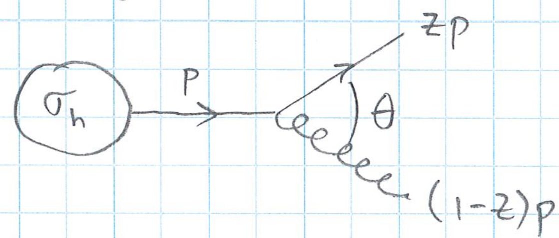
Hard gluon emission

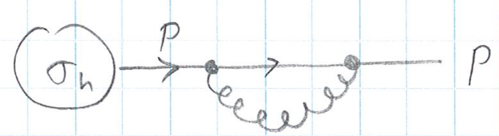


$C_F = \frac{4}{3}$ for 3 colors.

$$\sigma_{h+g} \approx \sigma_h \frac{\alpha_s C_F}{\pi} \frac{dE}{E} \frac{d\theta^2}{\theta^2} = \sigma_h \frac{\alpha_s C_F}{\pi} \frac{dz}{1-z} \frac{dk_t^2}{k_t^2}$$

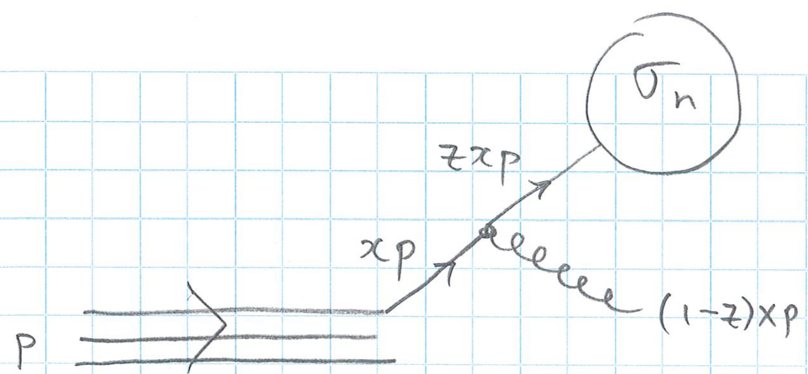
$$E = (1-z)p$$
$$k_t = E \sin\theta = E\theta \qquad (\theta \ll 1)$$

We must subtract the contribution to the cross section that is indistinguishable from virtual gluon emission



$$\sigma_{g+h} + \sigma_{v+h} = \frac{\alpha_s C_F}{\pi} \underbrace{\int_0^{Q^2} \frac{dk_t^2}{k_t^2}}_{\text{divergent}} \underbrace{\int \frac{dz}{1-z} \left(\sigma_h(zp) - \sigma_h(p) \right)}_{\text{finite}}$$

We do not expect perturbative QCD to work below some scale, $\mu^2 \approx 1\ \text{GeV}^2$ so we cut off the first integral to make it finite.



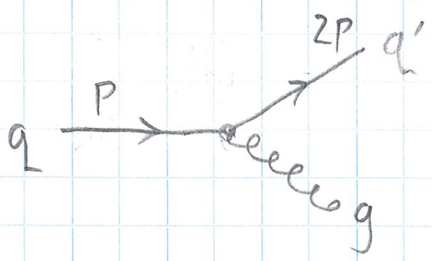
Leading order $\sigma_0 = \int dx\, \sigma_h(xp)\, q(x,\mu^2)$

First order

$$\sigma_1 \approx \frac{\alpha_s C_F}{\pi} \underbrace{\int_{\mu^2}^{Q^2} \frac{dk_t^2}{k_t^2}}_{\text{large but Finite}} \underbrace{\int \frac{dx\,dz}{1-z}\left(\sigma_h(zxp) - \sigma_h(xp)\right) q(x,\mu^2)}_{\text{Finite}}$$

μ^2 is called the factorization scale.

μ^2 is somewhat arbitrary. We need to predict how $q(x,\mu^2)$ will evolve as a function of this factorization scale.



$$q'(x) \sim \int \frac{dz}{1-z} \int_{\mu^2}^{Q^2} \frac{dk_t^2}{k_t^2}\, q(x/z)$$

$$\sim \log Q^2 \int \frac{dz}{1-z}\, q(x/z)$$

$$\frac{dq(x,\mu^2)}{d\log Q^2} \sim \int \frac{dz}{1-z}\, q(x/z,\mu^2)$$

This expresses how q will evolve with $\log Q^2$ due to gluon emission.

But gluons can also split into quarks

$$\frac{d}{d\log Q^2}\begin{pmatrix} q \\ g \end{pmatrix} = \begin{pmatrix} P_{q\leftarrow q} & P_{q\leftarrow g} \\ P_{g\leftarrow q} & P_{g\leftarrow g} \end{pmatrix} \otimes \begin{pmatrix} q \\ g \end{pmatrix}$$

Splitting functions

$$P_{q\leftarrow q} = C_F\left(\frac{1+z^2}{1-z}\right) \qquad C_F = \frac{4}{3}$$

$$P_{g\leftarrow q} = C_F\left(\frac{1+(1-z)^2}{z}\right)$$

$$P_{q\leftarrow g} = T_R\left(z^2+(1-z)^2\right) \qquad T_R = \frac{1}{2}$$

$$P_{g\leftarrow g} = 2C_A\left(\frac{z}{1-z} + \frac{1-z}{z} + z(1-z)\right) + \delta(1-z)\cdots$$

$$C_A = 3$$

These allow us to evolve the parton densities to different Q^2.

Once $f_{q/p}$ or $f_{g/p}$ is known or measured at one Q^2 it can be evolved to different Q^2.

Evolution in $\log Q^2$: DGLAP

Evolution at small x : BFKL

Example of using pdf sets: LHAPDF

http://arxiv.org/abs/1412.7420

This is an interface to many different attempts to analyze DIS data and extrapolate pdf's to different Q^2, x.

Example: CT14 nnlo

Check the sum rules:

$$\int_0^1 \frac{1}{x}\left(x u(x) - x\bar{u}(x) \right) dx \sim 1.93$$

$$\int_0^1 \frac{1}{x}\left(x d(x) - x\bar{d}(x) \right) dx \sim 0.96$$

$$\sum_i \int_0^1 x\, q_i(x)\, dx = 0.446$$

$$\sum_i \int_0^1 x\, \bar{q}_i(x)\, dx = 0.100$$

$$\int_0^1 x\, g(x)\, dx = 0.440$$

$$\sum = 0.986$$