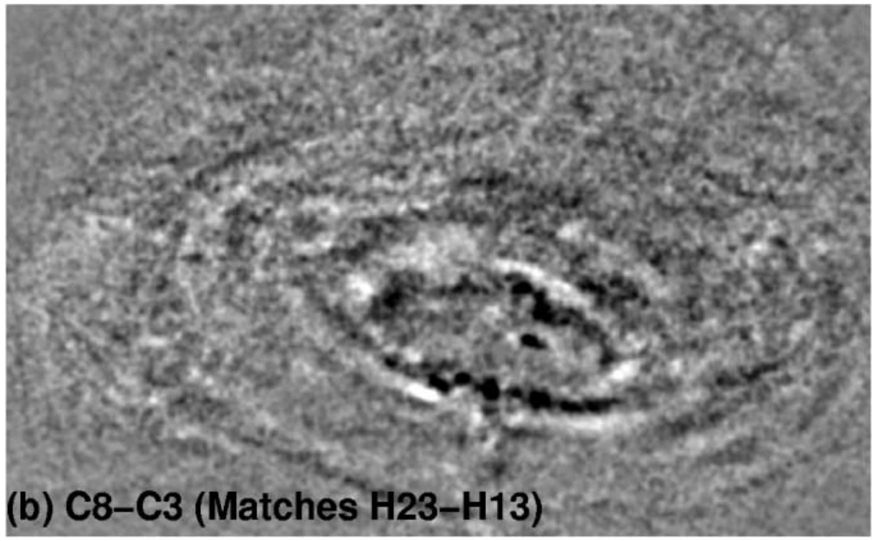
Over-producing polar column, outshines knot 1

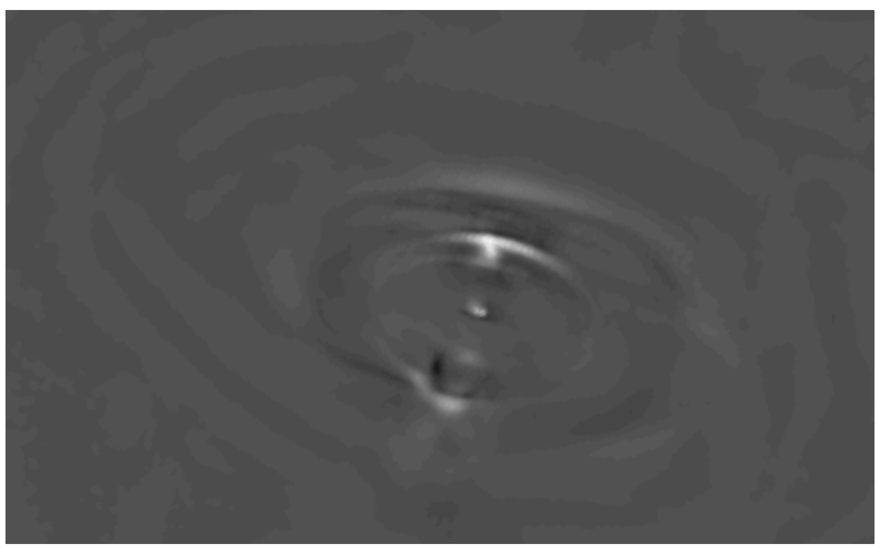
Recipe B

Good resemblance with Hubble observations of Crab

But: No jet in (hard) Xray?

Wisps





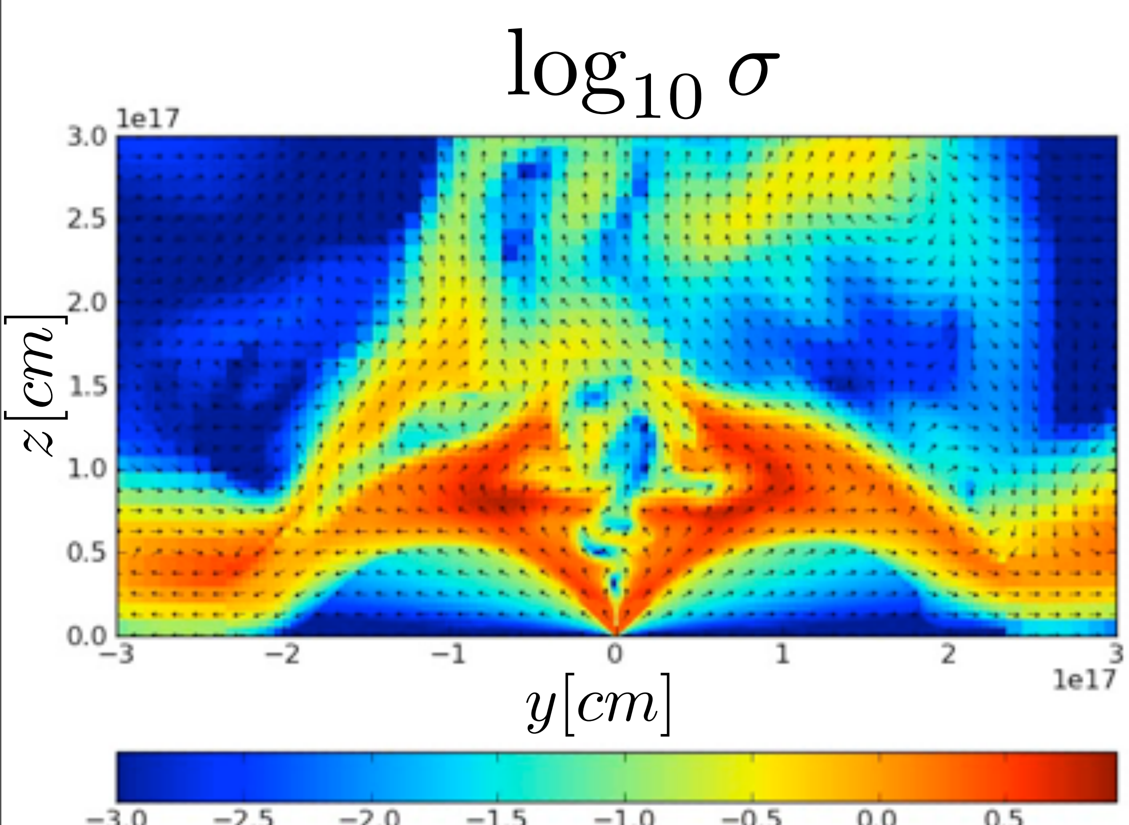
100 day Chandra difference image by Hester et al (2002) and synthetic version

