Nonthermal particle acceleration in relativistic plasma turbulence

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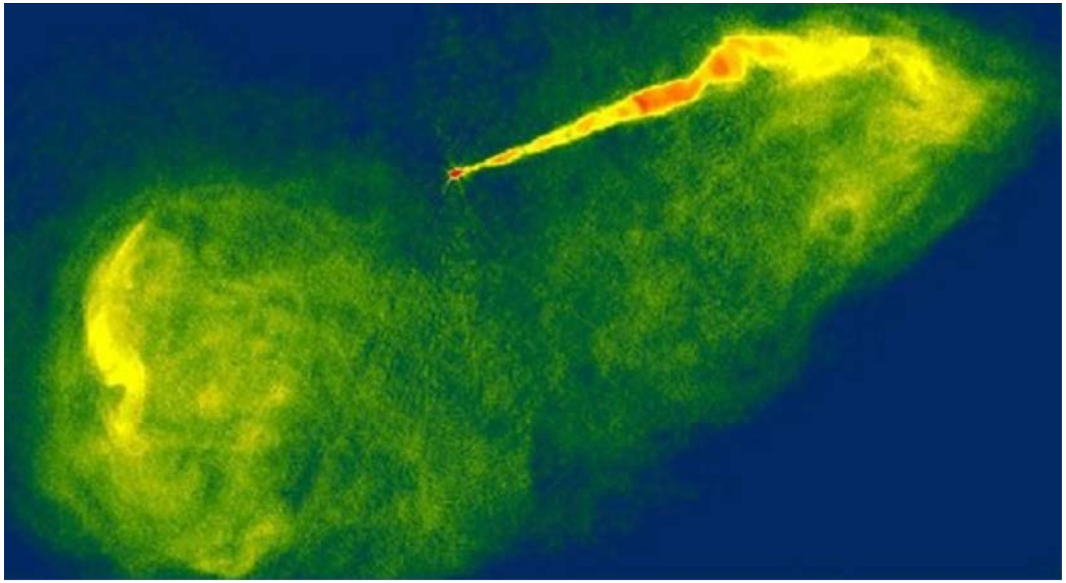
Purdue workshop, 5/7/2018

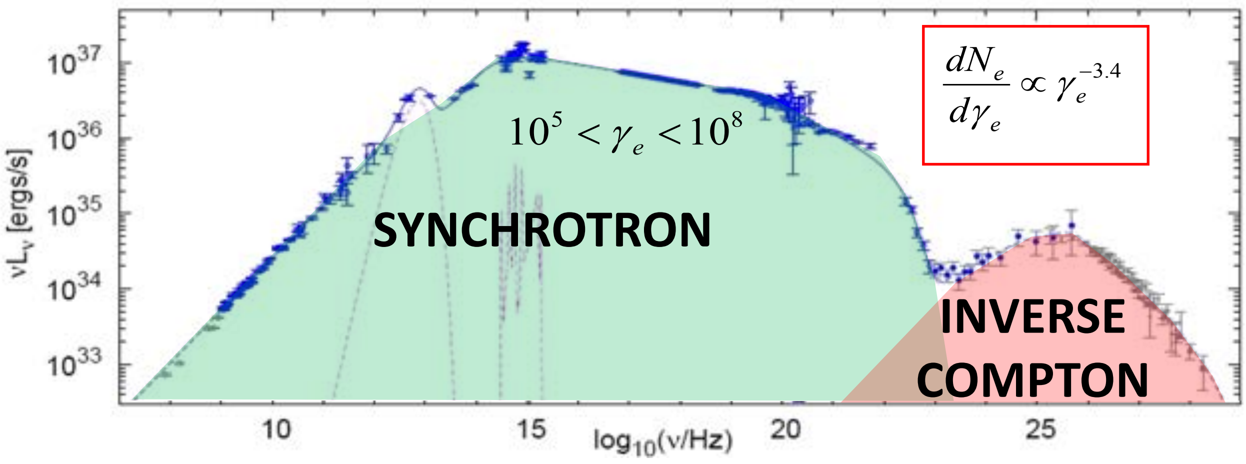


Relativistic, collisionless plasma turbulence

- Essentially unexplored regime of turbulence
- Ubiquitous in high-energy astrophysical systems (AGN, PWN, GRB, etc.)
- Prototypical problem for nonthermal particle acceleration
- Viable with first-principles particle-in-cell (PIC) simulations

1. What are the statistical properties of relativistic kinetic turbulence?
2. Is turbulence an efficient and viable particle accelerator?

Crab nebula spectrum (Meyer et al 2010)



Turbulent particle acceleration

- Many theories exist for turbulent particle acceleration in collisionless plasmas, but all require significant assumptions, few are self-consistent
  - First-order Fermi acceleration (converging flows)
  - Second-order Fermi acceleration (diffusive scattering by waves)
  - Shocks (in highly compressible case)
  - Magnetic reconnection in intermittent current sheets
- Past numerical studies are often in test particle limit, more complex geometries (e.g., instability-driven), or non-relativistic regime
- PIC codes open problem to first-principles examination
- Important questions include:
  - Is particle acceleration efficient (significant nonthermal population, hard distribution)?
  - Is particle acceleration viable (does it scale to large system sizes)?
  - What are mechanisms of acceleration?

Numerical simulations

- Driven turbulence with PIC code *Zeltron* (Cerutti+ 2013)
- Initialize thermal plasma, apply large-scale driving (TenBarge+ 2014)
- Relativistically hot pair plasma: $T_e/m_e c^2 \sim \bar{\gamma}/3 \sim 100$
- Uniform guide field: $B_0 \sim \delta B_{\rm rms}$
- Two physical parameters:

1) Magnetization (ratio of magnetic energy to particle energy):

$$\sigma \equiv \frac{B_{\rm rms}^2}{4\pi n_0 \bar{\gamma} m_e c^2} \qquad \frac{\delta v}{c} \sim \frac{v_A}{c} = \sqrt{\frac{\sigma}{\sigma + 4/3}}$$

Focus on $1/4 \leq \sigma \leq 4$

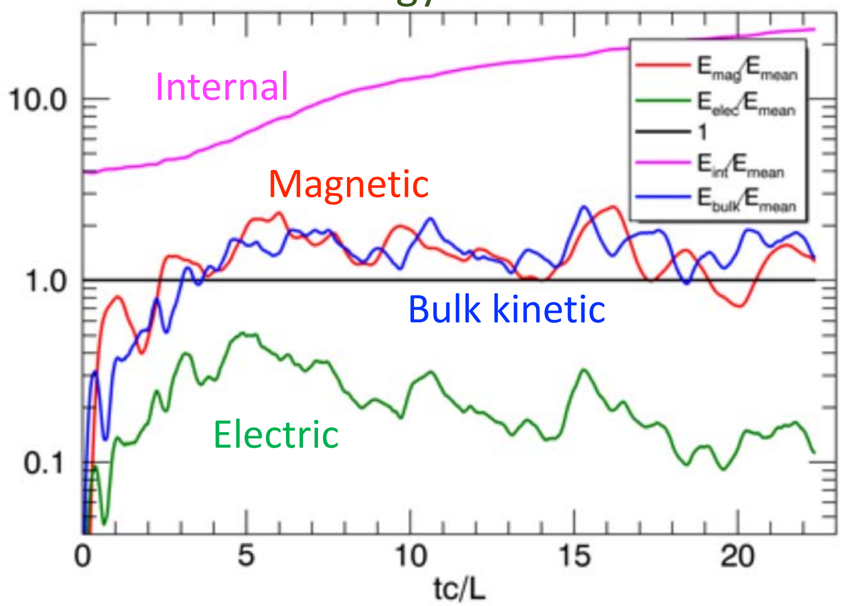
2) System size (ratio of driving scale to particle Larmor scale):

$$L/2\pi\rho_e \to 163 \qquad \rho_e = \frac{\bar{\gamma} m_e c^2}{e B_{\rm rms}}$$

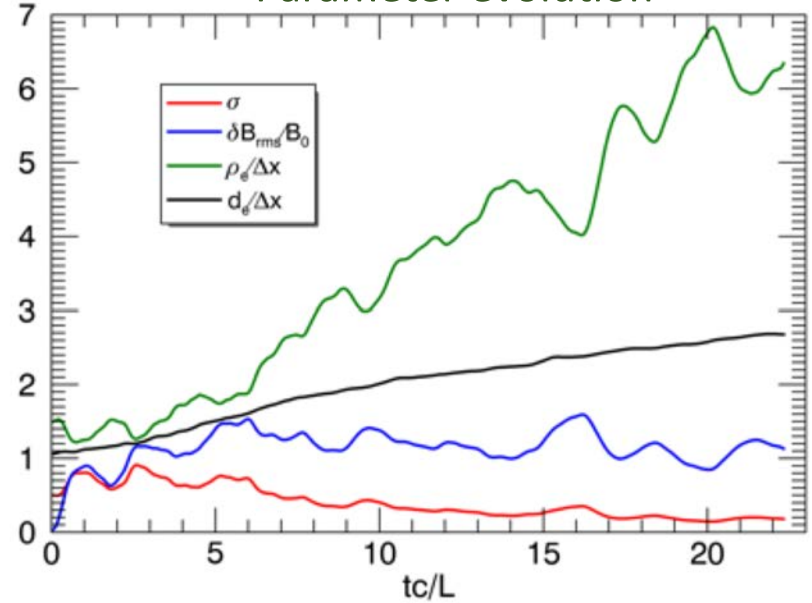
(256^3 - 1536^3 lattice)

