Contrasting Energy Scales of the Reentrant Integer Quantum Hall States

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(Dated: March 6, 2012)

We report drastically different onset temperatures of the reentrant integer quantum Hall states in the second and third Landau levels. This finding is in quantitative disagreement with the Hartree-Fock theory of the bubble phases which is thought to describe these reentrant states. Our results indicate that the association of the reentrant states in the second and third Landau levels with the various types of bubble phases is likely different than predicted.

The quantized motion of electrons confined to two dimensions and subjected to a magnetic field B results in a set of equidistant and degenerate energy levels called Landau levels (LL). By changing the Landau level occupancy, also called the Landau level filling factor ν, different ground states of a two-dimensional electron gas (2DEG) can be accessed. Of these ground states the fractional quantum Hall states (FQHS) [1] have now been observed in electrons confined to GaAs/AlGaAs [2], Si/SiGe [3], AlAs/AlGaAs [4], GaN/AlGaN heterostructures [5], and most recently graphene [6] and ZnO/MgZnO [7].

Certain fragile ground states such as the reentrant integer quantum Hall states (RIQHS), however, have only been observed in the cleanest 2DEGs forming in GaAs/AlGaAs [8–14]. The RIQHSs have been associated with the theoretically predicted bubble phases [15–20]. According to these theories, the guiding centers of the Landau orbitals in weak magnetic fields cluster into so called electron bubbles and, furthermore, the bubbles order into a periodic lattice. Such a bubble phase can be thought of as a triangular charge density wave with an internal degree of freedom, i.e. with either one or several electrons per lattice node. The one electron bubble phase is identical to the Wigner solid [15].

The observed signatures in dc trasport [8–11] and microwave response [12–14] of the RIQHSs forming in different LLs are similar and, furthermore, are consistent with the bubble interpretation of these states. A notable difference between the RIQHSs is, however, the disparity in their numbers. Indeed, when three or more LLs are populated, four RIQHSs were found in each Landau level [8, 9]. In contrast, when there are only two LLs populated, a total of eight such states form [10]. Theories cannot predict the number of RIQHS. They can, however, determine the type of bubbles energetically favored at any given ν. In the third Landau level (TLL) Hartree-Fock [17, 20], exact diagonalization [18], and density matrix renormalization group calculations [19] find two electrons per bubble. In contrast, Hartree-Fock calculations extended to the second Landau level (SLL) find both two electron and one electron bubble phases [20]. These predictions so far could not be tested.

In this Letter we report measurements of the onset temperatures of the RIQHSs in the third LL and provide quantitative comparisons with those forming in the SLL as well as with the predicted bubble phases. We find that, when measured in units of Coulomb energy, the onset temperatures of the RIQHSs of the TLL are more than 4 times higher than those of the RIQHSs of the SLL. This result is at odds with cohesive energy calculations obtained within the Hartree-Fock approximation and it indicates that the assignment of the RIQHSs to the various bubble phases is likely different than predicted [20].

We measured a 30 nm wide GaAs/AlGaAs quantum well sample with a density $n = 2.8 \times 10^{11}\text{cm}^{-2}$ and mobility $18 \times 10^6\text{cm}^2/\text{Vs}$ grown at Purdue. The low frequency magnetotransport measurements were performed at dilution refrigerator temperatures while our sample was immersed into a liquid He-3 bath [21]. The He-3 bath facilitates cooling of the sample and it enables B-field independent temperature measurements by the use of a quartz tuning fork viscometer.

In Fig.1 we show the magnetoresistance R_{xx} and the Hall resistance R_{xy} plotted against B and filling factor $\nu = nh/eB$ in the SLL and TLL. It is important to appreciate that a completely filled orbital Landau level is spin-split into two distinct energy levels and, hence, its filling factor is $\nu = 2$. Therefore the SLL correspond to filling factors $2 < \nu < 4$ while the TLL to $4 < \nu < 6$.

The well known integer quantum Hall states are seen in Fig.1 as plateaus in R_{xy} quantized to h/ie^2, with $i = 2, 3, 4, 5$, and 6. Each of these plateaus straddle the corresponding integer filling factor $\nu = i$. As B is varied, R_{xy} deviates from these plateaus. There are, however, other regions for which R_{xy} returns to an integer quantization but, in contrast to the plateaus of the integer quantum Hall states, these plateaus develop at ranges of ν which do not contain any integer values. These features define the RIQHSs [8–11]. As an example, the RIQHS labeled $R2c$ in Fig.1 has $R_{xy} = h/3e^2$ and it stretches be-

