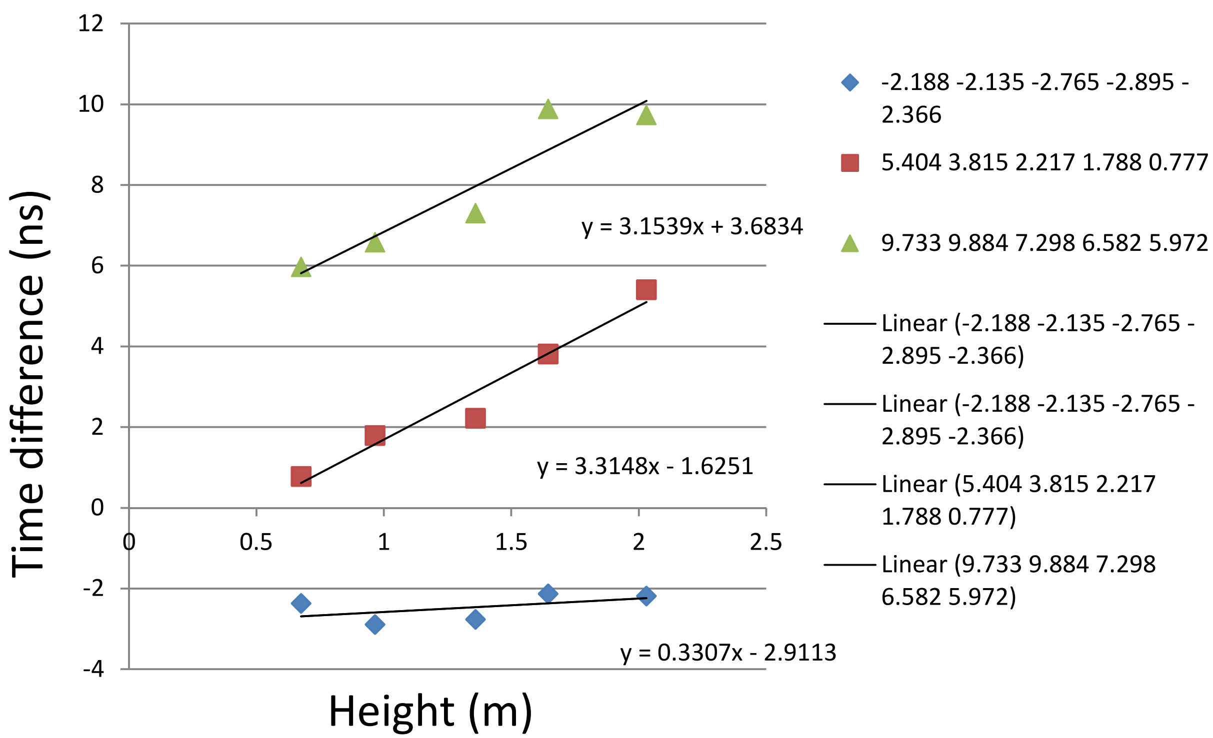
Discussion

- Yesterday we concluded that the coincidence rate was caused by “rays of very great penetrating power entering our atmosphere from above”
- We found that the rate depended (slightly) on the amount of material above us
- We measured the speed of the muons passing through the detector
  - It was close to the speed of light, $c = 3 \times 10^8\, m/s$
- We measured the muon lifetime
  - It was around $2\ \mu s$

Discussion



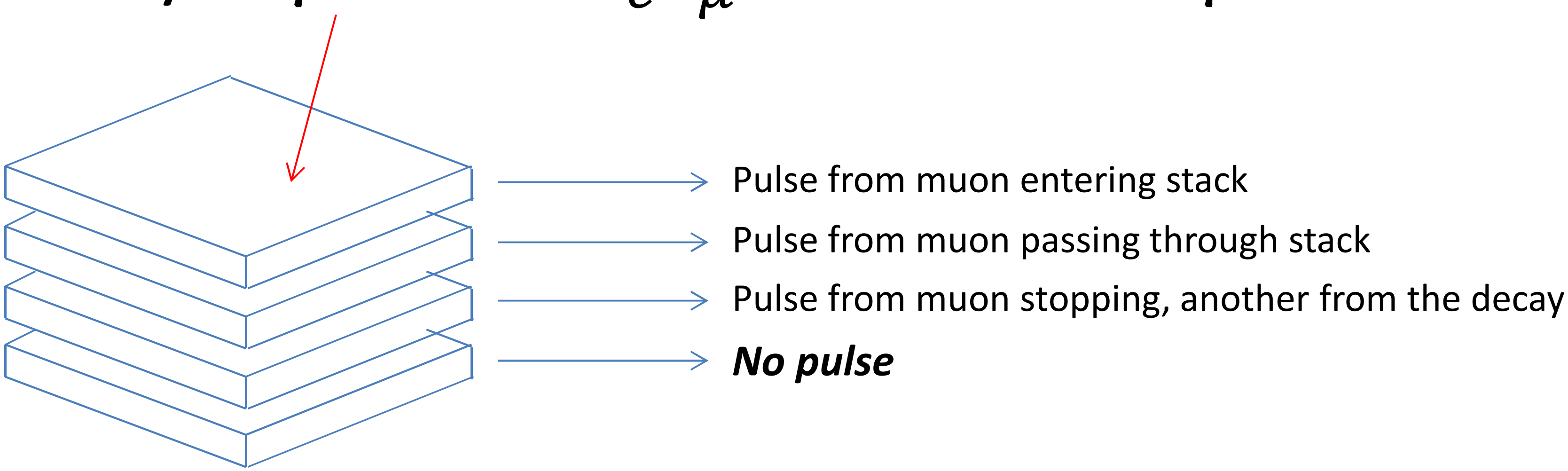
Speed measured using counters 1 and 3: $v_{13} = \left(\frac{1}{3.315}\right) m/ns = 3.0 \times 10^8\ m/s$

