Relativistic Accretion onto Neutron Stars

and some things about black hole magnetospheres

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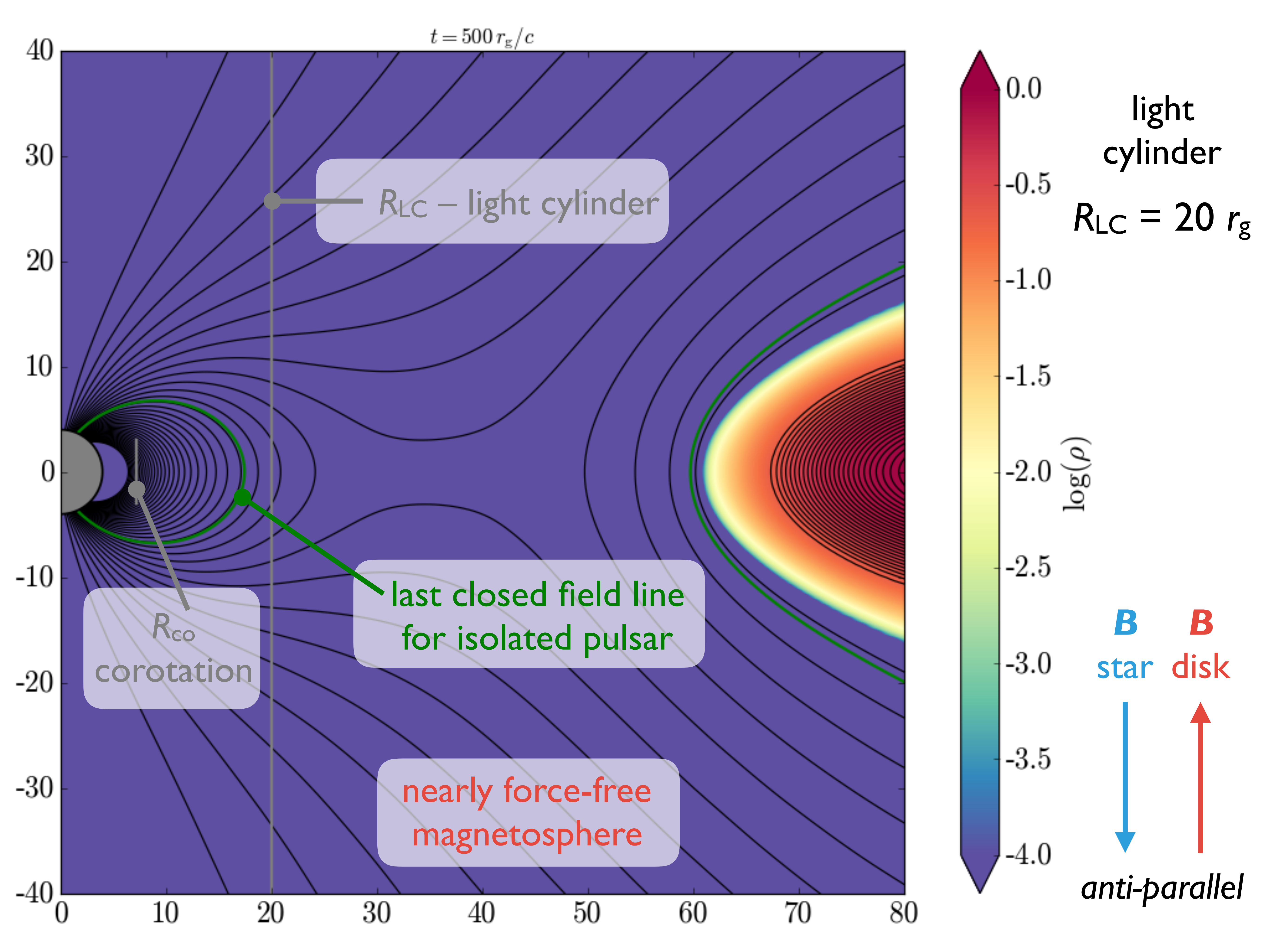
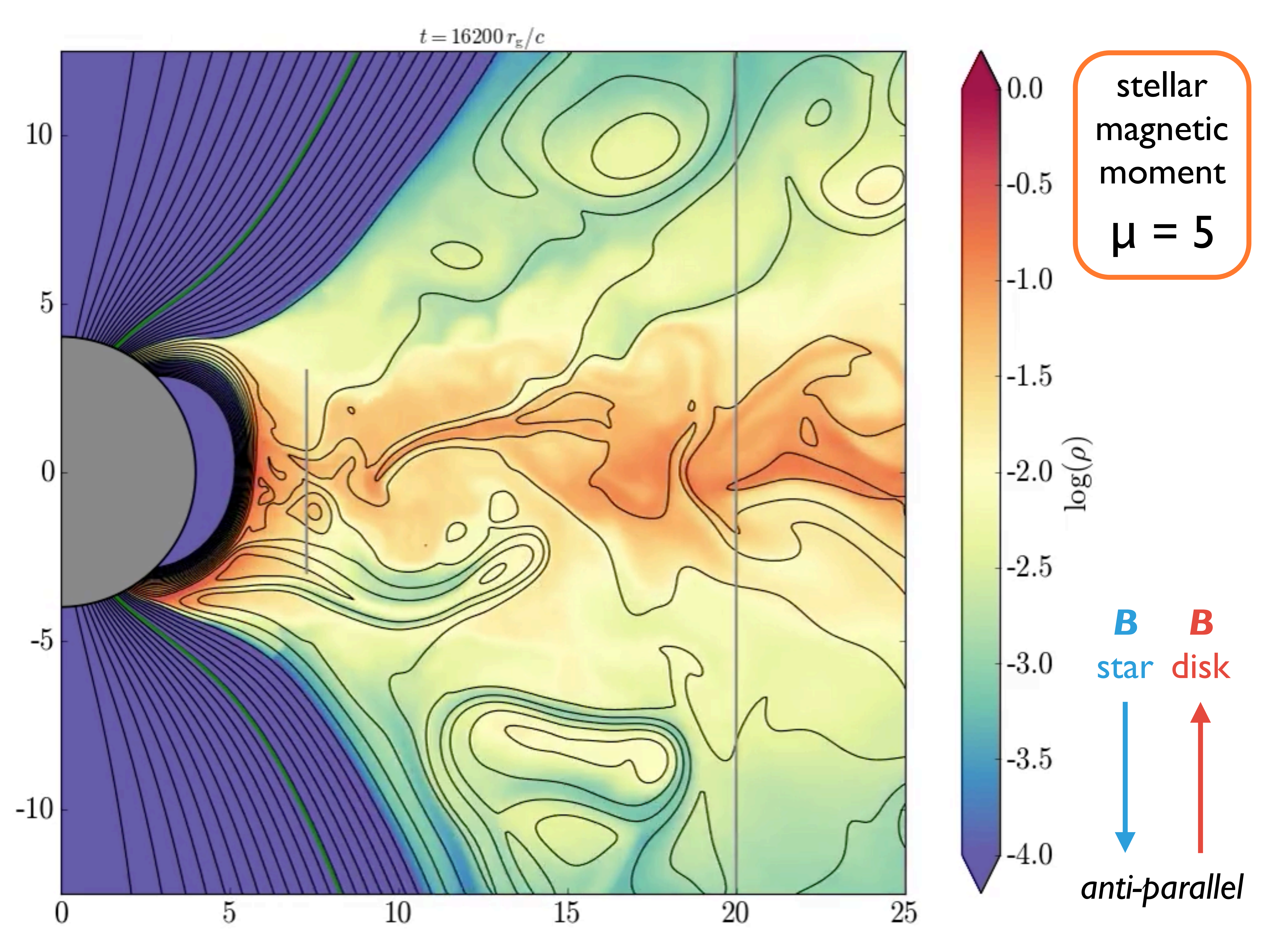
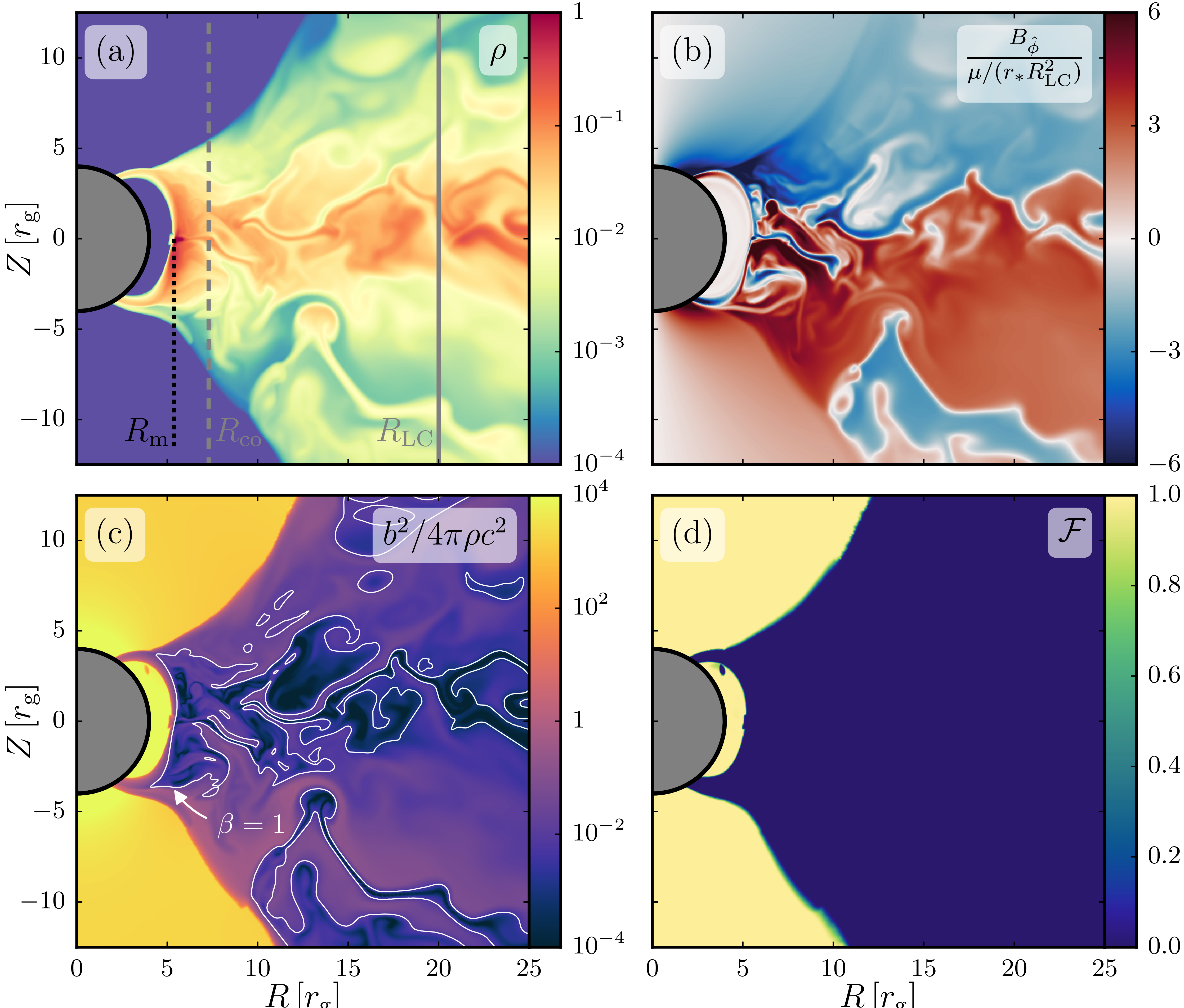


$t = 500\, r_g/c$
R_{LC} – light cylinder
last closed field line
for isolated pulsar
R_{co}
corotation
nearly force-free
magnetosphere
log(ρ)
light
cylinder
R_{LC} = 20 r_g
B
star
B
disk
anti-parallel

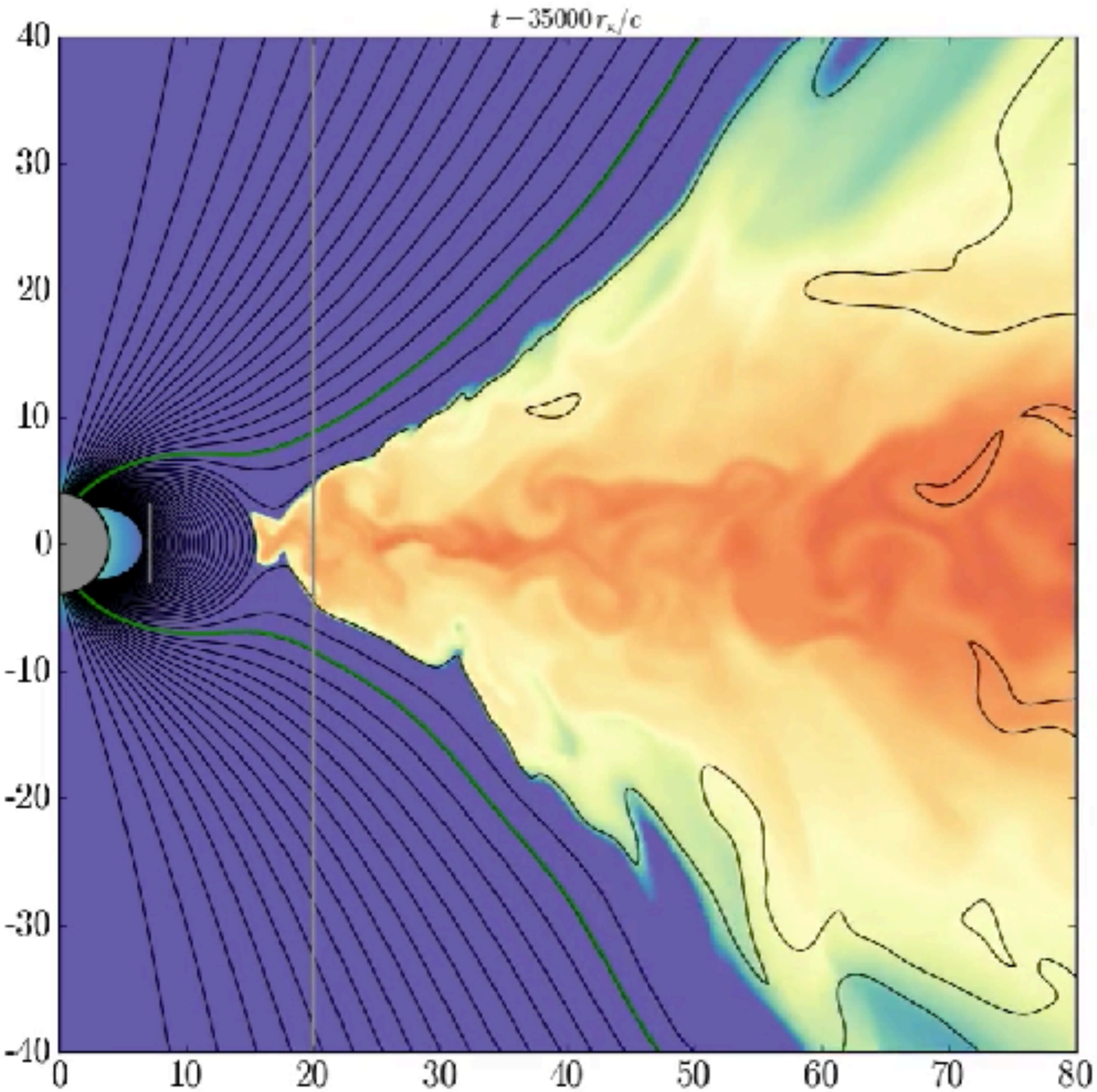


$t = 16200\, r_g/c$
10
5
0
-5
-10
0
5
10
15
20
25
0.0
-0.5
-1.0
-1.5
-2.0
-2.5
-3.0
-3.5
-4.0
$\log(\rho)$
stellar magnetic moment
μ = 5
B star
B disk
anti-parallel