Evolution of simulations

Energy evolution $\sigma_0 = 0.5$



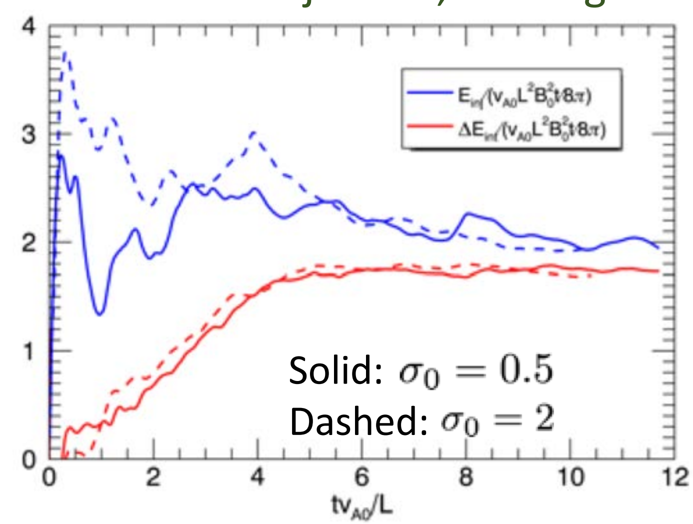
Parameter evolution

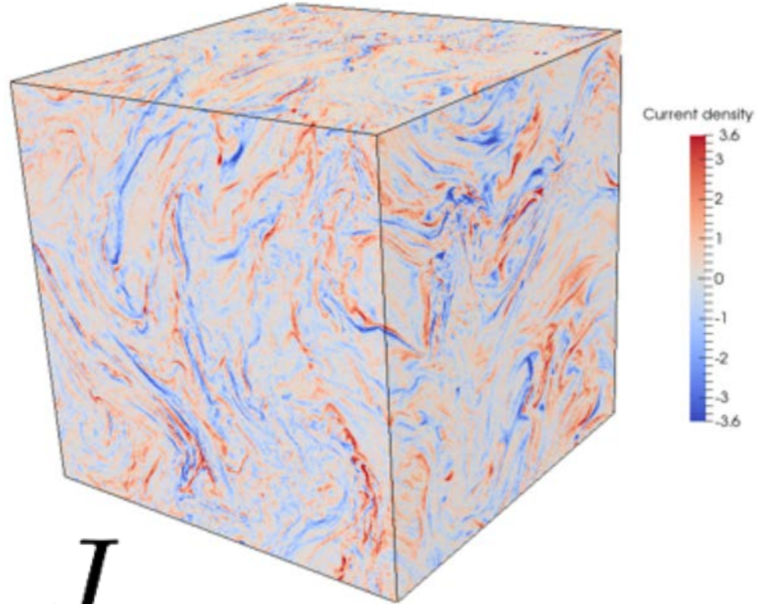


Time-dependent plasma parameters due to heating:

$$\langle \gamma \rangle \sim \gamma_0 \left(1 + \frac{1}{2} \eta_{\rm inj} \sigma_0 \frac{v_{A0} t}{L} \right)$$