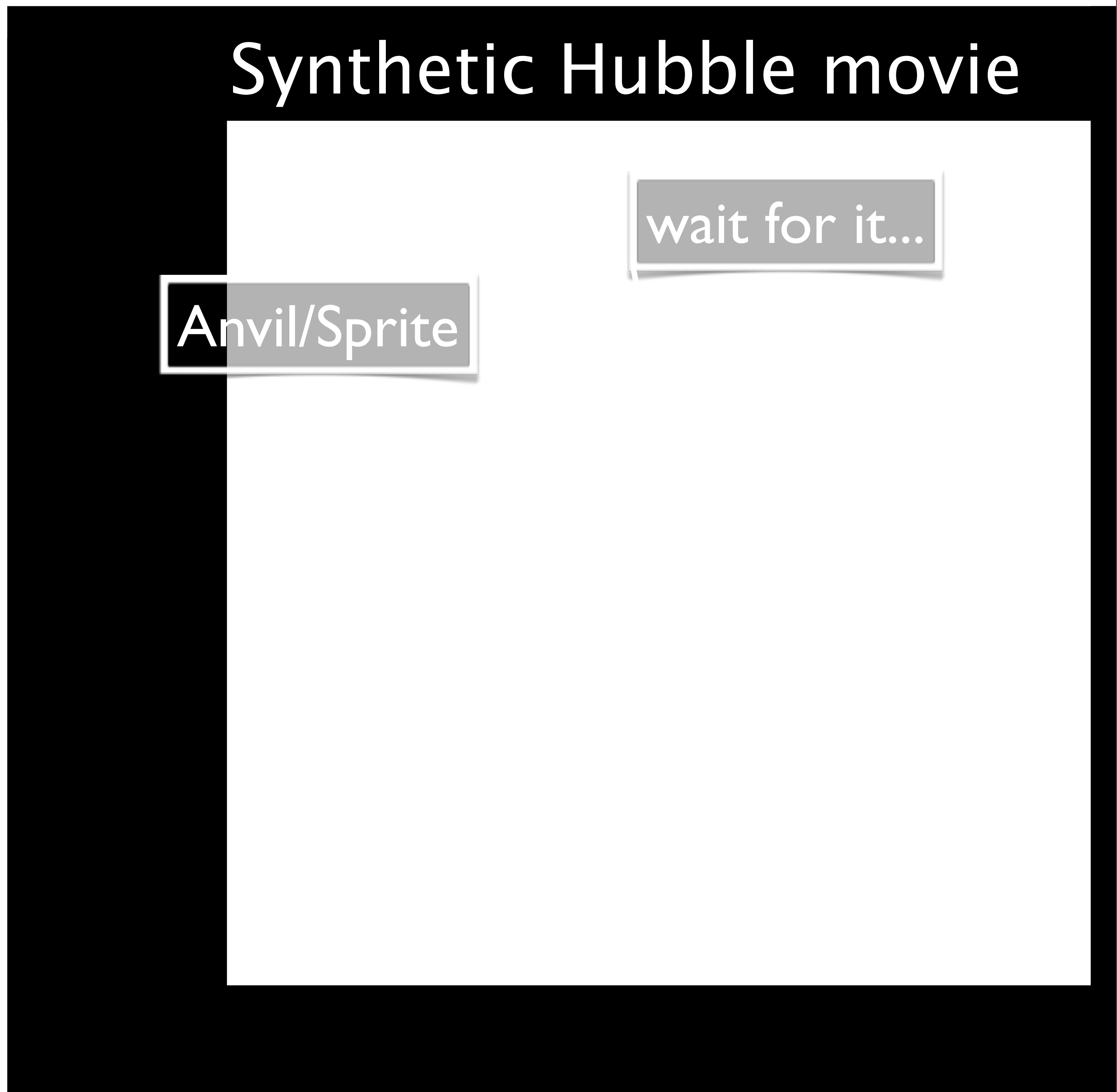
Synthetic Hubble movie

square root filtered intensity ~40 days between frames (1year total)

Anvil

- Highly variable feature at the base of the jet
- Tempting: candidate for γ-ray Flares (Tavani et al 2011, Abdo et al 2011)?
- Not seen in 2D simulations



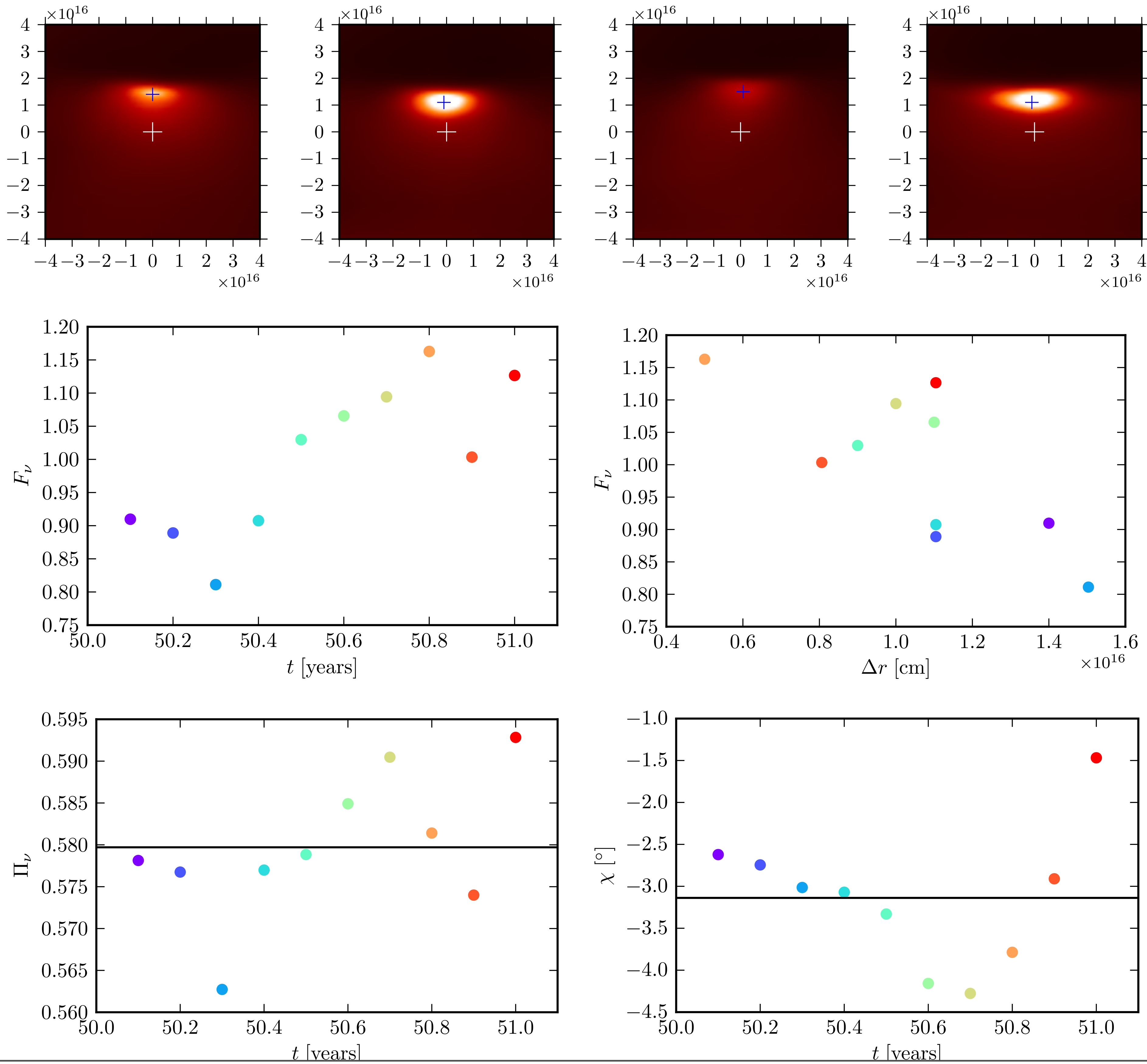


square root filtered intensity ~40 days between frames (1year total)

Compare with Moran et al. (2013)