KP & Tchekhovskoy 2017

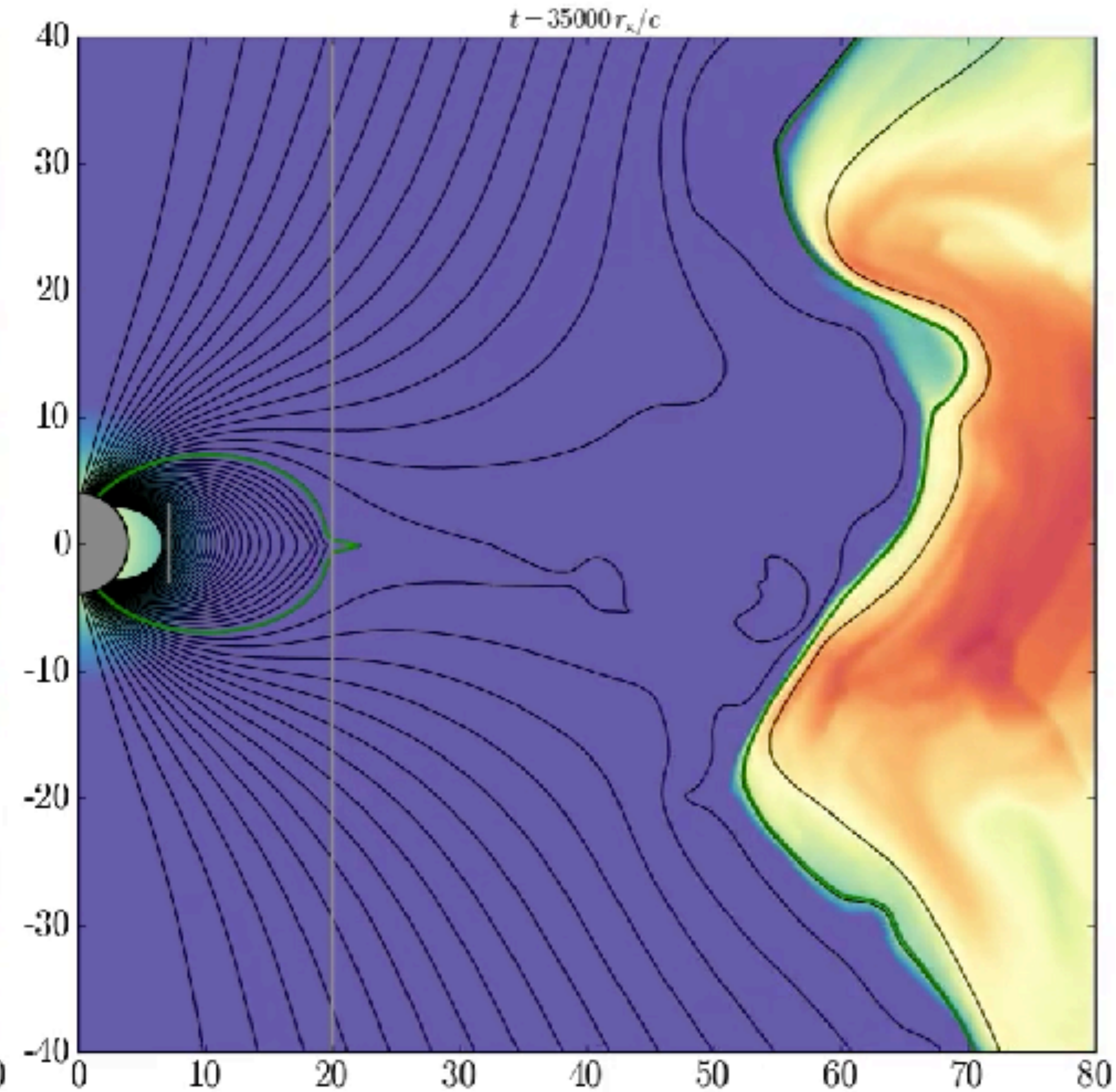


μ = 80

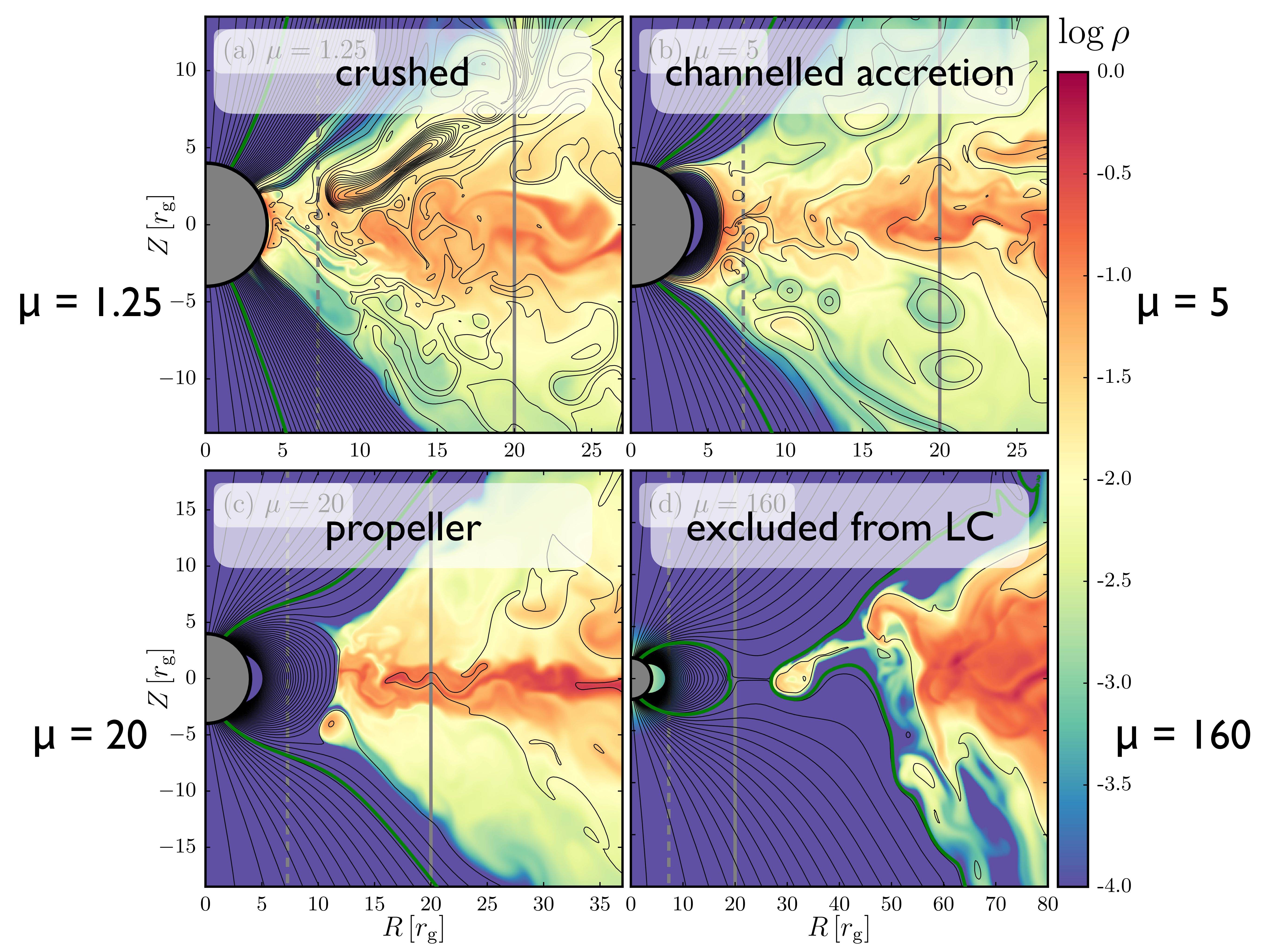


40
30
20
10
0
-10
-20
-30
-40
0
10
20
30
40
50
60
70
80

μ = 160



40
30
20
10
0
-10
-20
-30
-40
0
10
20
30
40
50
60
70
80



(a) $\mu = 1.25$
crushed
(b) $\mu = 5$
channelled accretion
(c) $\mu = 20$
propeller
(d) $\mu = 160$
excluded from LC
log ρ
0.0
-0.5
-1.0
-1.5
-2.0
-2.5
-3.0
-3.5
-4.0
μ = 1.25
μ = 5
μ = 20
μ = 160
$Z\,[r_g]$
$R\,[r_g]$

Average values of:

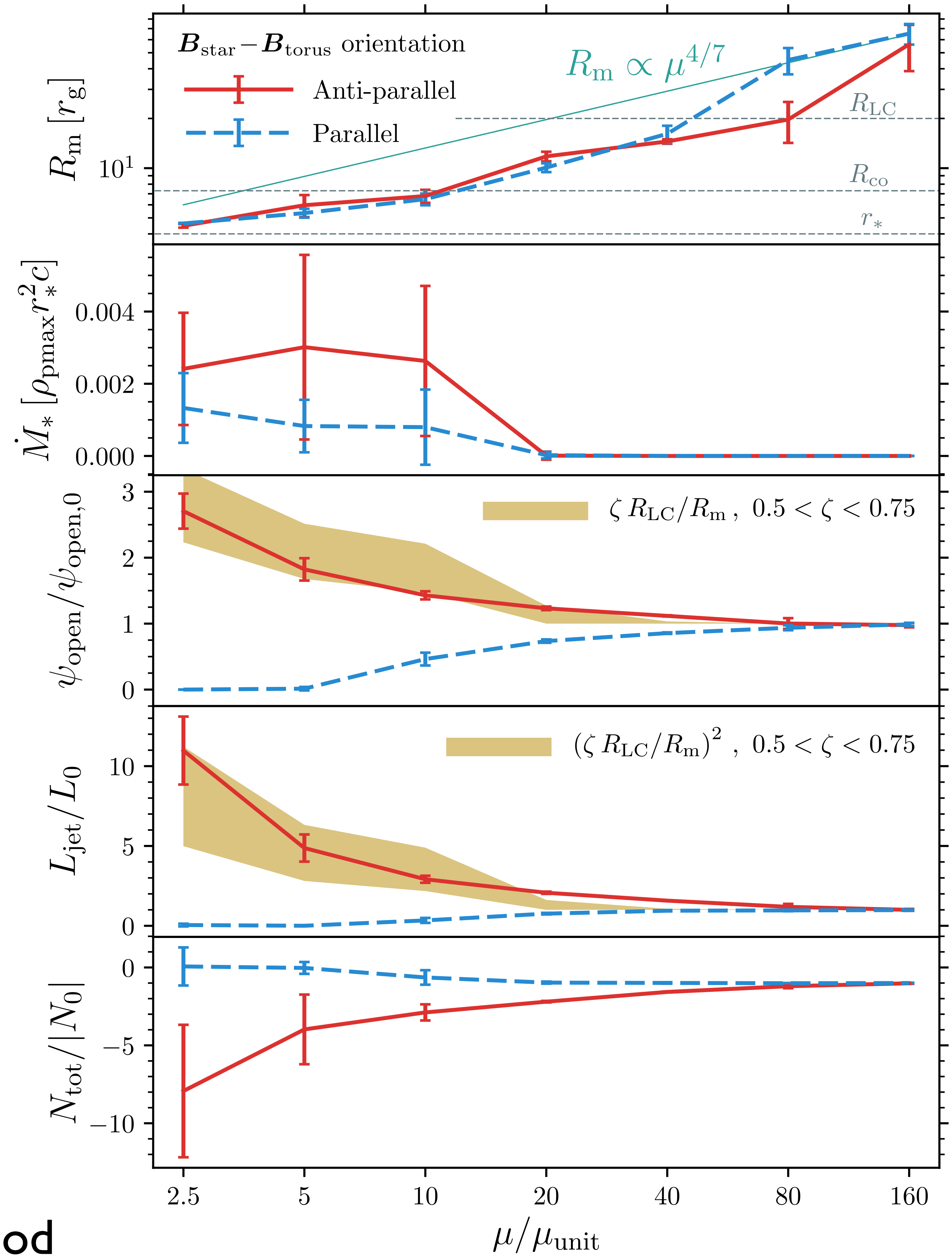
magnetospheric radius

accretion rate

open flux

jet power

stellar torque



KP & Tchekhovskoy 2017

vertical bars: standard deviation over averaging period

NS jet for short-GRBs in mergers?

Inferred jet power of GW170817 ~ 10^{49}-10^{50} erg/s

Simple jet model from previous slide:

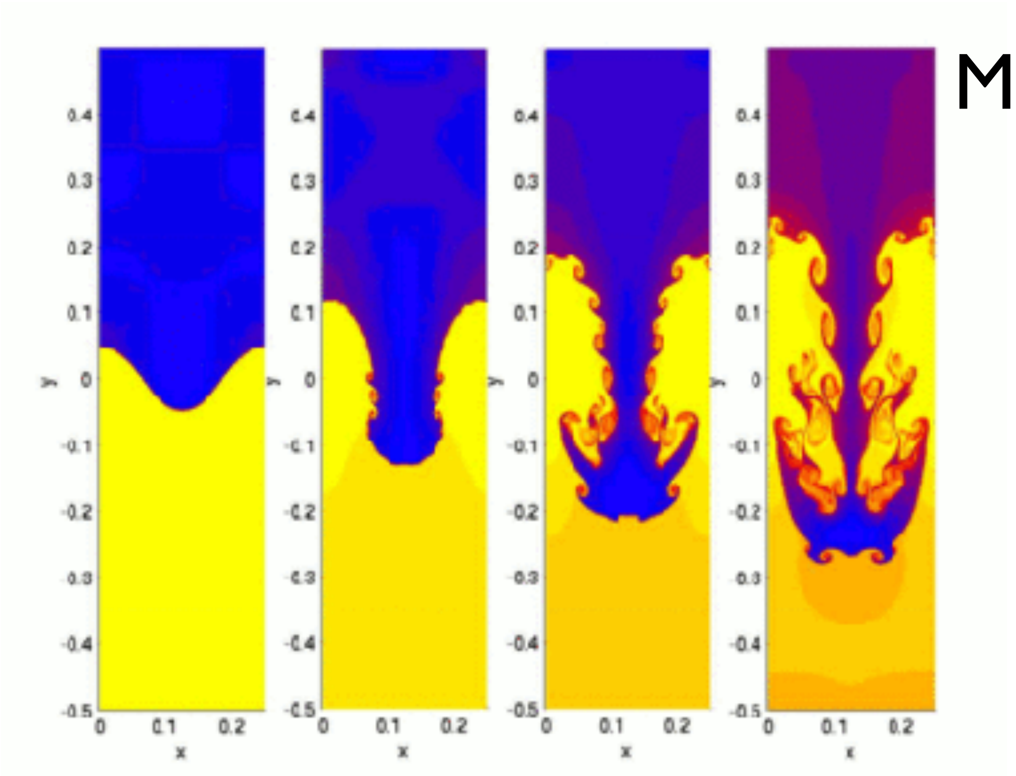
$$L_{\rm jet} \sim 10^{52} \left(\frac{B_*}{10^{15}\,{\rm G}}\right)^{6/7} \left(\frac{\dot{M}}{{\rm M}_\odot\,{\rm s}^{-1}}\right)^{4/7} \left(\frac{\nu}{\rm kHz}\right)^2 \,{\rm erg\,s}^{-1}$$

Most extreme GRMHD simulation has $L_{\rm jet} \approx 11\,L_0$

$$\rightarrow L_{\rm jet} \approx 6.4 \times 10^{50} \left(\frac{B_*}{10^{15}\,{\rm G}}\right)^2 \left(\frac{\nu}{\rm kHz}\right)^4 \,{\rm erg\,s}^{-1}$$

3D simulations

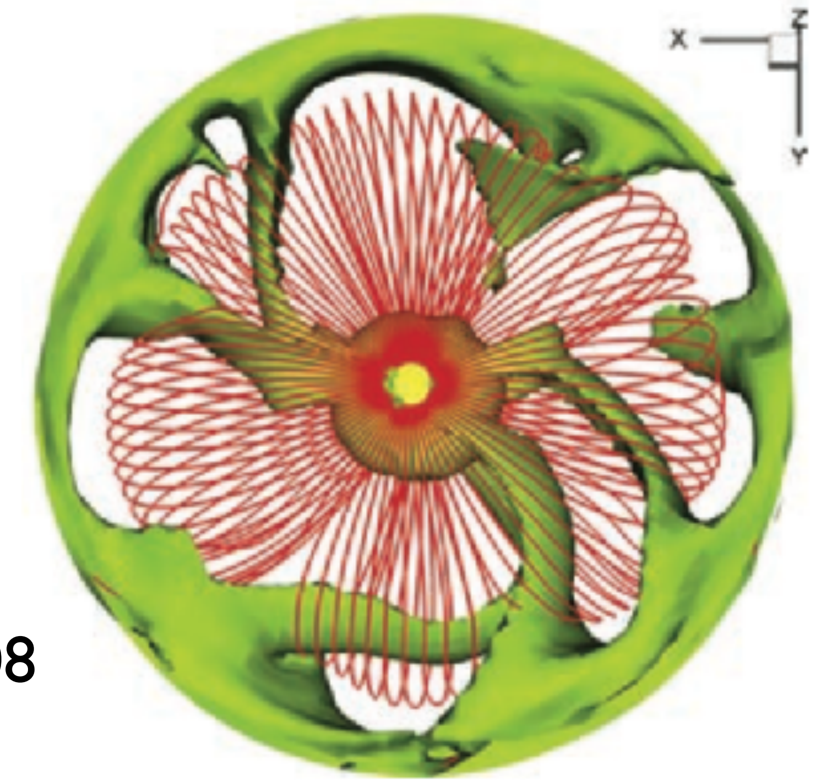
1. Need 3D for realistic MRI turbulent dynamo
2. Interchange instability: accretion through closed-field region