Normalized injection, heating rate

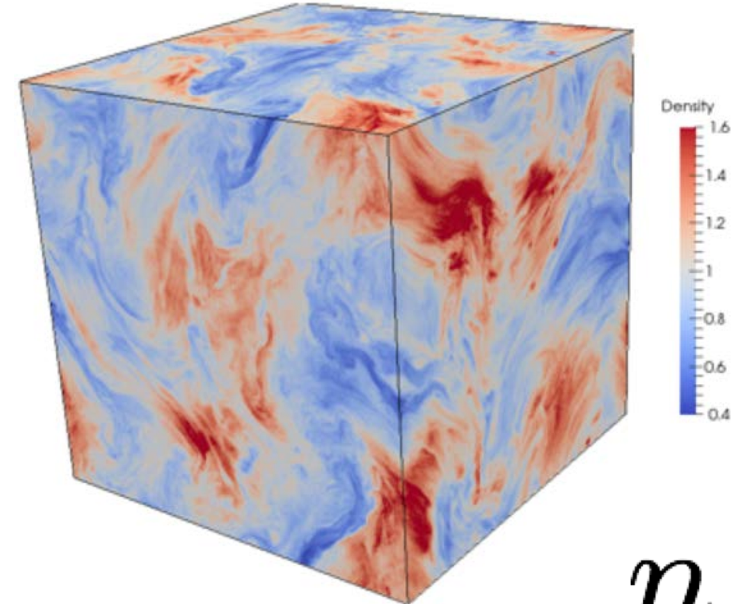




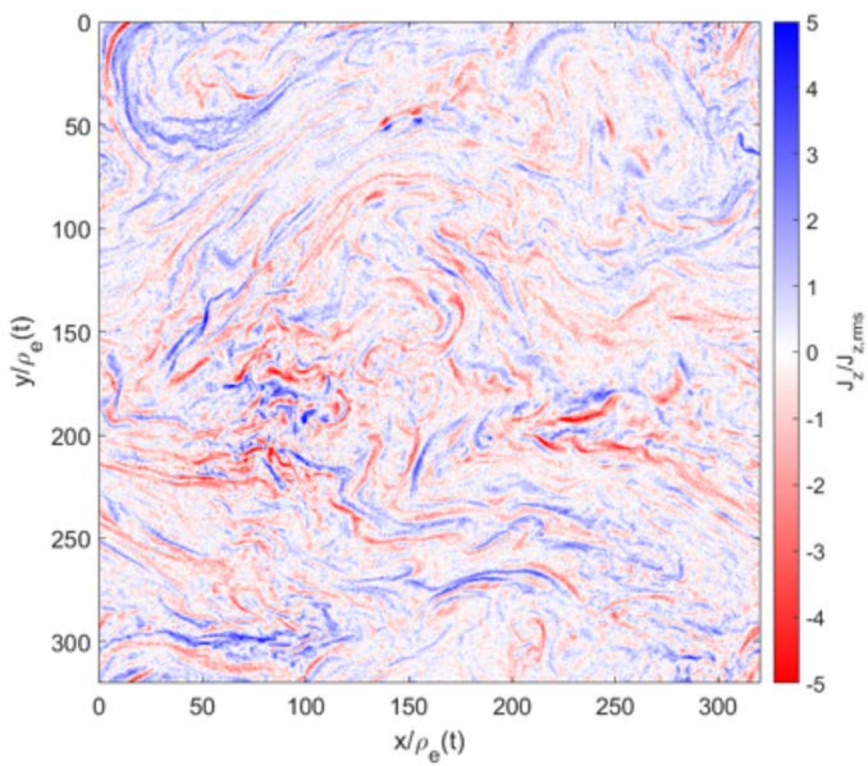
J_z

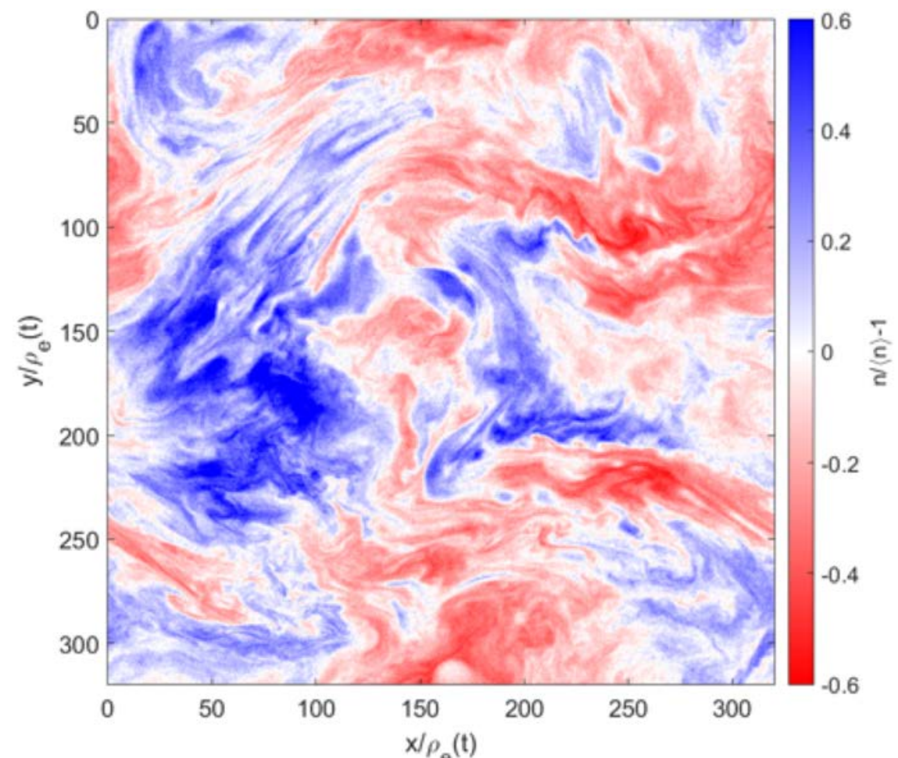
200 billion particles
1024^3 cells

$\sigma_0 = 0.5$



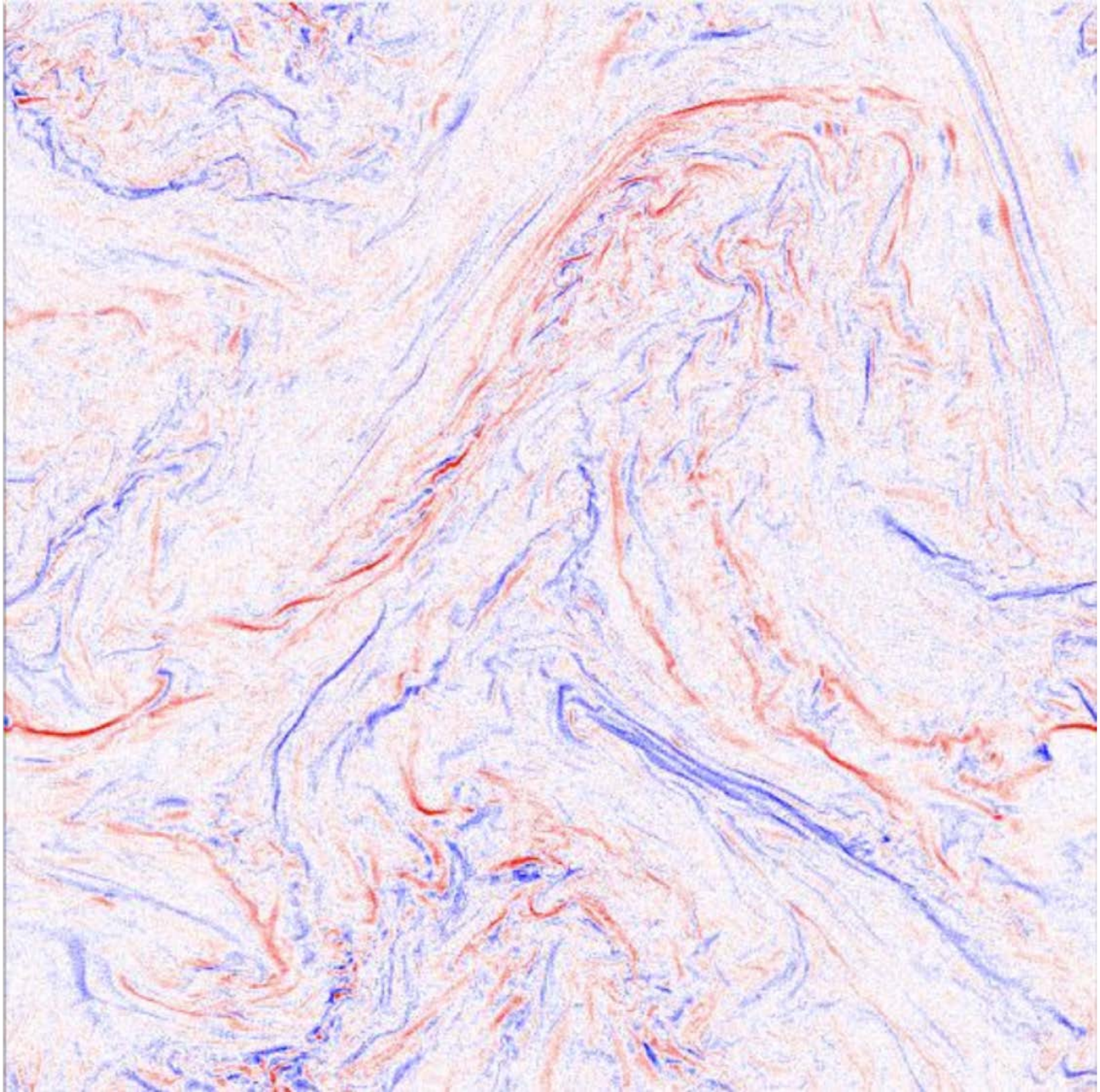
n





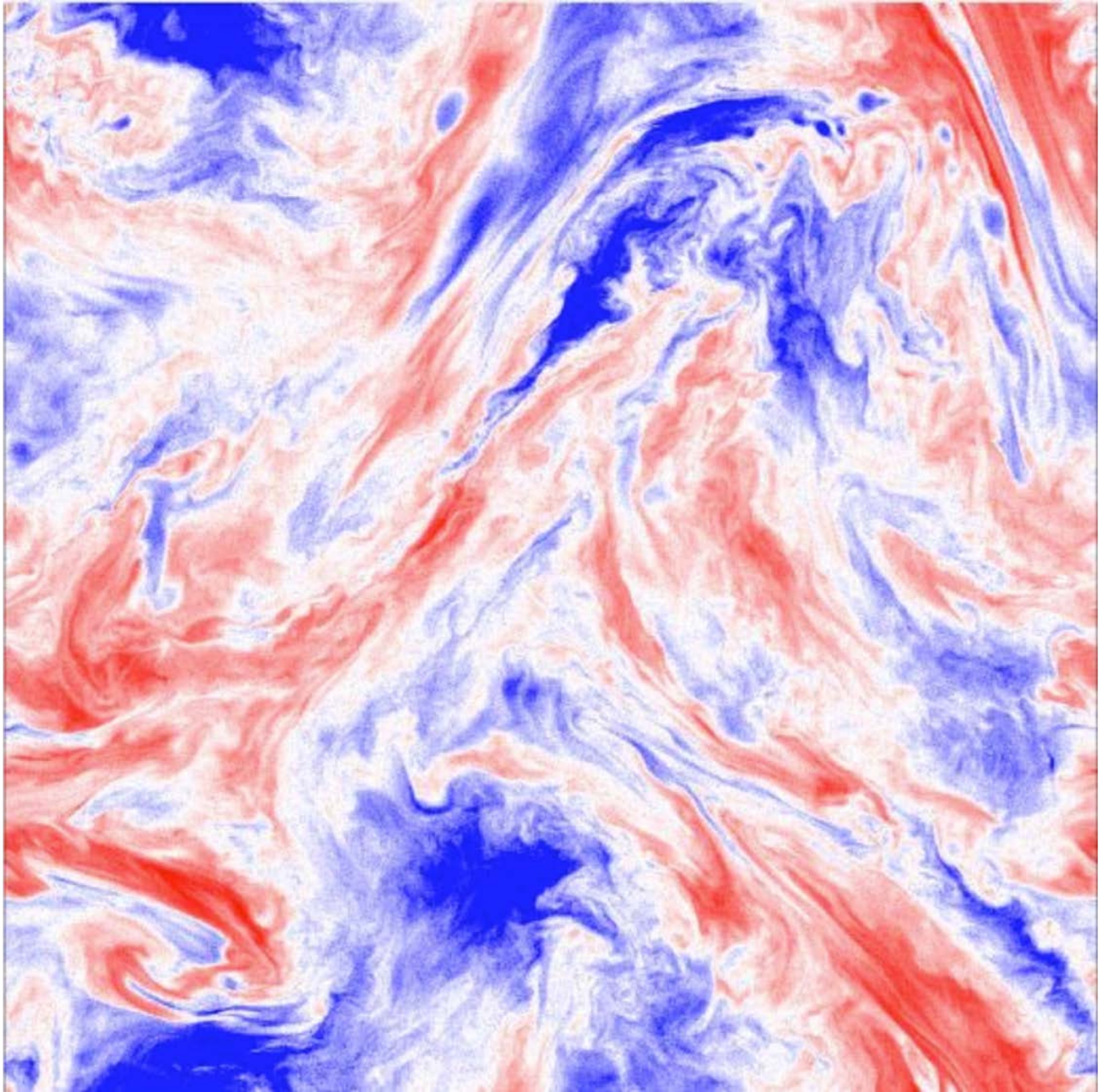
Fixed time fly through

J_z



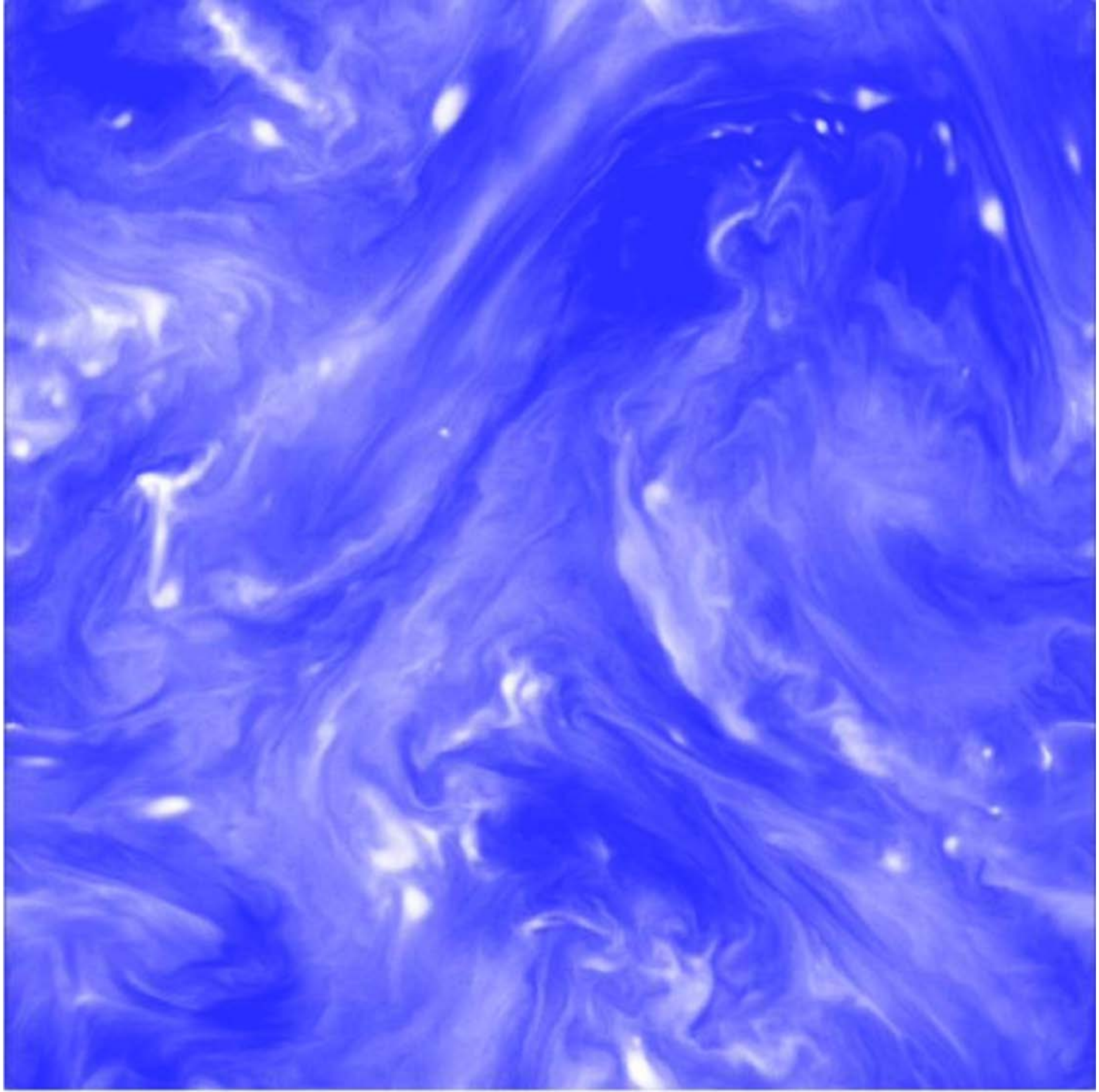
Fixed time fly through

δn



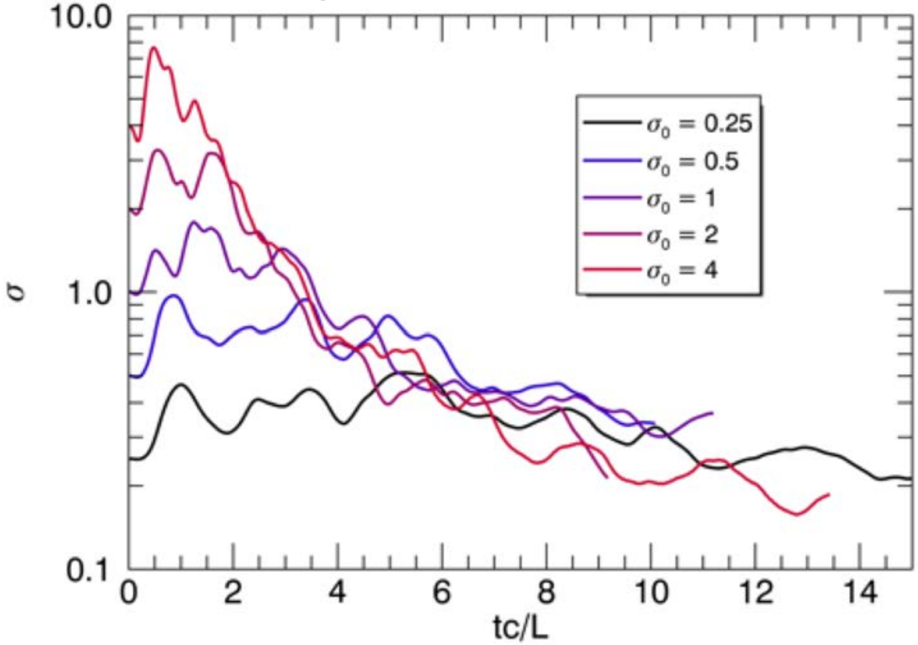
Fixed time fly through

B^2

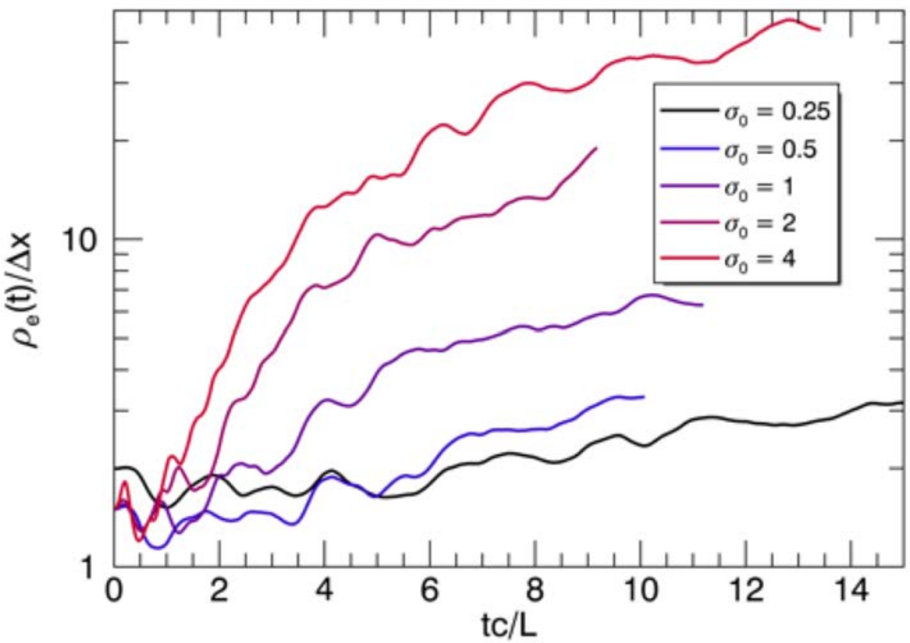


Magnetization scan

Magnetization evolution



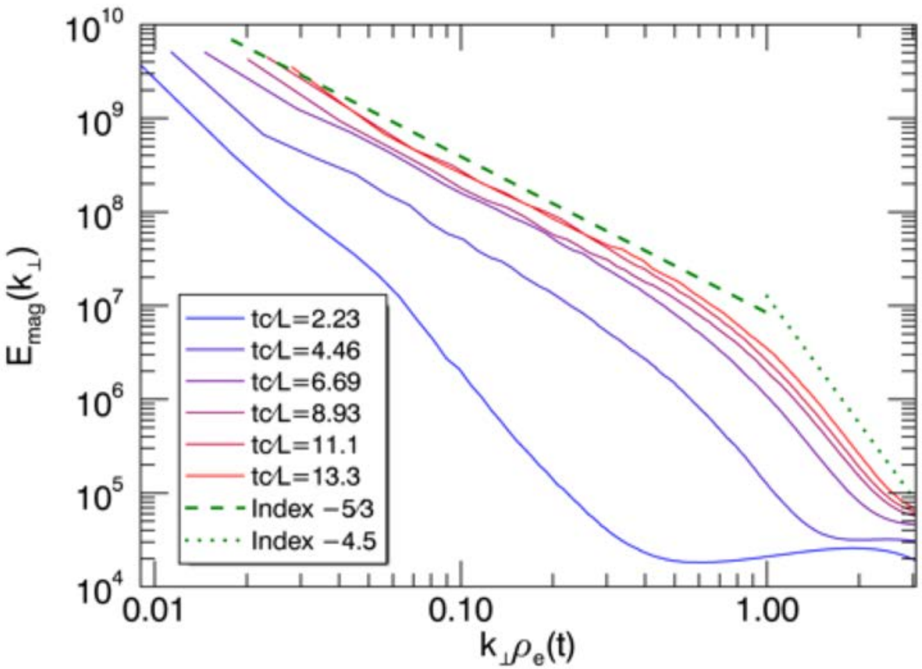
Larmor scale



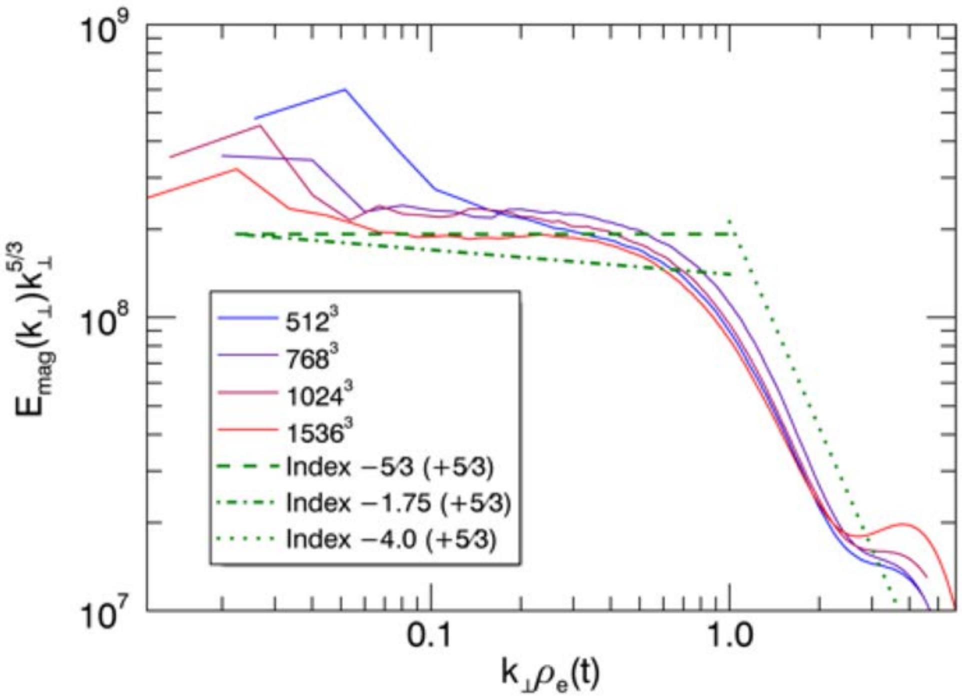
- Magnetization drops below unity for all cases (high sigma unsustainable)
