

# ***Relativistic Nonthermal Particle Acceleration in Magnetic Reconnection***

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# OUTLINE:

## Theory of NTPA In Magnetic Reconnection

- Physical Picture and Assumptions.
- Empirical (numerical) Knowledge Base.
- Key Ingredients of the Kinetic Equation:
  - Acceleration by reconnection electric field;
  - Magnetization by reconnected magnetic flux: (*power-law slope*)
  - Trapping in plasmoids: (*high-energy cutoff*)
- Effects of Guide magnetic Field
- Conclusions
- (Radiative Turbulence)

# Physical Picture: Reconnection

- Plasmoid-dominated magnetic reconnection plasmoid chain, characterized by a plasmoid distribution function  $F(w)$  and a cumulative distribution  $N(w)$ :

$$N(w) = \int_w^{w_{\max}} F(w') dw'$$

- Reconnection may or not be relativistic.

- Reconnection rate:  $E_{\text{rec}} = \beta_{\text{rec}} B_0 = \varepsilon V_A B_0 / c = \varepsilon \beta_A B_0$

$$V_A = c \beta_A = c \frac{\sqrt{\sigma_h}}{\sqrt{1 + \sigma_h}}$$

- “Hot” magnetization:  $\sigma_h = B_0^2 / (4\pi n h)$ ;  $h$  = relativistic enthalpy per particle.

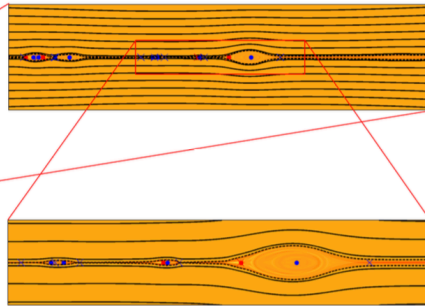
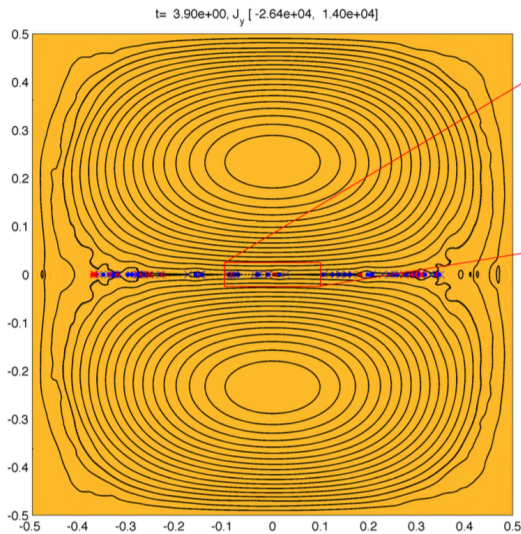
rel. limit:  $\sigma_h \gg 1 \rightarrow V_A \sim c$

non-rel. limit  $\sigma_h \ll 1 \rightarrow \beta_A = V_A / c \sim \sigma_h^{1/2} \ll 1$ .

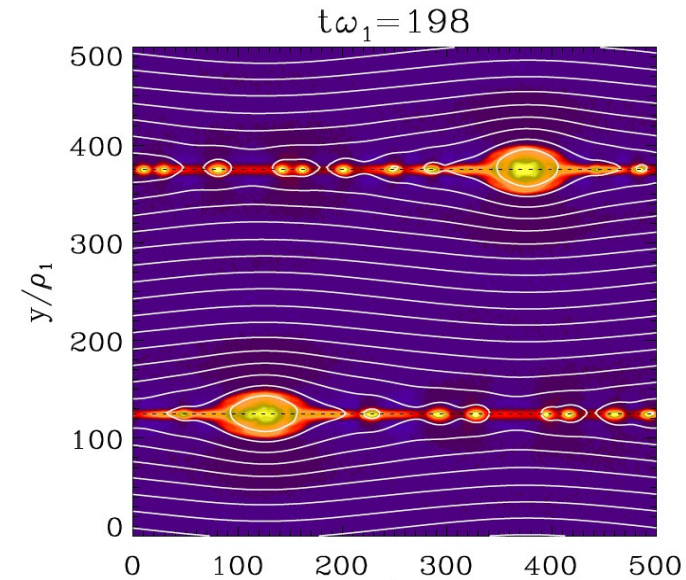
- I will also use cold  $\sigma = B_0^2 / (4\pi n_b m c^2)$

- But particles under consideration are ultra-relativistic.