Magnetic Rayleigh-Taylor / Interchange

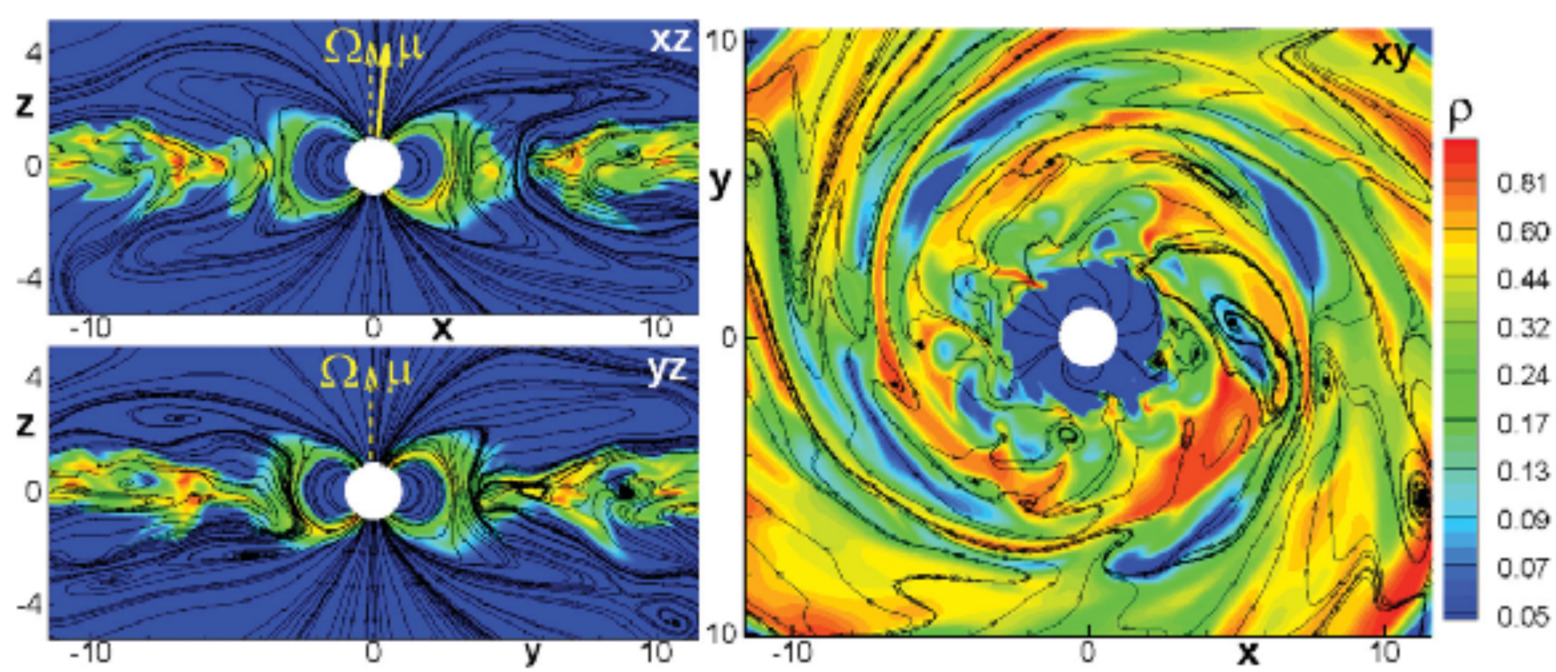
alpha-prescription resistive MHD

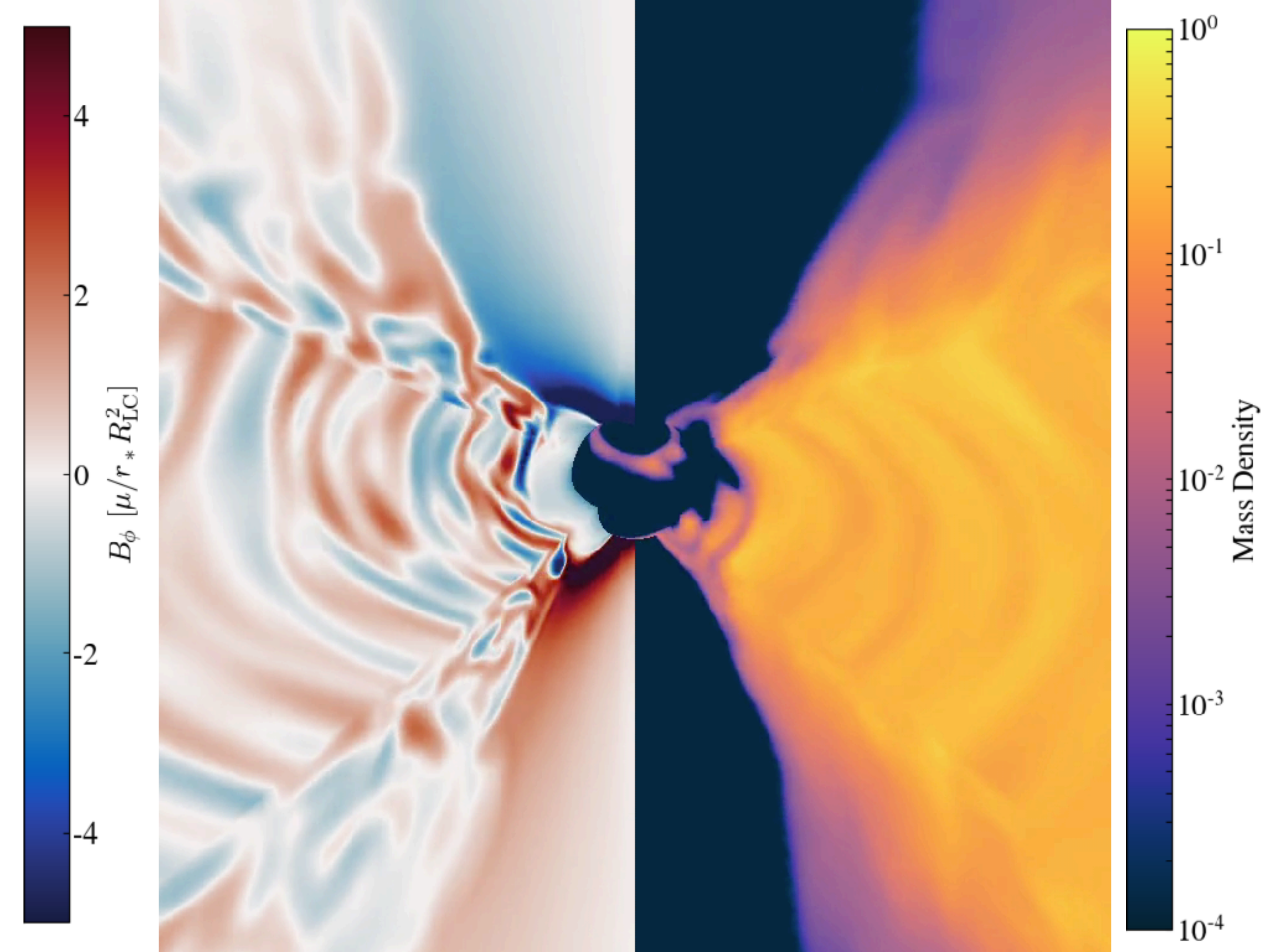
Kulkarni & Romanova 2008



Ideal MHD/MRI disc

Romanova+ 2012

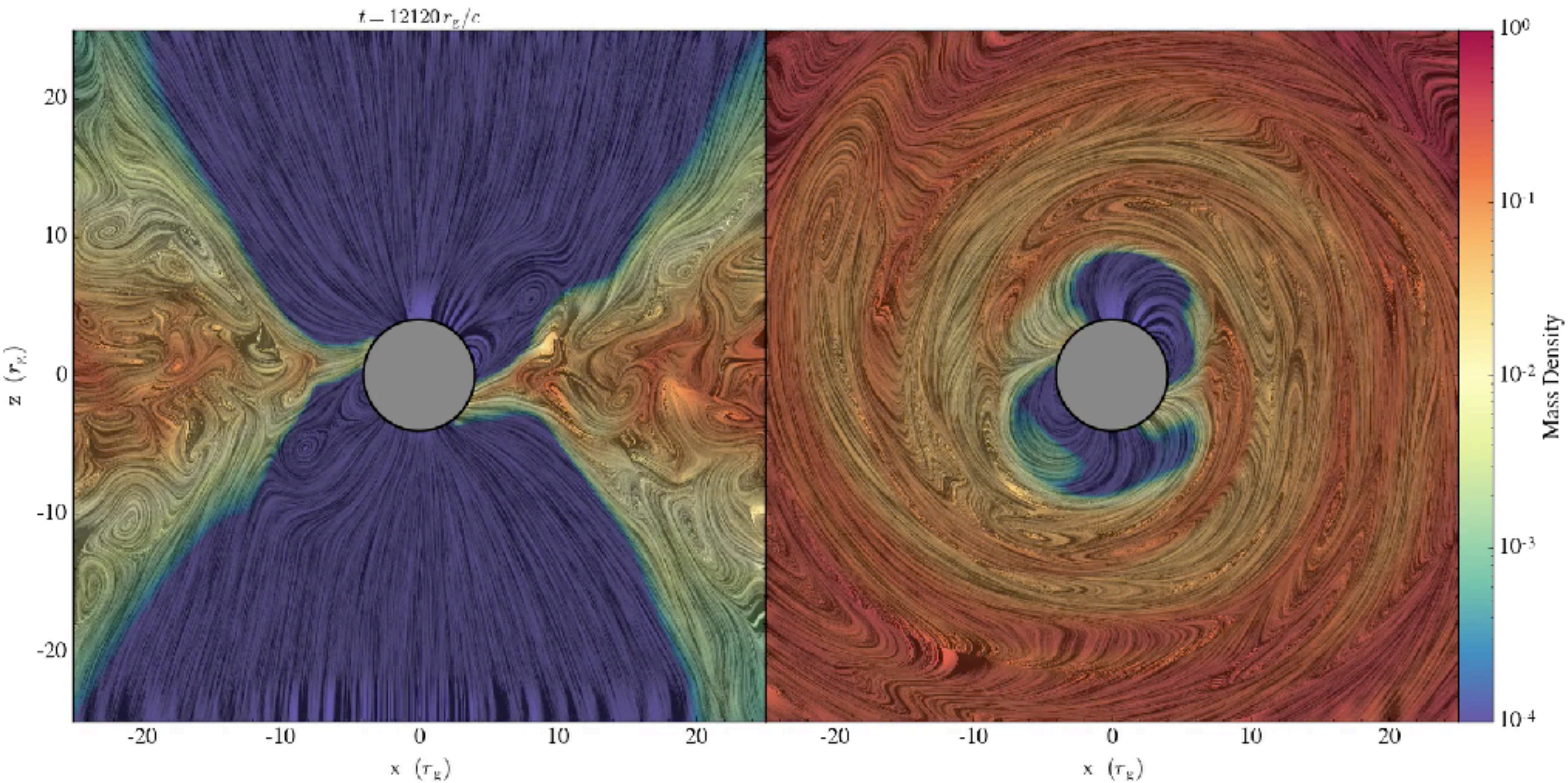




4
2
0
-2
-4
$B_\phi\ [\mu/r_*R_{\rm LC}^2]$
10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
Mass Density

stellar magnetic moment: μ = 10

$\chi = 45^\circ$



Accreting neutron stars

First (general-) relativistic simulations of pulsar accretion

Four regimes: crushed / accreting / propeller / excluded from light cylinder

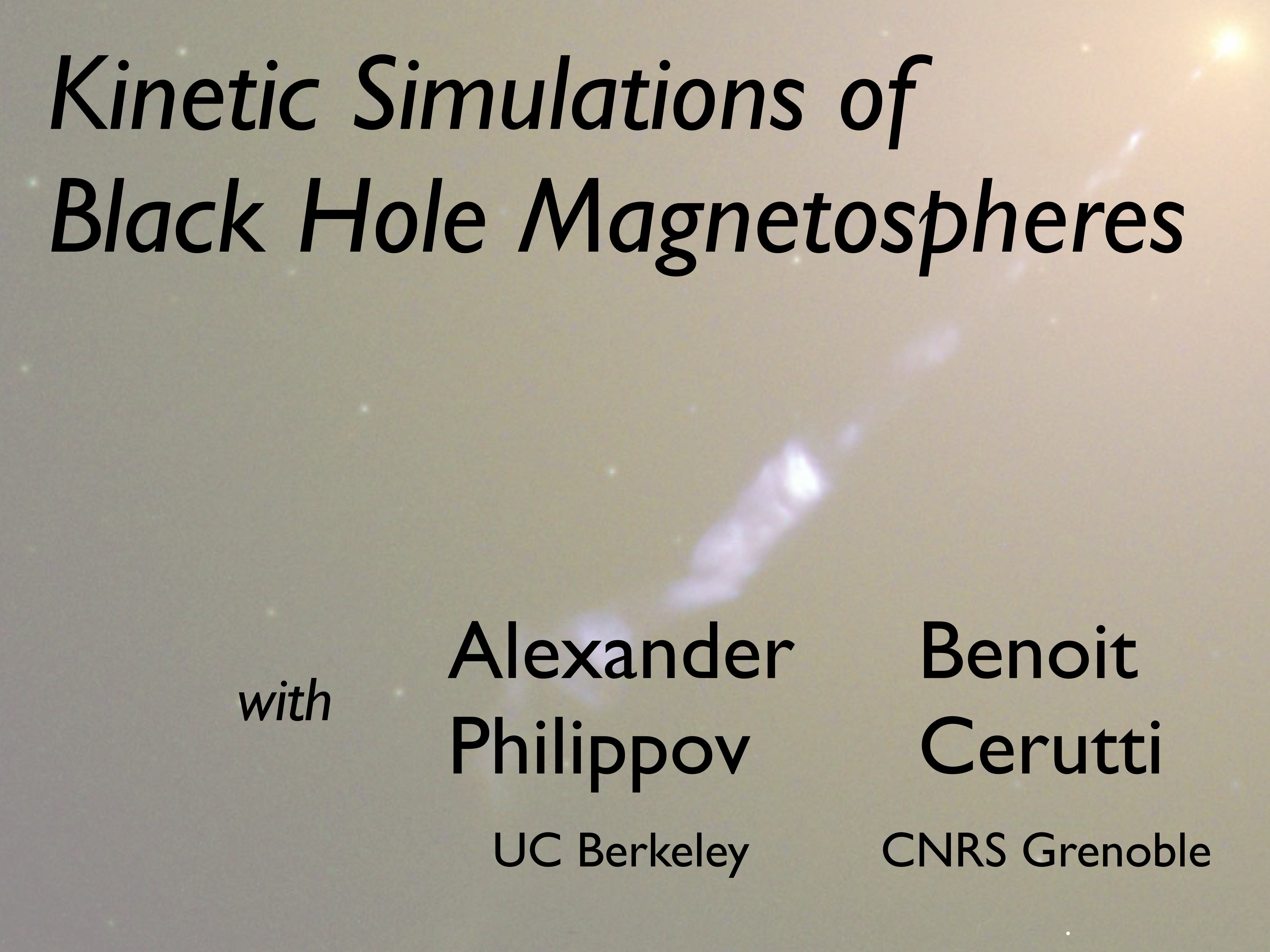
Efficient flux opening — weak star-disc magnetic coupling

— relativistic jets from millisecond pulsars

Force-free & MHD simulations support simple model for torques & jets

3D is important (realistic turbulence & interchange instability)

7 movies of axisymmetric runs on YouTube: link at **1708.06362** arXiv listing



Kinetic Simulations of Black Hole Magnetospheres

with

Alexander Philippov
UC Berkeley

Benoit Cerutti
CNRS Grenoble

Collisionless Black Hole Plasmas

Highly relativistic jets

Low-luminosity accretion

Accretion disc X-ray coronae

Electrostatic gaps

General-relativistic calculation: much of the action near the horizon

e.g. coronal X-ray sources within ~ 10 r_g

jet launching in ergosphere

gaps within a few r_g

Equations in curved spacetime: 3+1 ADM form

Maxwell's equations

$$\partial_t \boldsymbol{D} = \nabla \times \boldsymbol{H} - \boldsymbol{J} \qquad \nabla \cdot \boldsymbol{D} = \rho_{\mathrm{e}}$$

$$\partial_t \boldsymbol{B} = -\nabla \times \boldsymbol{E} \qquad \nabla \cdot \boldsymbol{B} = 0$$