- Larmor radius grows in time (inertial range diminishes in time)

Magnetic energy spectrum

Spectrum evolution (1024^3) $\sigma_0 = 0.5$



Compensated, averaged spectrum



Inertial range converged for 768^3 and larger $(L/2\pi\rho_{e0} \gtrsim 80)$

MHD range -5/3 index (Goldreich & Sridhar 1995, Thompson & Blaes 1998)

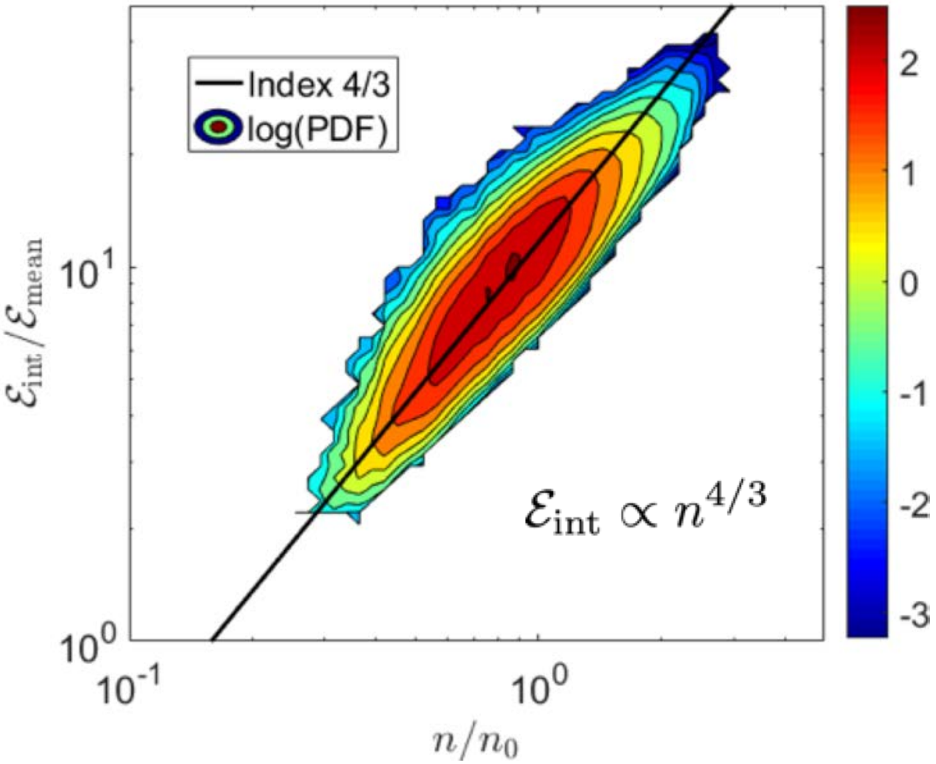
(similar to MHD turbulence simulations with modest guide field)

Kinetic range -4.5 index (kinetic cascade?, Schekochihin+ 2009)

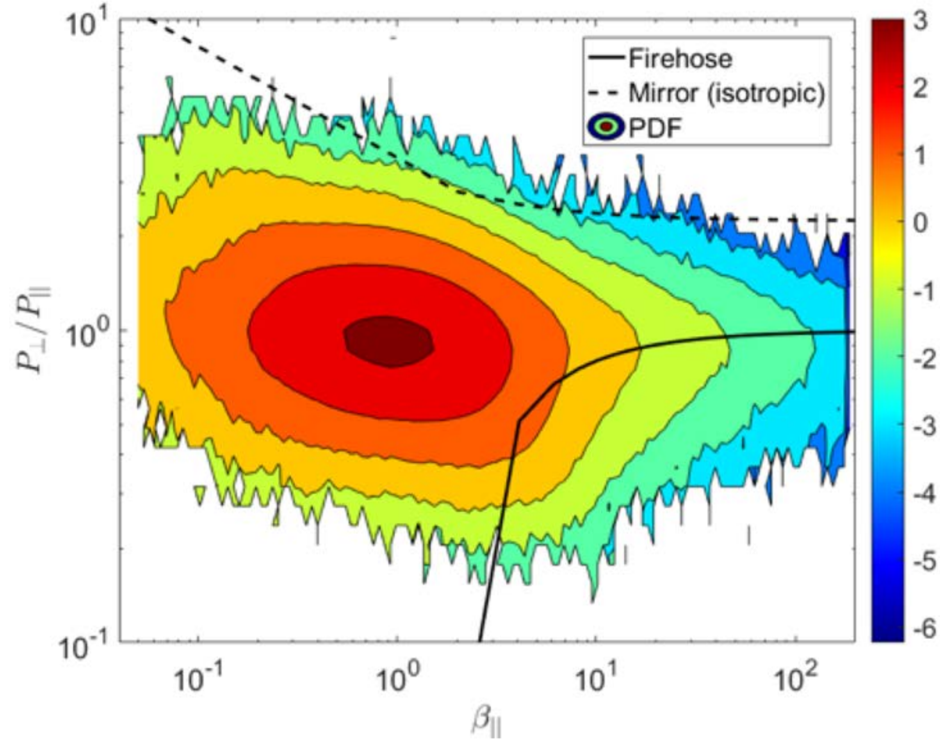
Equation of state

- Ultra-relativistic ideal gas from first principles
- Approximate signatures of mirror, firehose instabilities in pressure anisotropy