# Self-similar hierarchical plasmoid chain



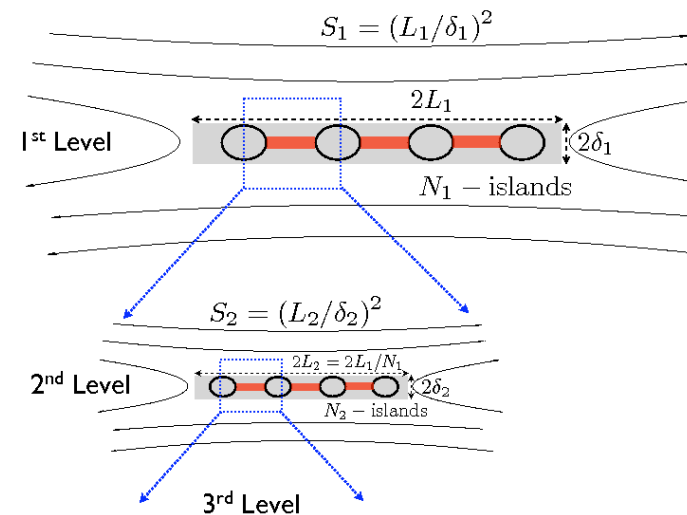
Bhattacharjee et al. 2009



Cerutti et al. 2013

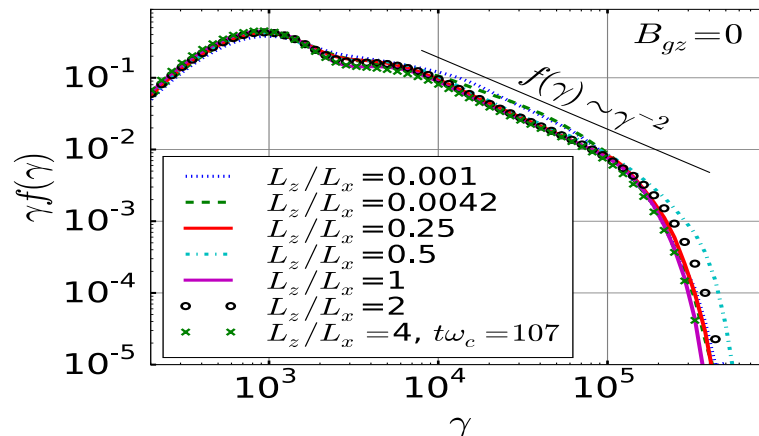
Two Phases of Reconnected Magnetic Field:

- of order  $B_1 = \epsilon B_0 = 0.1 B_0$  in current sheets
- of order  $B_0$  in plasmoids



# Dimensionality

- Real world is 3D.
- However, recent PIC simulations of relativistic pair-plasma reconnection (Werner & Uzdensky 2017, also Sironi & Spitkovsky 2014, Guo et al. 2015) show that 2D and 3D give the same NTPA.



- (However, for non-relativistic electron-ion plasma reconnection, 2D and 3D NTPA may be different, see Dahlin et al. 2015-2017.)
- Thus here we will consider 2D reconnection.
- Direction perpendicular to current sheet: particles confined to the sheet by reconnecting magnetic field  $\rightarrow$  consider only motion along the sheet.

# Basic Picture

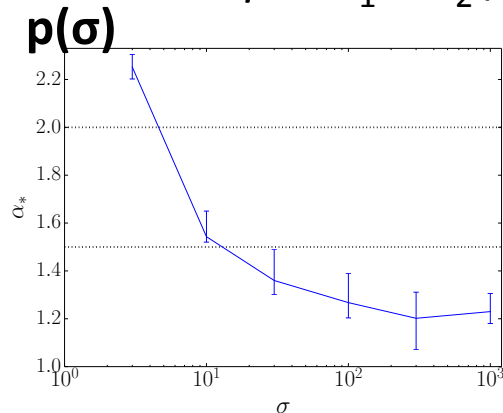
- While they are unmagnetized, energetic particles are accelerated by the main reconnection electric field  $E_{\text{rec}}$ .
- This steady acceleration proceeds until a particle is captured by the reconnected magnetic field.
- Reconnected magnetic field is bimodal:
  - $\sim B_1 \sim \epsilon B_0 \sim 0.1 B_0$  in inter-plasmoid current layers;
  - $\sim B_0$  in circularized plasmoids [with size distribution  $F(w)$ ]
- We will treat magnetization in  $B_1$  and trapping in plasmoids separately.

# Empirical Numerical Properties of Reconnection-driven relativistic NTPA

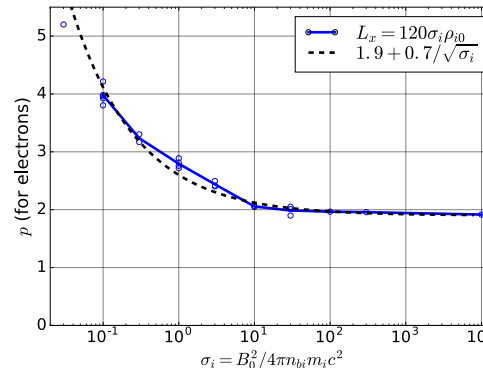
## Power-law index $p$

- $p$  scales with  $\sigma_h$  as

$$p \sim C_1 + C_2 / \sigma_h^{1/2}$$



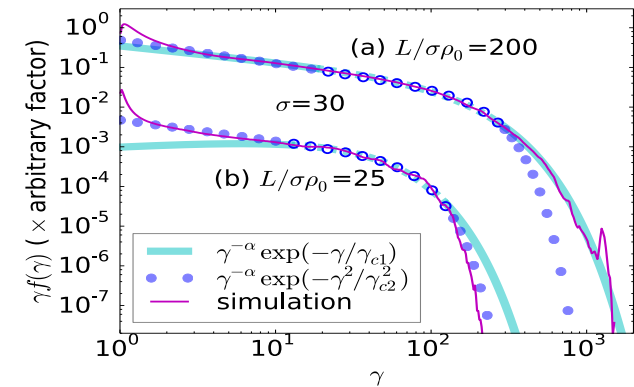
pair plasma:  
Werner et al. 2016



electron-ion plasma:  
Werner et al. 2018

## High-energy cutoff

(Werner et al. 2016)



$$f(\gamma) = \frac{dN}{d\gamma} \propto \gamma^{-\alpha} \exp\left(-\gamma/\gamma_{c1} - \gamma^2/\gamma_{c2}^2\right)$$