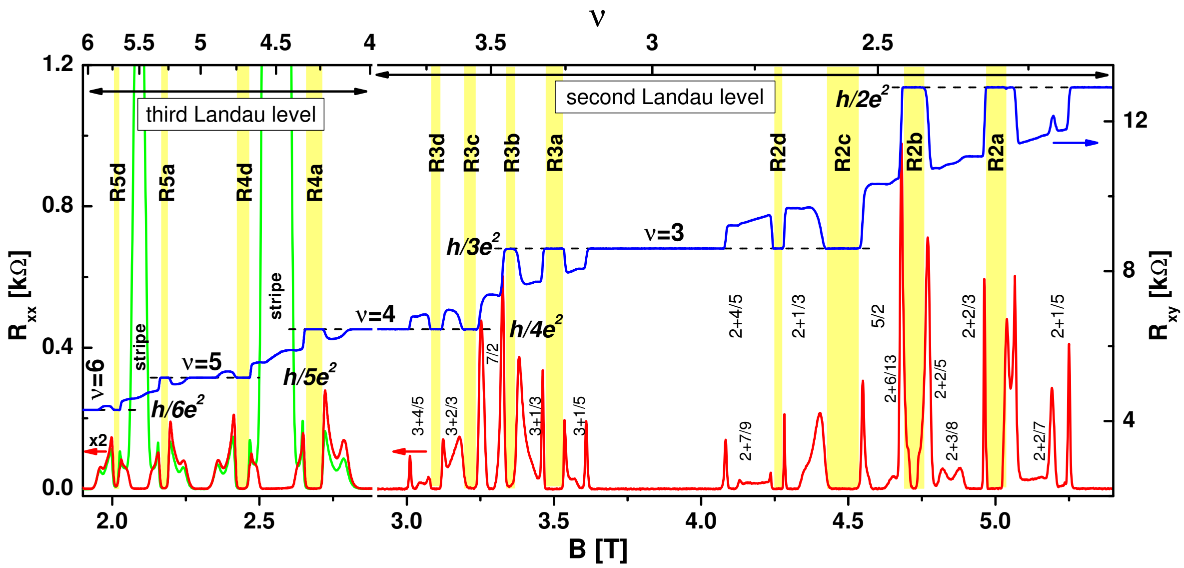

FIG. 1. The magnetoresistance in the second ($2 < \nu < 4$) and third ($4 < \nu < 6$) Landau levels as measured at 6.9 mK and 77mK, respectively. RIQHSs are marked by shaded stripes and FQHSs by their filling factors. In the TLL the two R_{xx} traces shown are measured along mutually perpendicular directions and, for clarity, are magnified by a factor two.

tween $2.54 < \nu < 2.60$, a region which does not contain any integers. Quantization of R_{xy} is accompanied by a vanishing R_{xx}. Althogether, in the SLL there are eight RIQHS labeled *R2a*, *R2b*, *R2c*, *R2d*, *R3a*, *R3b*, *R3c*, and *R3d* while in the TLL there are only four of such states labeled *R4a*, *R4d*, *R5a*, and *R5d*. These RIQHSs are shaded in Fig.1.

In Fig.1 we also identify anisotropic ground states called stripe phases [15, 17] in the vicinity of $\nu = 4.5$ and 5.5 [8, 9], a very strong FQHS at $\nu = 5/2$ [22] with a gap of 0.50 K, and we discern developing FQHSs at $\nu = 2 + 6/13$, $2 + 2/7$, $2 + 7/9$, and $2 + 3/8$ [23, 24]. We also report a new feature in the Hall resistance at $B = 5.196$ or $\nu = 2.214$. This feature is a clear deviation from the classical Hall line and it may signal the development of another RIQHS.

Inspite the disparity in their numbers, the RIQHSs in the SLL and high LLs share common features in the quantized reentrant transport [8–11] and microwave response [12–14]. In the following we establish two additional common transport signatures of the RIQHSs in the SLL and TLL: spikes flanking the vanishing regions of the R_{xx} versus B curves and a peak in the temperature dependent R_{xx}. These findings further stregthen the argument that the RIQHSs of different LL are similar ground states.

One similarity between the RIQHSs in the SLL and TLL is the presence of two sharp spikes in the flanks of the vanishing R_{xx} versus B curves, i.e. the shaded areas of Fig.1. We have recently reported such spikes in the SLL [25], and now we observe them in the TLL. We also find a similar temperature evolution of these spikes. The T dependence of the *R4a* state in the TLL, shown in Fig.2, shares the following pattern with that of RIQHSs in the SLL [25]: at the lowest temperatures there are two spikes of finite resistance flanking the vanishing R_{xx}, with increasing T these two spike merge into a single peak, and this peak dissapears into a smooth background with a further increase in T. We define the center of a RIQHS as the location ν_c at which the extent of the vanishing R_{xx} plateau is nearly zero. For example, the curve at 128 mK of Fig.2 exhibits a *R4a* state of nearly zero width at $\nu_c = 4.287$. The partial filling factor ν_c^* is the decimal part of ν_c, and values for the various RIQHSs are summarized in Table.I.