Speed measured using counters 1 and 4: $v_{14} = \left(\frac{1}{3.154}\right) m/ns = 3.2\ \times 10^8\ m/s$

Close to the speed of light, but we didn't take into account the propagation of uncertainties in the measurements

Muon Decay Trigger

- We want to identify events where a muon stops in one of the scintillators and then decays… $\mu^+ \rightarrow e^+\nu_e\bar{\nu}_\mu$ with $\tau = 2.2\ \mu s$



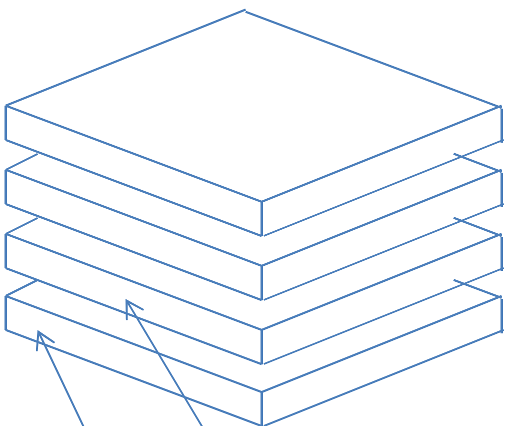
Require 3-fold coincidence
GATE WIDTH = 10,000 ns
PIPELINE DELAY = 20 ns

This isn't *exactly* what we want because it triggers on any 3 channels, but the trigger rate is low enough that we can examine each event to see if it is just the top three channels with pulses.

Muon Lifetime (delayed coincidence)

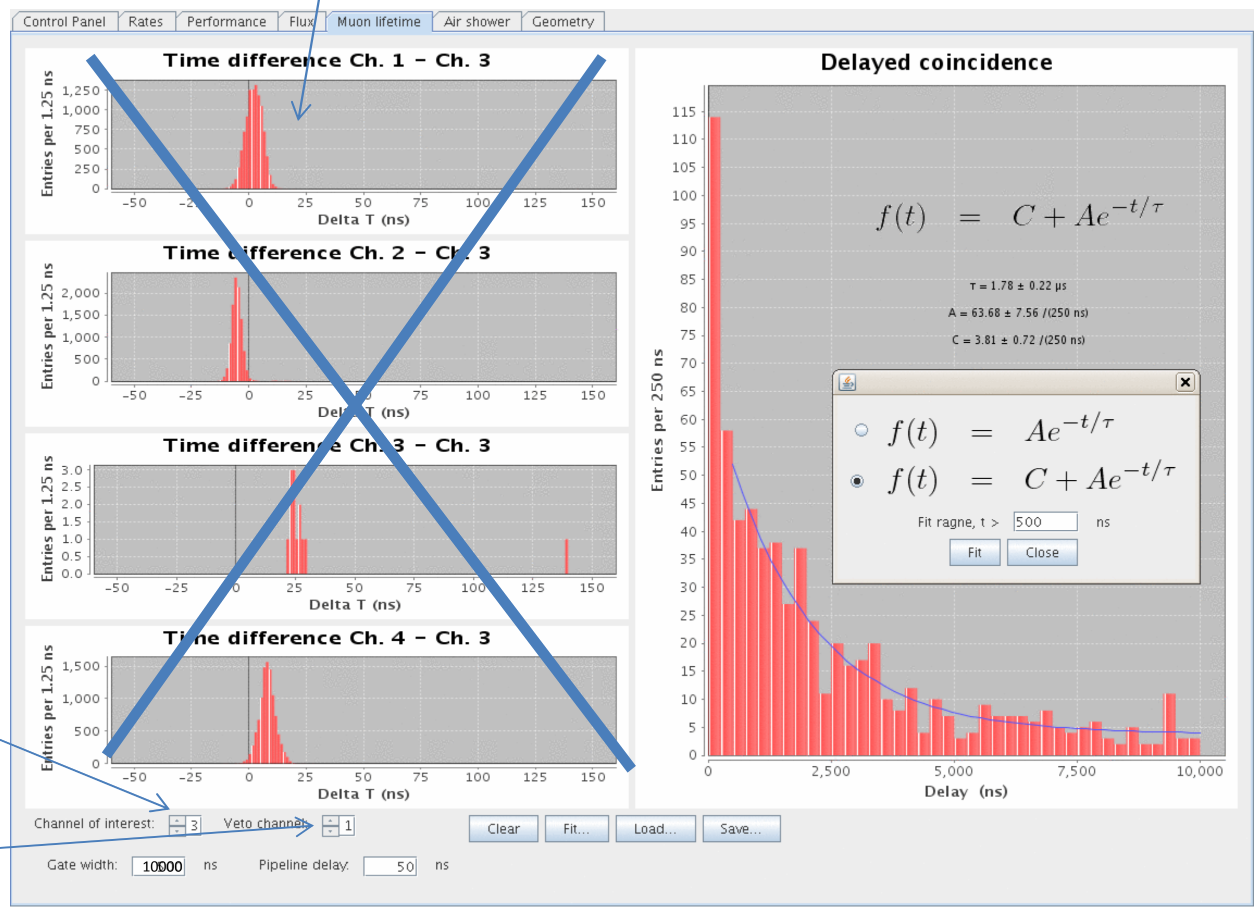
Remember to set the gate width and use a 3-fold coincidence…

