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Jet Clustering Algorithms.

At hadron colliders, the longitudinal boost of an event (ie, along the beam axis) is unknown a priori.

Rapidity is defined

$$y = \frac{1}{2} \log\left(\frac{E + p_z}{E - p_z}\right)$$

and is related to other kinematic variables:

$$E = m_T \cosh y$$
$$p_z = m_T \sinh y$$
$$m_T^2 = m^2 + p_x^2 + p_y^2 \quad \text{(transverse mass)}$$

careful! Other definitions exist!)

Also, $y = \log\left(\frac{E + p_z}{m_T}\right) = \tanh^{-1}\left(\frac{p_z}{E}\right)$

A boost along the z-axis with velocity β will transform y:

$$y \to y' = y - \tanh^{-1}(\beta)$$

Thus, differences in y are invariant:

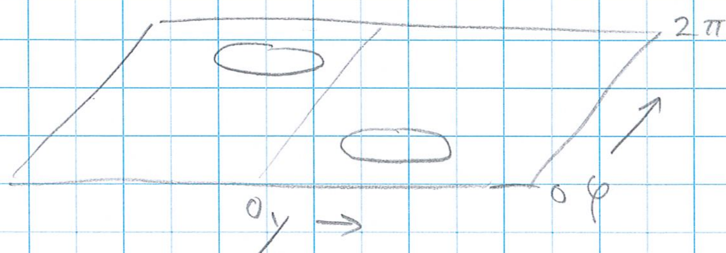
$$\Delta y = y_1 - y_0 \longrightarrow \Delta y' = \Delta y.$$

Transverse momenta are invariant under boosts and angles in the transverse plane are invariant under rotations.

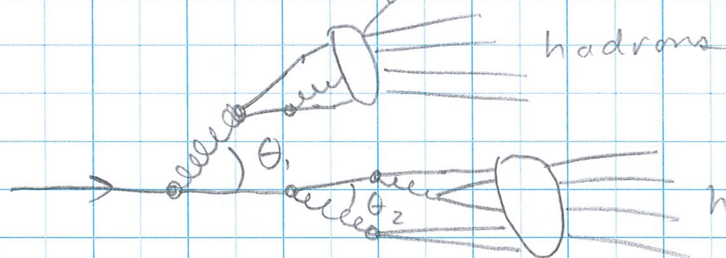
Also, $y \approx -\log \tan \theta/2 \equiv \eta$

$\sinh \eta = \cot \theta \Rightarrow \theta = 45° \longrightarrow \eta = 0.88$

It is useful to imagine unrolling the φ coordinate and plotting distributions in φ and y:



Energy or track activity can be visualized as clusters in the y-φ plane. These roughly correspond to early stage parton activity:



$$\theta_1 > \theta_2 \quad \text{(angular ordering)}.$$

Also, matrix elements calculate multiparton final states ($n \leq 4$ or so)

Because momentum is conserved, if we could sum up the 4-vectors of the final state hadrons, then we would know the 4-vector of the initial state parton.

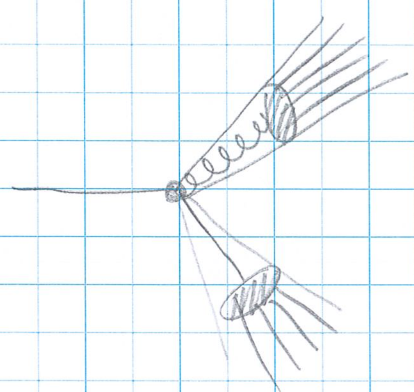
How do we resolve the different jet activity in an event?