Constitutive relations:

$$\boldsymbol{E} = \alpha \boldsymbol{D} + \boldsymbol{\beta} \times \boldsymbol{B}$$

$$\boldsymbol{H} = \alpha \boldsymbol{B} - \boldsymbol{\beta} \times \boldsymbol{D}$$

Particle equations of motion

$$\frac{\mathrm{d}x^i}{\mathrm{dt}} = \frac{\alpha}{m\Gamma} p^i - \beta^i$$

$$\frac{\mathrm{d}p_i}{\mathrm{d}t} = -m\Gamma \partial_i \alpha + p_j \partial_i \beta^j - \frac{\alpha}{2\Gamma m} \partial_i (\gamma^{lm}) p_l p_m + q \left\{ \alpha D_i + \epsilon_{ijk} (v^j + \beta^j) B^k \right\}$$

GRPIC 1. – fields

$$\partial_t \boldsymbol{D} = \nabla \times \boldsymbol{H} - \boldsymbol{J}$$

$$\partial_t \boldsymbol{B} = -\nabla \times \boldsymbol{E}$$

3+1 ADM form

$$\nabla \cdot \boldsymbol{D} = \rho_{\mathrm{e}}$$

$$\nabla \cdot \boldsymbol{B} = 0$$

$$\boldsymbol{E} = \alpha \boldsymbol{D} + \boldsymbol{\beta} \times \boldsymbol{B}$$

$$\boldsymbol{H} = \alpha \boldsymbol{B} - \boldsymbol{\beta} \times \boldsymbol{D}$$

Komissarov 2004

particles determine current density $\boldsymbol{J}$:

$$\boldsymbol{J} = \alpha \boldsymbol{j} - \rho \boldsymbol{\beta}$$

set directly by $v^i = u^i/u^t$

FIDO-measured current density

GRPIC 1. – fields

$$\partial_t \boldsymbol{D} = \nabla \times \boldsymbol{H} - \boldsymbol{J}$$

$$\partial_t \boldsymbol{B} = -\nabla \times \boldsymbol{E}$$

3+1 ADM form

$$\nabla \cdot \boldsymbol{D} = \rho_{\mathrm{e}}$$

$$\nabla \cdot \boldsymbol{B} = 0$$

$$\boldsymbol{E} = \alpha \boldsymbol{D} + \boxed{\boldsymbol{\beta} \times \boldsymbol{B}}$$

$$\boldsymbol{H} = \alpha \boldsymbol{B} - \boxed{\boldsymbol{\beta} \times \boldsymbol{D}}$$

Komissarov 2004

particles determine current density $\boldsymbol{J}$: $\boldsymbol{J} = \alpha \boldsymbol{j} - \rho \boldsymbol{\beta}$

set directly by $v^i = u^i/u^t$ (→ $\boldsymbol{J}$)

FIDO-measured current density (→ $\boldsymbol{j}$)

Start of timestep, know:

$B^n, D^{n+1/2}$ and $B^{n-1}, D^{n-1/2}$

p^n, x^n

(i) evolve ***B***

"Trapezoidal leapfrog" for shift-dependent term

$$\bar{B}^{n-1/2} = \frac{1}{2}(B^n + B^{n-1})$$

$$\bar{D}^n = \frac{1}{2}(D^{n+1/2} + D^{n-1/2})$$

$$\tilde{E}^n = \alpha \bar{D}^n + \beta \times B^n$$

$$\tilde{B}^{n+1/2} = \bar{B}^{n-1/2} - \Delta t \nabla \times \tilde{E}^n$$

Use half-step auxiliary *B* to find final ***B***.

$$E^{n+1/2} = \alpha D^{n+1/2} + \beta \times \tilde{B}^{n+1/2}$$

$$B^{n+1} = B^n - \Delta t \nabla \times E^{n+1/2}$$

(ii) particles

$$\bar{B}^{n+1/2} = \frac{1}{2}(B^n + B^{n+1})$$

$p^n, x^n \longrightarrow p^{n+1}, x^{n+1}$ using $\bar{B}^{n+1/2}, D^{n+1/2}$

$$J^{n+1} = J(x^{n+1}, p^{n+1})$$

(iii) evolve ***D***

Find $D^{n+3/2}$ using J^{n+1} & the trapezoidal leapfrog for $\beta \times D$

- Goes to (energy conserving) leapfrog as $\beta \rightarrow 0$
- Gives better energy conservation for nonzero shift
- One particle push per timestep

GRPIC 2. – particles

Start from Hamiltonian: $H = \pi_i v^i - L$

conjugate momentum $\pi_i = p_i + qA_i$

with kinetic momentum p_i and $v^i = \frac{\mathrm{dx^i}}{\mathrm{dt}}$

Lagrangian $L = -m\alpha/\Gamma + qA_j v^j + qA_t$

Hamilton's equations give

$$\frac{\mathrm{d}x^i}{\mathrm{d}t} = \frac{\alpha}{m\Gamma}p^i - \beta^i$$

$$\frac{\mathrm{d}p_i}{\mathrm{d}t} = -m\Gamma\partial_i\alpha + p_j\partial_i\beta^j - \frac{\alpha}{2\Gamma m}\partial_i(\gamma^{lm})p_l p_m + q\left\{\alpha D_i + \epsilon_{ijk}(v^j + \beta^j)B^k\right\}$$

gravitational acceleration

~ extrinsic curvature

Lorentz force

Solve entirely in spherical coordinates

How should you solve these things?

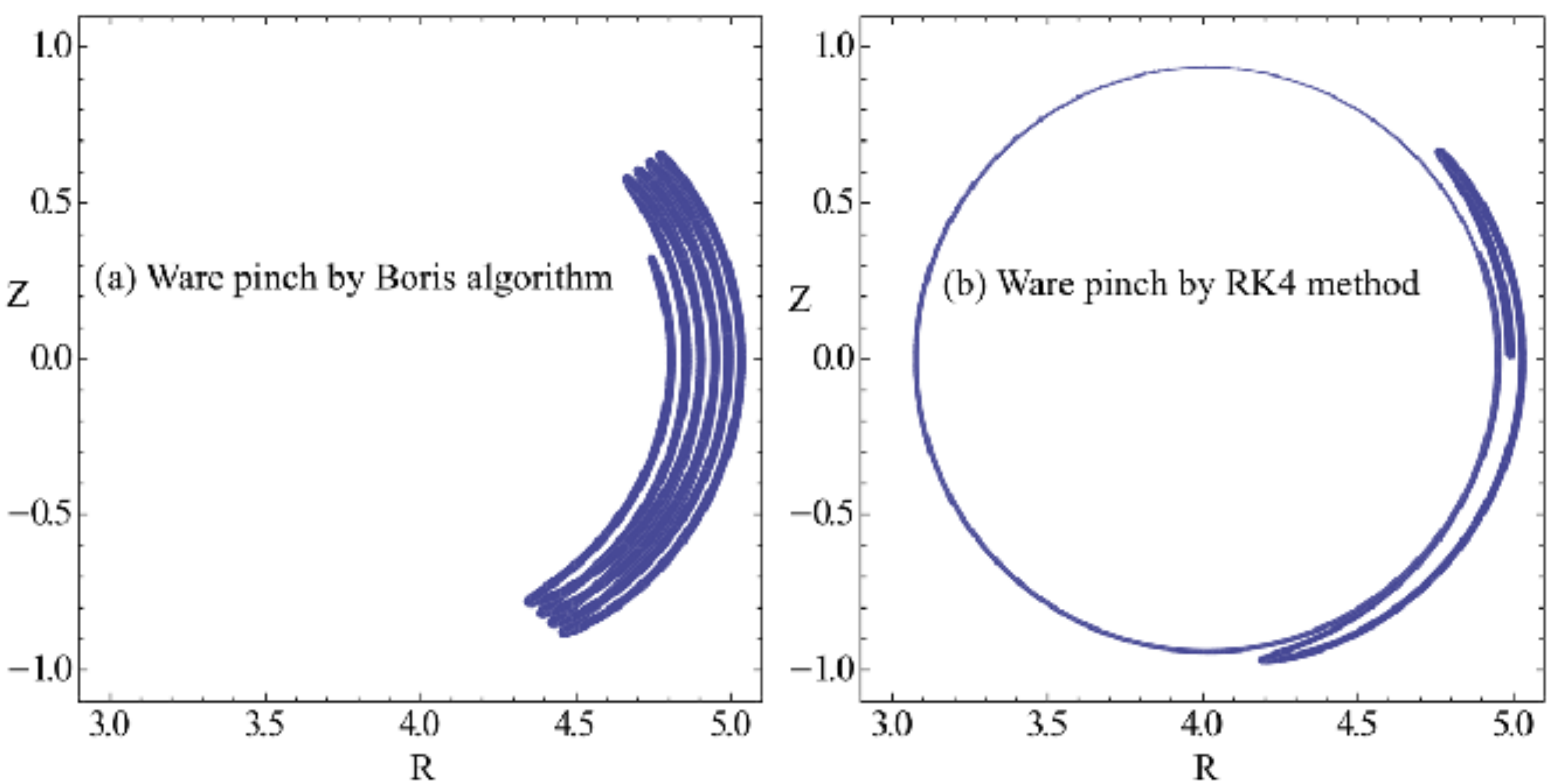
Ideally want **symplectic** integrator

preserves symplectic two-form: $s_{\mu\nu} = x_\mu \wedge p_\nu$

very good energy stability

Flat spacetime: often use **Boris push** for Lorentz force

not symplectic, but volume preserving: $|x_\mu \wedge p_\nu|$ is maintained



Qin+ 2013

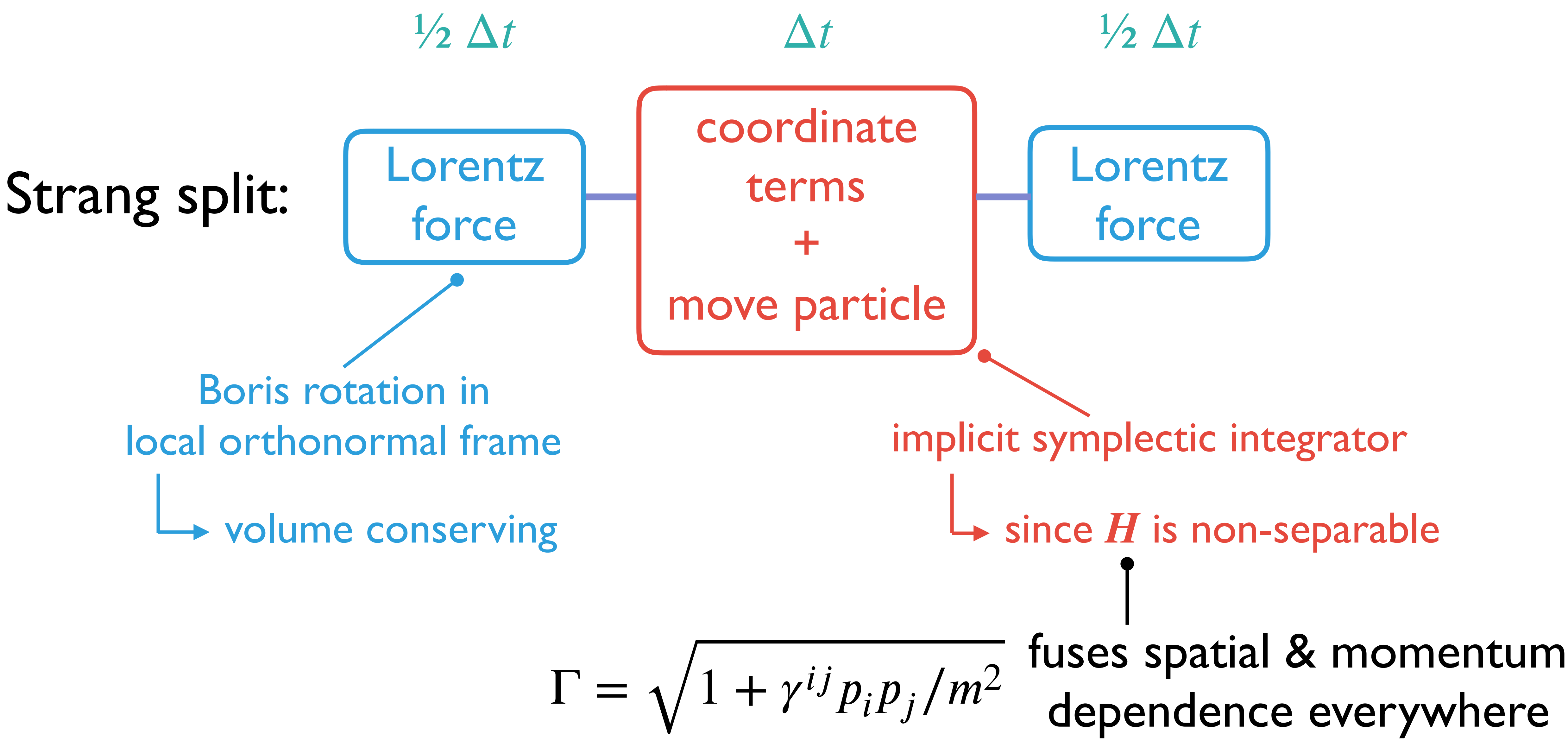
"Why is Boris Algorithm So Good?"

Particle integrator scheme

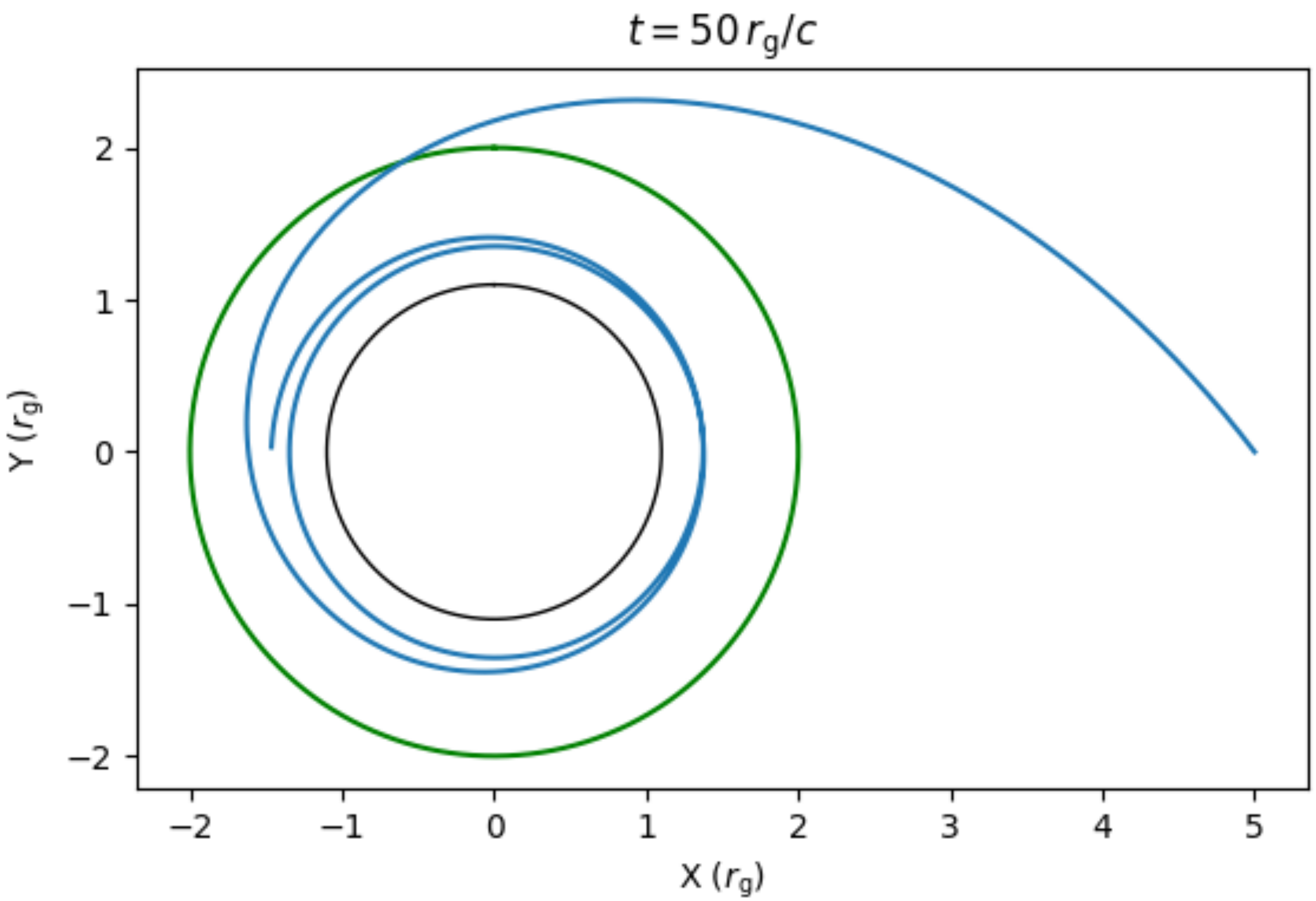
Requirements:

1. conserve phase-space volume, $|\boldsymbol{x} \wedge \boldsymbol{p}|$

2. time-symmetric

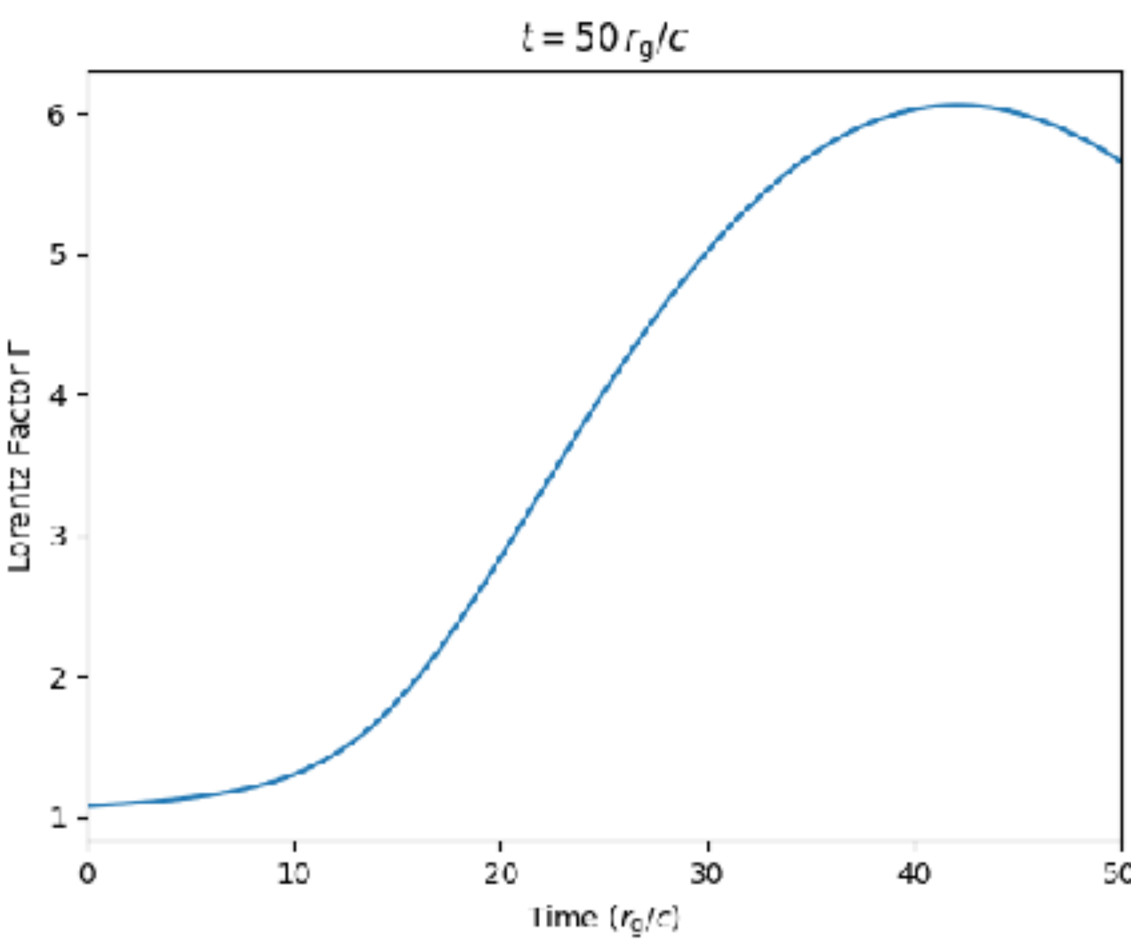


Geodesic tests — $\boldsymbol{B} = 0$



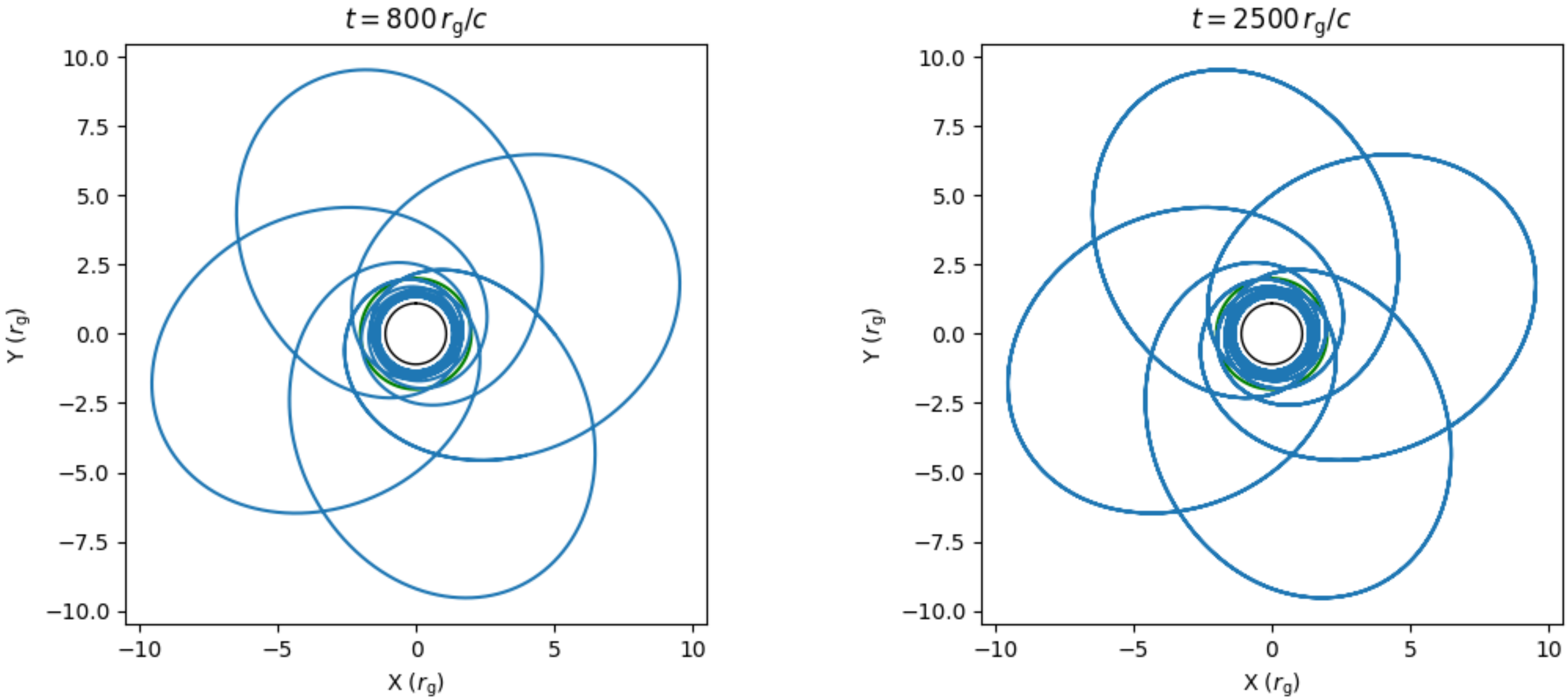
Kerr black hole
$a = 0.995$

Prograde orbit w/
$L = u_\varphi = 2$
$E = -u_t = 0.915082$



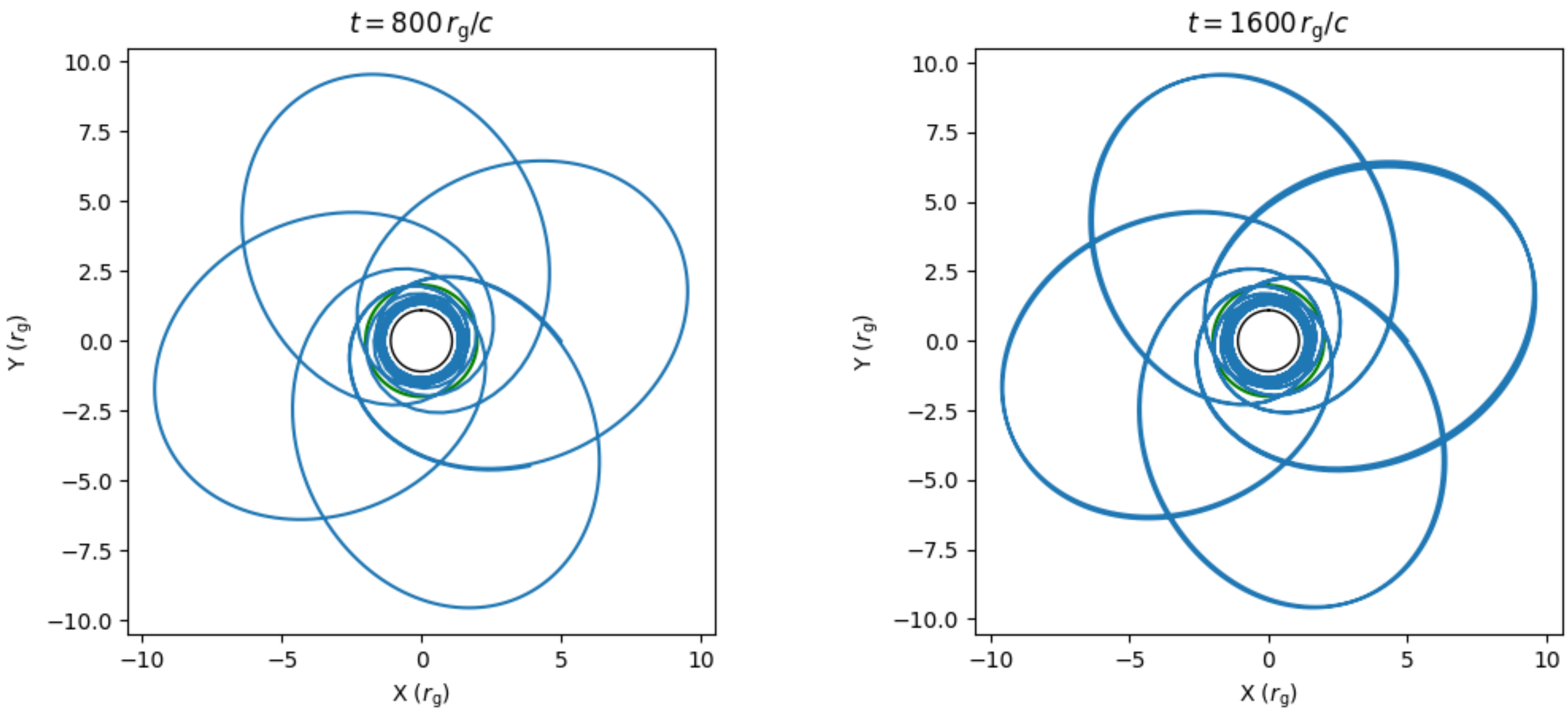
Symplectic integrator (implicit midpoint)
with dt = 0.1 r_g/c

