On the anomalous torque acting on the rotating magnetized sphere

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Radio pulsars

30 years – first pulsar publication

25 years – theory of the pulsar radio emission

20 years – book

Two points

- No magnetodipole radiation
- Anomalous torque

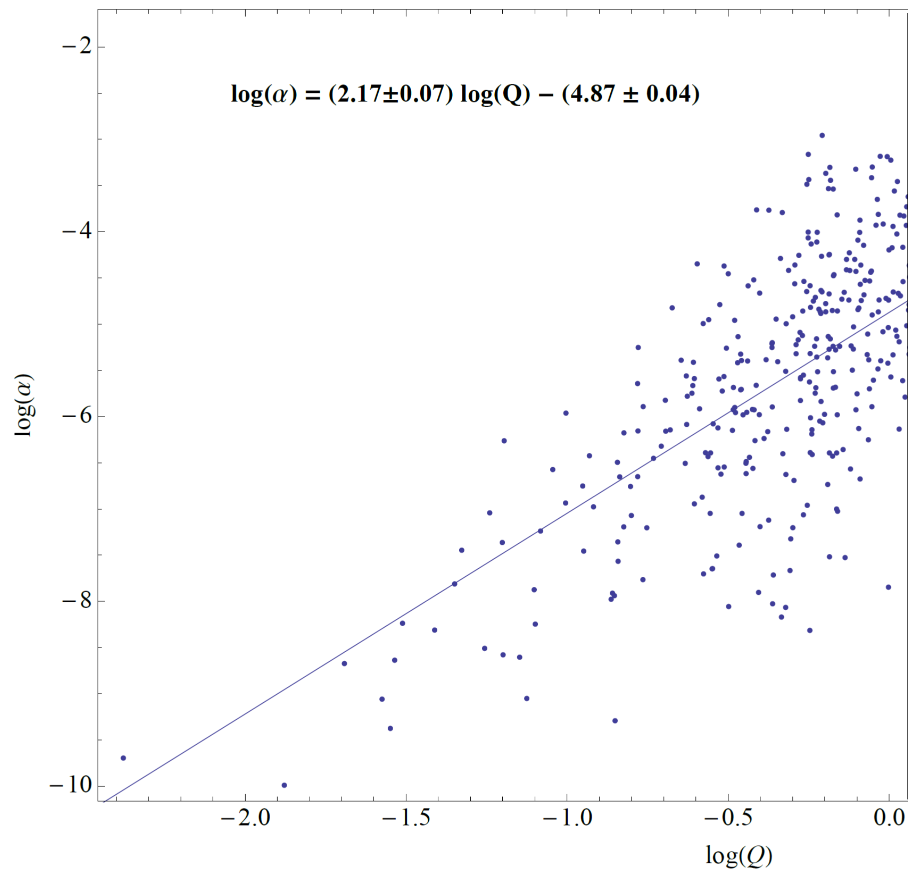
Very briefly

- Modern numerical simulations confirm the absence of the magnetodipole radiation.
- Anomalous torque acting on the body depends on the angular momentum of the electromagnetic field inside the body. Hence, it depends on its internal structure.

One prediction

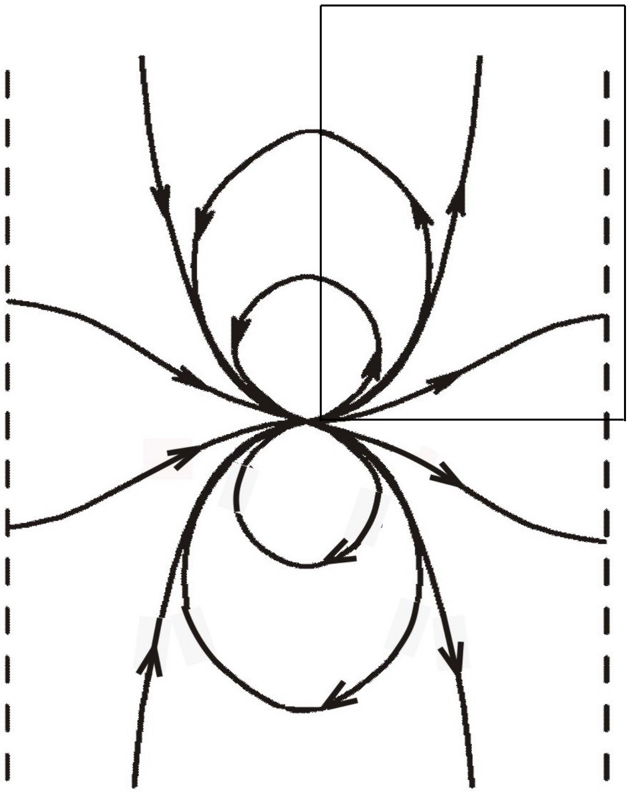
- $Q = 2\, P^{11/10}\, \dot{P}_{-15}^{\;-4/10}$
- $H/R_0 \sim Q\,, \quad r_{in}/R_0 \sim Q^{7/9}$
- $W_{part}/W_{tot} \sim Q^2$

$\alpha = L_{rad}/W_{tot}$



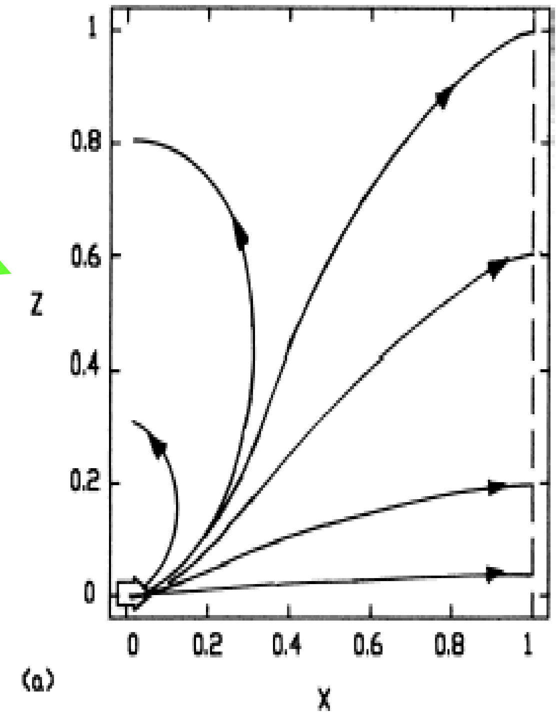
No magnetodipole radiation

Orthogonal rotator, $I = 0$



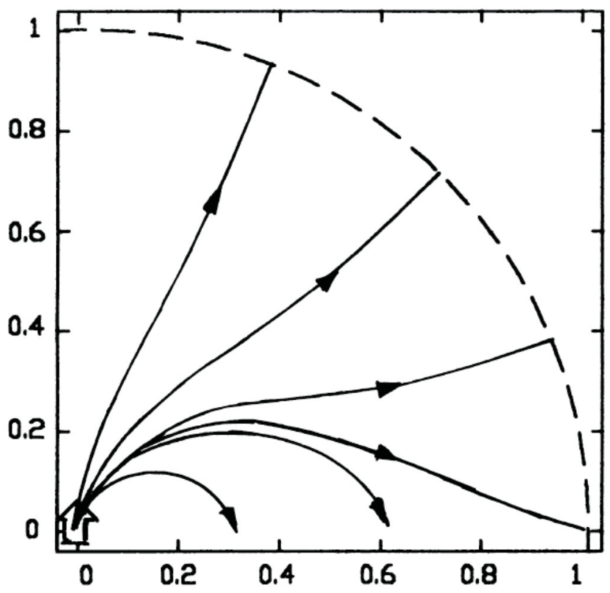
$\chi = 90^\circ$

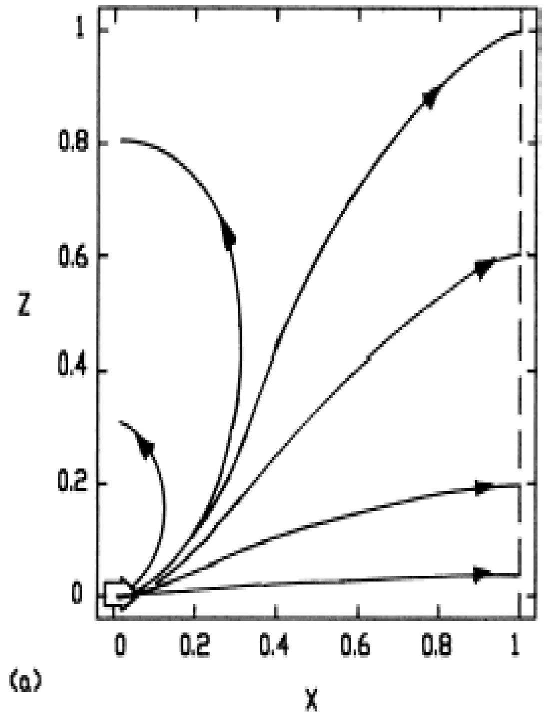
VB, A.V.Gurevich, Ya.N.Istomin, JETP, **58**, 235 (1983)



L.Mestel, P.Panagi, S.Shibata, MNRAS, **309**, 388 (1999)

Orthogonal rotator, $I = 0$

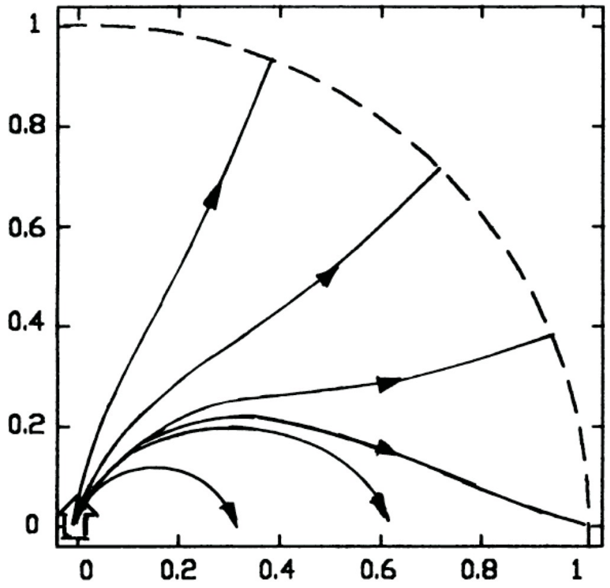




L.Mestel, P.Panagi, S.Shibata, MNRAS, **309**, 388 (1999)