**Two** high-energy cutoffs:

- $\exp(-\gamma/\gamma_{c1})$ ;  $\gamma_{c1} \sim 4\sigma$ ,
- $\exp[-(\gamma/\gamma_{c2})^2]$ ;  $\gamma_{c2} \sim 0.1 L/\rho_0$

$$(\rho_0 = m_e c^2/e B_0)$$

(guide magnetic field suppresses NTPA, see below)

# High-Energy Power-Law Cutoff

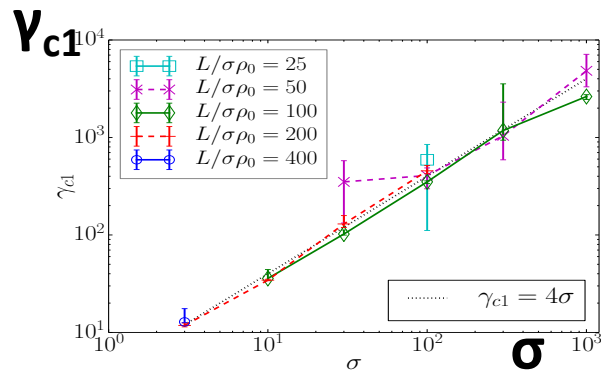
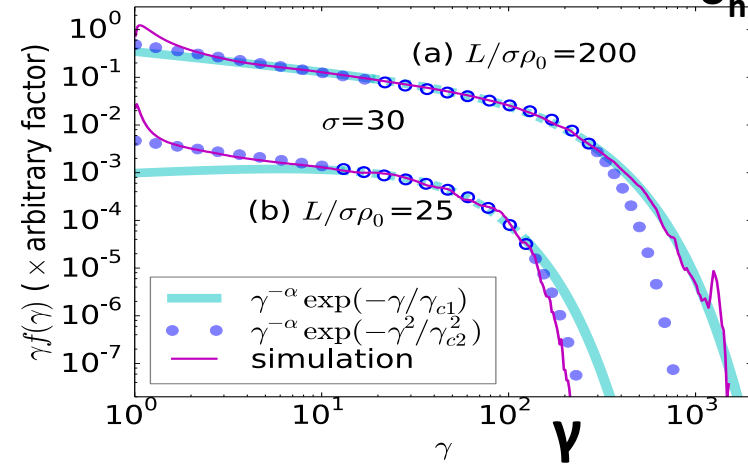
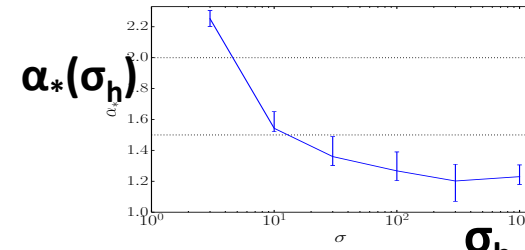
(Werner, Uzdensky, Cerutti, Nalewajko, Begelman, ApJL 2016)

$$f(\gamma) = \frac{dN}{d\gamma} \propto \gamma^{-\alpha} \exp(-\gamma/\gamma_{c1} - \gamma^2/\gamma_{c2}^2)$$

**Two** high-energy cutoffs:

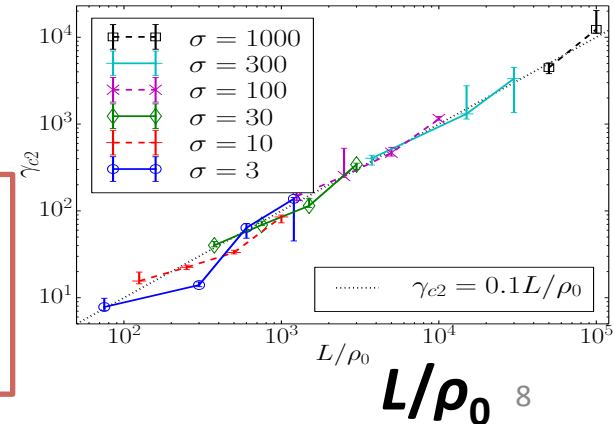
- $\exp(-\gamma/\gamma_{c1})$ ;  $\gamma_{c1} \sim 4\sigma$ ,  $\gamma_{c1} \sim 10 \langle \epsilon \rangle$
- independent of  $L$ ;