Geodesic tests — $\boldsymbol{B} = 0$

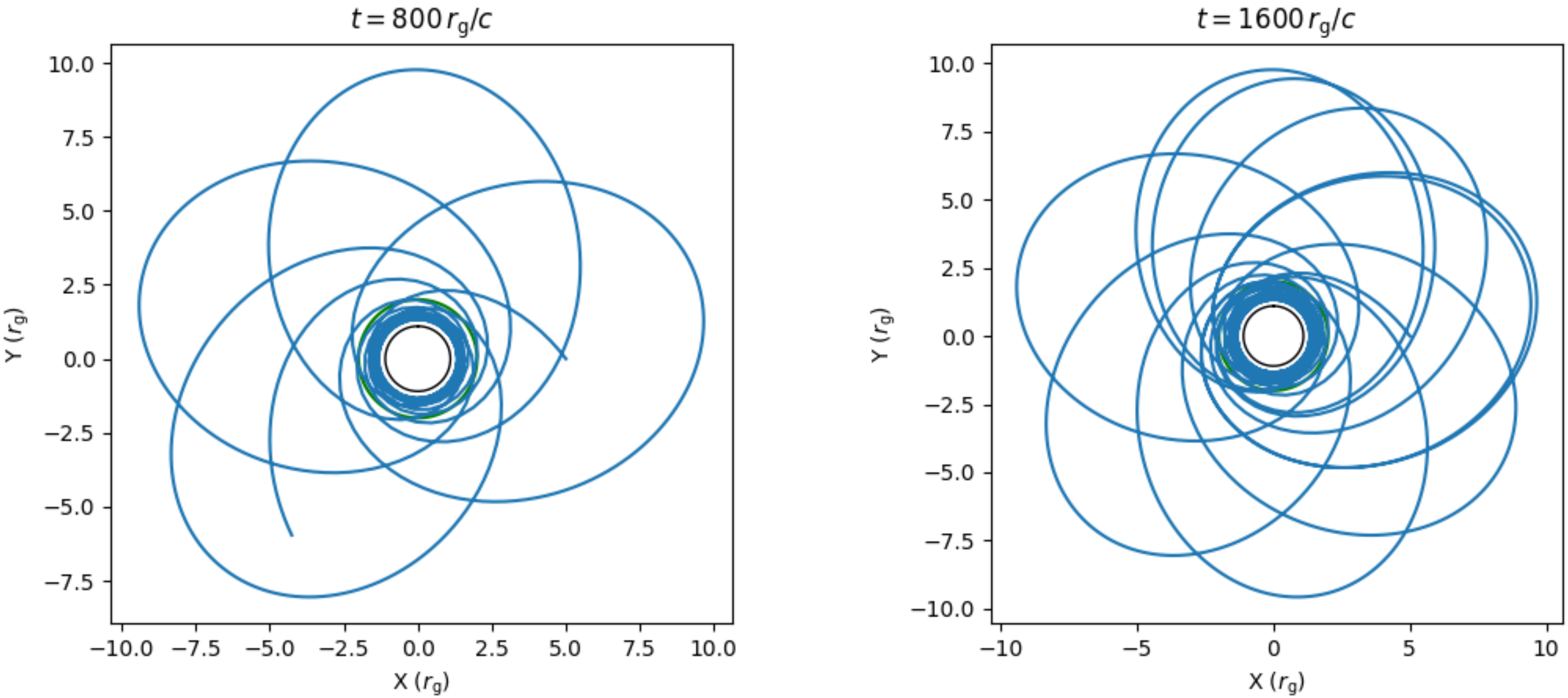


Periodic orbits: Levin & Perez-Giz 2008

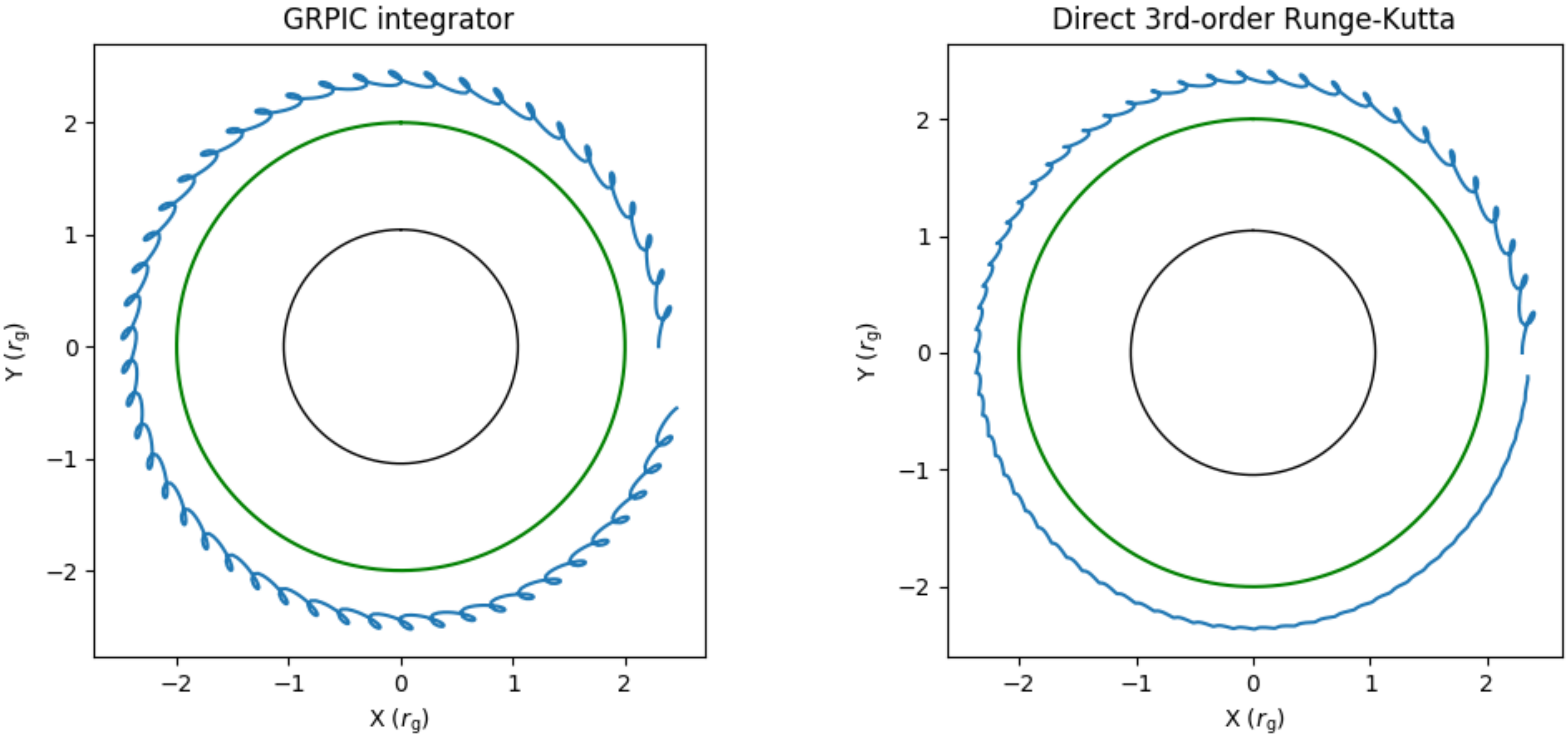
Symplectic integrator @ dt = r_g/c



3rd-order Runge-Kutta @ dt = r_g/c



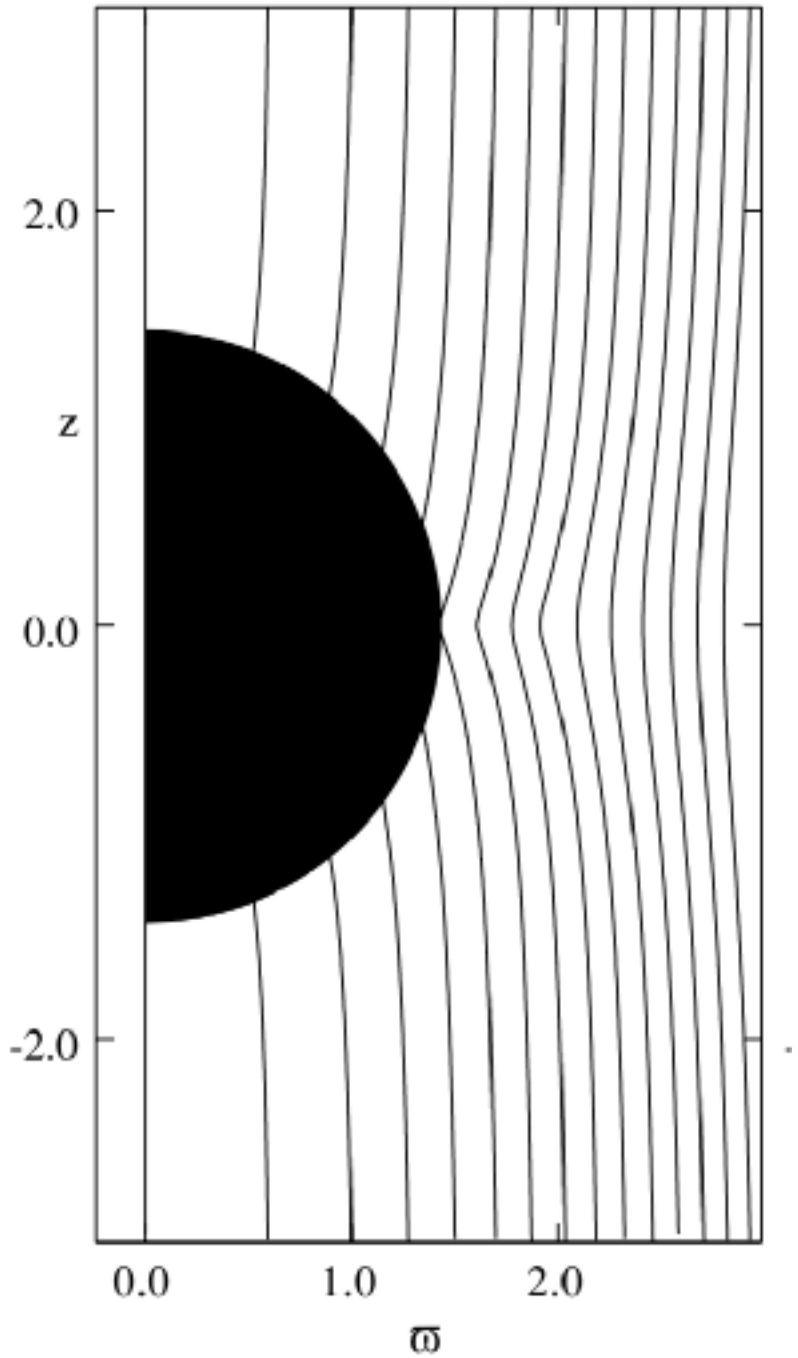
Add $\boldsymbol{B}$ – particle in Wald vacuum field



Kerr metric: a = 0.999

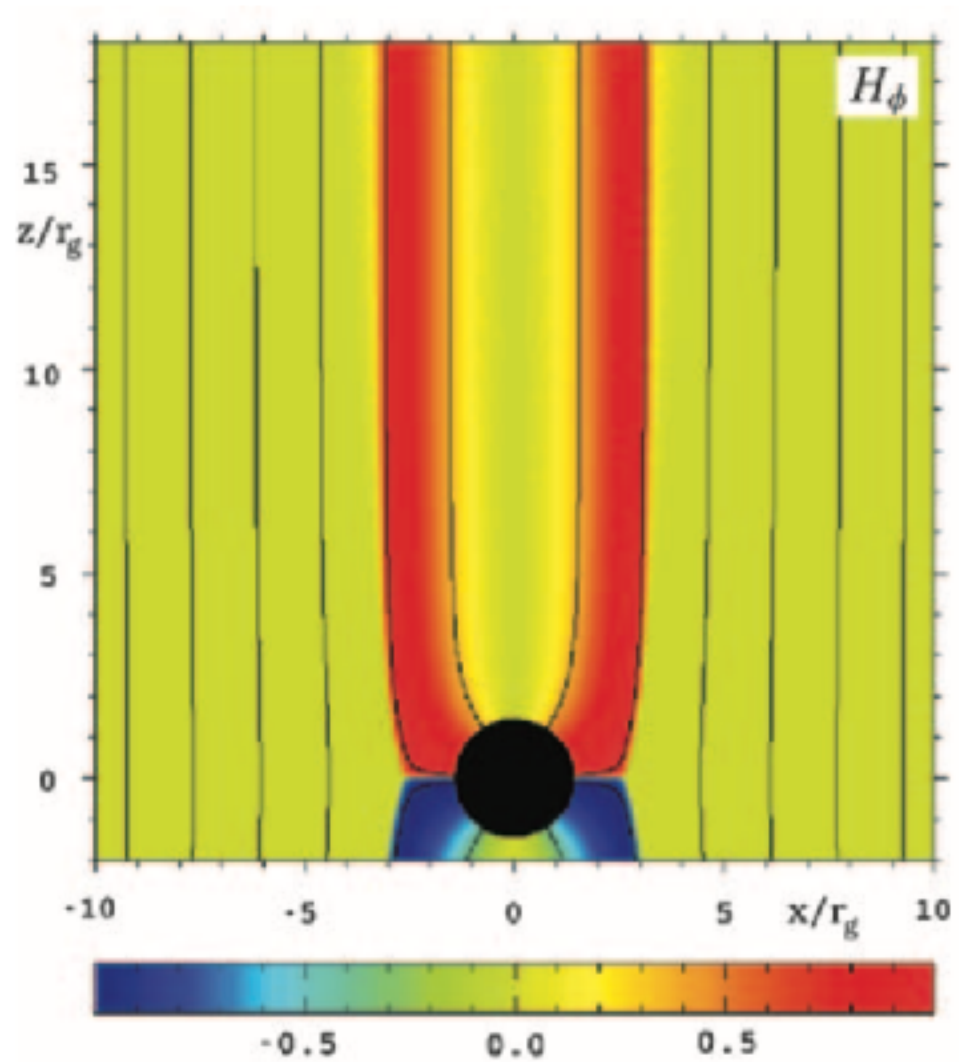
equatorial plane

A first problem: “magnetospheric Wald”



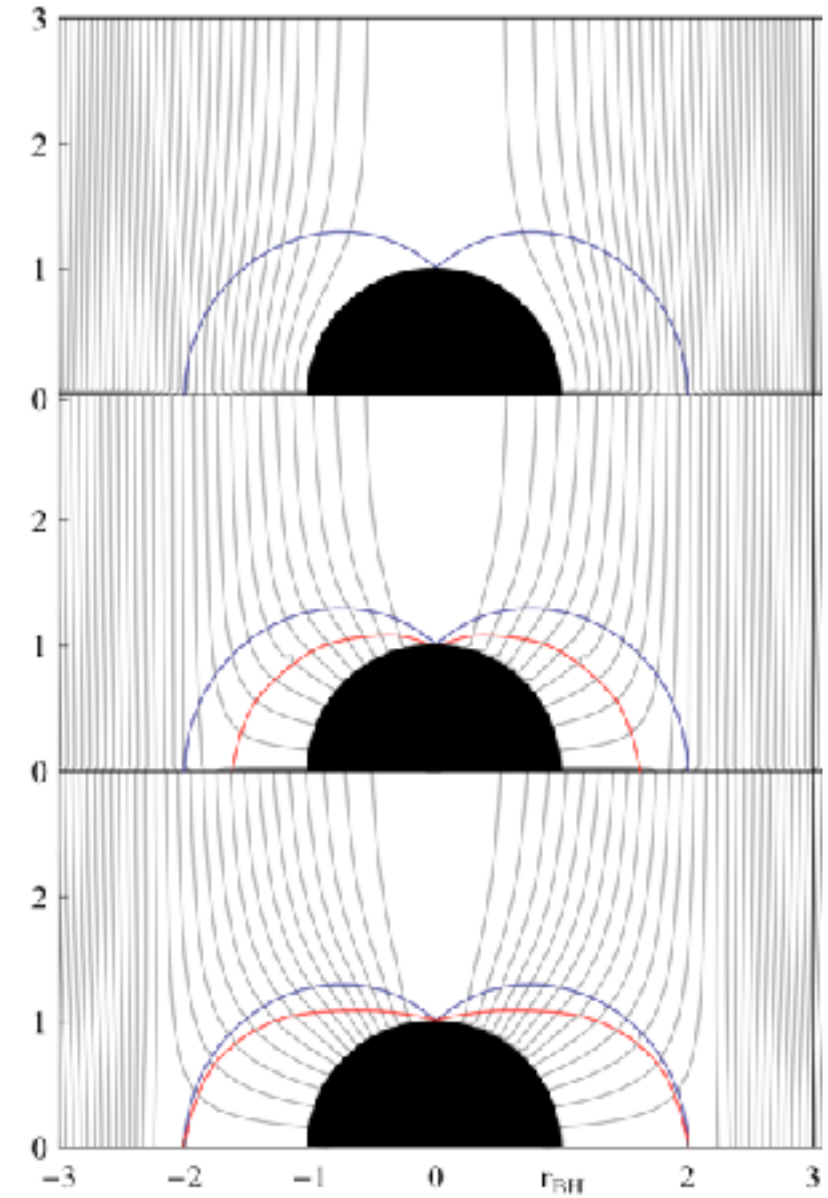
force-free

Komissarov 2004



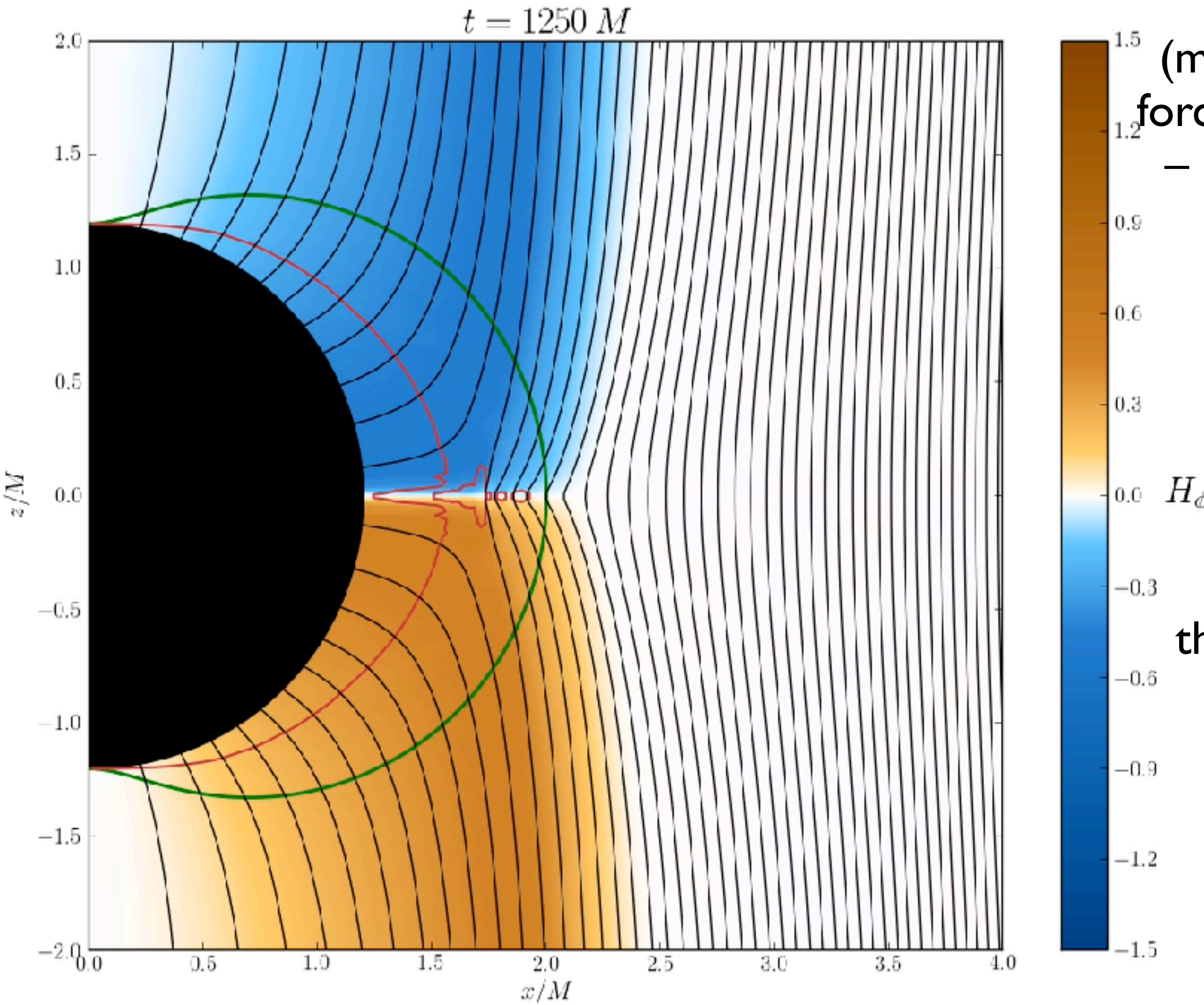
MHD

Komissarov 2005



Grad-Shafranov

Nathanail & Contopoulos 2014



(mildly resistive) force-free solution – Phaedra code

Energy flux through horizon

$S_H \sim 0.18\ B_0^2$

Setup

Fiducial quantities

$r_{L,0} = 10^{-3}\, r_g \longrightarrow B_0$ Uniform field strength at infinity

$n_0 = \dfrac{\Omega_H B_0}{4\pi c e} \longrightarrow \sigma_0 \approx 2000$

+ synchrotron cooling $P_{cool} \propto \left(\dfrac{\gamma}{\gamma_{sync}}\right)^2$