NO magnetodipole radiation

VB, A.V.Gurevich, Ya.N.Istomin, JETP **58**, 235 (1983)

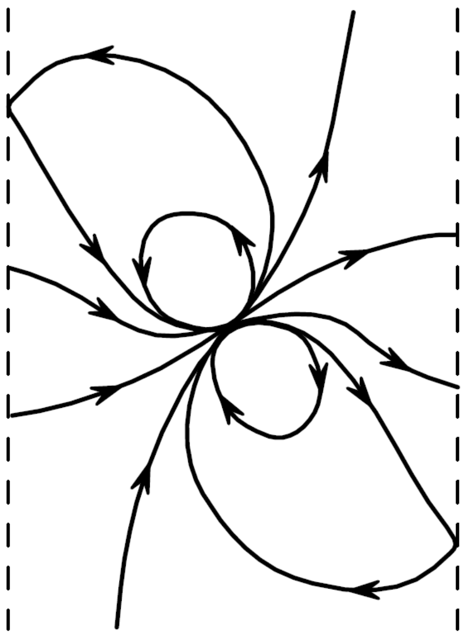


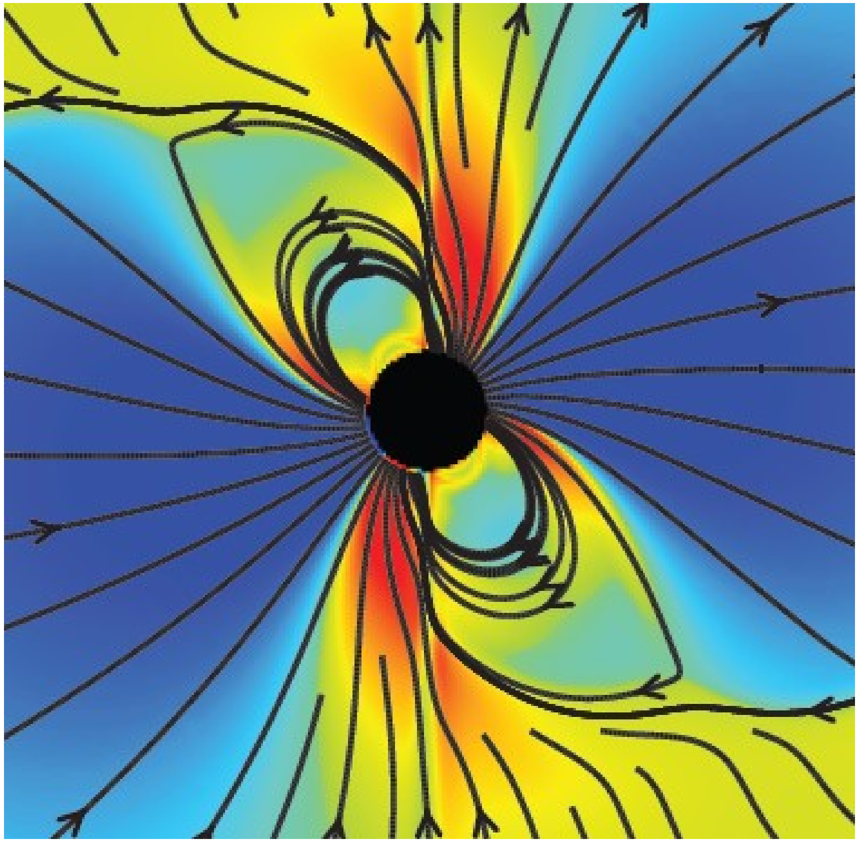
No toroidal magnetic field at the light cylinder (for zero longitudinal current)

– $$B_\varphi \propto (1 - x_r^2)^2$$

no electromagnetic flux through the light cylinder.

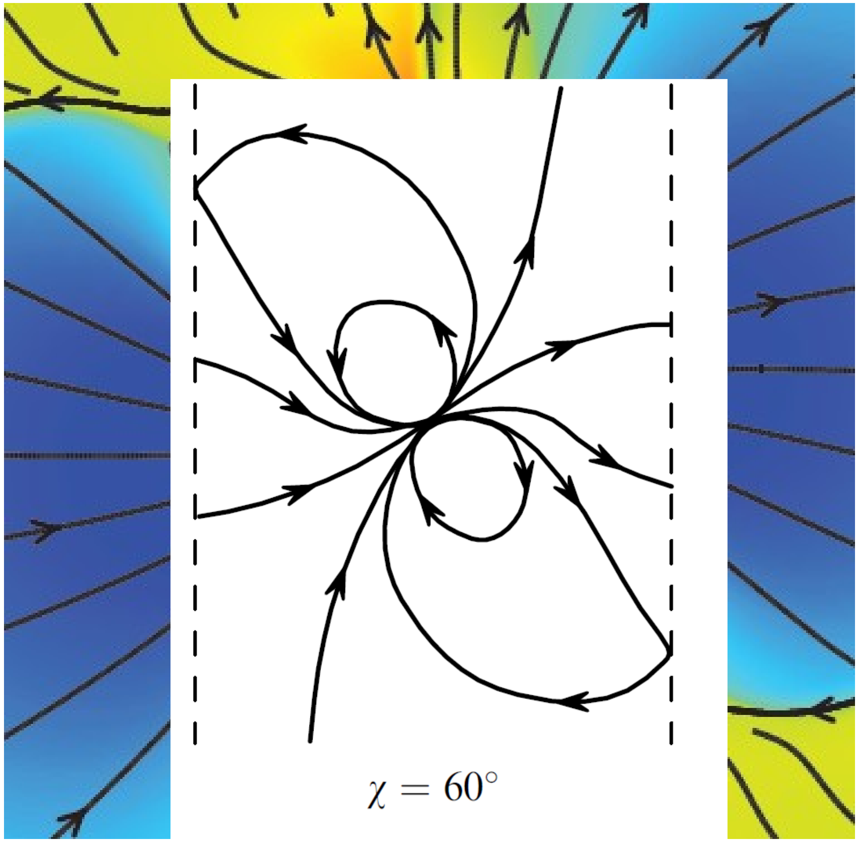
Incline rotator


$\chi = 60^\circ$



A.Tchekhovskoy,
A.Spitkovsky, J.Li,
arxiv.org/pdf/1211.2803.pdf

Incline rotator



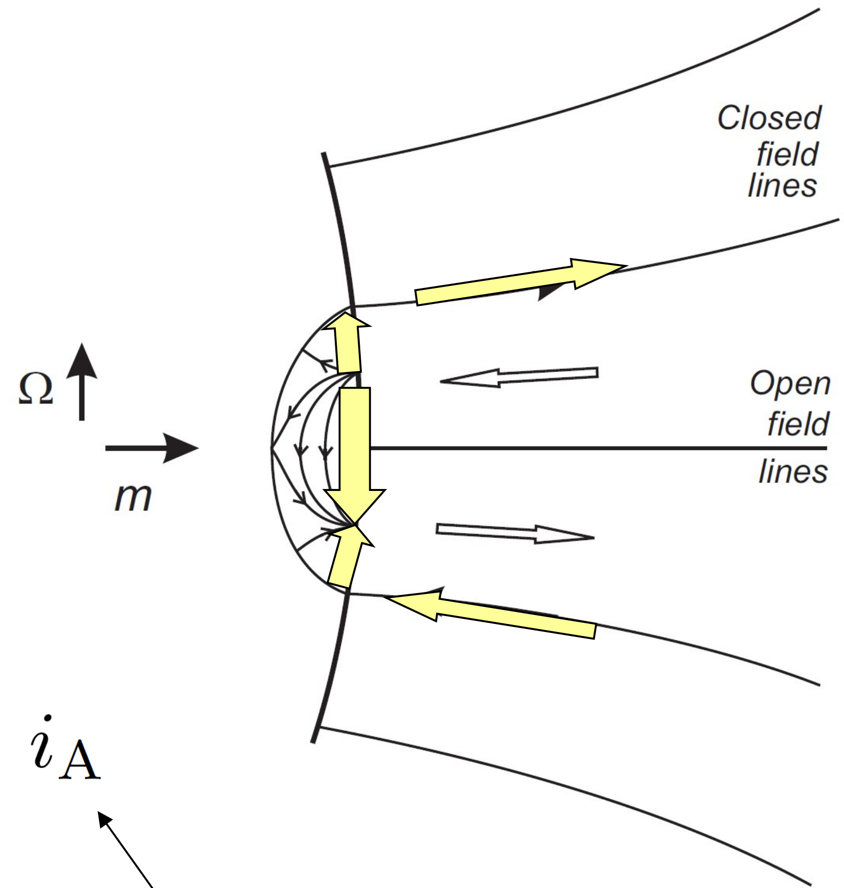
A.Tchekhovskoy,
A.Spitkovsky, J.Li,
arxiv.org/pdf/1211.2803.pdf

Orthogonal rotator

VB, A.V.Gurevich, Ya.N.Istomin, JETP **58**, 235 (1983)

$$j_{\rm GJ} \approx \frac{\Omega B}{2\pi} \cos\theta$$

$$\mathbf{K} = \frac{1}{c}\int [\mathbf{r} \times [\mathbf{J}_{\rm s} \times \mathbf{B}]]\mathrm{d}S$$



$$W_{\rm tot} = c_{\perp}\frac{B_0^2\Omega^4 R^6}{c^3}\left(\frac{\Omega R}{c}\right) i_{\rm A}$$