- $\exp[-(\gamma/\gamma_{c2})^2]$ ;  $\gamma_{c2} \sim 0.1 L/\rho_0$
  - independent of  $\sigma$ .
- available voltage drop  
 $\epsilon_{\max} \sim e E_{\text{rec}} L \sim 0.1 e B_0 L$   
 (Hillas, extreme acceleration)



$$(\rho_0 = m_e c^2 / e B_0) \quad \gamma_{c2}$$

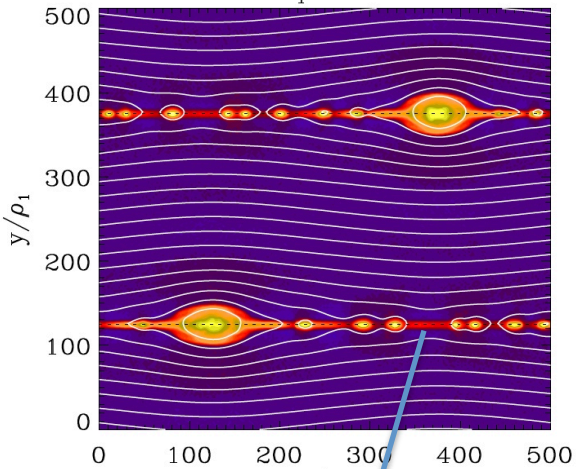
Large-system regime:  
 $(\gamma_{c1} < \gamma_{c2})$ :  
 $L/\rho_0 > 40 \sigma$



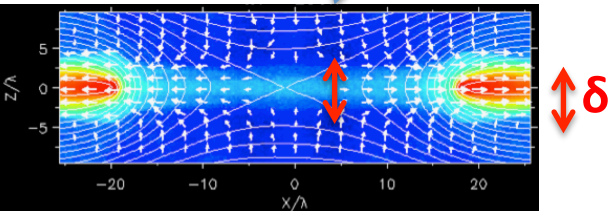


# Why is there a $\gamma_c \approx 4\sigma$ cutoff?

$t\omega_1 = 198$



Cerutti et al. 2013



Zenitani & Hoshino 2001

$$(\rho_0 = m_e c^2 / e B_0)$$

$$\sigma = B_0^2 / (4\pi n m c^2)$$

- Cutoff comes from small laminar *elementary inter-plasmoid layers* at the bottom of the plasmoid hierarchy (marginally stable to tearing).
- Particles are accelerated in these layers but then become **trapped inside plasmoids**.
- Cutoff:  $\gamma_c = e E_{\text{rec}} l / m_e c^2 \approx 0.1 e B_0 l / m_e c^2 = 0.1 l / \rho_0$ .
- Layers are marginally stable to tearing  $\rightarrow l \sim 100 \delta$ .
- Layer thickness:  $\delta \approx \rho \langle \gamma \rangle = \langle \gamma \rangle \rho_0 \approx (\sigma / 3) \rho_0$ .
- Thus,  $l / \rho_0 \approx 100 \delta / \rho_0 \approx 30 \sigma \Rightarrow \gamma_0 = 3 \sigma$ .

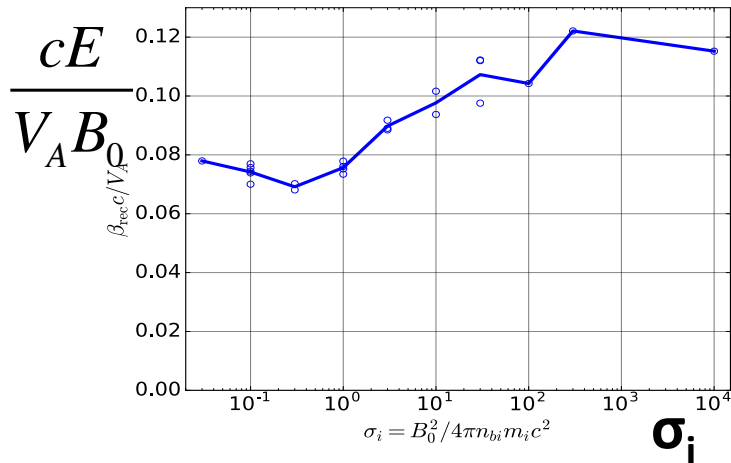
Further particle acceleration is possible, e.g., in plasmoid mergers, but this 2nd-stage reconnection acceleration occurs with lower sigma and smaller L.