A second shared feature of the RIQHSs in the SLL and TLL is the similar R_{xx} and R_{xy} versus T curves measured at a fixed ν. In Fig.3 we show such curves for the *R4a* and *R4d* states of the TLL in close vicinity to their respective central filling factors. As the temperature is increased the Hall resistance undergoes a change from the nearest integer quantized value to the classical Hall value $B/ne = h/\nu e^2$. Simultaneously with the sharp change in R_{xy} the longitudinal resistance R_{xx} exhibits extremely sharp peaks of width at half height of only 10 mK for the *R4a* state. We have recently reported similar depen-

TABLE I. Central filling factors ν_c^* and onset temperatures T_c of the RIQHSs measured.

	$R2a$	$R2b$	$R2c$	$R2d$	$R3a$	$R3b$	$R3c$	$R3d$	$R4a$	$R4d$	$R5a$	$R5d$
ν_c^*	0.300	0.438	0.568	0.700	0.288	0.430	0.576	0.713	0.287	0.714	0.286	0.714
T_c[mK]	45.3	29.8	39.9	29.5	38.1	25.4	31.0	25.5	145	125	111	100

dences of both R_{xy} and R_{xx} of the RIQHSs in the SLL of a higher density sample and have interpreted the peak temperature as the onset temperature T_c of the RIQHSs [25]. We hereby accertain that the presence of a peak in the R_{xx} versus T curves accompanied by a sharp transition of R_{xy} from the classical Hall to a quantized value is not specific to the SLL, but is also a property of the RIQHSs forming in the TLL.

In the following we compare the RIQHSs to each other and also to the predicted bubble phases. Inspecting Table.I we find that $R2a$, $R3a$, $R4a$, and $R5a$ develop at similar partial filling factors. Indeed, $\nu_c^*|_{R4a} = \nu_c^*|_{R5a} = \nu_c^*|_{R3a}$ within our measurement error of ± 0.003 and are quite close to $\nu_c^*|_{R2a}$. This is seen in Fig.4 as a clustering of these states near $\nu^* \approx 0.29$. The similar locations for these RIQHSs are suggestive of a common origin. Theory, however, favors one-electron bubbles for $R2a$ and $R3a$ [20] and two-electron bubbles for $R4a$ and $R5a$ [17–20]. It is, therefore, counterintuitive why the RIQHSs in different LLs but forming at similar partial filling factors should be different types of bubbles.

As a further test we examine the energy scales of the RIQHSs. Data in Table.I shows that the onset temperatures of RIQHSs in the TLL are substantially higher than those in the SLL. This result is consistent with early data in which no RIQHSs were observed in the SLL but the RIQHSs in the TLL were well developed [8, 9]. Quantitative comparisons of these RIQHSs, however, have not yet been possible. We find that the reduced onset temperatures $t_c = k_B T_c / E_c$ of the RIQHSs in the TLL are more than a factor 4 larger than those of the SLL. Here $E_c = e^2/4\pi\epsilon l_B$ is the Coulomb energy and $l_B = \sqrt{\hbar/eB}$ the magnetic length.

The melting temperature of an electron solid was found to be proportional to its cohesive energy [26]. In analogy we assume that, within the bubble interpretation, the onset temperature of a RIQHS is a measure of its cohesive energy. We note that a strict proportionality between these two quantities is not expected to hold [15]. We find that the reduced onset temperatures shown in Fig.4 are more than 2 orders of magnitude smaller than the calculated reduced cohesive energies $e_{\rm coh} = E_{\rm coh}/E_c$ of the bubble phases. We attribute this difference to disorder effects which are not included in the Hartee-Fock estimations [15, 17, 20].

Within the SLL, the measured energy scales of $R2a$ and $R2b$ compare surprizingly well with the theoretical ones: $t_c^{R2a}/t_c^{R2b} = 1.5$ and $e_{\rm coh}^{R2a}/e_{\rm coh}^{R2b} \approx 1.2$ [20]. The energy scale of the $R4a$ state of the TLL is, however, disproportionately larger than any of the RIQHSs in the SLL. The most striking disagreement is between

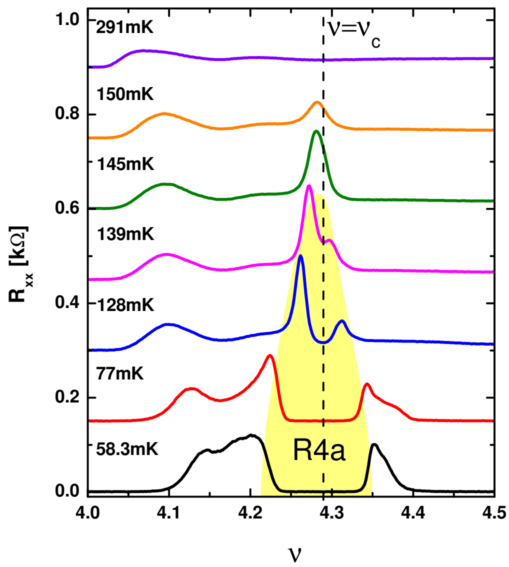

FIG. 2. The evolution with temperature of the $R4a$ RIQHS of the third Landau level. For clarity traces are shifted by 150Ω relative to another and the reentrant region is shaded.