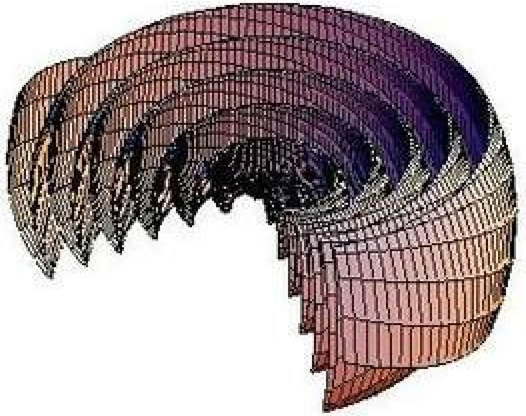
$i_{\rm A} = j_{||} / \rho_{\rm GJ} c$

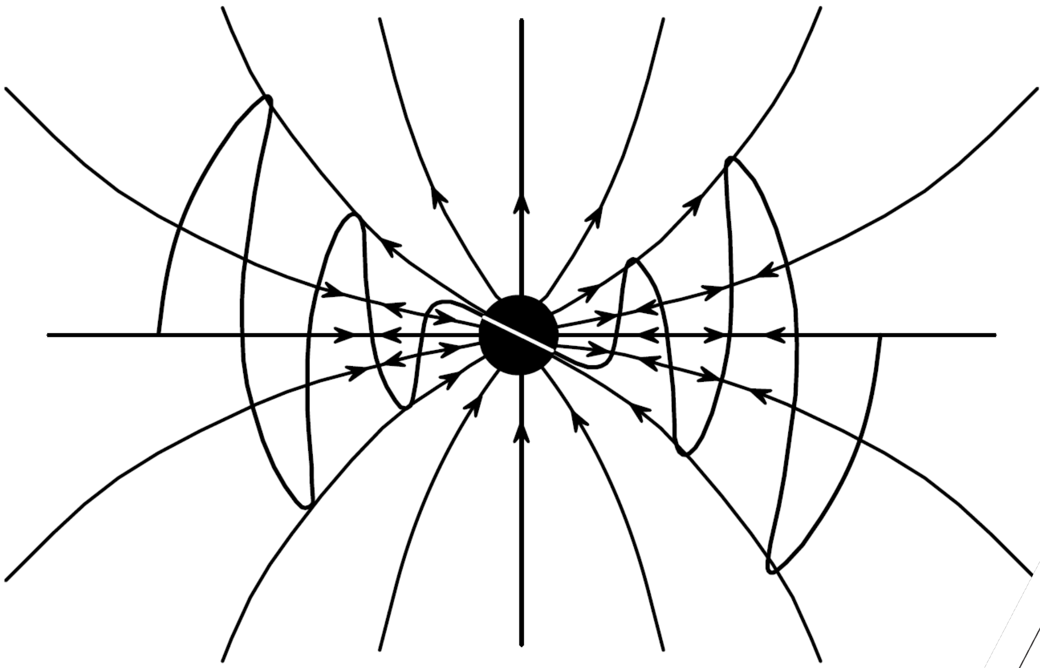
BGI – main results

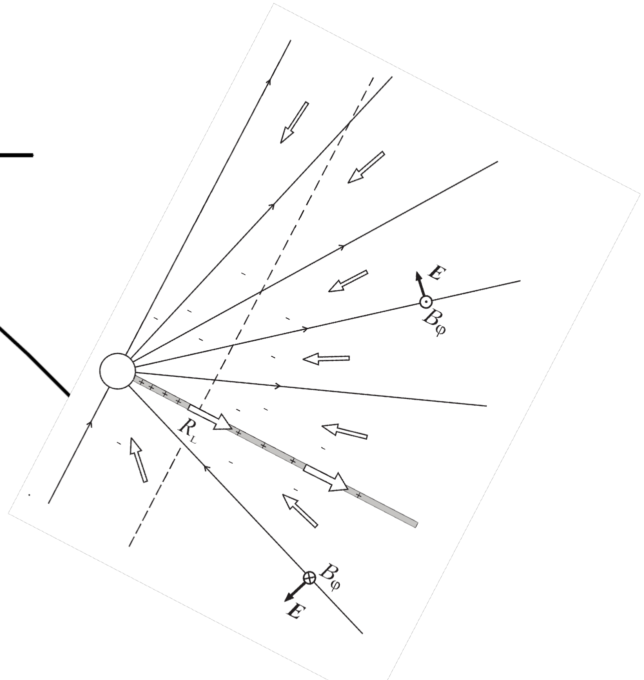
- No magnetodipole radiation
- **<u>All</u>** energy losses are to be connected with current losses
- Smaller energy losses for orthogonal rotator
- Inclination angle evolves to 90 deg.
- Existence of the back electric current along the separatrix

Orthogonal rotator – inclined split monopole

S.V.Bogovalov, A&A, **349**, 1017 (1999)

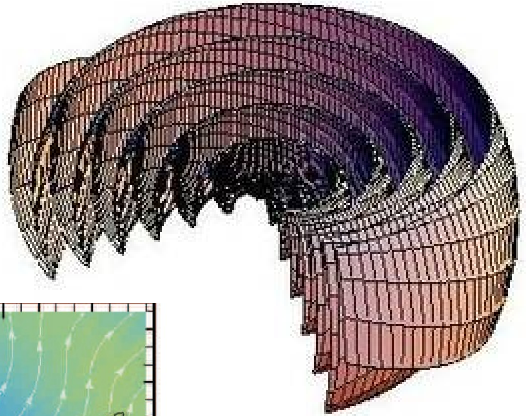


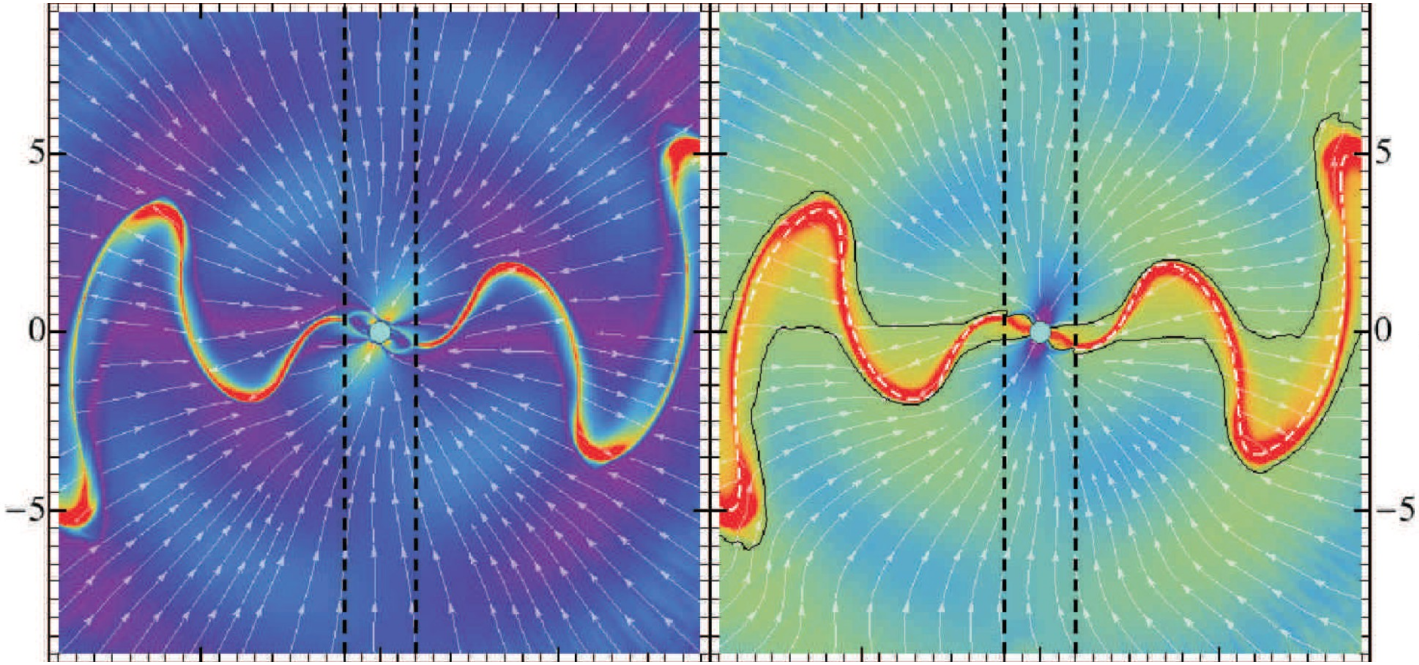




Orthogonal rotator – numerical

I.Contopoulos et al, (2012)





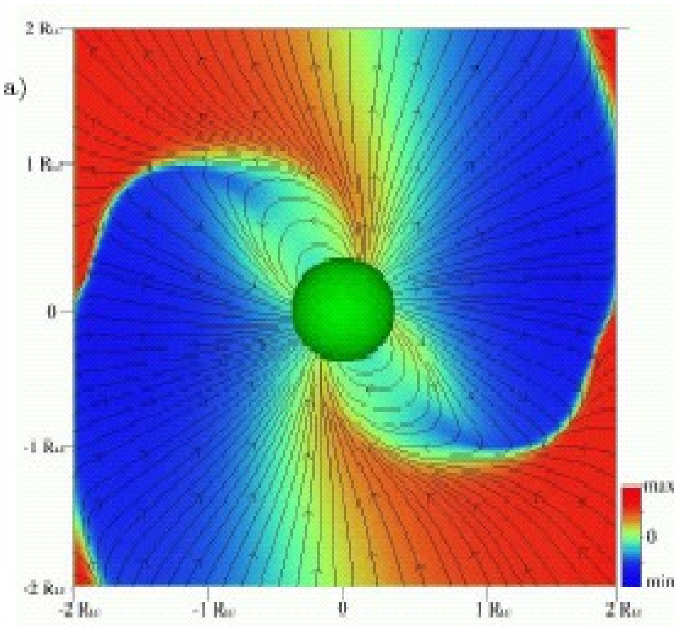
$$\Phi = \sin\alpha \sin\theta \sin(\varphi - \Omega t + \Omega r/c) + \cos\theta \cos\alpha$$