Equation of state



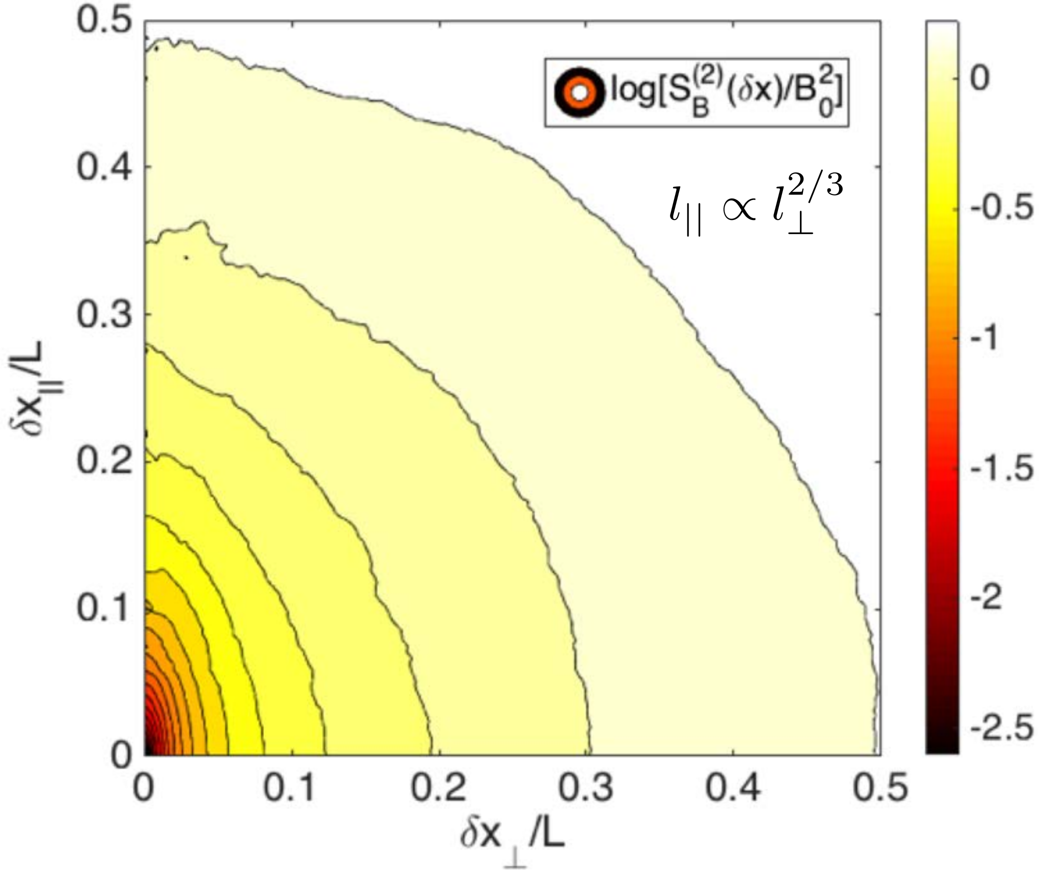
Pressure anisotropy



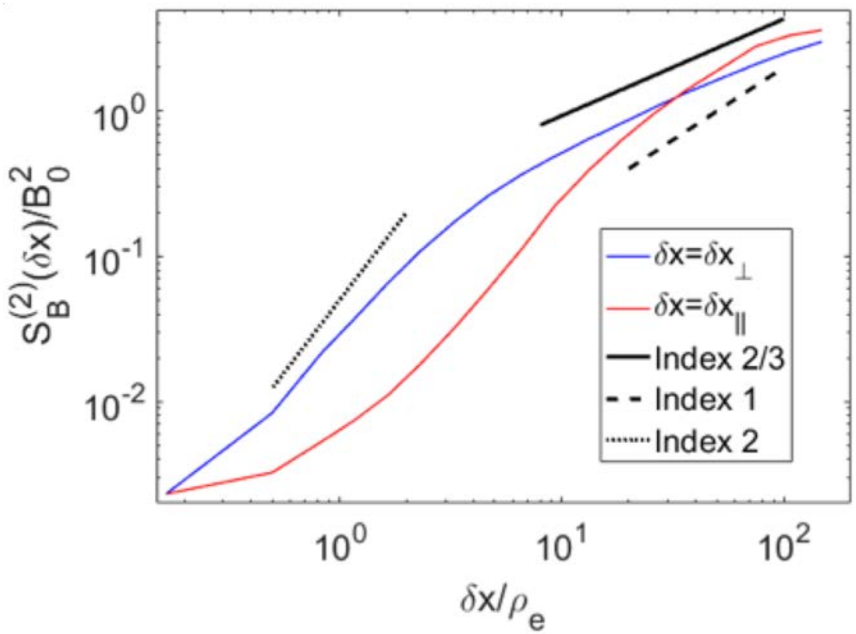
Thresholds from Chou & Hau 2004

Turbulence anisotropy

Structure function anisotropy (1024^3)



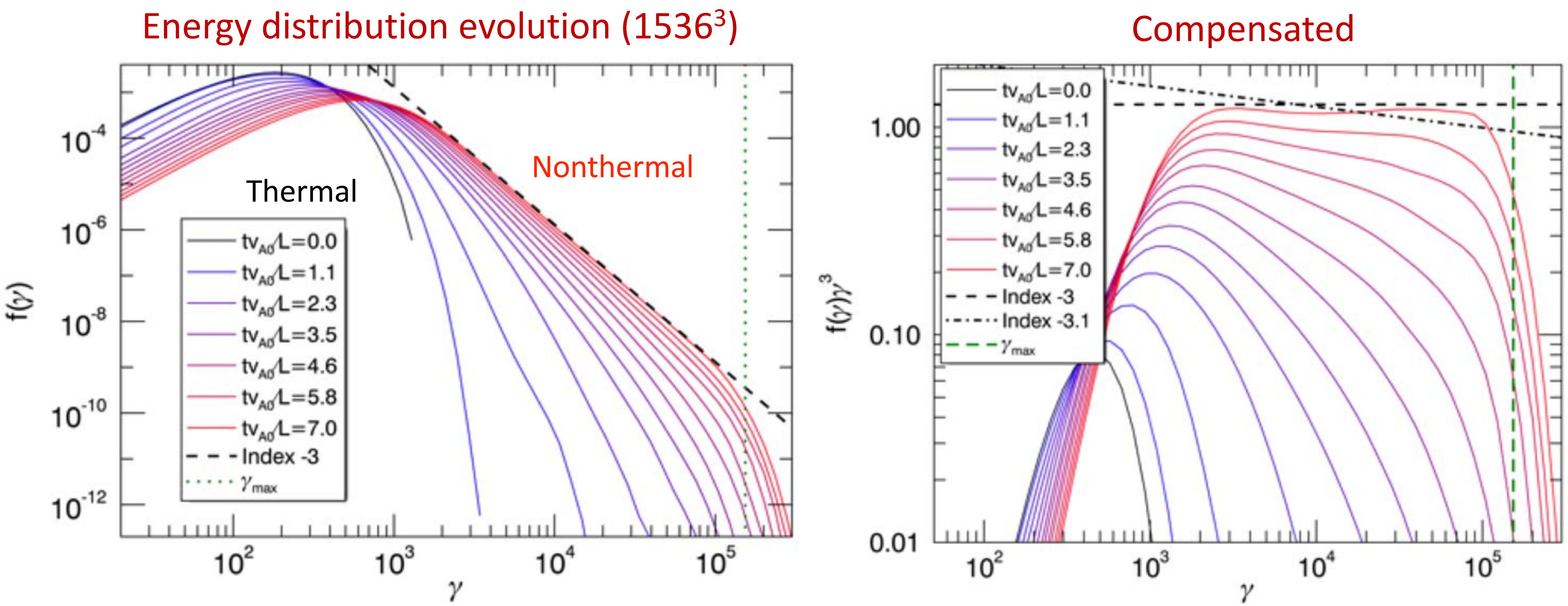
Structure function scaling



$$S_B^{(q)}(\delta \boldsymbol{x}, t) = \langle |\boldsymbol{B}(\boldsymbol{x} + \delta \boldsymbol{x}, t) - \boldsymbol{B}(\boldsymbol{x}, t)|^q \rangle_{\boldsymbol{x}}$$

Scale-dependent local-field anisotropy consistent with MHD critical balance (Goldreich & Sridhar 1995, Cho & Vishniac 2000): $\frac{\delta B_l}{l_\perp} \sim \frac{B_0}{l_\parallel} \implies l_\parallel \propto l_\perp^{2/3}$

Nonthermal particle acceleration



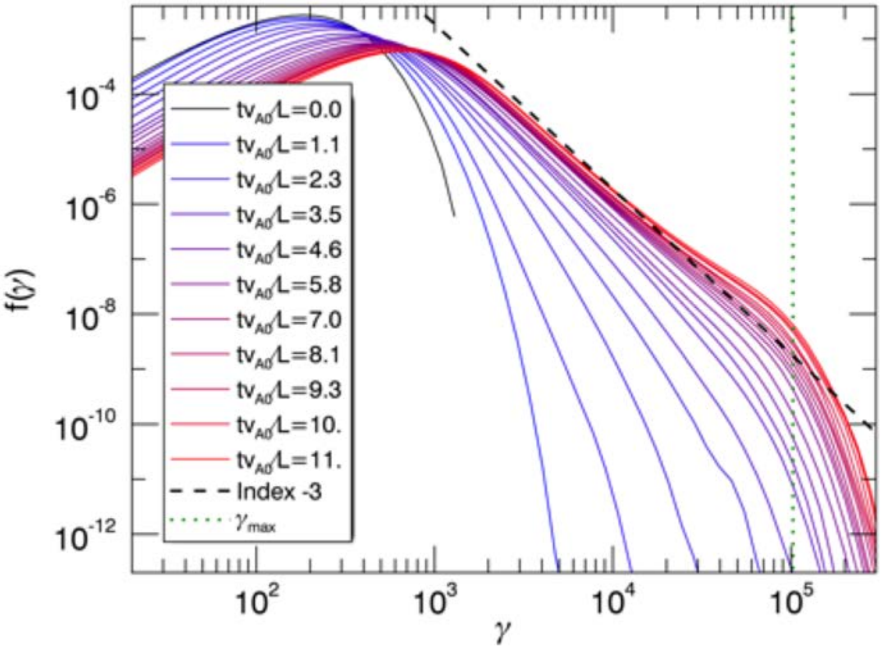
Power law tail: $f(\gamma) \sim \gamma^{-\alpha}$ $(\gamma = E/m_e c^2)$