Ignore these for now…



Channel of interest

Veto channel



Muon Lifetime

- In about an hour you should get 5-10 muon decay events.
  - Leave it running over night or over the weekend for better statistics
- The fit panel lets you chose whether to consider the possibility of a background component distributed uniformly in time.
- You can skip the early data since these might come from air showers.
  - How does the fitted lifetime depend on these assumptions?
  - How much data would you need to measure the muon lifetime with a precision of 10%?
  - How about 1%?

Muon Lifetime

- Fundamental subatomic particles don't have any internal structure (as far as we know).
- They don't have parts that wear out, or pieces that break.
- If none of their properties change with time, why do they decay?
  - Blame Quantum Mechanics…

Quantum Mechanics

- Quantum Mechanics lets you answer the following question:
  - If a “system” is in a particular “state” at some initial time t_0, what is the probability of finding it in some other “state” at a later time, t?
- The hard part is defining what we mean by “state” and “system” and how to do the calculation…

Muon Decay

- In the case of muon decay, the “system” is the muon and whatever it decays into
  - We can ignore the scintillator, the electronics, the people in the room
- The initial state is “muon”
- The final state is “no more muon”
- Quantum mechanics lets us calculate the probability that a system in the “muon” state is found in the “no more muon” state after time t.
- The “probability per unit time” is a constant.

Muon Decay

- Statistical interpretation:
  - If we started with a sample of N muons, and the probability per unit time that they would decay was Γ then the change in the number of muons, during time Δt would be

$$\Delta N = -N\Gamma\Delta t$$

  - We can make a differential equation:

$$\frac{\Delta N}{\Delta t} = -N\Gamma \Rightarrow \frac{dN}{dt} = -N(t)\Gamma$$

  - A solution is $N(t) = N_0 e^{-\Gamma t}$ or $N(t) = N_0 e^{-t/\tau}$
- The “lifetime” is the reciprocal of the decay probability per unit time…