Split monopole – main results

- No magnetodipole radiation
- Energy losses do not depend on the inclination angle
- Existence of the current sheet separating magnetic fluxes
- No information about the inclination angle evolution

Orthogonal rotator – numerical

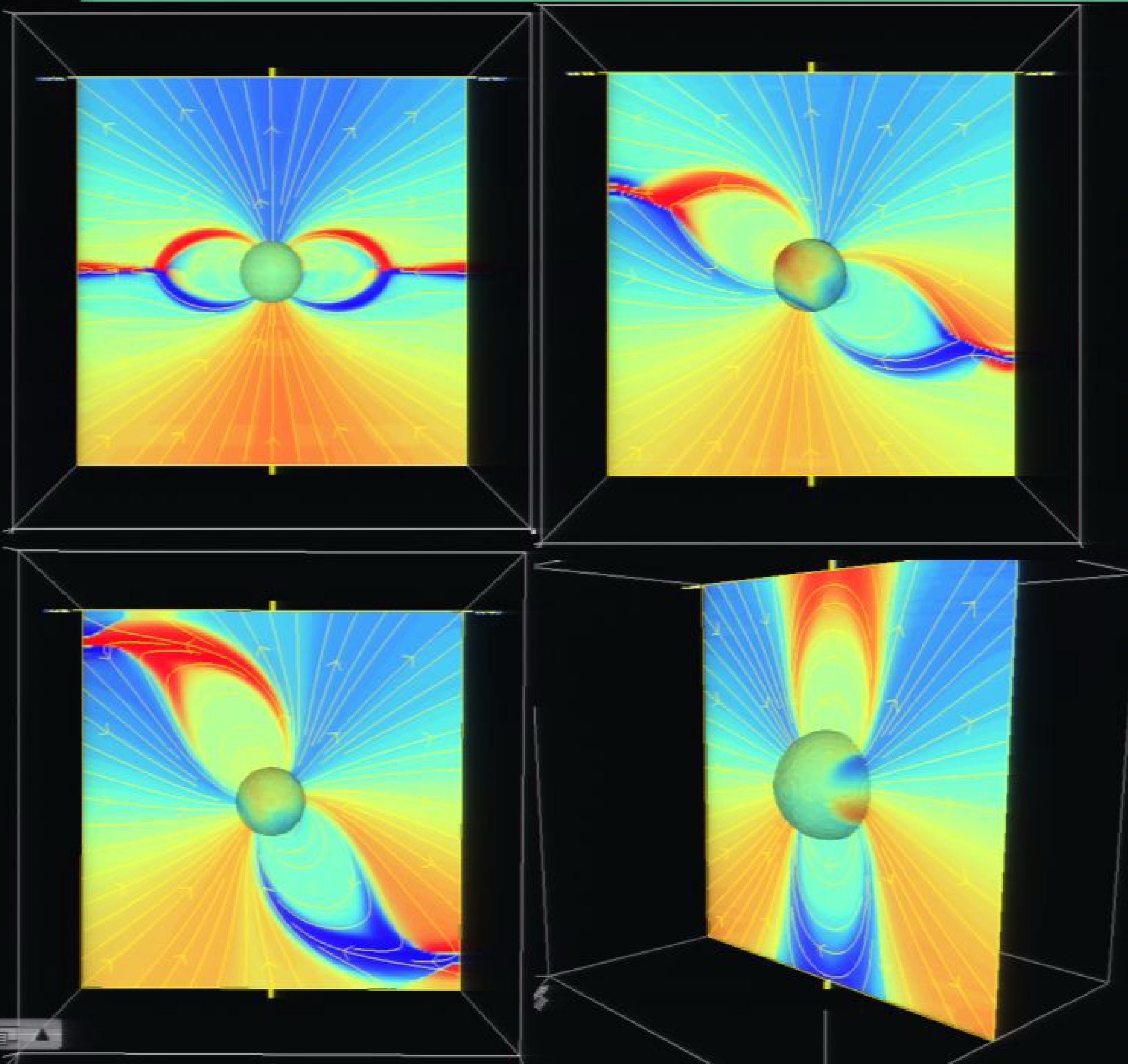
A.Spitkovsky, ApJ Lett., **648**, L51 (2006)



$$W_{tot} \approx \frac{1}{4}\frac{B_0^2\Omega^4 R^6}{c^3}\left(1+\sin^2\chi\right)$$

- No magnetodipole radiation
- Back electric current along the separatrix
- Smaller energy losses for axisymmetric rotator

Magnetospheric currents

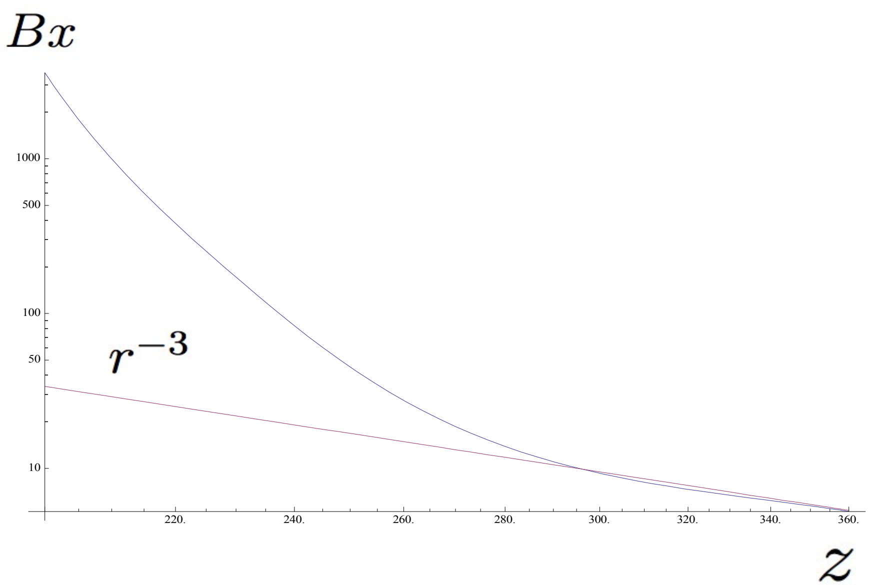


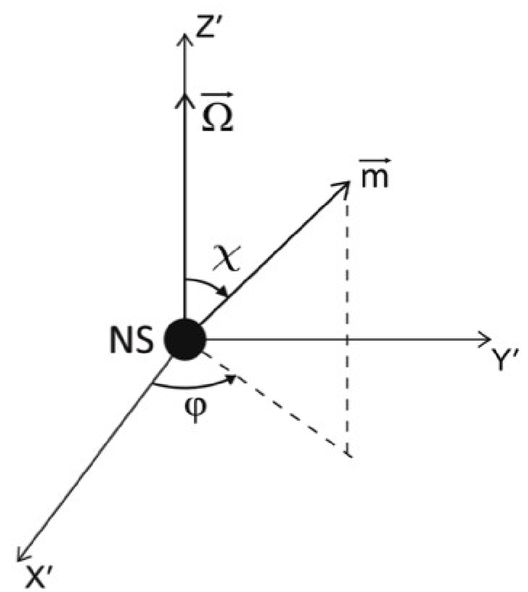
Oppositely flowing currents can occupy the same open flux tube. Does this have any obervational implications?

There is always a null-current field line in the open zone.

Spitkovsky solution, $\chi = 60^{o}$

No magnetodipole radiation





In vacuum $B_x = \dfrac{\ddot{d}}{cr}$

VB, YA.N.Istomin, A.A.Philippov, Phys. Uspekhi, **56**, 164 (2013)

Current losses again

$$K_{\parallel} = -\frac{B_0^2\Omega^3 R^6}{c^3}\left[c_{\parallel} i_{\rm S} + \mu_{\parallel}\left(\frac{\Omega R}{c}\right)^{1/2} i_{\rm A}\right],$$

$$K_{\perp} = -\frac{B_0^2\Omega^3 R^6}{c^3}\left[\mu_{\perp}\left(\frac{\Omega R}{c}\right)^{1/2} i_{\rm S} + c_{\perp}\left(\frac{\Omega R}{c}\right) i_{\rm A}\right]$$

$$J_r\frac{{\rm d}\Omega}{{\rm d}t} = K_{\parallel}\cos\chi + K_{\perp}\sin\chi,$$

$$J_r\Omega\frac{{\rm d}\chi}{{\rm d}t} = K_{\perp}\cos\chi - K_{\parallel}\sin\chi.$$

VB, A.V.Gurevich, Ya.N.Istomin, JETP **58**, 235 (1983)

Current losses again

$$
\begin{aligned}
K_{\|} &= -\frac{B_0^2 \Omega^3 R^6}{c^3}\left[c_{\|} i_{\mathrm{S}} + \mu_{\|}\left(\frac{\Omega R}{c}\right)^{1/2} i_{\mathrm{A}}\right], \\
K_{\perp} &= -\frac{B_0^2 \Omega^3 R^6}{c^3}\left[\mu_{\perp}\left(\frac{\Omega R}{c}\right)^{1/2} i_{\mathrm{S}} + c_{\perp}\left(\frac{\Omega R}{c}\right) i_{\mathrm{A}}\right]
\end{aligned}
$$

$$
\begin{aligned}
J_r \frac{\mathrm{d}\Omega}{\mathrm{d}t} &= K_{\|}^{(A)} + \left[K_{\perp}^{(A)} - K_{\|}^{(A)}\right]\sin^2\chi, \\
J_r \Omega \frac{\mathrm{d}\chi}{\mathrm{d}t} &= \left[K_{\perp}^{(A)} - K_{\|}^{(A)}\right]\sin\chi\cos\chi.
\end{aligned}
$$