# Relativistic $e-i$ reconnection: Key PIC Sims Results I

Werner et al., MNRAS 2018; arXiv:1612.04493

**Systematic 2D PIC study:**  
(no guide field,  $m_i/m_e=1836$ )

**Reconnection rate:**  $v_{in}/c = E$

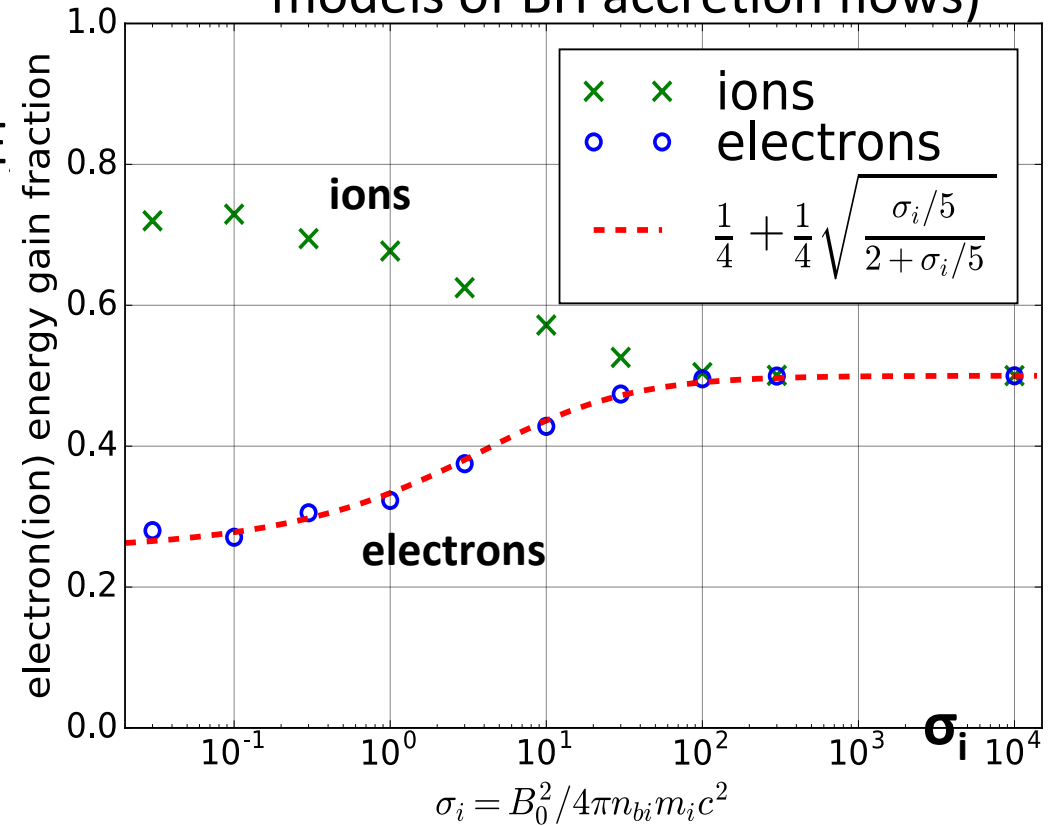


$$V_A = c \frac{B_0}{\sqrt{4\pi n_i m_i c^2 + B_0^2}} = c \frac{\sqrt{\sigma_i}}{\sqrt{1 + \sigma_i}}$$

D. Uzdensky

## Energy partitioning between electrons and ions

(useful prescription for GRMHD models of BH accretion flows)



**In semi-relativistic case ions gain 3 times more energy than electrons.**

5/8/2018

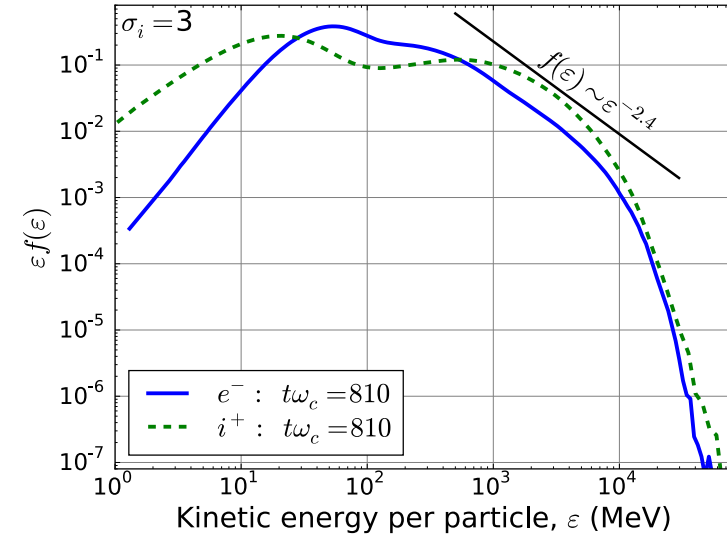
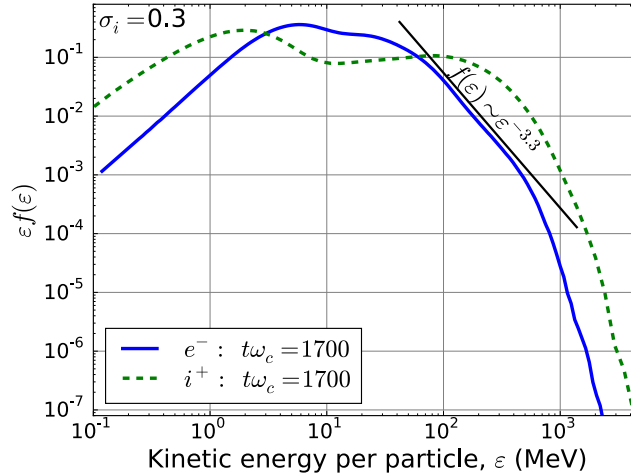
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# Relativistic $e-i$ reconnection: Key PIC Sims Results II