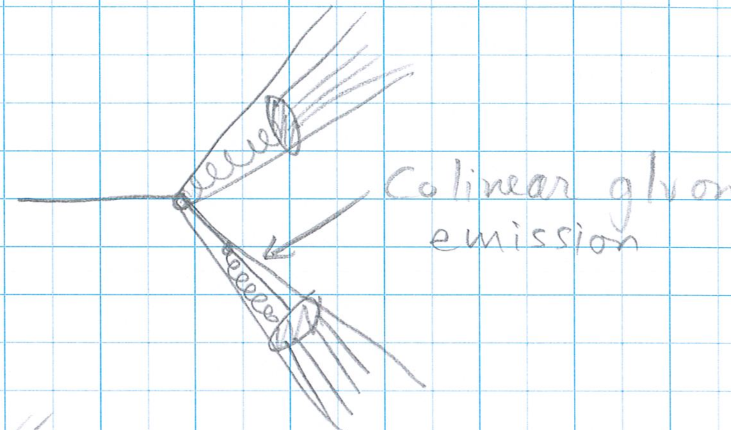
Jet-finding/clustering algorithms.

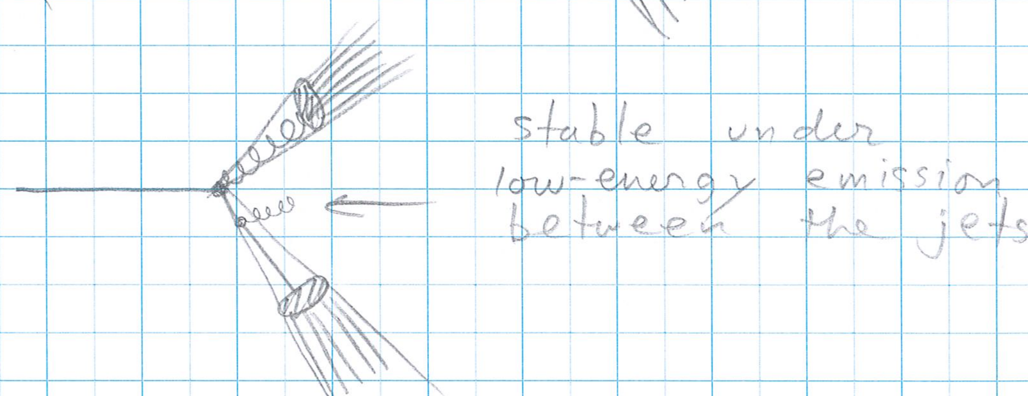
Requirements:

1. Colinear safety. The same jet should be found if a particle in the jet splits into two colinear particles.

2. Infrared safety. The same jets should be found if soft emission occurs.







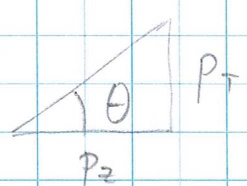
Inputs to jet-finding algorithms:

1. Calorimeter energy: E_i, φ_i, η_i

   Energy is measured. Calorimeter segmentation determines φ_i, η_i.

   Calorimeter energy can be clustered. Adjacent energy deposits can be added together and (φ_i, η_i) could be calculated from the weighted mean.

2. Tracking information: p_x, p_y, $p_z = p_T \sinh\eta$



$$\tan\theta = \frac{p_T}{p_z} \qquad p_z = p_T \cot\theta$$

3. Monte Carlo generator parton information:

   exact 4-vectors of each parton.

Two main types of algorithms:

- Cone Algorithms
- Sequential clustering Algorithms.

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Iterative Cone Algorithm.

1. Find a seed (most energetic particle in the event)

2. Find all particles within $\Delta R = \sqrt{\Delta\varphi^2 + \Delta\eta^2}$ of the seed. Sum up their momenta.

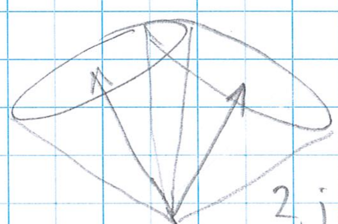
3. If the trial jet axis and the seed axis are identical (within some precision) then
   - Remove all particles from the event and proceed to cluster the remaining particles

   else
   - Repeat with the trial jet axis as the new seed.

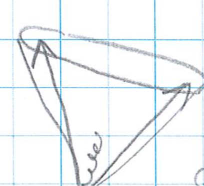
4. Iterate until no more seeds above threshold (eg 1 GeV) are left.