A.Philippov, A.Tchekhovskoy, J.Li

$$
\boxed{K_{\perp}^{(A)} = \left(\frac{\Omega R}{c}\right) i_{\mathrm{A}}}
$$

Main problem

- One-to-one correspondence

  χ evolves to 90 deg. if $W_{tot}(0) > W_{tot}(90)$

  χ evolves to 0 deg. if $W_{tot}(0) < W_{tot}(90)$

- For Spitkovsky et al low the asymmetrical current is to be (much) larger than GJ one

$$i_A > (\Omega R/c)^{-1}$$

Main problems

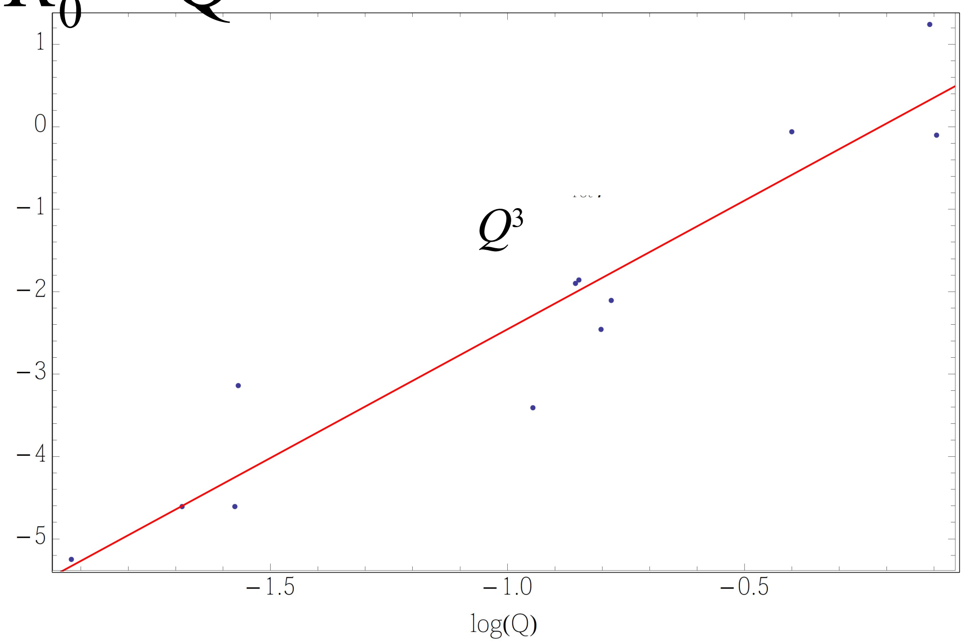
- For orthogonal rotator the longitudinal current is to be 10000 (!!!) times larger than local GJ one
- Heating problem.
- Magnetic field disturbance.

Heating problem

- $Q = 2\, P^{11/10}\, \dot{P}_{-15}^{-4/10}$

- $H/R_0 \sim Q\,, \quad r_{in}/R_0 \sim Q^{7/9}$

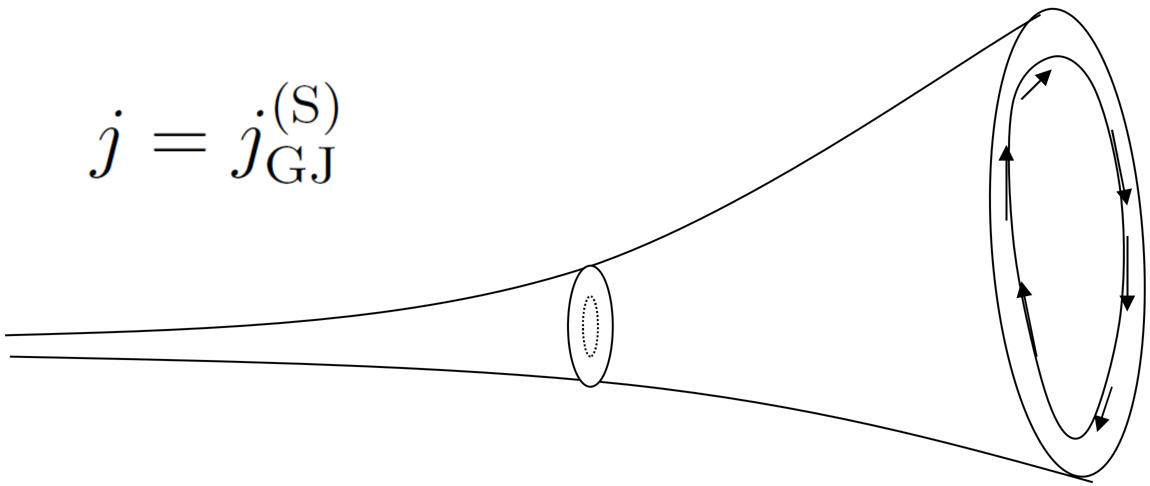
$L_X/W_{tot} \sim$ (



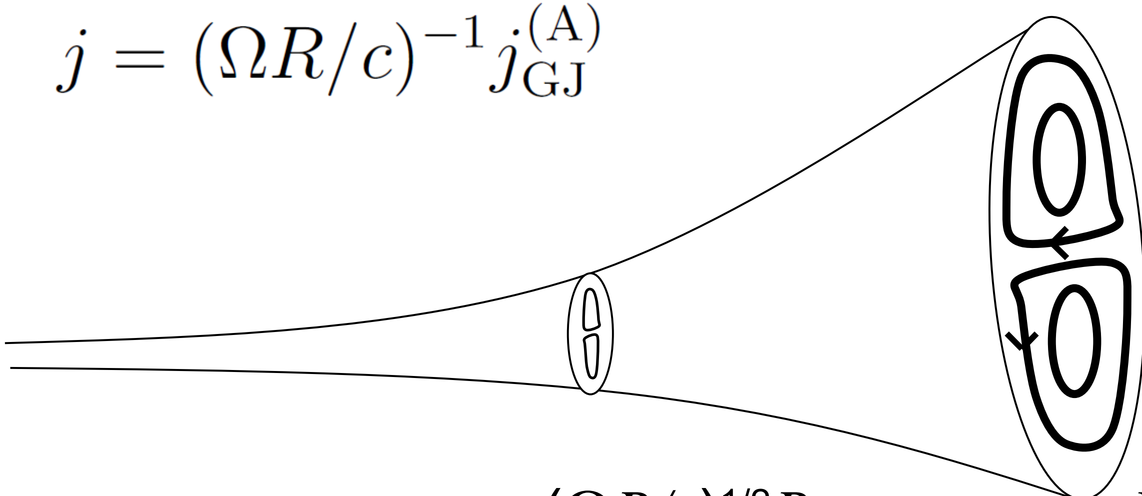
Istomin & Sobyanin (2010), Timokhin (2010), Timokhin & Arons (2013)

Magnetic field disturbance

$$j = j_{\mathrm{GJ}}^{(\mathrm{S})}$$



$$j = (\Omega R/c)^{-1} j_{\mathrm{GJ}}^{(\mathrm{A})}$$



Light surface

The longitudinal current is to be much larger than local GJ one.

If not, the light surface $|\boldsymbol{E}| = |\boldsymbol{B}|$ is to exist.

Prediction

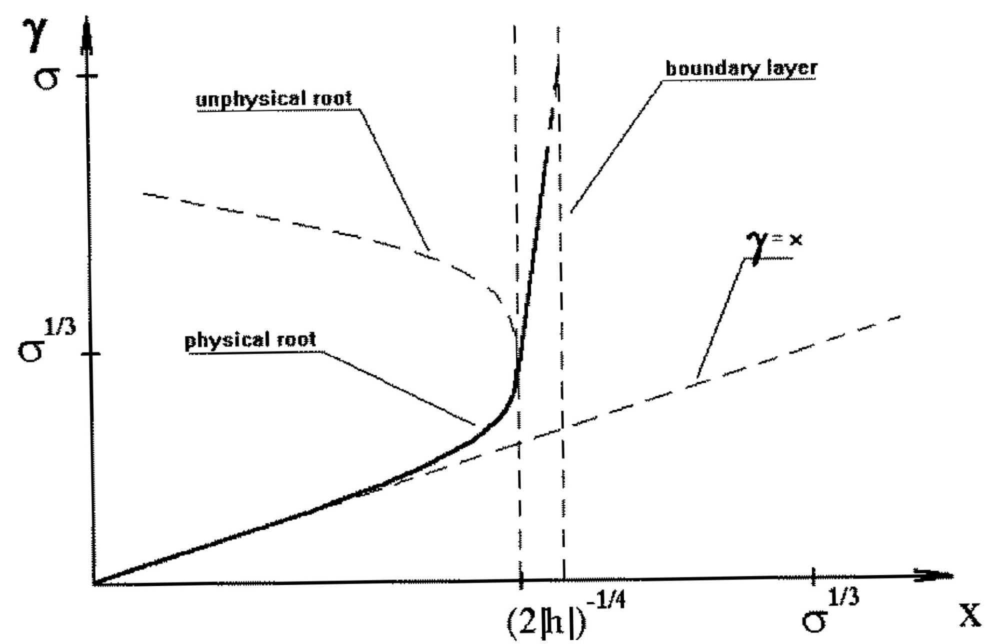
VB, A.V.Gurevich & Ya.N.Istomin, JETP, **85,** 235 (1983)
VB, R.R.Rafikov, MNRAS, **313**, 433 (2000)

- Narrow sheet $\delta r \sim R_L/\lambda$
- Effective particle acceleration up to $\gamma \sim \sigma$ (10^6 for Crab)