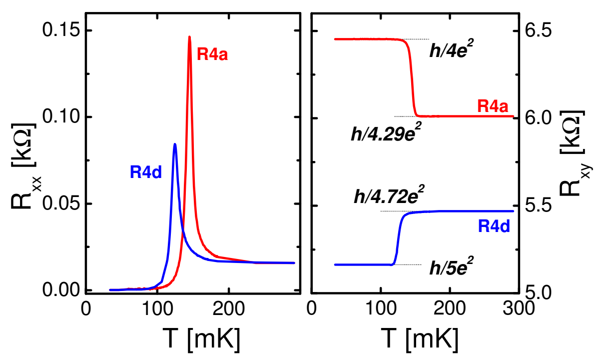

FIG. 3. The magnetoresistance R_{xx} and the Hall resistance R_{xy} of two RIQHSs in the third Landau level measured at $\nu = 4.29$ and $\nu = 4.72$.

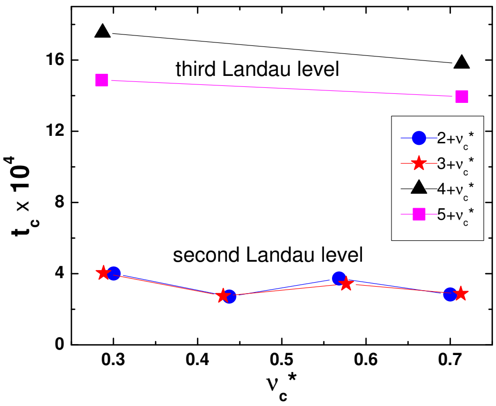

FIG. 4. The reduced onset temperatures $t_c = k_B T_c / E_c$ of the RIQHSs in the SLL and TLL plotted as function of the partial filling factor ν_c^*. Lines are guides to the eye.

the two-electron bubbles $R2b$ and $R4a$. The theory predicts similar cohesive energies $e_{\rm coh}^{R4a}/e_{\rm coh}^{R2b} \approx 1.2$ [20], while we measure a large difference in the onset temperatures $t_c^{R4a}/t_c^{R2b} = 6.4$. We also find $e_{\rm coh}^{R4a}/e_{\rm coh}^{R2a} \approx 1$ [20], while $t_c^{R4a}/t_c^{R2a} = 4.3$. Another difference between theory and experiment is the recently reported electron-hole asymmetry of the onset temperatures of the RIQHSs in the SLL [25]. As seen in Fig.4, this electron-hole asymmetry is also observed in the current sample and it manifests itself in a non-monotonic dependence of t_c on ν_c^* [25].

Taken together, we conclude that there are clear quantitative inconsistencies between the measured and calculated energy scales of the RIQHSs. One scenario which could account for our onset temperature data is that, contrary to the theory [20], all of the RIQHSs in the SLL are bubble phases of the same type and those in the TLL are bubbles of a different kind.

We cannot, however, discard the possibility that the RIQHSs of the second and third LL are the same type of bubble phases. This scenario is contrary to the theory as at least some bubbles in the SLL are predicted to be of different type than those in the TLL. The large difference in onsets could be caused by an effect dependent on LL occupancy. Because of the presence of one extra filled LL, screening of the disorder potential in the TLL is expected to be more effective than that in the SLL [15, 27, 28]. The substantially larger onsets of the RIQHSs in the TLL as compared to those in the SLL could thus be a consequence of smoother effective disorder potentail in the TLL which results from screening of one extra filled LL.

There is a recent report of a developing RIQHS in the lowest LL [29]. The relationship of this state and those in higher LLs is not understood at this time.

The newly reported common features in the transport of the RIQHSs both in the TLL and SLL, together with the reentrant behavior and radiofrequency response, supports the idea that the RIQHSs belong to the same family of ground states irrespective of the LL they form in. These features are consistent with the bubble interpretation of these phases. We found, however, that the very different energy scales of the RIQHSs in different LLs are inconsistent with quantitative predictions of the theory of the bubbles. This disagreement is suggestive of an assignment of the RIQHSs to bubble phases different than that proposed by the theory.

This work was supported by the DOE BES grant DE-SC0006671 and we acknowledge useful discussions with M. Fogler and Y. Lyanda-Geller.

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