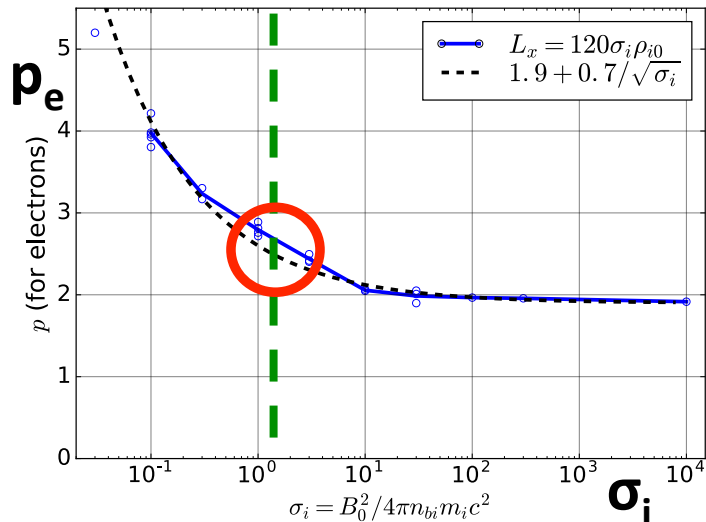
## Particle Acceleration:

- electrons: ✓

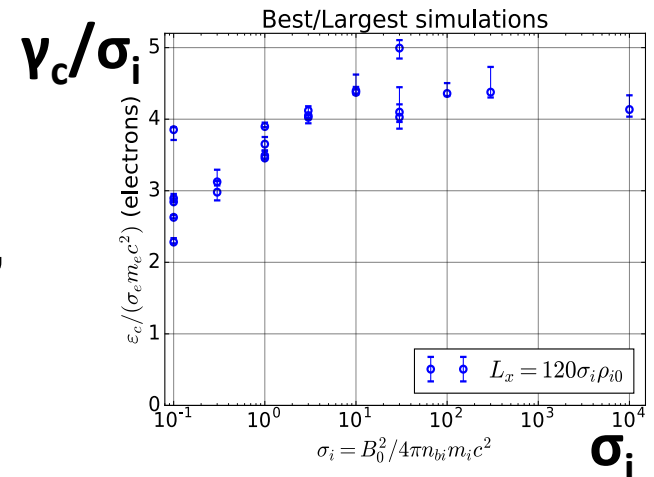
- ions ?



## Electron power-law index $p$ and cutoff $\gamma_c$ :



Observed blazar  $p_e \sim 2-3$  imply  $\sigma_i \sim 1$ , consistent with expectations from MHD jet models.



**Werner et al. 2018**

# Kinetic Equation

$$\partial_t f(\gamma, t) = -\partial_\gamma(\dot{\gamma}_{\text{acc}} f) - \frac{f(\gamma)}{\tau_{\text{magn}}(\gamma)} - \frac{f(\gamma)}{\tau_{\text{tr}}(\gamma)}$$

## Key Ingredients:

- Acceleration by main reconnection electric field:

$$\dot{\gamma}_{\text{acc}} = eE_{\text{rec}}c/m_e c^2 = \epsilon\beta_A\Omega_0$$

- independent of  $\gamma$  ...

- Magnetization by reconnected B-field (--> power-law):

$$\tau_{\text{mag}}(\gamma) \sim \ell_{\text{mag}}/c \sim \epsilon^{-1}\gamma\Omega_0^{-1}$$

- Trapping by large [ $w > \rho_L(\gamma)$ ] plasmoids ( $\rightarrow$  cutoff):

$\tau_{\text{tr}}$  controlled by plasmoid distribution function (below)

# Steady State Kinetic Equation

- Since  $\dot{\gamma}_{acc}$  is independent of  $\gamma$ , we get

$$\dot{\gamma}_{acc} \frac{df}{d\gamma} = -\frac{f(\gamma)}{\tau(\gamma)}$$

- Solution:

$$f(\gamma) = C \exp\left(-\frac{1}{\dot{\gamma}_{acc}} \int \frac{d\gamma}{\tau(\gamma)}\right)$$

where  $\frac{1}{\tau(\gamma)} = \frac{1}{\tau_{magn}(\gamma)} + \frac{1}{\tau_{trap}(\gamma)}$

# Magnetization by Reconnected Field and NTPA power law

- Energetic particle passes right through small plasmoids.
- Distance a particle travels before being magnetized

$$l_{\text{mag}}(\gamma) \sim \rho_L(\gamma, B_1) = (B_0/B_1) \rho_L(\gamma, B_0) = \epsilon^{-1} \rho_0 \gamma$$

- magnetization time-scale  $\tau_{\text{mag}}(\gamma) \sim l_{\text{mag}}/c \sim \epsilon^{-1} \gamma \Omega_0^{-1}$

- Balance magnetization with acceleration in kinetic eqn:

- power-law solution  $f(\gamma) \sim \gamma^{-p}$

- power-law index:  $p = p(\sigma_h) \sim \frac{1}{\beta_A} = \sqrt{\frac{1 + \sigma_h}{\sigma_h}}$

– ultra-rel ( $\sigma_h \gg 1$ ):  $p \rightarrow \text{const}$  (cf. Zenitani & Hoshino 2001)

– non-rel. case ( $\sigma_h \gg 1$ ):  $p \sim \sigma_h^{-1/2}$  (c.f., Werner et al. 2018)

# Plasmoid Chain I: single power-law

- Energetic particles can be trapped in *large* plasmoids when  $w = w(\gamma) = \rho_L(\gamma) = \rho_0 \gamma$  ( $\rho_0 = m_e c^2 / e B_0$ )
- **High-energy cutoff** is controlled by plasmoid distribution function  $F(w) = - dN/dw$ .