Variability of Knot

$\alpha = 45^\circ, \ \sigma_0 = 1$



- optical intensity (linear scale) of the knot measured ~ 1 month apart
- Flux as a function of time and as function of displacement
- Unresolved polarisation degree and direction of the Knot
- Consistent with Komissarov & Lyubarsky (2004)
- Significant flux variability ~20%
- Closer in <-> brighter
- Stable polarisation signal at a degree of 60%

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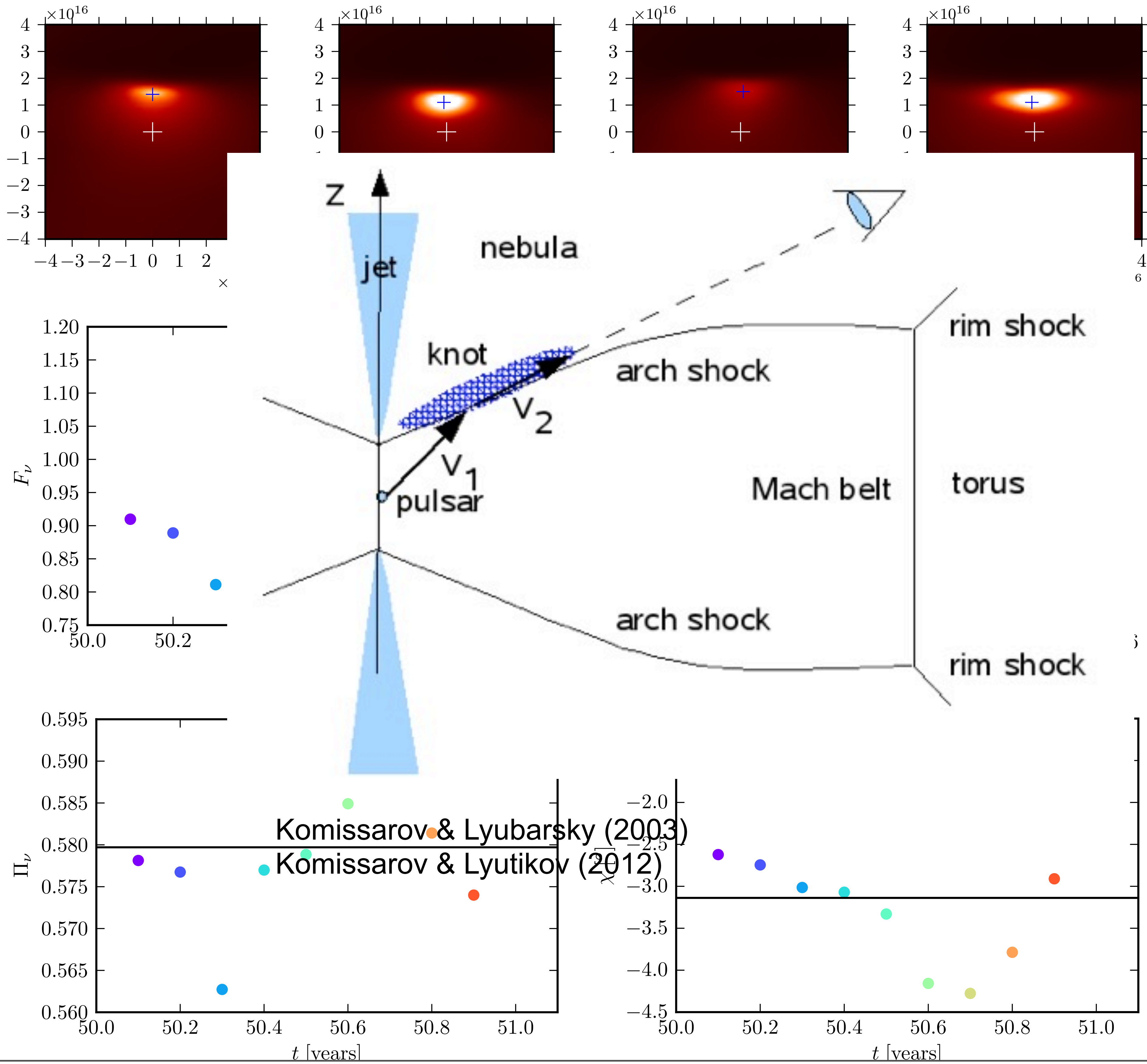