Spans from mean energy $\langle \gamma \rangle$ to system-size limited energy $\gamma_{\max} = LeB/2mc^2$

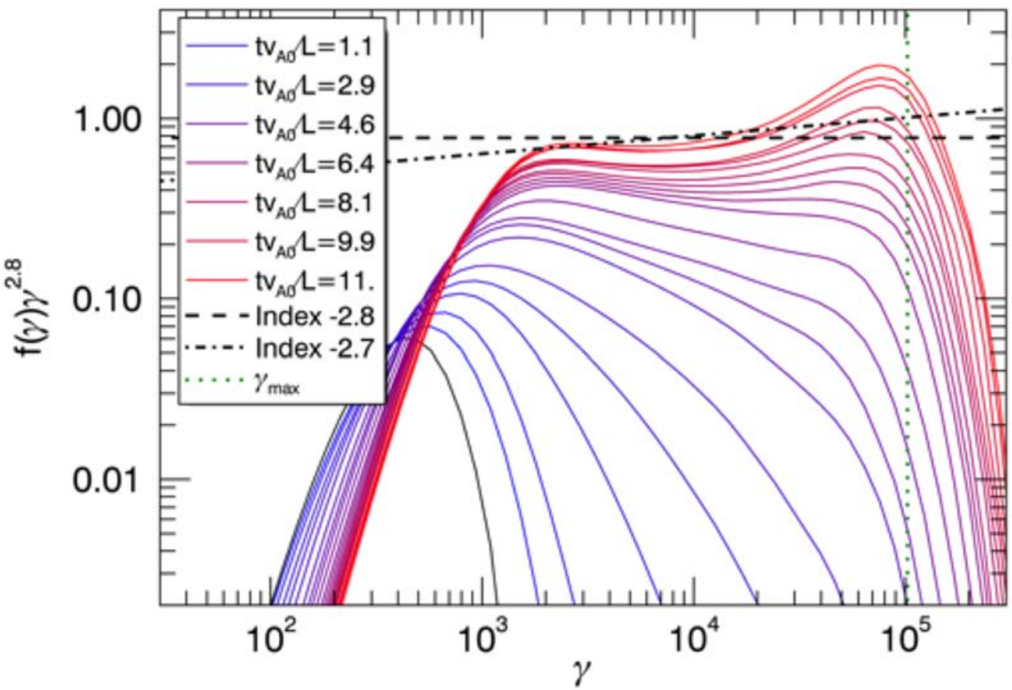
$(\rho_e \sim L/2)$

Late-time evolution: pileup

Long 1024^3 run



Compensated

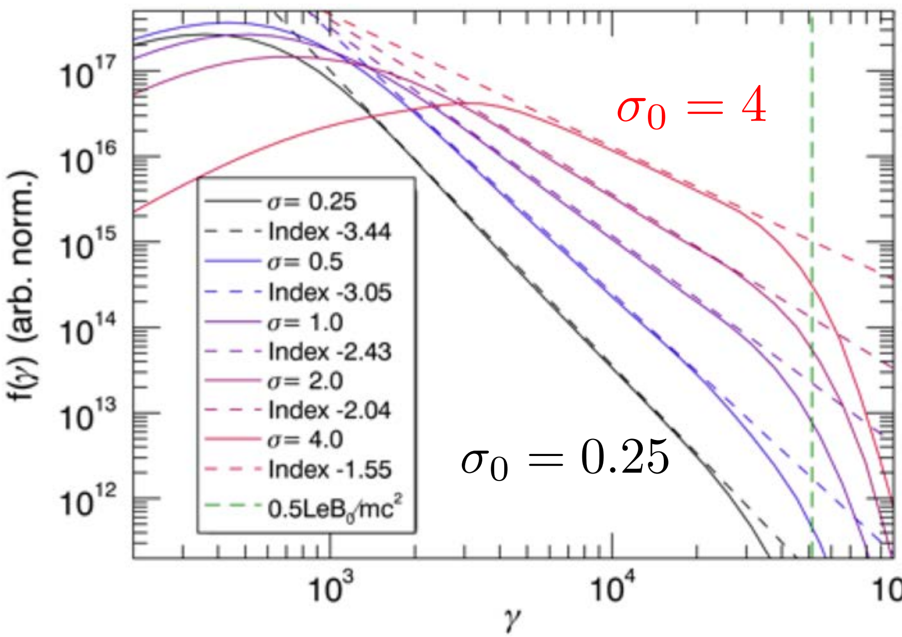


Particle pileup at system-size energy limit: $\gamma_{\rm max} = LeB/2mc^2$

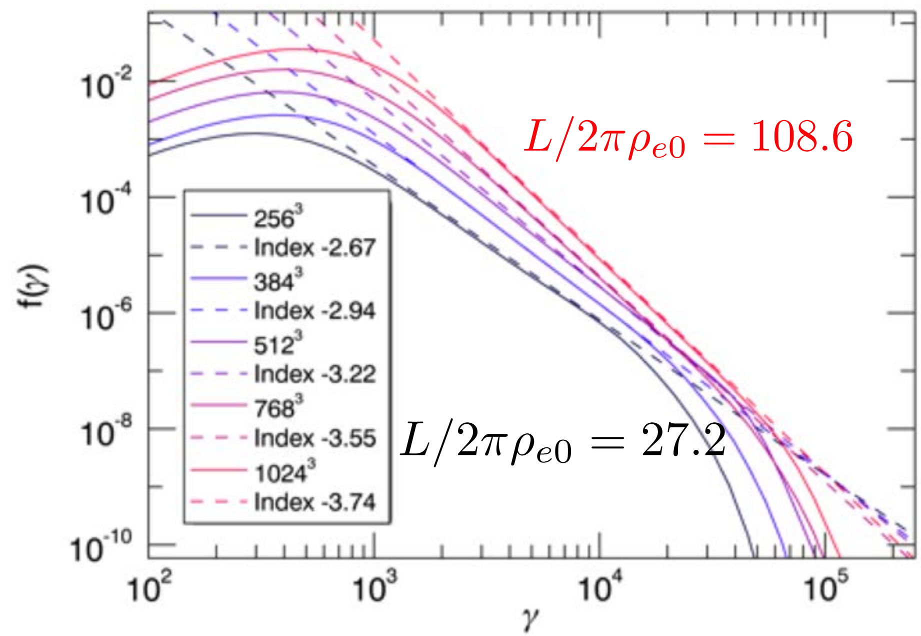
Inflection point appears in distribution, some ambiguity in index (return to this!)

Parameter dependence

Magnetization scan



System size scan (partial)

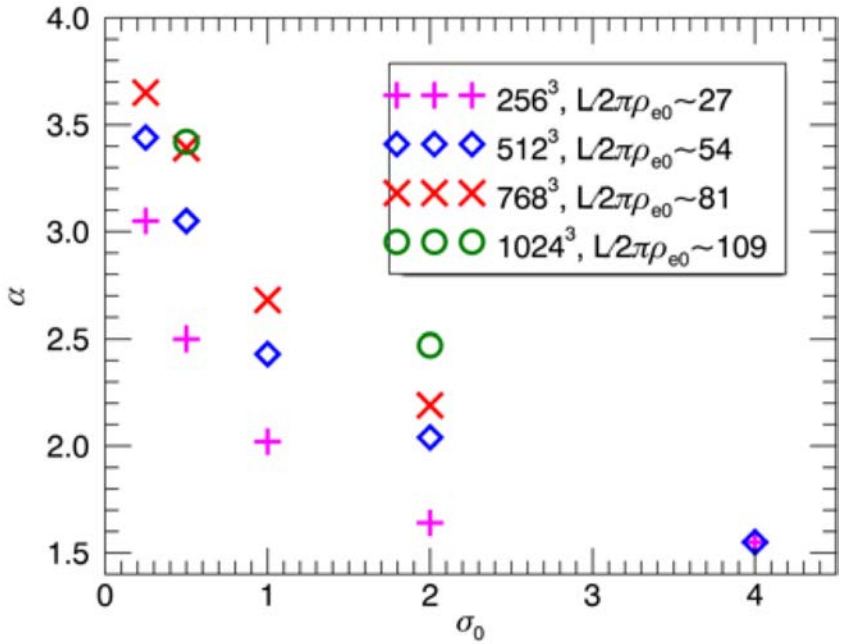


Hardens with increasing magnetization (relativistic motions, efficient acceleration)

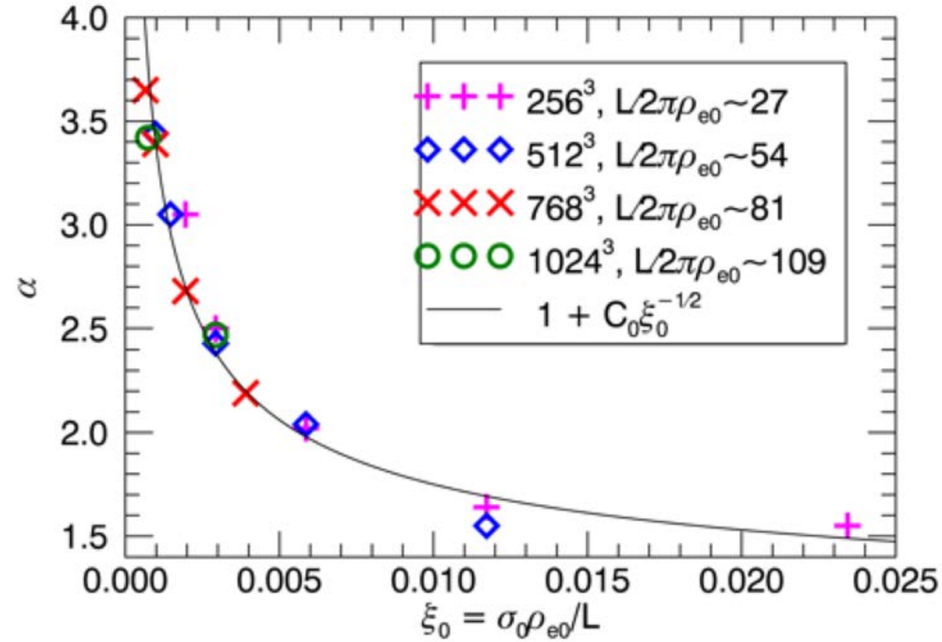
Softens with increasing system size (possible issues with convergence?)

An empirical formula?