Injection

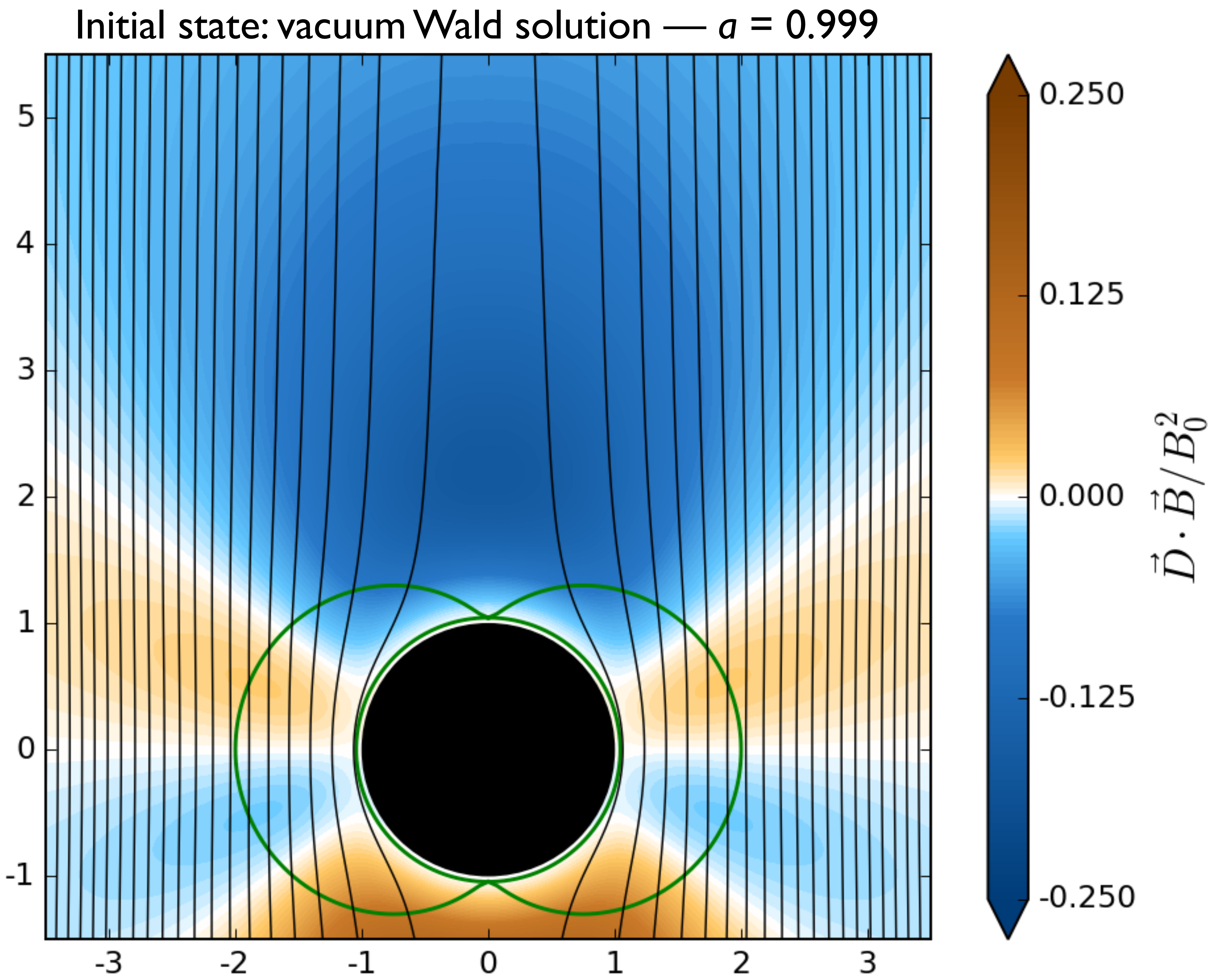
If $\dfrac{\vec{D}\cdot\vec{B}}{B_0^2}$ > threshold ~ 0.01 AND σ > threshold ~ 200 :

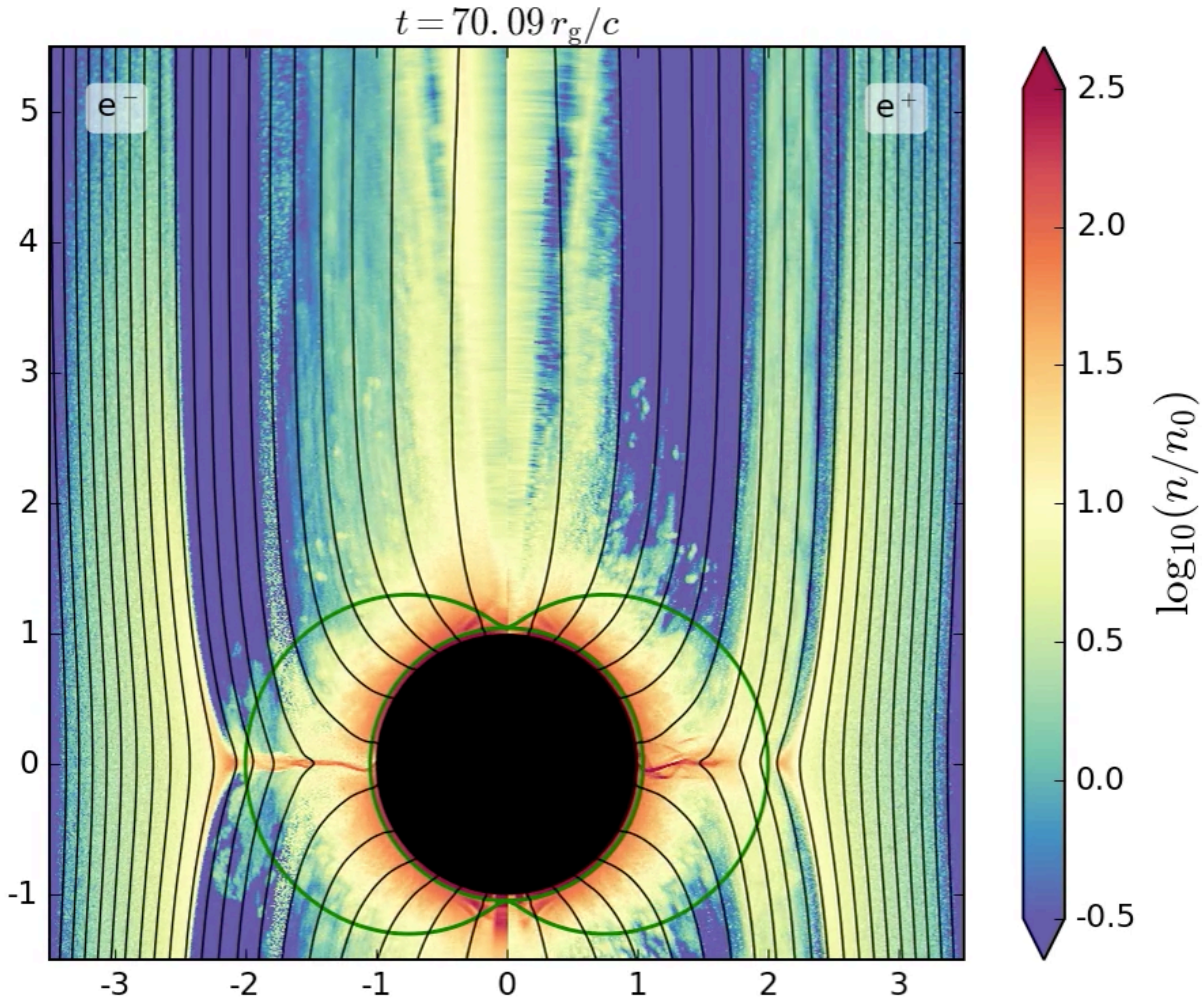
inject particles $n \propto |\vec{D}\cdot\vec{B}|$

Initial Conditions

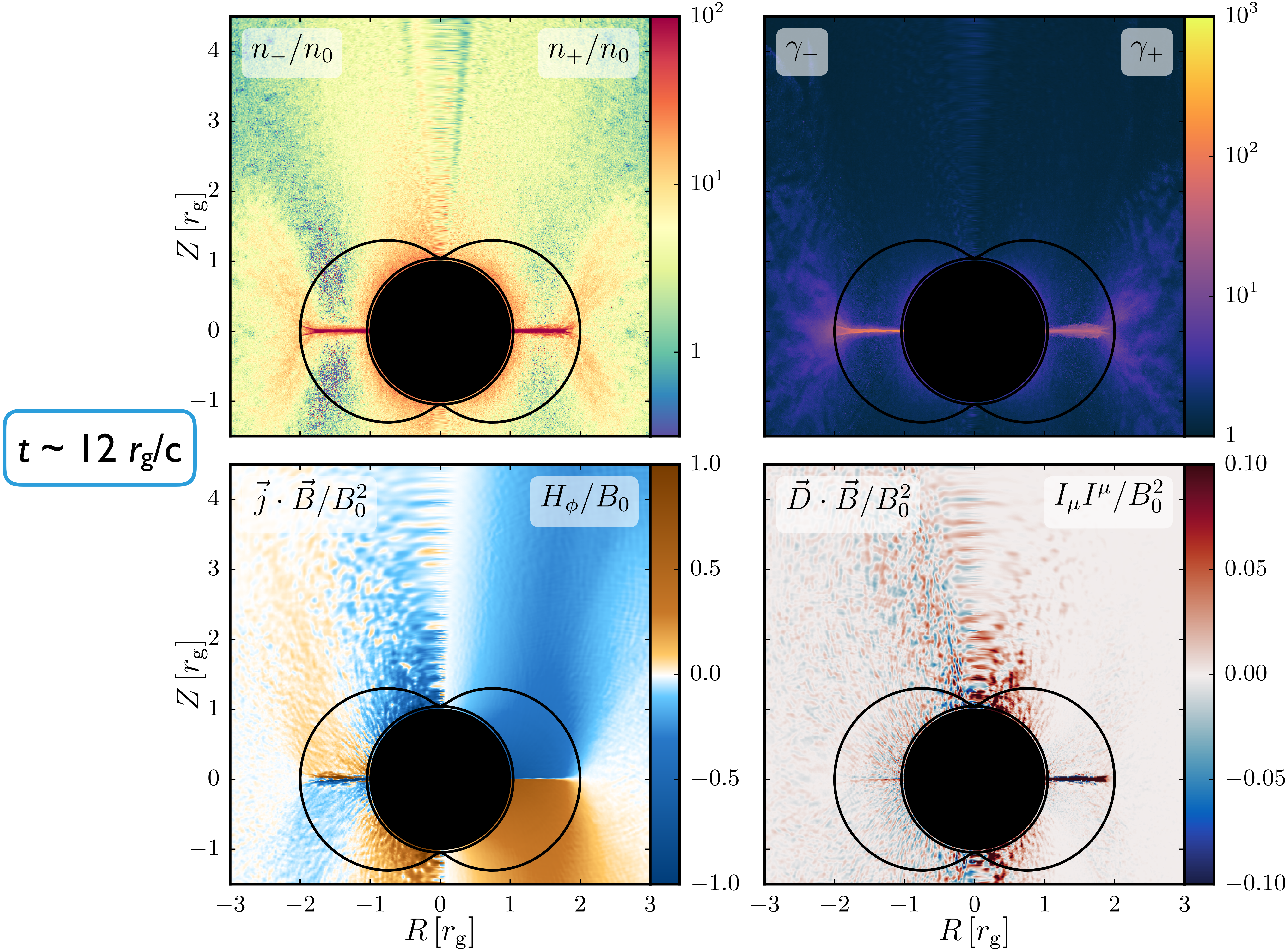
Vacuum steady state: $A_\mu = m_\mu + 2a k_\mu$ $\qquad m^\mu = \partial_\phi$

Wald 1974 $\qquad k^\mu = \partial_t$

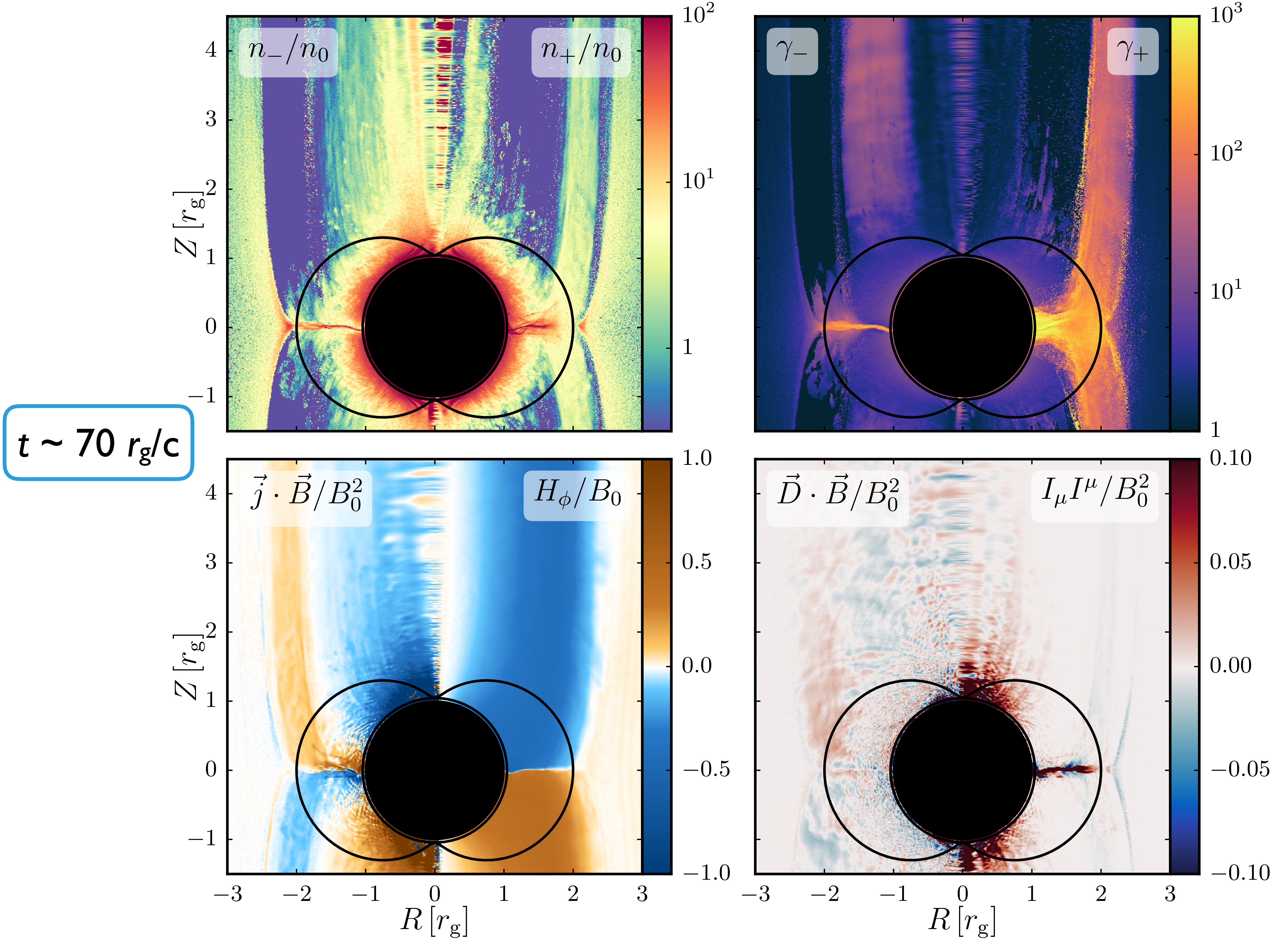

Initial state: vacuum Wald solution — a = 0.999
5
4
3
2
1
0
-1
-3
-2
-1
0
1
2
3
0.250
0.125
0.000
-0.125
-0.250
$\vec{D}\cdot\vec{B}/B_0^2$



$t = 70.09\, r_g/c$
e⁻
e⁺
5
4
3
2
1
0
-1
-3
-2
-1
0
1
2
3
2.5
2.0
1.5
1.0
0.5
0.0
-0.5
$\log_{10}(n/n_0)$