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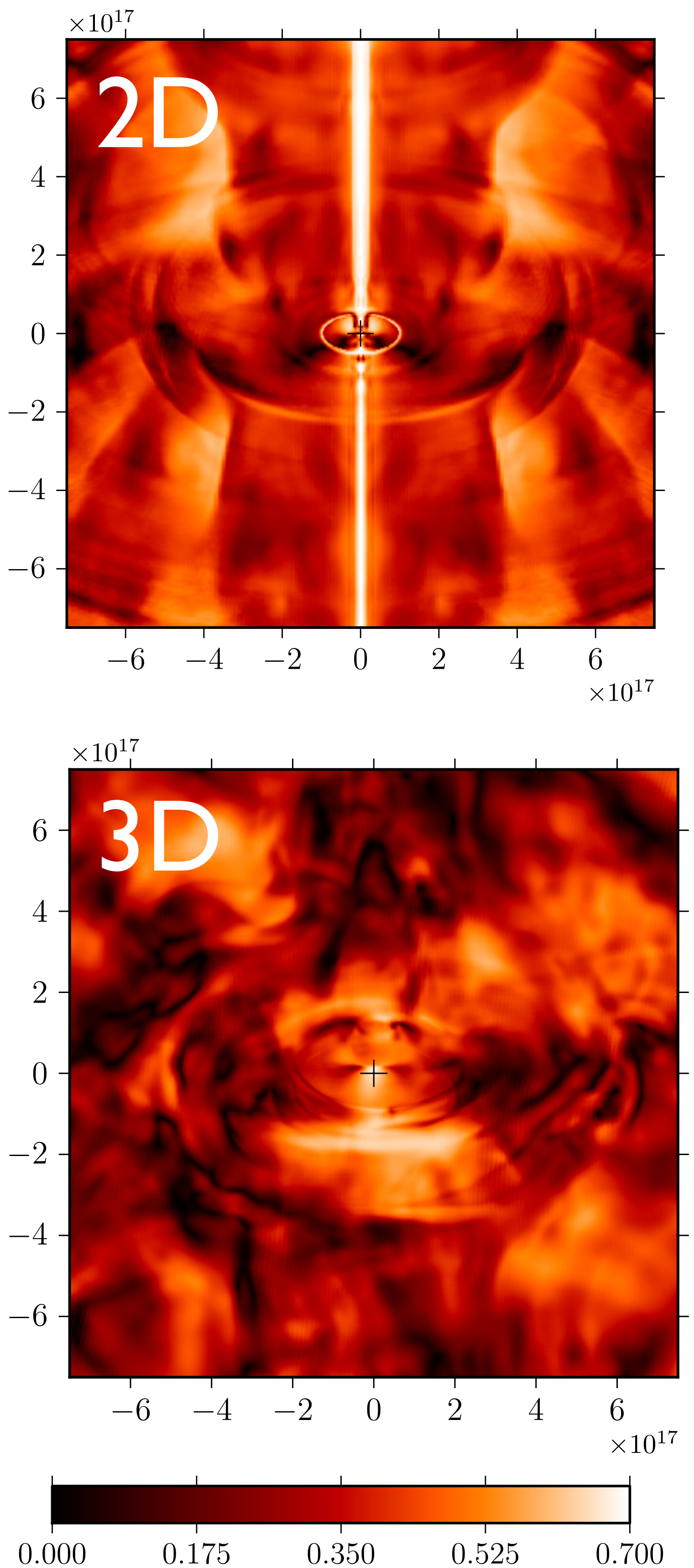
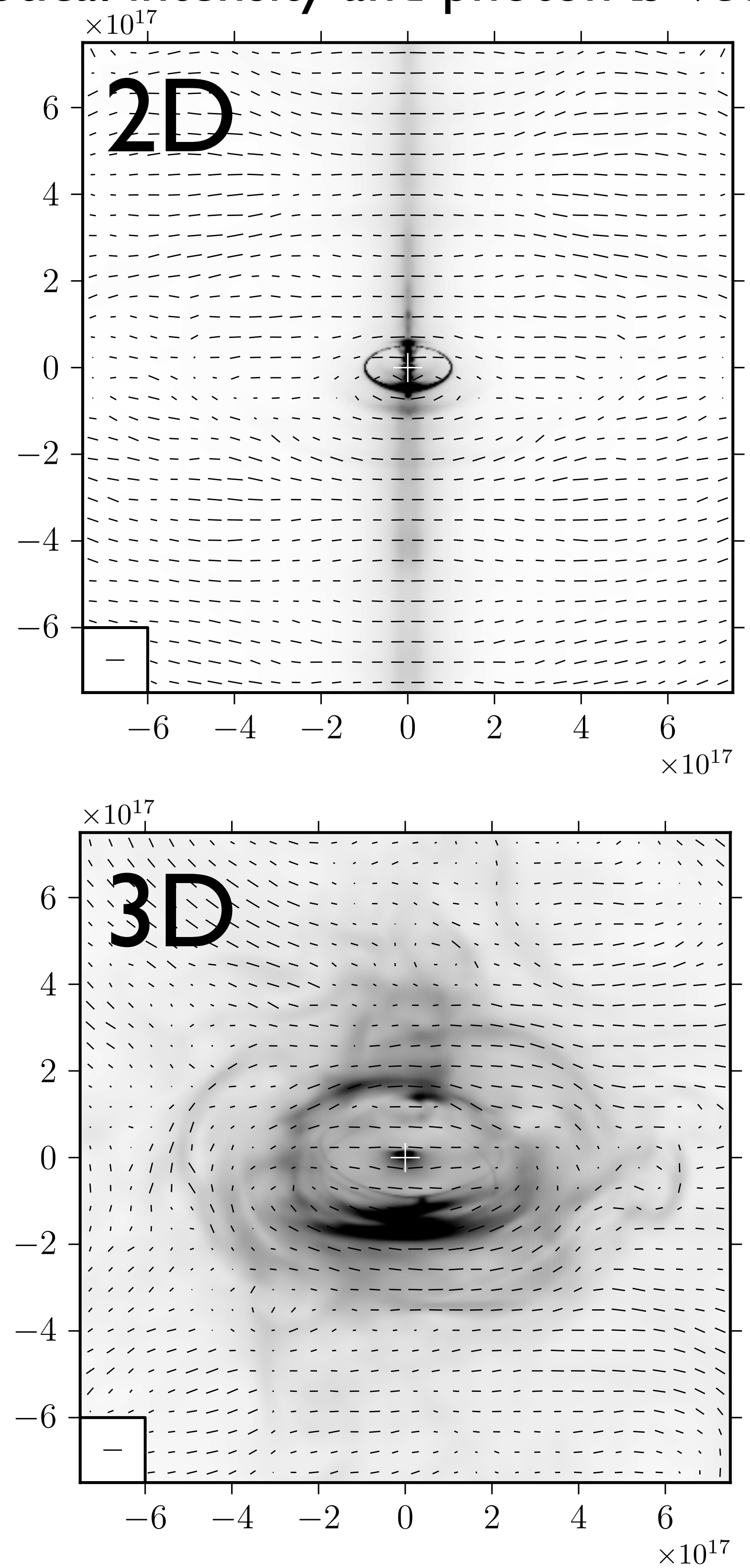
$\alpha = 45^\circ,\ \sigma_0 = 1$ linear polarisation

Optical polarisation degree



Optical intensity and photon **b**-vectors



- Features in intensity are highly polarised
- Photon b-vectors align with wisps
- Indicative of toroidal magnetic field in torus, also in 3D
- Polarisation stays aligned with wisps as they deform
- Unresolved polarization degree ~34%
- “Randomization” not fast enough to wipe out freshly injected toroidal plasma

Conclusions

- Three mechanisms involved in the solution of the sigma problem:
    - Striped Wind (here: assumption)
    - Nebula turbulence (field randomization)
    - Turbulent magnetic dissipation in the nebula
- 3D RMHD models for Crab with $\sigma_0=3$ (>1)
- MHD model of Crab can explain many observed features: shock variability, jet, torus, wisps, knot1, robust in 3D!
- The jets form downstream of the termination shock where the magnetic hoop stress causes collimation of the flow lines that pass through the shock at intermediate latitudes.
- Jets don't drill through the nebula bubble, z-pinch magnetohydrostatic configurations obtained in 2D unphysical! Total nebula pressure mostly uniform.
- Illuminating the jet (v up to 0.7c) might require particle acceleration in addition to the striped wind region at the termination shock.
- Polar beam in our simulations becomes (kink-) unstable early on
    - Origin of the Anvil/Sprite feature? Flares?
- Knot 1 variability compatible with recent Hubble observations analyzed by Moran et al. (2013)
    - As knot flux increases, polarization remains stable, degree 0.6, **e**-direction along rotation axis
    - knot flux correlated with position: brighter states <-> knot closer to pulsar
- Toroidal wisps seen also in 3D simulation, inner nebula synchrotron polarization indicates toroidal field

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  - X-ray jet in simulations too faint
  - Average optical polarisation degree too high (~34% vs. ~9% observed) and intensity contrast torus/nebula also too high

local 3D simulations

- Simulate only part of domain around equator:

  $\Delta\theta = \Delta\phi = \pi/4 \qquad r \in [0.05, 10]\mathrm{Ly} \qquad n_r \times n_\theta \times n_\phi = 2048 \times 80 \times 80$

- Use periodic boundary conditions in φ, Θ

- **Anti-periodic** boundaries in Θ for the magnetic field

- Inject PW as before

- Capture termination shock, dissipation region

- Now focus on long-term evolution:

  - Turbulent magnetic dissipation

  - RT-instability of contact

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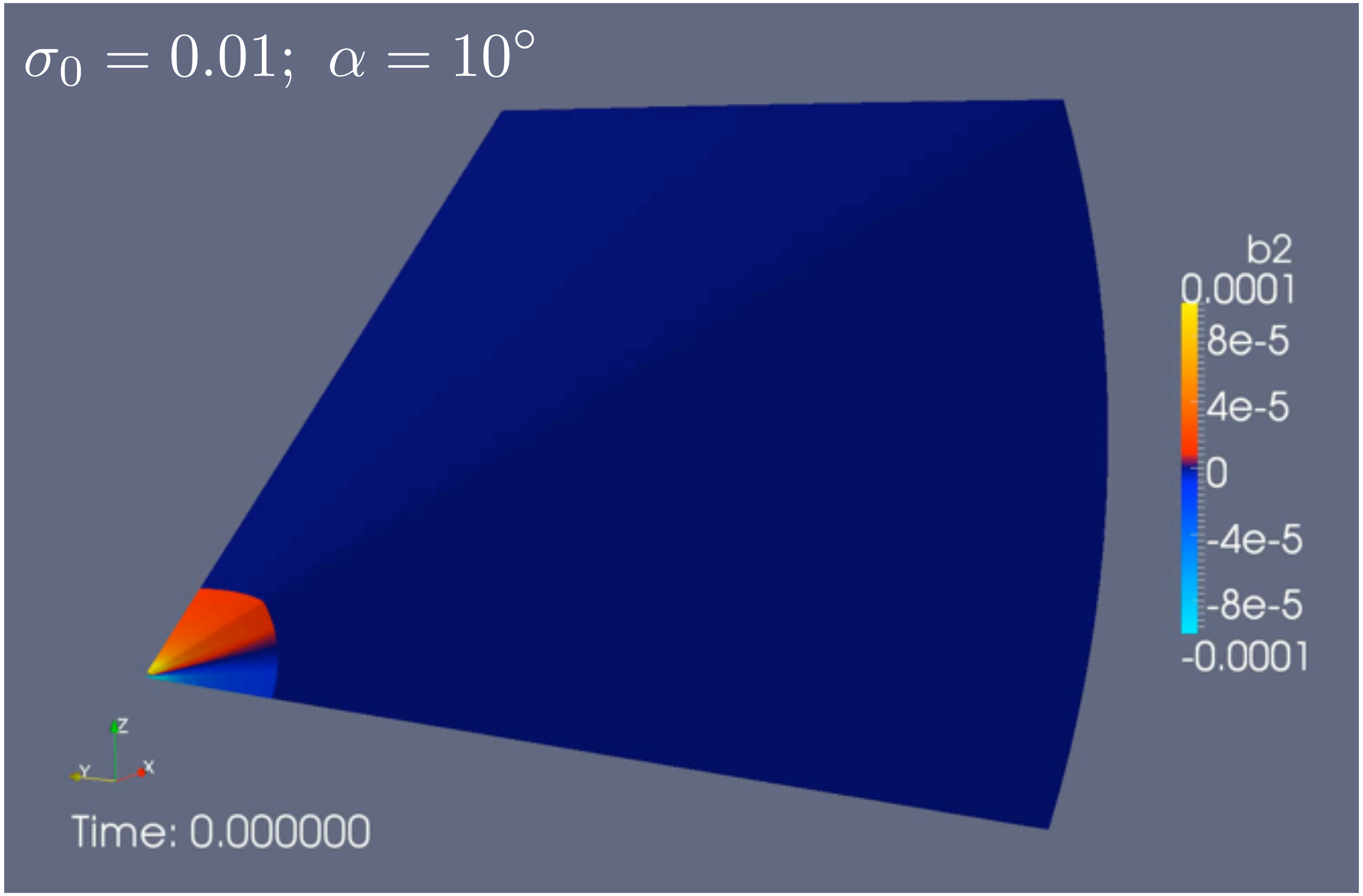