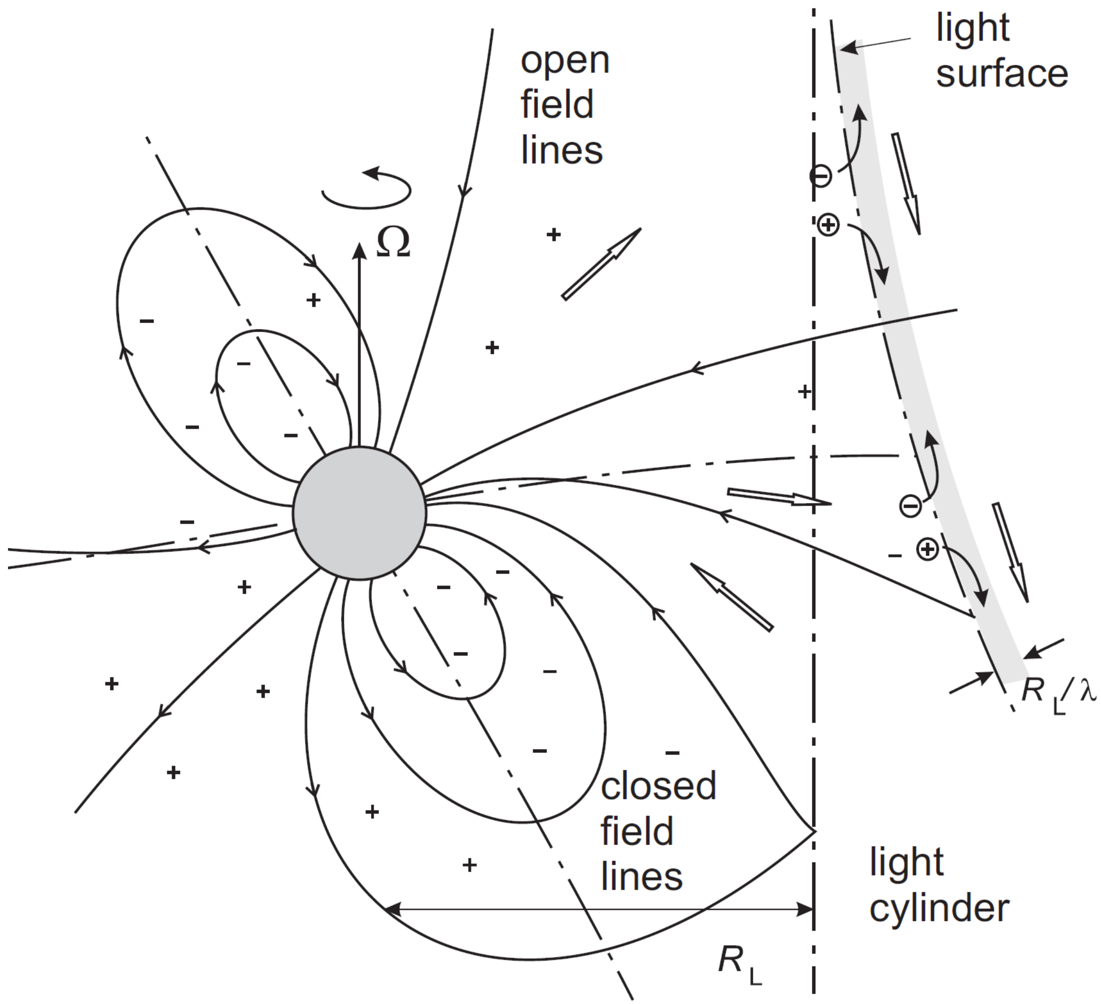
$$\lambda = \frac{n^{(\mathrm{lab})}}{n_{\mathrm{GJ}}}$$

$$\sigma = \frac{\Omega^2 \Psi_{\mathrm{tot}}}{8\pi^2 c^2 \mu \eta}$$

maximum bulk Lorentz-factor



Prediction_



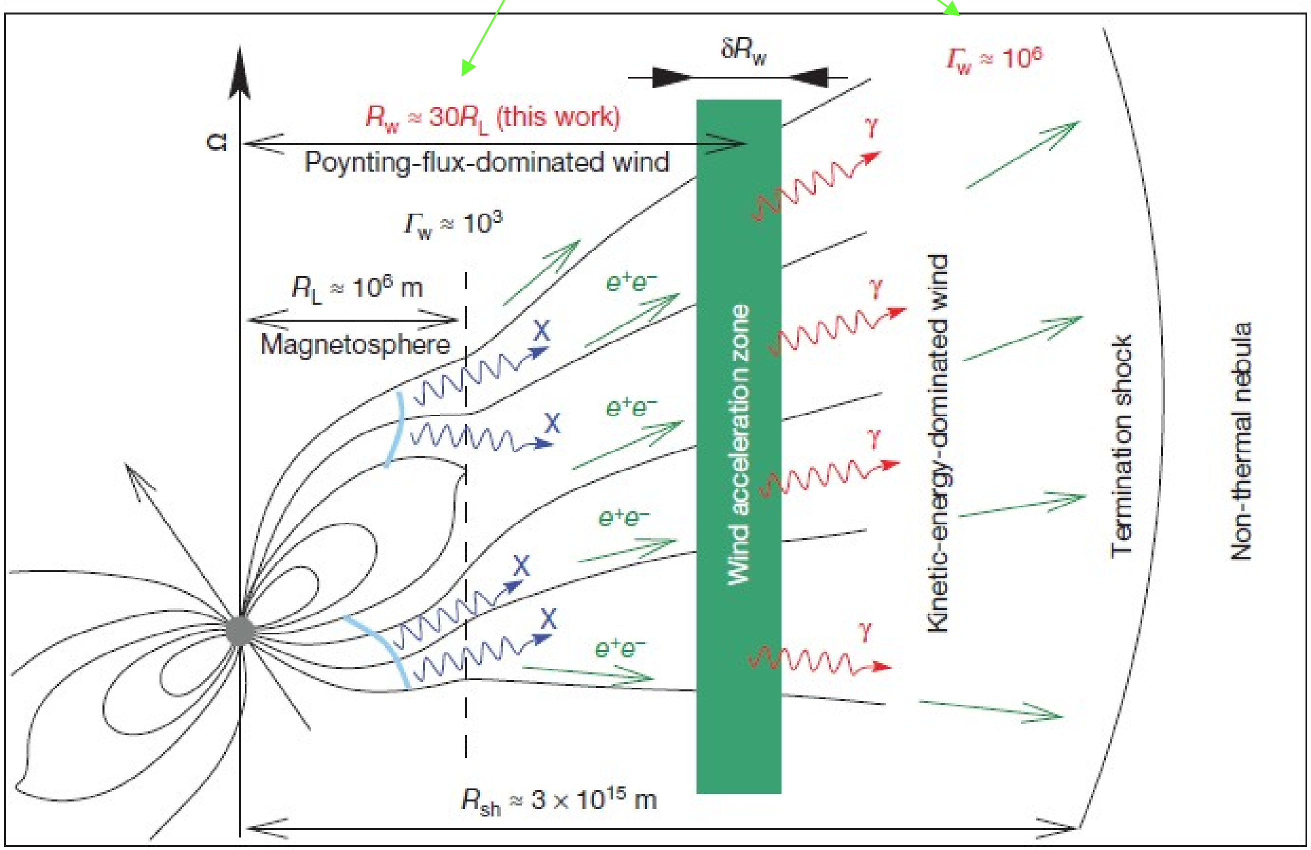
open
field
lines
light
surface
Ω
R_L/λ
closed
field
lines
light
cylinder
R_L

LETTER

doi:10.1038/nature10793

Abrupt acceleration of a 'cold' ultrarelativistic wind from the Crab pulsar

F. A. Aharonian[1,2], S. V. Bogovalov[3] & D. Khangulyan[4]



A problem

Can the pair creation process generate large enough longitudinal current to avoid the formation of the light surface?

The main point of view – YES.

Our point of view – NO.

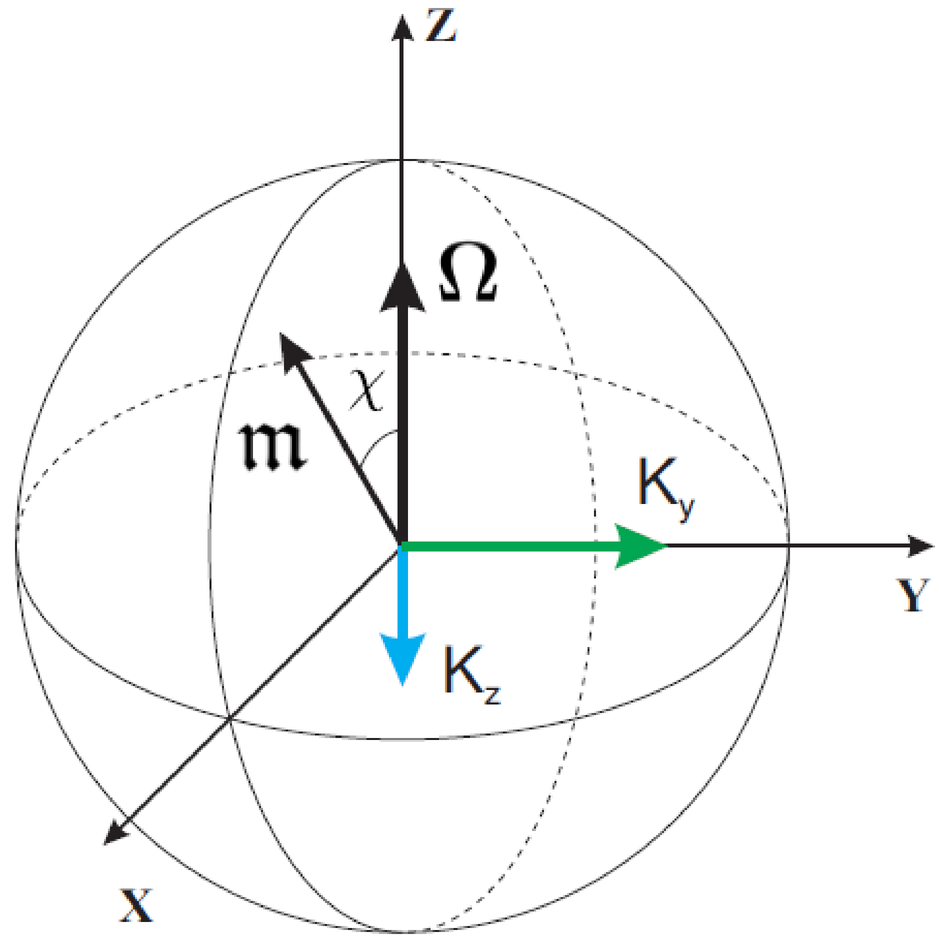
A problem

Can the pair creation process generate large enough longitudinal current to avoid the formation of the light surface?