Index vs parameters



Rescaled



Hardens with increasing magnetization, as in reconnection (Werner et al. 2016-17)

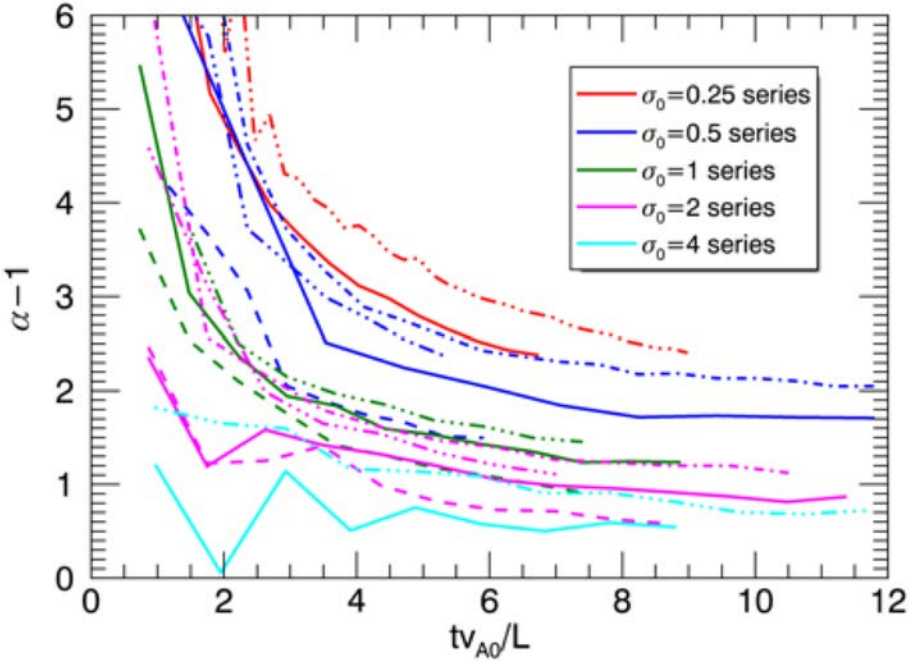
Softens with increasing size, inefficient acceleration for large astrophysical systems?

$$\alpha \sim 1 + C_0 \left(\frac{L}{\rho_{e0}\sigma_0} \right)^{1/2} \qquad C_0 \approx 0.075$$

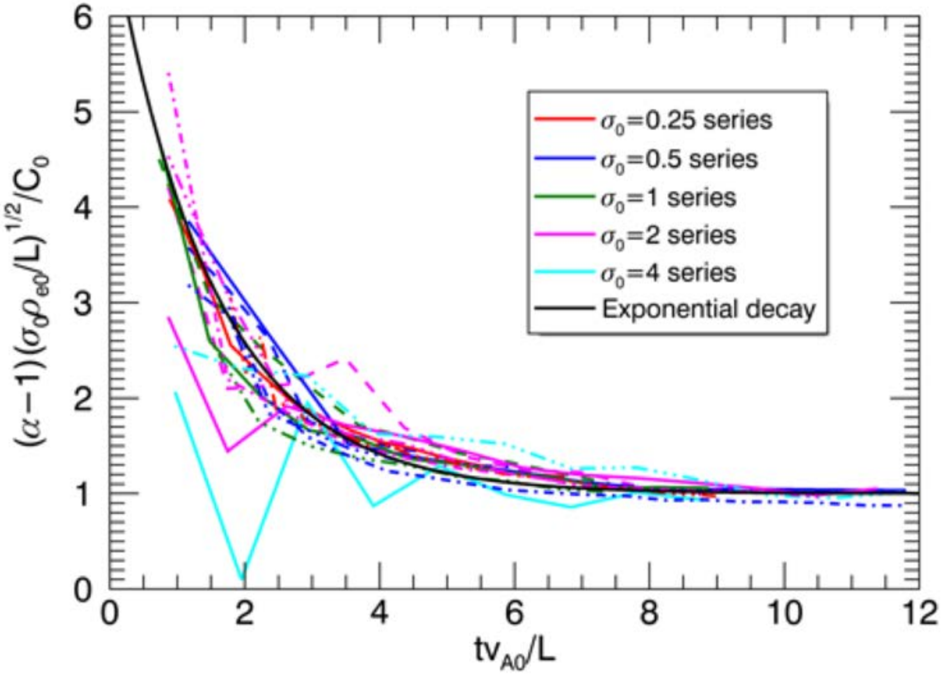
Zhdankin, Werner, Uzdensky & Begelman PRL 2017

Time evolution of index

Index vs time



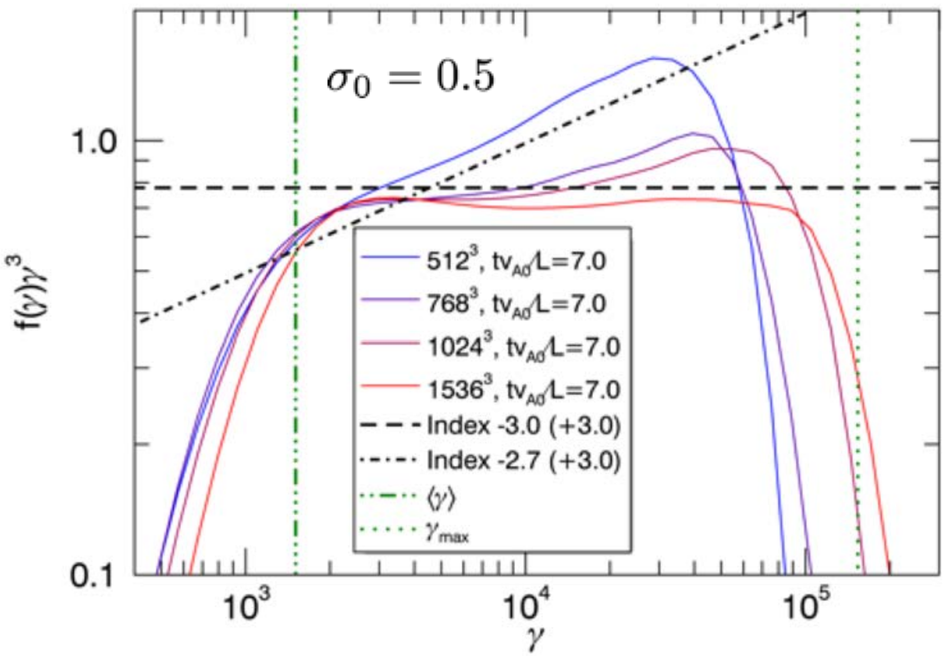
Rescaled by empirical formula



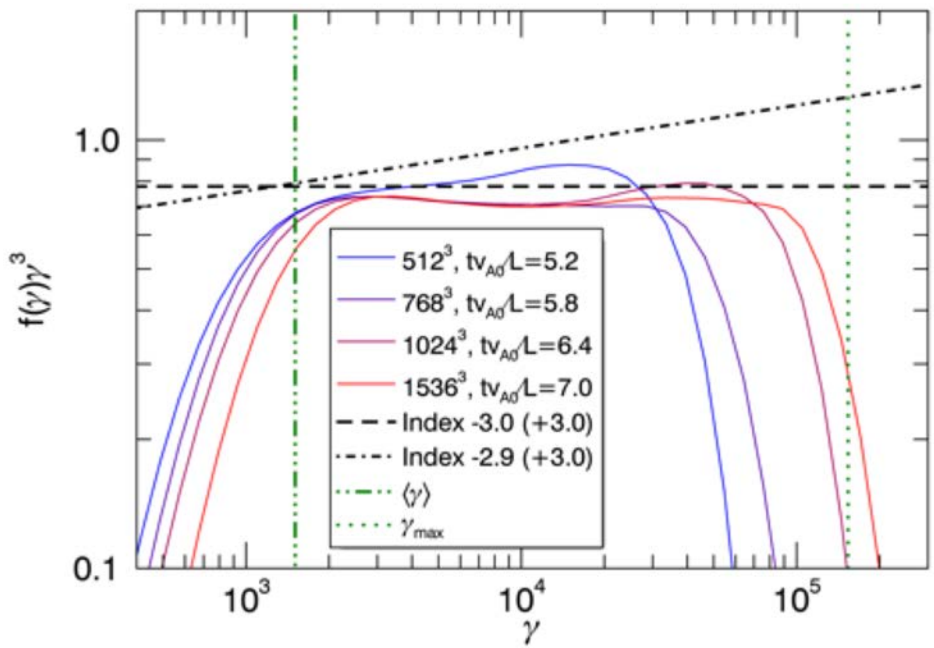
Empirical formula rescales the index evolution reasonably well (for small/intermediate simulations)

System-size dependence: recent perspective

Distributions at fixed time



Distributions at log times



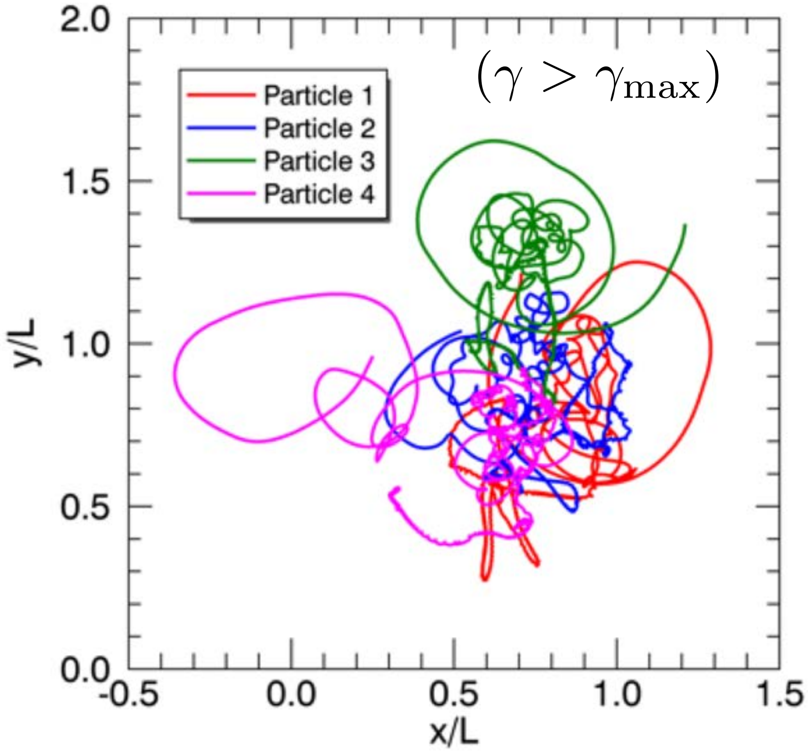
At fixed time, weak system-size dependence up to 1536^3 $(L/2\pi\rho_{e0} \sim 160)$

At times scaling logarithmically with system size, apparent convergence is obtained for 768^3 and larger $(L/2\pi\rho_{e0} \gtrsim 80)$