- $\tau_{tr} = \lambda_{pl}(w)/c$

where  $\lambda_{pl}(w)$  is separation between plasmoids of size  $w$ :

- $\lambda_{pl}(w) = L/N(w)$
- Thus  $\tau_{tr} = L/c N(w)$

# Single-Power-law plasmoid chain

- Consider for illustration:  $F(w) \sim w^{-\alpha}$  for  $w < w_{\max}$
- Cumulative distribution:  $N(w) \sim (w/w_{\max})^{1-\alpha}$

[where  $N(w_{\max}) = 1$ ]

$$\tau_{\text{trap}}^{-1}(\gamma) \sim \frac{c}{L} N[w(\gamma)] \sim \frac{c}{L} \left( \frac{\gamma}{\gamma_{\max}} \right)^{1-\alpha}, \quad \alpha \neq 1$$

where  $\gamma_{\max} = w_{\max}/\rho_0$

- Special case  $\alpha = 2$ :  $\tau_{\text{trap}} \sim \gamma$  (same as for magnetization - later)
- Trapping rate overtakes magnetization at

$$\gamma_c = \gamma_{\max} \left( \frac{w_{\max}}{\epsilon L} \right)^{\frac{1}{\alpha-2}}.$$

- For  $w_{\max} \sim \epsilon L$ :  $\gamma_c \sim \gamma_{\max}$



# Establishing Cutoff

- Balancing acceleration against trapping in plasmoids:

$$f(\gamma) \sim \exp \left[ -\frac{w_{\max}}{\epsilon \beta_A L (2 - \alpha)} \left( \frac{\gamma}{\gamma_{\max}} \right)^{2 - \alpha} \right]$$

- Special case  $\alpha \sim 1$  (ignoring log-corrections)

$$f(\gamma) \sim \exp \left[ -\frac{\rho_0}{L} \frac{1}{\beta_A \epsilon} \gamma \right] = \exp(-\gamma / \gamma_{c1})$$

- where  $\gamma_{c1} \equiv \beta_A \epsilon \frac{L}{\rho_0} \simeq \beta_A \frac{w_{\max}}{\rho_0}$  (can be  $< \gamma_{\max}$ , if  $\beta_A < 1$ )

- Special case  $\alpha=2$ : no cutoff but another power-law:

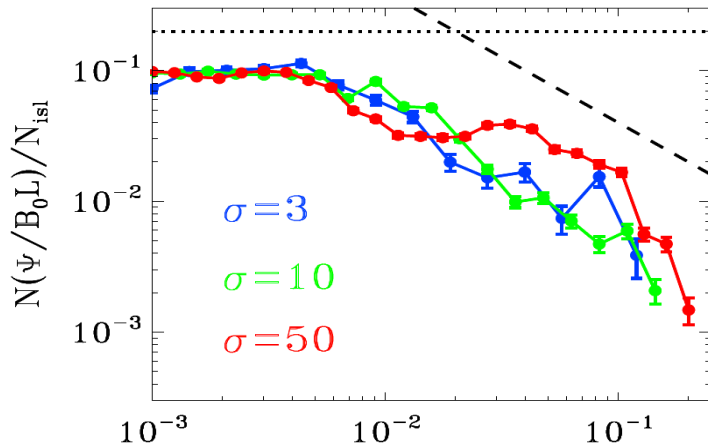
$$p \simeq \frac{1}{\epsilon \beta_A} \frac{w_{\max}}{L}$$

- For  $w_{\max} \sim \epsilon L$ :  $p \sim \beta_A^{-1}$  (~same as without plasmoids)
- Combined with magnetization:  $\rightarrow$  steeper (by x2) power law.

# Plasmoid Chain II:

## Realistic double-power-law

- Real simulations show double-power-law plasmoid distributions (Loureiro et al. 2012, Huang et al. 2013, Sironi et al. 2016, Petropoulou et al. 2018)



$$F \sim w^{-\alpha_1}, \quad w < w_c$$

$$F \sim w^{-\alpha_2}, \quad w_c < w < w_{\max}$$

$$N(w < w_c) = N_c \left( \frac{w}{w_c} \right)^{1-\alpha_1}$$

$$N(w > w_c) = \left( \frac{w}{w_{\max}} \right)^{1-\alpha_2}$$

Most likely:  $\alpha_1 \approx 1$ ,  $\alpha_2 \approx 2$

$$\gamma_{\text{cutoff}} = \gamma_c \left( \frac{w_c}{\epsilon \lambda_c} \right)^{\frac{1}{\alpha_1 - 2}} \sim \gamma_c = w_c / \rho_0$$

For  $\alpha_1 \approx 1$ :  $\gamma_{\text{cutoff}} = \gamma_c \left( \frac{\epsilon \lambda_c}{w_c} \right) \sim \gamma_c$

Important Question:

**What controls  $w_c$ ?**

**If  $\alpha_2 \approx 2$ : possible 2<sup>nd</sup> (steeper) power law above spectral break at  $\gamma_c$**

# Important Question:

## What controls $w_c$ ?

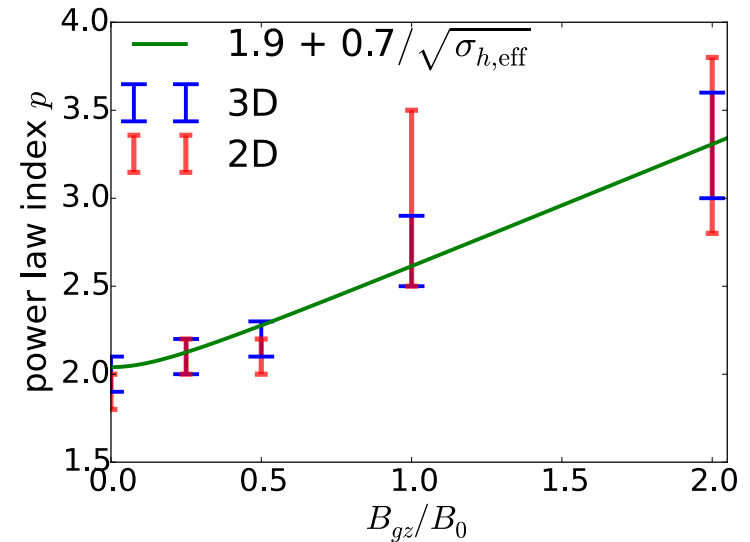
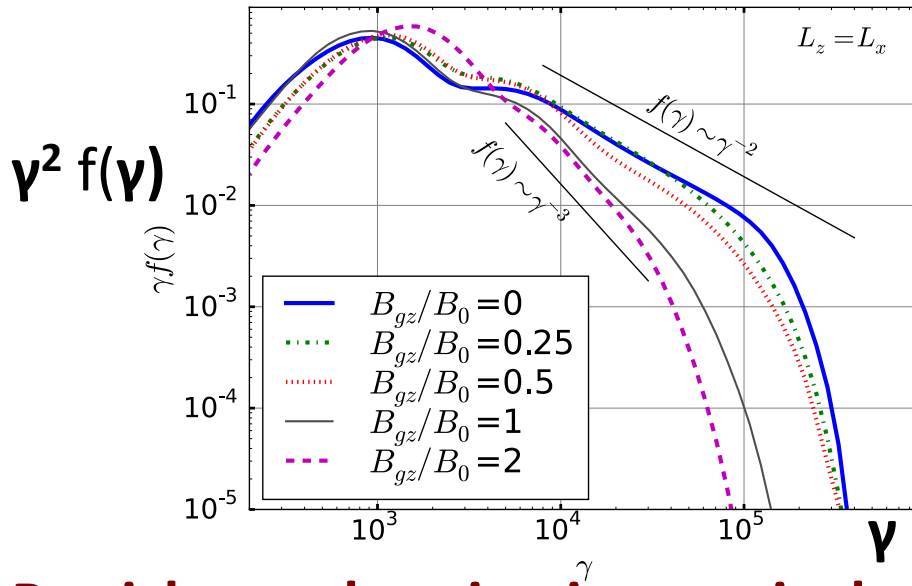
- Consider large-system regime:  $L \gg \delta \sim \langle \rho \rangle \sim \sigma \rho_0$
- The plasmoid distribution break size  $w_c$  may be anywhere between microscopic  $\sim \sigma \rho_0$  and macroscopic  $\sim L$ .
- If  $w_c \sim \sigma \rho_0$ , then  $\gamma_c \sim \sigma$ , e.g.,  $\gamma_c \approx 4\sigma$  (Werner et al. 2016)
- If  $w_c \sim L$ , then  $\gamma_c \sim L/\rho_0$  -- “extreme” (Hillas) acceleration limit.

# Effect of Guide Magnetic Field

(Werner & Uzdensky 2017 ApJL)

Particle spectra for different  $B_z/B_0$ :

$L_z/L_x = 1$



**Particle acceleration is negatively affected by strong guide field.**

Explanation: Guide field's *inertia*: Guide-field needs to be advected out of the layer with the plasma, so guide field's inertia contributes  $B_{gz}^2/4\pi$  to the enthalpy in the denominator of  $\sigma_h$ :

$$\sigma_{h,\text{eff}} = B_0^2 / (B_{gz}^2 + 4\pi nh)$$

-- reduces in-plane  $V_A$ , hence reconnection outflow speed, hence  $E_{\text{rec}}$

# Conclusions

Relativistic Nonthermal Particle Acceleration (NTPA) in reconnection is an interplay of:

– steady acceleration by reconnection electric field;

checked by “escape” from acceleration zone:

- (1) magnetization by general reconnected magnetic field  $B_1 \sim 0.1 B_0 \rightarrow$  power-law index

$$p \sim (E_{\text{rec}}/B_1)^{-1} \sim 1/\beta_A \sim [1+\sigma_h]/\sigma_h^{1/2}$$

- (2) capture/trapping by plasmoids with  $w \sim \rho_L(\gamma) = \gamma \rho_0$ .  
 $\rightarrow$  high-energy cutoff.

- Cutoff depends on plasmoid-distribution function, e.g., for  $\alpha=1$  it is simple exponential with  $\gamma_c = w_{\text{max}}/\rho_0$ .

- Realistic double-power-law  $F(w)$ : cutoff at break size  $w_c$  and

- possible steeper power law (if  $\alpha_2 \approx 2$ ) above  $w_c$

• Guide field's inertia  $B_{gz}^2/4\pi$  adds to enthalpy, reduces effective  $\sigma_h$