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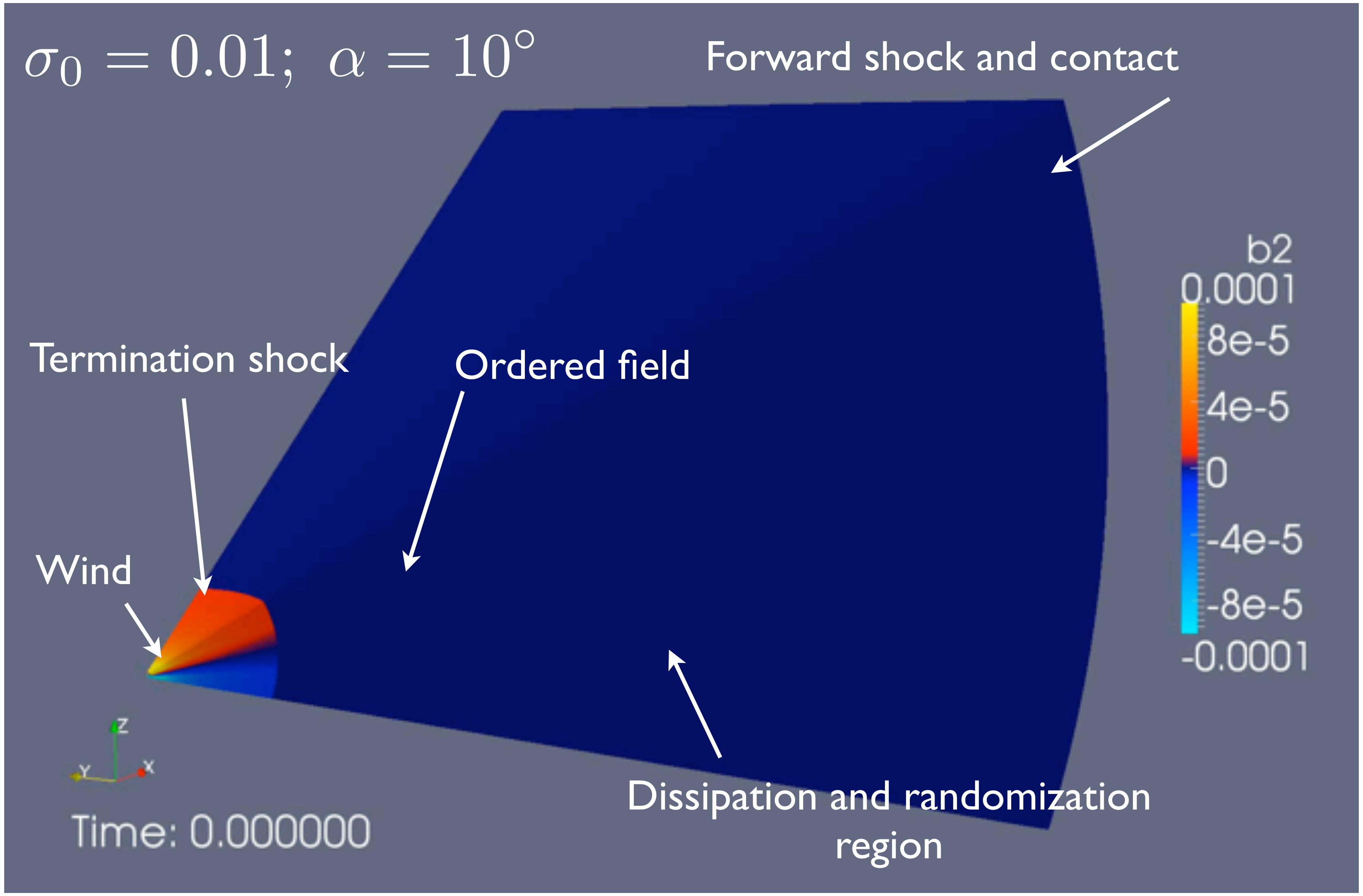
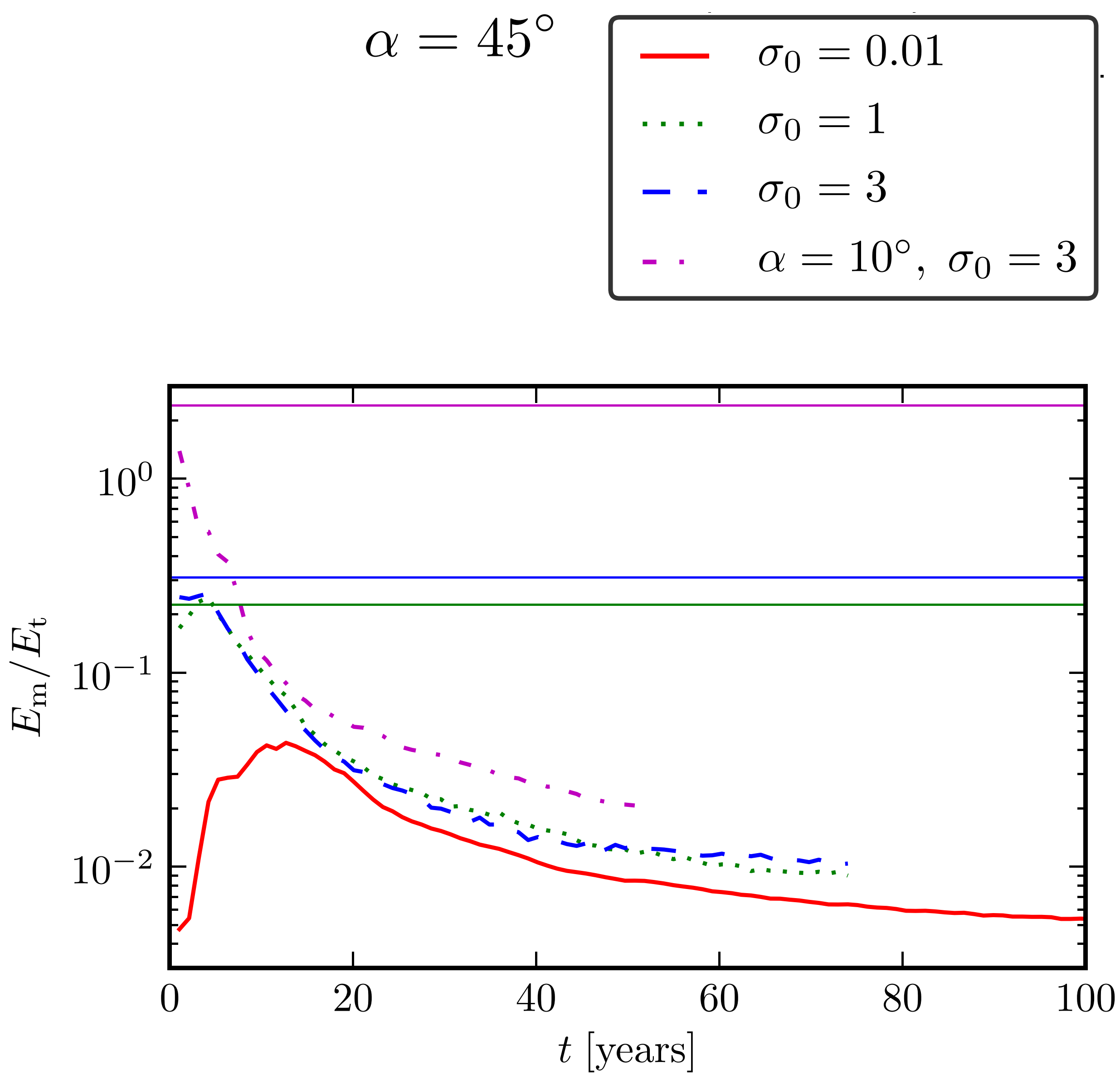
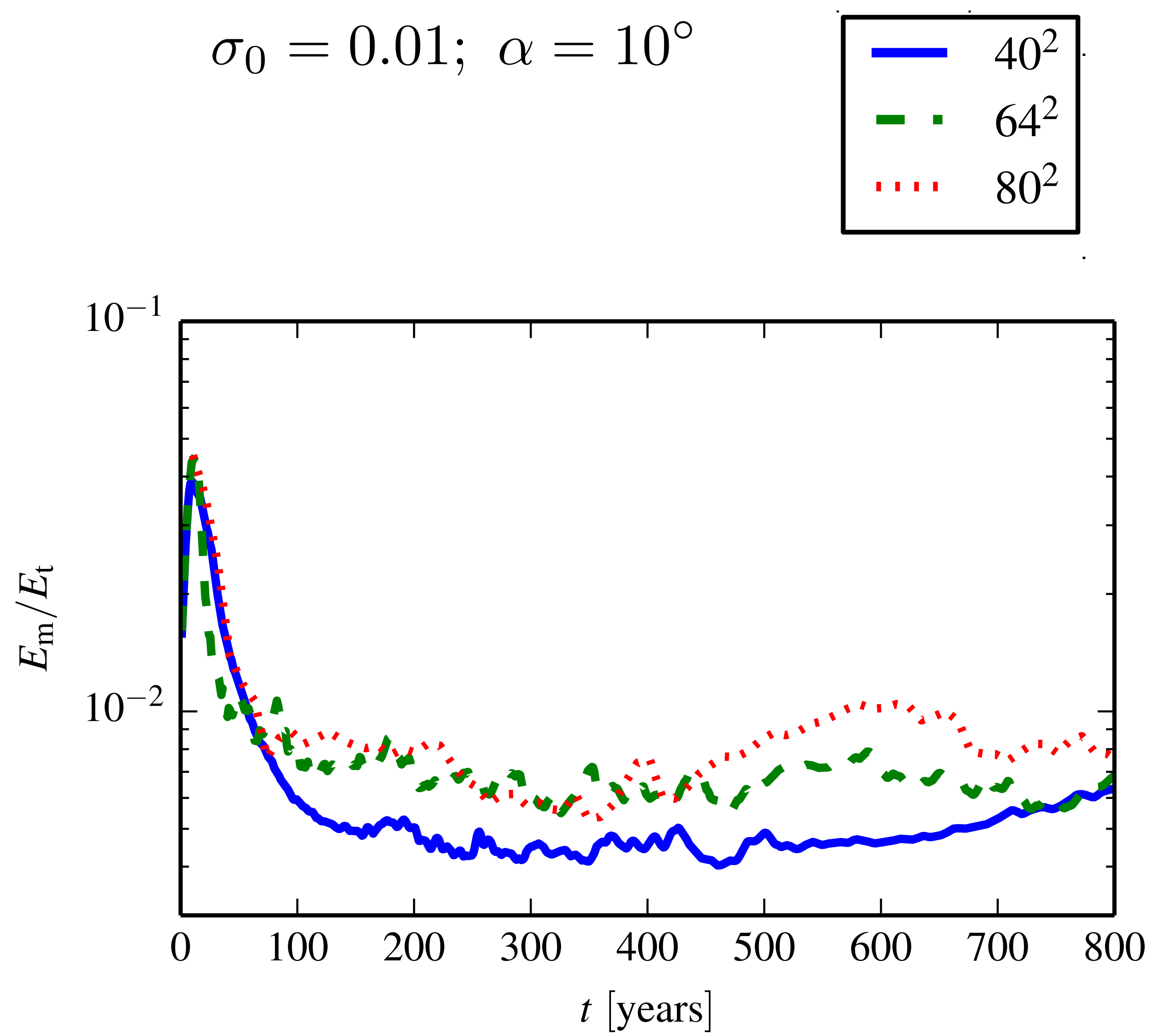