Muon Decay

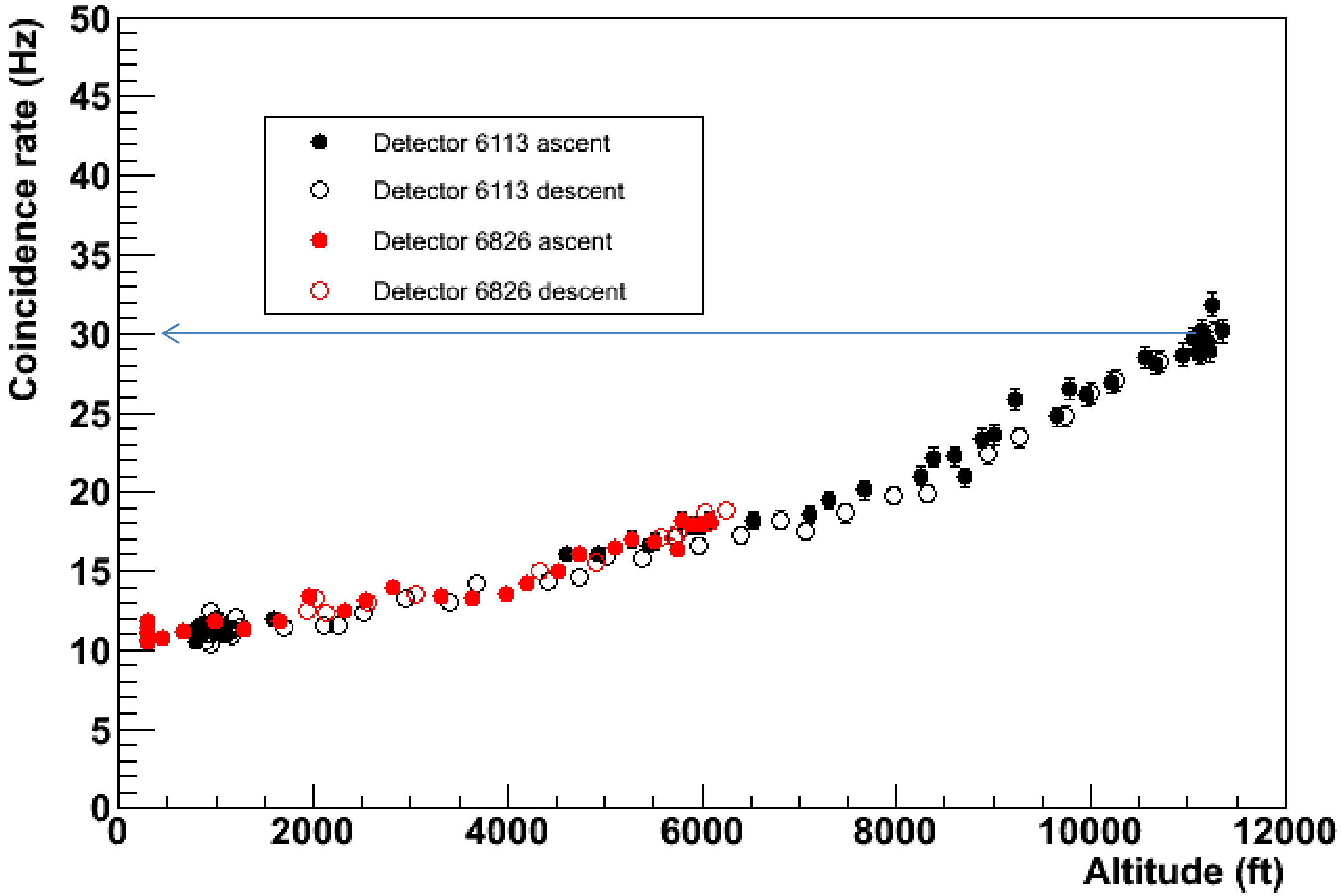
- The point is, that Quantum Mechanics lets us calculate the parameter τ which we call the lifetime.
- The number of muons in a sample decreases with time because they decay:

$$N(t) = N_0 e^{-t/\tau}$$

- We can even calculate the lifetime:

$$\tau = \frac{192\pi^3}{G_F^2 m_\mu^5} \approx 2.2\ \mu s$$

Rates as a function of altitude



The rate changes by a factor of 3 over a distance of about 11000 feet (3350 meters)

Decay Rates

- Can we explain the rate as a function of altitude?
  - We know the distance they travel: $d \approx 3000\ m$
  - We know how fast they are moving: $v \approx 3 \times 10^8\ m/s$

$$t = \frac{d}{v} = 10 \times 10^{-6}\ s$$

  - We know their decay rate: $\tau \approx 2 \times 10^{-6}\ s$
- It takes "5 lifetimes" to travel this distance…
  - The rate should be lower by $e^{-5} = 0.007$
- But we saw that the rate was only about 1/3 lower…

Special Relativity

- Through logical reasoning, Einstein worked out that "clocks" in a moving reference frame run slow (time dilation)
  - If we are watching the muon, which is moving very fast, its "clock" is running slow and it travels the distance in only 1 lifetime, not 5.
- Equivalently, distances in a moving reference frame are contracted (Lorentz contraction)
  - If we are a muon, then the distance we travel we need to travel has been contracted to 1/5 the altitude.

Lorentz Contraction/Time Dilation

- When muons move with velocity v, then we should observe

$$N(t) = N_0 e^{-t/\gamma\tau}$$

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \approx 5 \Rightarrow v \approx 0.98c$$

- This is "consistent" with what we measured
  - the velocity was very close to the speed of light.
- Special relativity is easy to observe – it isn't at all theoretical or abstract. You just need to study something that is moving really fast.