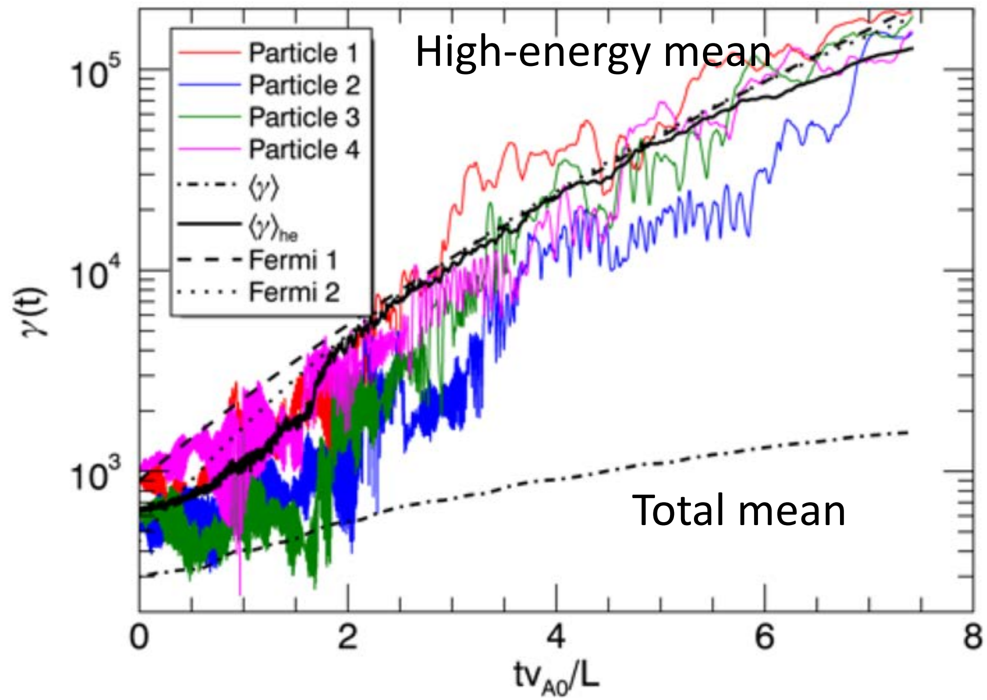
Proposal: nonthermal distributions are delayed in larger simulations

Tracked particles

Four most energetic particles



Particle energy evolution



High-energy particle evolution consistent with Fermi acceleration:

$$\frac{d\gamma}{dt} \sim \frac{\gamma}{\tau_{\rm acc}(t)}$$

First-order: $\tau_{\rm acc} \propto \dfrac{L}{v_A(t)}$

Second-order: $\tau_{\rm acc} \propto \dfrac{Lc}{v_A^2(t)}$

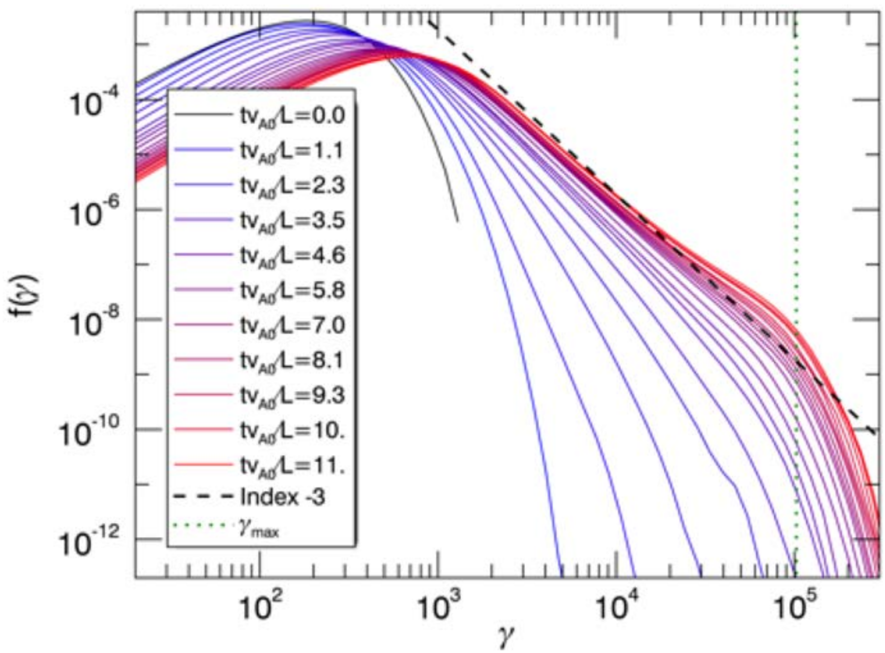
Implications of Fermi acceleration

Time for particles to reach system-size energy limit is logarithmic function of system size:

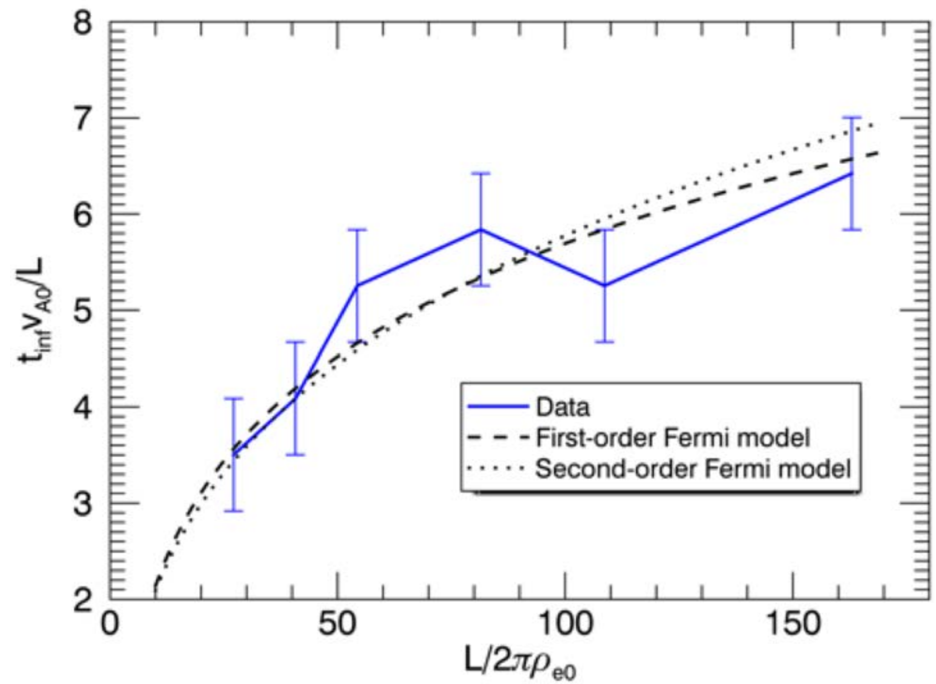
$$\gamma \sim \gamma_i \exp\left(t/\tau_{\rm acc}\right) \implies t_{\rm inf} \sim \tau_{\rm acc} \log\left(\gamma_{\rm max}/\gamma_i\right) \sim \tau_{\rm acc} \log L/\rho_{e0}$$

(generalize to time-dependent acceleration time)

Pileup forms at "inflection time"

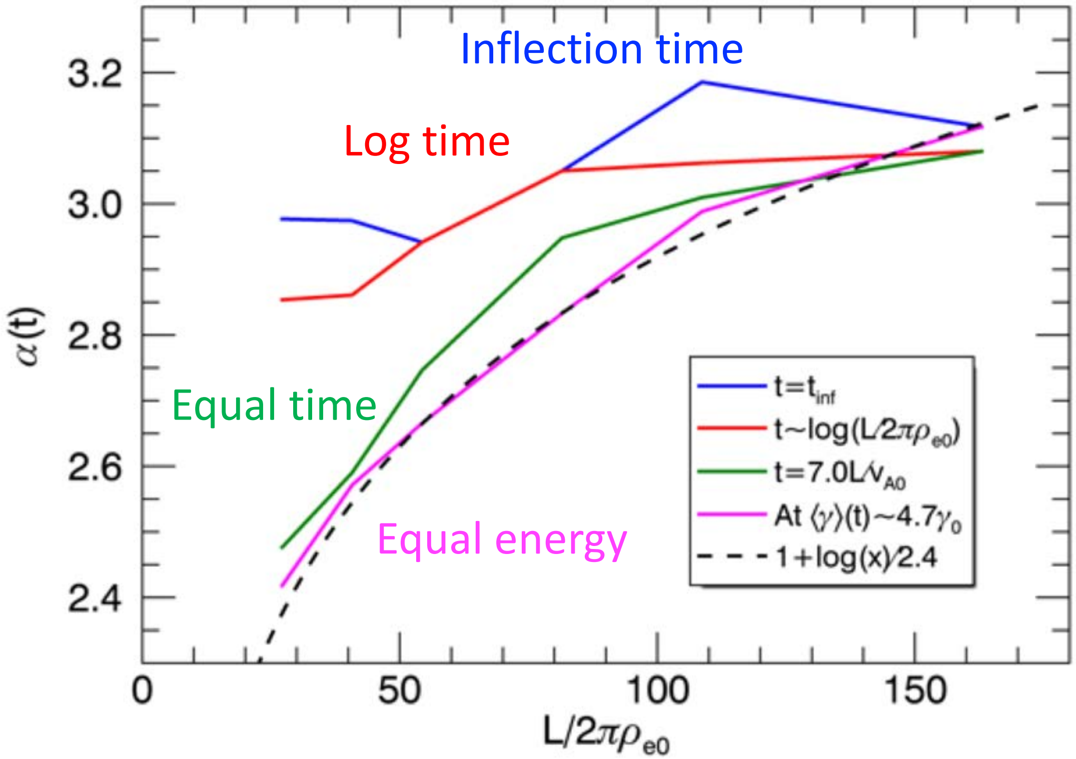


Inflection time vs system size



Convergence?

Index vs size: four perspectives



$$\alpha_{\mathrm{conv}} \approx 3.1$$

$$(\sigma_0 = 1/2)$$

No clear convergence when indices measured at equal times or at equal energy (logarithmic dependence rather than 1/2 power law?)

Convergence when measured at logarithmic times or at inflection time

Conclusions

- PIC simulations are ideal for exploring 3D relativistic turbulence
- MHD range is well reproduced (-5/3 power spectrum, etc.)
- May need to compare nonthermal distributions at logarithmic times
- Turbulence can be efficient *and* viable astrophysical particle accelerator

References:
Zhdankin, Werner, Uzdensky & Begelman PRL 2017
Zhdankin, Uzdensky, Werner & Begelman MNRAS 2018
Zhdankin, Uzdensky, Werner & Begelman in prep

+ more on the way