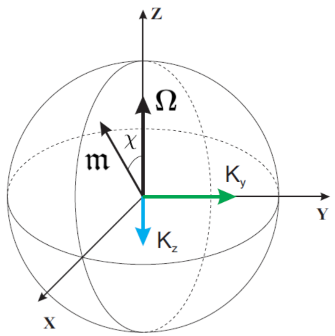
The main point of view – YES.

Our point of view – NOT (so clear).

Anomalous torque



Anomalous torque



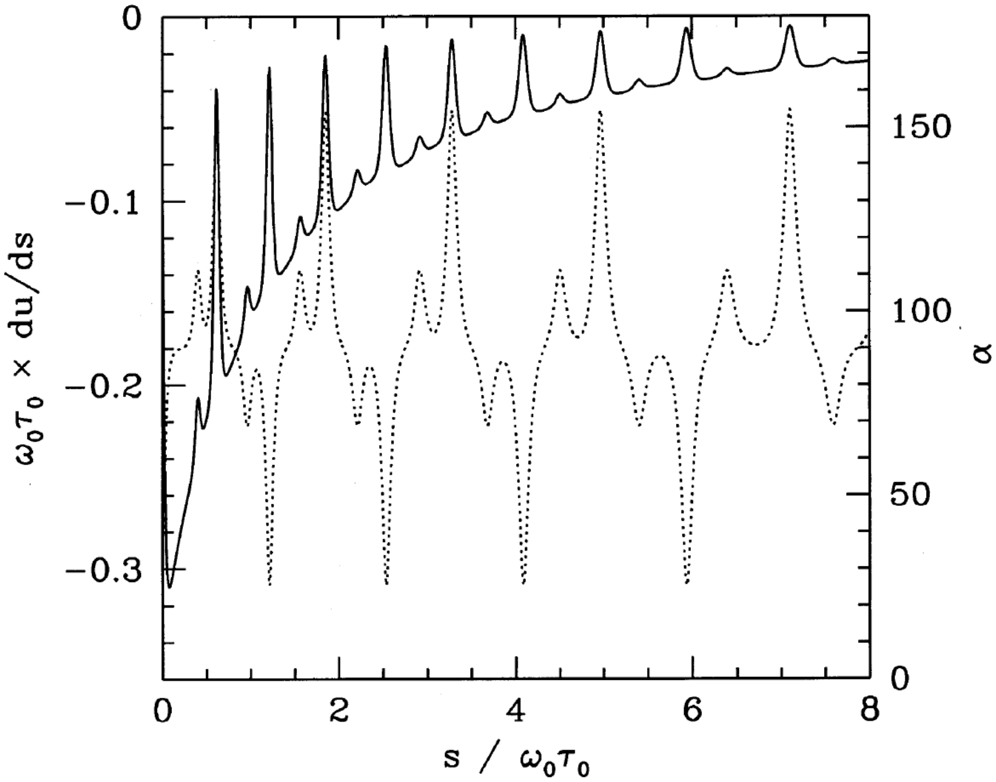
$$K_{z'} = \frac{2}{3}\frac{\mathfrak{m}^2}{R^3}\left(\frac{\Omega R}{c}\right)^3 \sin^2\chi$$

$$K_{x'} = \frac{2}{3}\frac{\mathfrak{m}^2}{R^3}\left(\frac{\Omega R}{c}\right)^3 \sin\chi\cos\chi$$

$$\boxed{K_{y'} = \xi\frac{\mathfrak{m}^2}{R^3}\left(\frac{\Omega R}{c}\right)^2 \sin\chi\cos\chi}$$

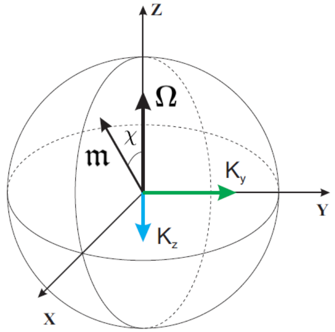
$\xi = 1$ (Davis & Goldstein 1970, Goldreich 1970)
$\xi = 1/5$ (Good&Ng 1985)
$\xi = 0$ (Michel 1991, Istomin 2005)
$\xi = 3/5$ (Melatos 2000)

Evolution



A.Melatos, MNRAS, **313**, 217 (2000),
D.Barsukov & A.Tsygan, MNRAS, **409**, 1077 (2010),
G.Beskin, A.Biryukov, S.Karpov, MNRAS,**420**, 103 (2012)

Anomalous torque



Main assumptions

- Rotation in vacuum
- Freezing-in condition inside the body

$$\mathbf{E} + \beta_\mathrm{R} \times \mathbf{B} = 0$$

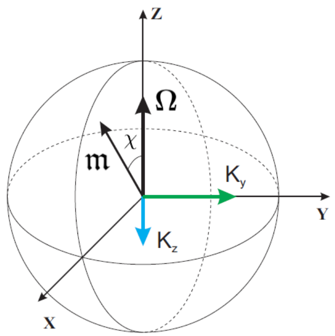
$$\beta_\mathrm{R} = \mathbf{\Omega} \times \mathbf{r}/c$$

- Corotation currents $\mathbf{j} = c\rho_\mathrm{e}\beta_\mathrm{R}$ only (hence, zero volume force)

$$\mathrm{d}\mathbf{F} = \rho_\mathrm{e}\mathbf{E}\,\mathrm{d}V + [\mathbf{j} \times \mathbf{B}]/c\,\mathrm{d}V$$

- Spherical body

Anomalous torque



Basic equation

$$\frac{\mathrm{d}\,\mathbf{L}_{\text{field}}}{\mathrm{d}t} + \mathbf{K}^M + \int [\mathbf{r} \times \mathbf{F}]\mathrm{d}V = 0$$

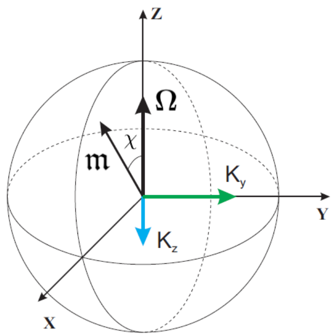
Electromagnetic angular momentum

$$\mathbf{L}_{\text{field}} = \int \frac{[\mathbf{r} \times [\mathbf{E} \times \mathbf{B}]]}{4\pi c}\mathrm{d}V$$

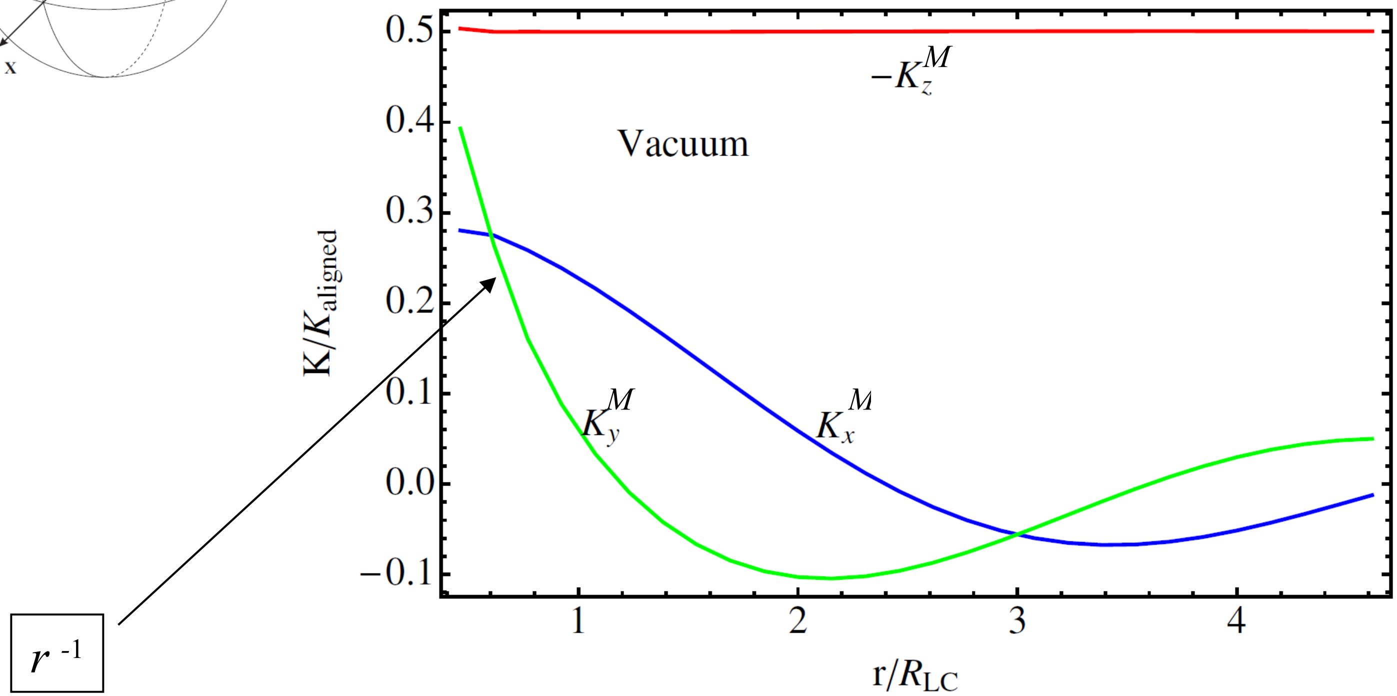
Flux $K_i^M = -\int \varepsilon_{ijk} r_j T_{kl} \mathrm{d}S_l$ (spherical surface)

$$\mathbf{K}_{y'}^M = \frac{R^3}{4\pi} \int \Big([\mathbf{n} \times \mathbf{B}]_{y'} (\mathbf{B}\mathbf{n}) + [\mathbf{n} \times \mathbf{E}]_{y'} (\mathbf{E}\mathbf{n}) \Big) \mathrm{d}o$$