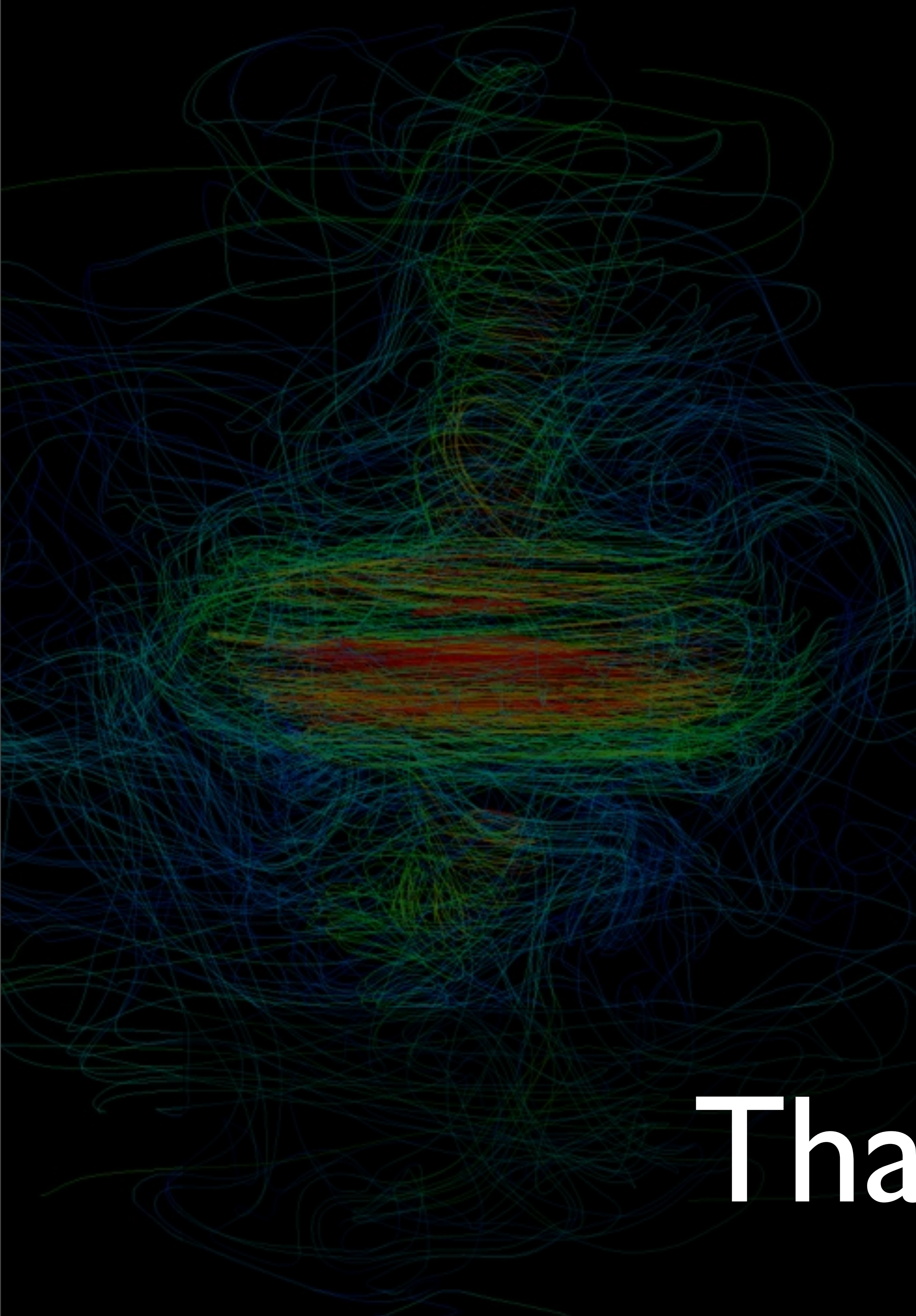
local 3D simulations - energetics

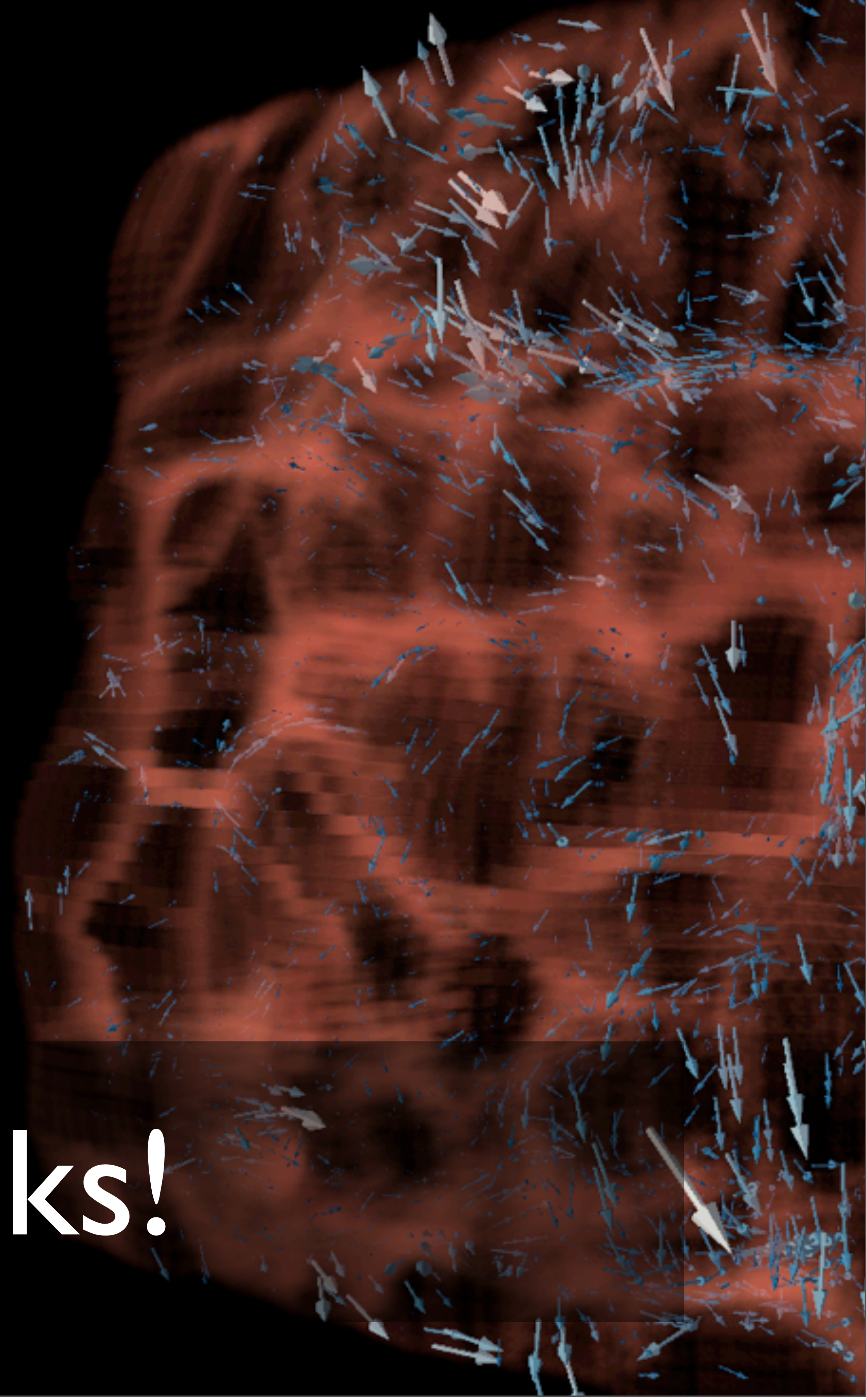
Global simulations:



Local simulations:



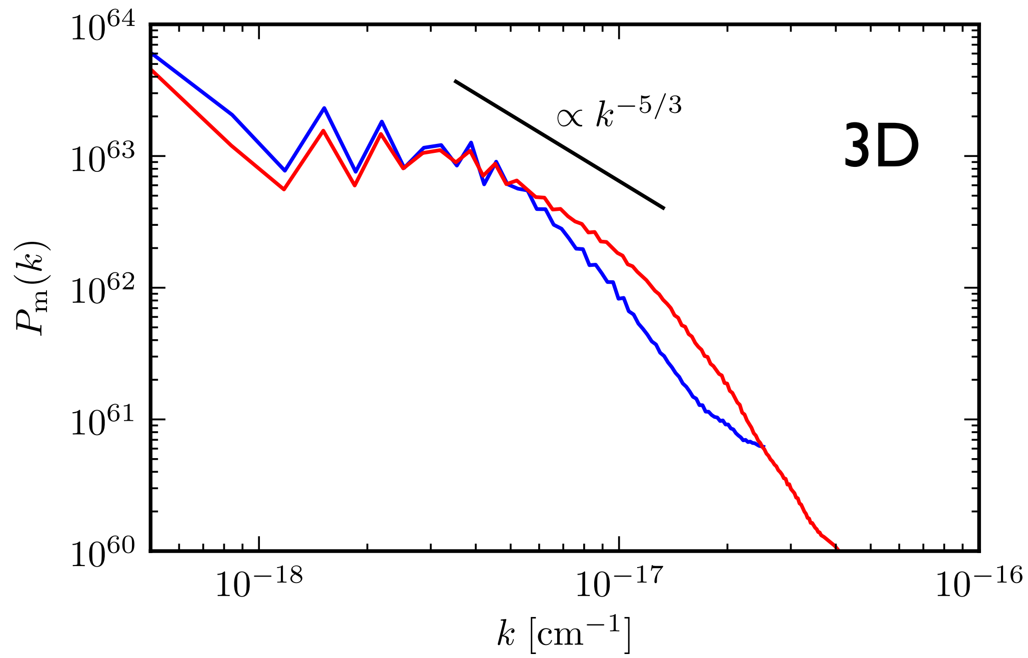


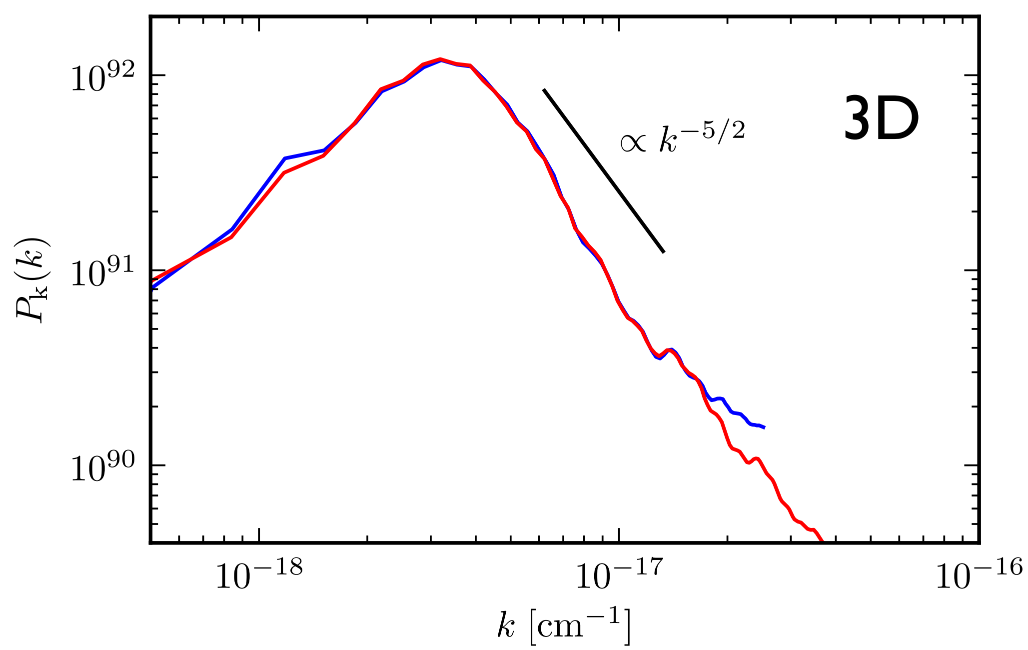


Thanks!

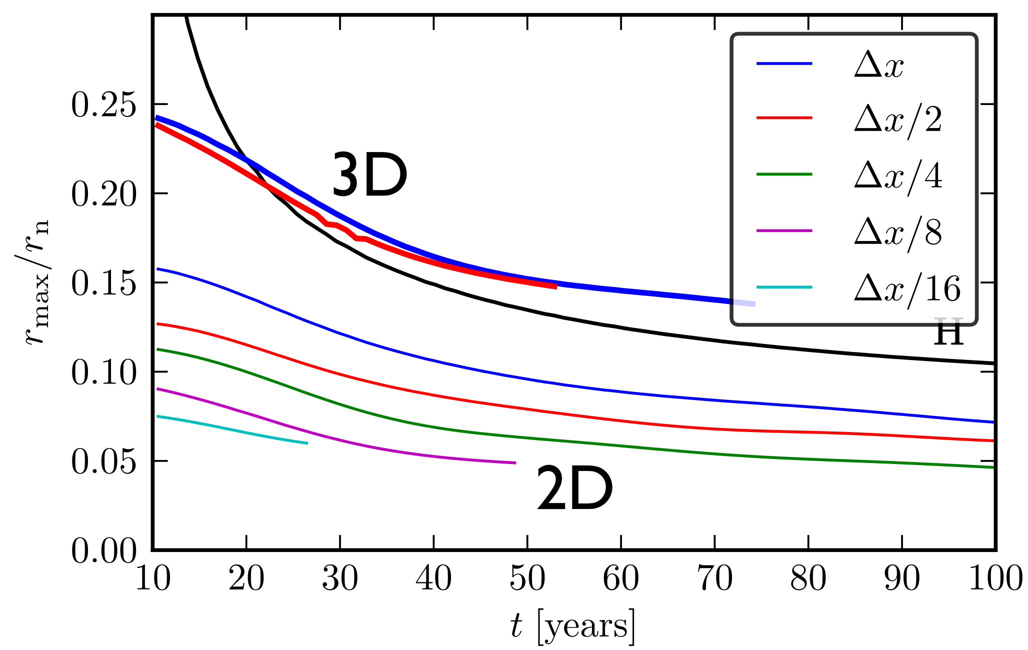
What about convergence?

Magnetic and kinetic Power spectra





Shock radii



Energetics