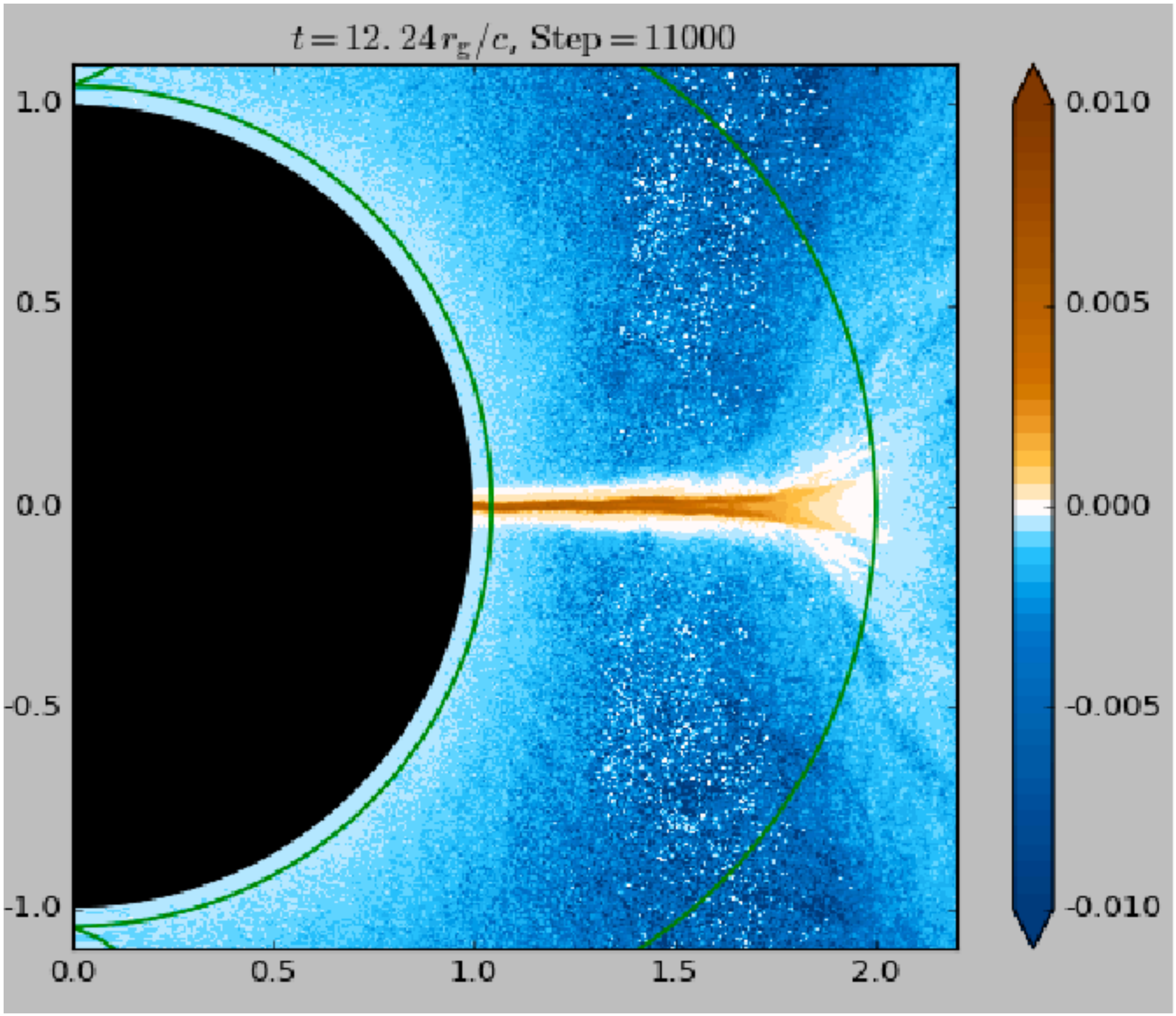


t ~ 12 r_g/c
n_-/n_0
n_+/n_0
γ_-
γ_+
$\vec{j}\cdot\vec{B}/B_0^2$
H_ϕ/B_0
$\vec{D}\cdot\vec{B}/B_0^2$
$I_\mu I^\mu/B_0^2$
$Z\,[r_\mathrm{g}]$
$R\,[r_\mathrm{g}]$
10^2
10^1
1
10^3
1.0
0.5
0.0
−0.5
−1.0
0.10
0.05
0.00
−0.05
−0.10



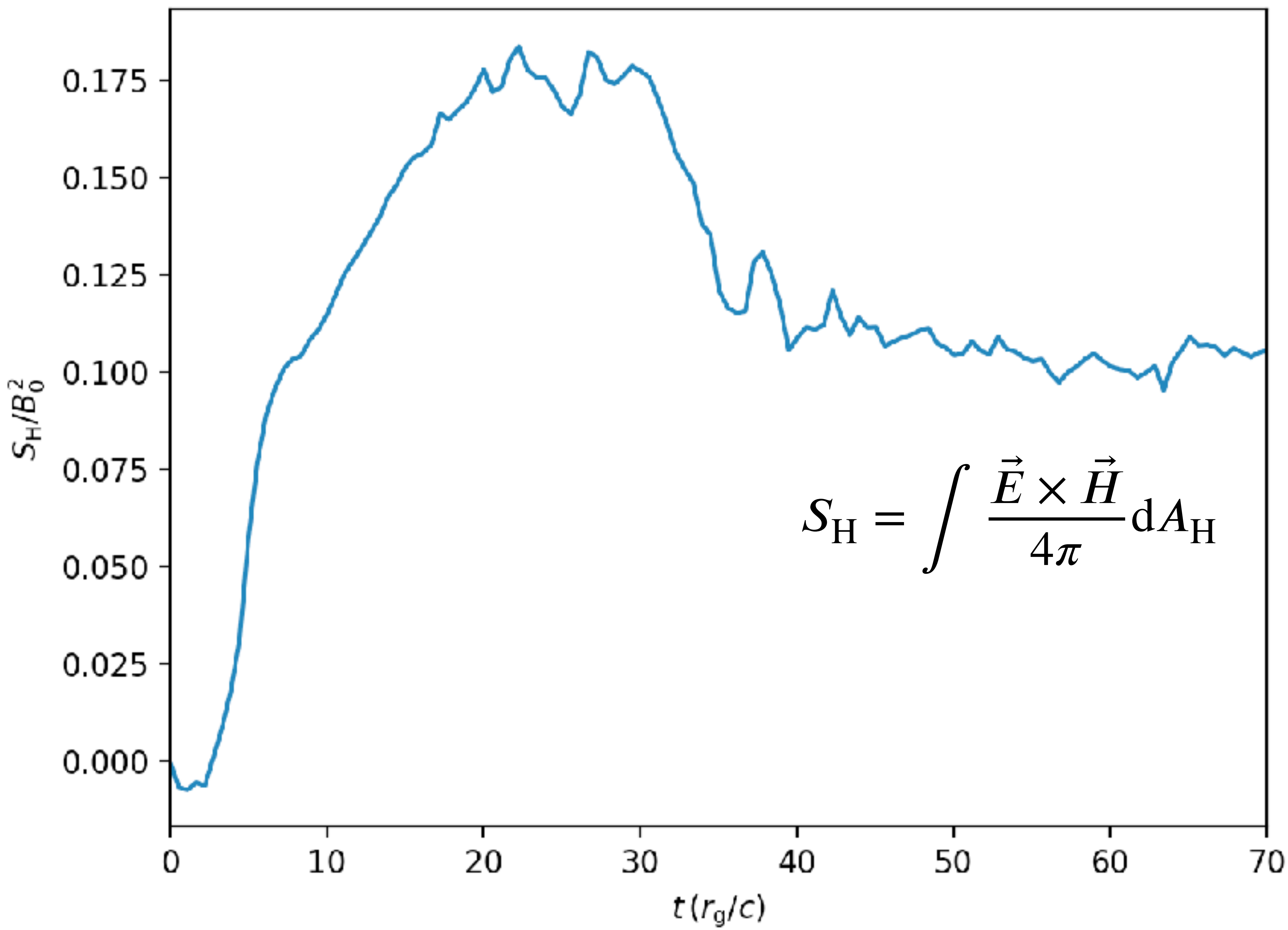
t ~ 70 r_g/c
n_-/n_0
n_+/n_0
γ_-
γ_+
$\vec{j}\cdot\vec{B}/B_0^2$
H_ϕ/B_0
$\vec{D}\cdot\vec{B}/B_0^2$
$I_\mu I^\mu/B_0^2$
$Z\,[r_\mathrm{g}]$
$R\,[r_\mathrm{g}]$

Penrose particles — electrons

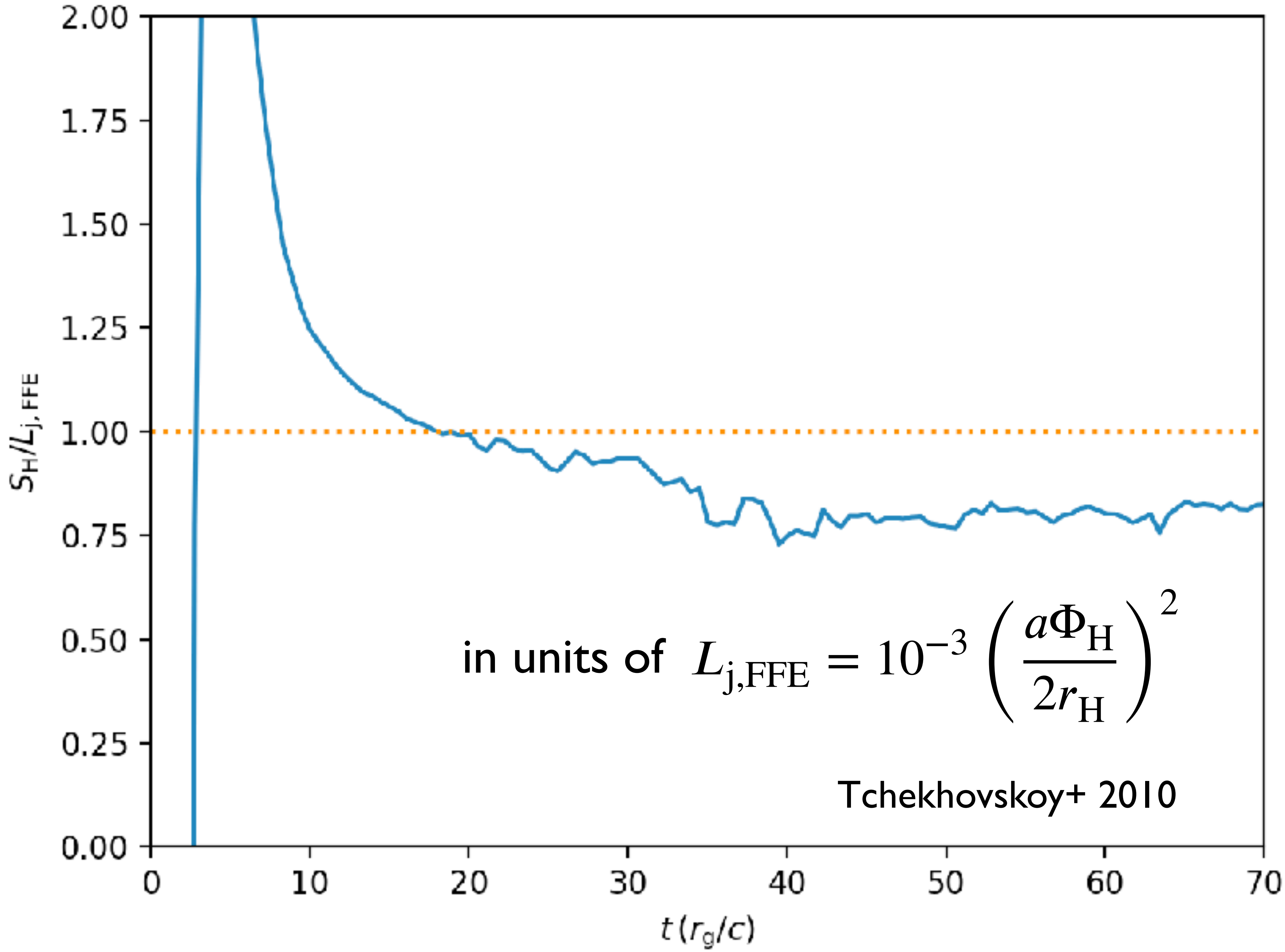


u_t

EM energy-at-infinity flux through horizon



EM energy-at-infinity flux through horizon



How important are the particles' curvature terms?

Maxwell's equations

$$\partial_t \boldsymbol{D} = \nabla \times \boldsymbol{H} - \boldsymbol{J} \qquad \nabla \cdot \boldsymbol{D} = \rho_{\mathrm{e}}$$

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Particle equations of motion

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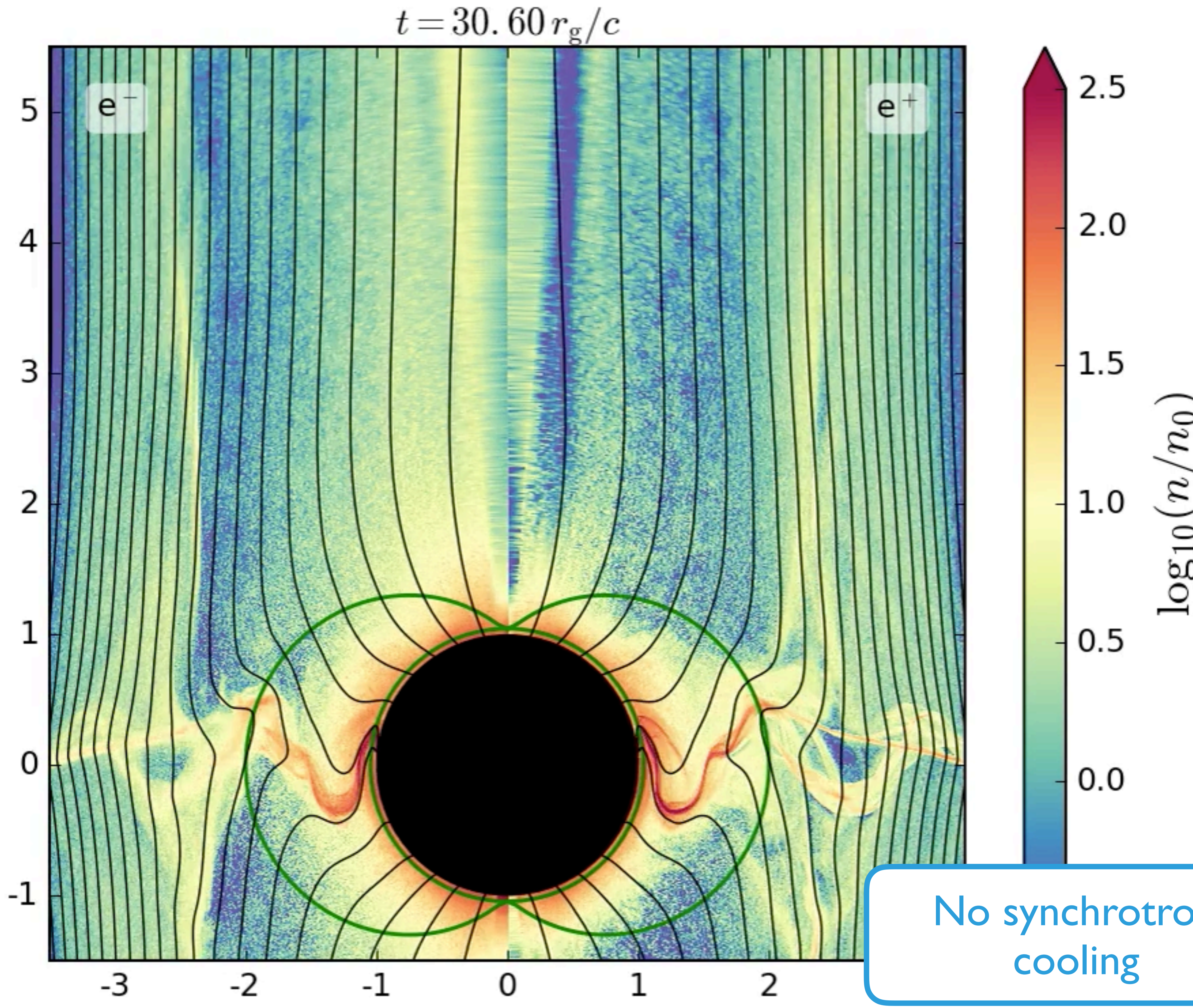
$$\frac{\mathrm{d}x^i}{\mathrm{d}t} = \frac{\alpha}{m\Gamma} p^i - \beta^i$$

Particle equations of motion

Set metric derivatives to **flat spacetime** values

$$\frac{\mathrm{d}p_i}{\mathrm{d}t} = -m\Gamma \partial_i \alpha + p_j \partial_i \beta^j - \frac{\alpha}{2\Gamma m} \partial_i (\gamma^{lm}) p_l p_m + q \left\{ \alpha D_i + \epsilon_{ijk} (v^j + \beta^j) B^k \right\}$$



No synchrotron cooling

Black hole magnetospheres

Can recover nearly force-free states

GRPIC isn't prohibitively expensive

Many interesting applications

May need to keep all terms in the particle pusher
if have energetic particles near horizon

Can include photon propagation with
the same geodesic integrator method