Anomalous torque

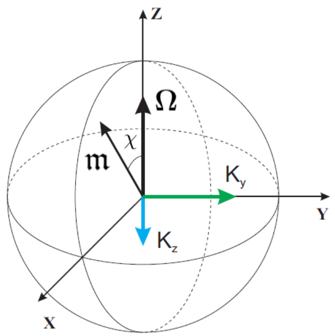


A.Philippov, A.Tchekhovskoy, J.Li,
http://arxiv.org/abs/1311.1513



$$\frac{\mathrm{d}\,\mathbf{L}_{\rm field}}{\mathrm{d}t} + \mathbf{K}^M + \cancel{\int [\mathbf{r} \times \mathbf{F}]\mathrm{d}V} = 0$$

Anomalous torque



Torque, acting on the body

$$\frac{\mathrm{d}\,\mathbf{L}_{\mathrm{mat}}}{\mathrm{d}t} = \int [\mathbf{r} \times \mathbf{F}]\mathrm{d}V$$

$$\mathrm{d}\mathbf{F} = \sigma_{\mathrm{e}}\mathbf{E}\,\mathrm{d}S + [\mathbf{I}_{\mathrm{S}} \times \mathbf{B}]/c\,\mathrm{d}S$$

$$\boxed{\mathbf{K} = \int \mathbf{r} \times \mathrm{d}\mathbf{F} = \frac{R^3}{4\pi}\int \Big([\mathbf{n} \times \{\mathbf{B}\}]\,(\mathbf{B}\mathbf{n}) + [\mathbf{n} \times \mathbf{E}]\,(\{\mathbf{E}\}\,\mathbf{n})\Big)\mathrm{d}o}$$

- for spherical body
- for highly conducting interior
- for corotation currents only

Anomalous torque

Calculations

- small parameter $\varepsilon = \dfrac{\Omega R}{c}$
- quasi-stationary $\varphi - \Omega t$

$$\begin{aligned}
\nabla \times (\mathbf{E} + \beta_{\mathrm{R}} \times \mathbf{B}) &= 0, \\
\nabla \times (\mathbf{B} - \beta_{\mathrm{R}} \times \mathbf{E}) &= \frac{4\pi}{c}\mathbf{j} - 4\pi\rho_e\beta_{\mathrm{R}}
\end{aligned}$$

$$\beta_{\mathrm{R}} = \boldsymbol{\Omega} \times \mathbf{r}/c$$

$$\mathbf{j} = c\rho_{\mathrm{e}}\beta_{\mathrm{R}}$$

$$\boxed{\begin{aligned}
\mathbf{E} + \beta_{\mathrm{R}} \times \mathbf{B} &= -\nabla\psi, \\
\mathbf{B} - \beta_{\mathrm{R}} \times \mathbf{E} &= \nabla h
\end{aligned}}$$

$$\longrightarrow \psi^{(1)} \longrightarrow E^{(1)}$$

$$\longrightarrow h^{(2)} \longrightarrow B^{(2)}$$

Anomalous torque

$$
\begin{aligned}
B_r^\perp &= \frac{|\mathfrak{m}|}{r^3}\sin\theta\,\mathrm{Re}\left(2-2i\frac{\Omega r}{c}\right)\exp\left(i\frac{\Omega r}{c}+i\varphi-i\Omega t\right),\\
B_\theta^\perp &= \frac{|\mathfrak{m}|}{r^3}\cos\theta\,\mathrm{Re}\left(-1+i\frac{\Omega r}{c}+\frac{\Omega^2 r^2}{c^2}\right)\exp\left(i\frac{\Omega r}{c}+i\varphi-i\Omega t\right),\\
B_\varphi^\perp &= \frac{|\mathfrak{m}|}{r^3}\,\mathrm{Re}\left(-i-\frac{\Omega r}{c}+i\frac{\Omega^2 r^2}{c^2}\right)\exp\left(i\frac{\Omega r}{c}+i\varphi-i\Omega t\right),\\
E_r^\perp &= 0,\\
E_\theta^\perp &= \frac{|\mathfrak{m}|\Omega}{r^2 c}\,\mathrm{Re}\left(-1+i\frac{\Omega r}{c}\right)\exp\left(i\frac{\Omega r}{c}+i\varphi-i\Omega t\right),\\
E_\varphi^\perp &= \frac{|\mathfrak{m}|\Omega}{r^2 c}\cos\theta\,\mathrm{Re}\left(-i-\frac{\Omega r}{c}\right)\exp\left(i\frac{\Omega r}{c}+i\varphi-i\Omega t\right).
\end{aligned}
$$

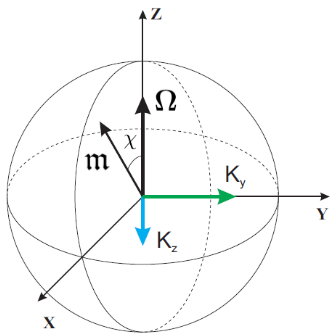
Anomalous torque

$$\begin{aligned} \mathbf{E} + \beta_{\mathrm{R}} \times \mathbf{B} &= -\nabla\psi, \\ \mathbf{B} - \beta_{\mathrm{R}} \times \mathbf{E} &= \nabla h \end{aligned}$$

Calculations

- Freezing-in condition $\psi^{(\mathrm{In})} = 0$
- Hence, no freedom in $E^{(1)}$
- Exactly corresponds to vacuum solution
- Freedom in $B^{(2)}$(free functions $h^{(2)}$)
- Result DOES NOT depend on this freedom

Anomalous torque



Torque, acting on the body

$$\mathbf{K} = \int \mathbf{r} \times \mathrm{d}\mathbf{F} = \frac{R^3}{4\pi} \int \Big([\mathbf{n} \times \{\mathbf{B}\}] (\mathbf{B}\mathbf{n}) + [\mathbf{n} \times \mathbf{E}] (\{\mathbf{E}\}\, \mathbf{n}) \Big) \mathrm{d}o$$

$$\boxed{K = K^M(R+0) - K^M(R-0)}$$

$$\xi = \frac{3}{5}$$

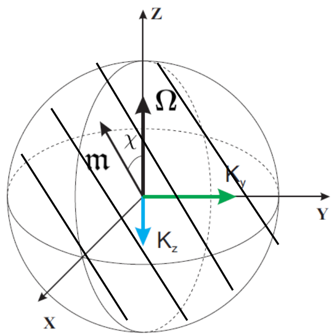
$$\mathbf{K}^M = -\frac{\mathrm{d}\,\mathbf{L}_{\mathrm{field}}}{\mathrm{d}t}$$

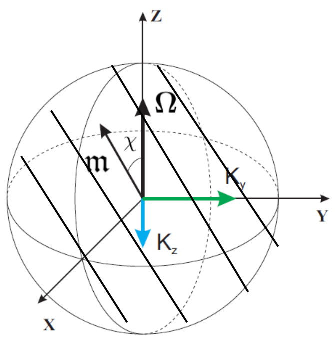
$$\mathbf{L}_{\mathrm{field}} = \int \frac{[\mathbf{r} \times [\mathbf{E} \times \mathbf{B}]]}{4\pi c} \mathrm{d}V$$

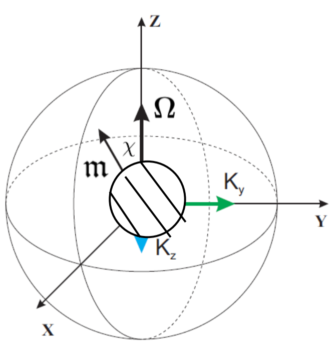
- Does not depend on the internal structure and freedom in $h^{(2)}$
- Obtained by A.Melatos (2000) for Deutsch solution

Depends on the internal structure

Anomalous torque







Three cases

- Homogeneously magnetized sphere

$$K_{y'} = \frac{1}{3}\frac{\mathfrak{m}^2}{R^3}\left(\frac{\Omega R}{c}\right)^2 \sin\chi\cos\chi$$

- Homogeneously magnetized hollow envelope

$$K_{y'} = \frac{31}{45}\frac{\mathfrak{m}^2}{R^3}\left(\frac{\Omega R}{c}\right)^2 \sin\chi\cos\chi$$

- Homogeneously magnetized core $R_{\text{in}} < R$

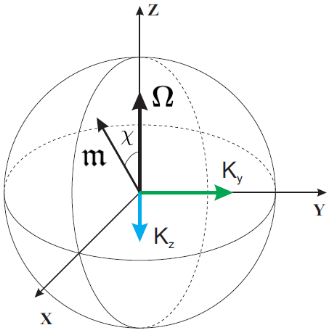
$$K_{y'} = \left(\frac{8}{15} - \frac{1}{5}\frac{R}{R_{\text{in}}}\right)\frac{\mathfrak{m}^2}{R^3}\left(\frac{\Omega R}{c}\right)^2 \sin\chi\cos\chi$$

Comparison with previous results

$\xi = 1$ (Davis & Goldstein 1970, Goldreich 1970)
$\xi = 1/5$ (Good&Ng 1985)
$\xi = 0$ (Michel 1991, Istomin 2005)
$\xi = 3/5$ (Melatos 2000)

$\xi = 1$ – electric current only
$\xi = 0$ – outer sphere only
$\xi = 3/5$ – stress for $r = R+0$

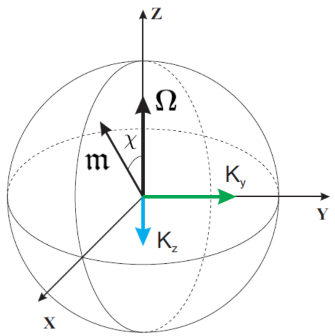
Anomalous torque



Main conclusion

Anomalous torque acting on the body depends on the internal structure of the electromagnetic field inside the body:

Anomalous torque



More conclusions

- Nonzero angular momentum L_{field} of the electromagnetic field exist only in Ω^2 order.
- In Ω^3 order it vanishes. It implies no problem in calculating the braking torque.
- Hence, energy losses can be determined via the flux $K_i = -\int \varepsilon_{ijk} r_j T_{kl} \mathrm{d}S_l$