Some problems.

- not IR safe

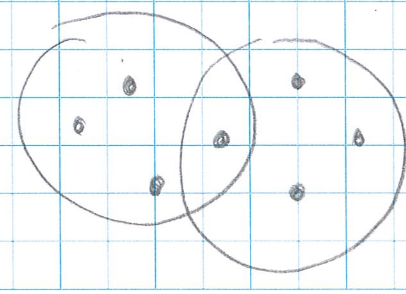


2 jets

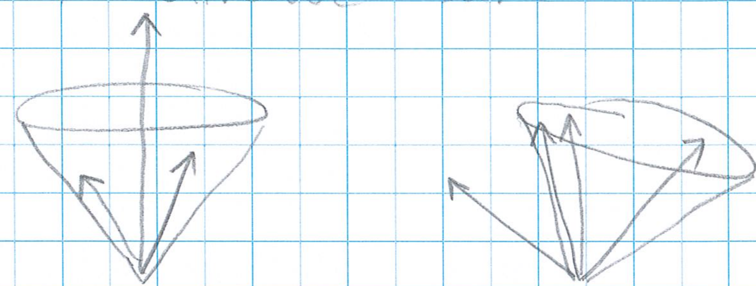


one jet.

Overlap:



Not Colinear safe:



Various remidies have been proposed

- Seedless jet clustering algorithms
- Splitting / merging algorithms

$k_\perp$ Algorithms:

1. For each cluster, define

$$d_i = p_{T_i}^2$$

For each pair of clusters, define

$$d_{ij} = \min\left(p_{T_i}^2, p_{T_j}^2\right) \frac{\Delta R_{ij}^2}{D^2}$$

$$\Delta R_{ij}^2 = (y_i - y_j)^2 + (\varphi_i - \varphi_j)^2$$

$D \approx 1$ is a parameter.

2. Let $d_{min} = \min(\text{all } d_i, \text{ all } d_{ij})$

3. If d_{min} is a d_{ij} then cluster i and j into a new cluster with

$$E_{ij} = E_i + E_j$$
$$\vec{P}_{ij} = \vec{P}_i + \vec{P}_j$$

4. If d_{min} is a d_i then remove from the list of clusters and add to the list of jets.

Variants:

Let $d_{ij} = \min\left(k_{T_i}^{2p}, k_{T_j}^{2p} \right) \frac{\Delta R_{ij}}{D}$

$p = 1$ is the k_T clustering algorithm.

$p = -1$ is the anti-k_T algorithm.

d_{ij} is determined by a hard cluster and the algorithm will merge low energy clusters with high energy ones before clustering low energy clusters together.

Other practical considerations:

Combining calorimeter and tracking information (particle flow)

Pileup and underlying event subtraction

Correction for Jet energy response/scale.

The k_T/anti-k_T algorithms are IR and colinear safe and can be applied to partons in a well defined way.

Event Shape Variables.

- Primarily used at e^+e^- and ep colliders.

Example:

Thrust, $T = \max\limits_{\hat{n}} \frac{\sum |\vec{p}_i \cdot \hat{n}|}{\sum |\vec{p}_i|}$

Leading order $e^+e^- \to q\bar{q}$ has $T = 1$.

Hard gluon emission $e^+e^- \to q\bar{q}g$ has $T < 1$.

Sphericity tensor $S_{\alpha\beta} = \frac{\sum_i p_{i\alpha} p_{i\beta}}{\sum |\vec{p}_i|^2}$

$\alpha = x, y, z$.

Principal axes $\hat{n}_1, \hat{n}_2, \hat{n}_3$ with eigenvalues $Q_1 < Q_2 < Q_3$, normalized so that $Q_1 + Q_2 + Q_3 = 1$.

Sphericity, $S = \frac{3}{2}(1 - Q_3) = \frac{3}{2}(Q_1 + Q_2)$

$S = 0$ for 2-jet event

$S = 1$ for isotropic