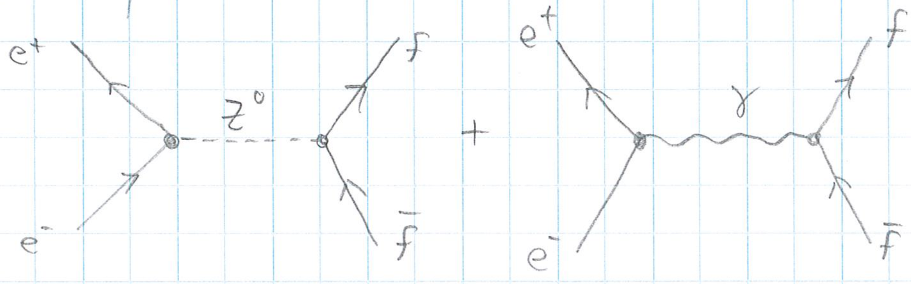
Physics at particle colliders.

Consider first a lepton collider, such as LEP.

LEP operated from 1989 to 1996 at $\sqrt{s} = 91$ GeV and up to $\sqrt{s} = 209$ GeV by 2000.

Physics processes:



$$-i\mathcal{M}_{LL} = \frac{ie^2}{s}\left(1 + r\, c_L^e c_L^f\right) \bar{v}_L^e \gamma^\mu u_R^e \; \bar{u}_L^f \gamma_\mu v_R^f$$

and $-i\mathcal{M}_{LR}$ is similar.

$$r = \frac{\sqrt{2}\, G_F M_Z^2}{s - M_Z^2 + i\Gamma_Z M_Z}\left(\frac{s}{e^2}\right)$$

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4s}\left(A_0\left(1+\cos^2\theta\right) + A_1 \cos\theta\right)$$

$$A_0 = \frac{1}{4}\Big(|1 + r c_L^e c_L^f|^2 + |1 + r c_L^e c_R^f|^2 + |1 + r c_R^e c_L^f|^2 + |1 + r c_R^e c_R^f|^2 \Big)$$

$$A_1 = \frac{1}{2}\Big(|1 + r c_L^e c_L^f|^2 - |1 + r c_L^e c_R^f|^2 + |1 + r c_R^e c_R^f|^2 - |1 + r c_R^e c_L^f|^2 \Big)$$

$$c_L^e = -\frac{1}{2} + \sin^2\theta_W$$
$$c_R^e = \sin^2\theta_W$$
etc.

For lepton colliders, the colliding particles are fundamental, with no substructure.

When be beams collide along the z-axis the initial transverse momentum is zero and the collision products will be uniformly distributed in φ.

If beams have equal and opposite momentum, eg

$$E_+ = 45.5 \text{ GeV} \qquad p_z^+ = 45.5 \text{ GeV}/c$$
$$E_- = 45.5 \text{ GeV} \qquad p_z^- = -45.5 \text{ GeV}/c$$

then the center of mass frame is also the lab frame since $\Sigma p_z = 0$.

The scattering angle θ with respect to the z-axis is well defined and is directly related to the matrix element.

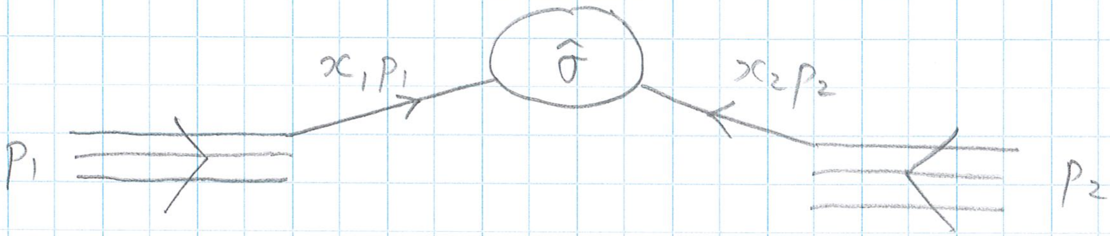
If the colliding particles were not fundamental eg, protons, then the centre of mass frame of the colliding partons would not coincide with the lab frame due to their internal motion within the proton.

Parton model:

In the relativistic limit, the partons are frozen into pancakes. Each carries a fraction x of the proton's momentum.

$$\sigma = \int dx_1 f_{q/p}(x_1) \int dx_2 f_{\bar{q}/p}(x_2)\, \hat{\sigma}(x_1 p_1, x_2 p_2)$$

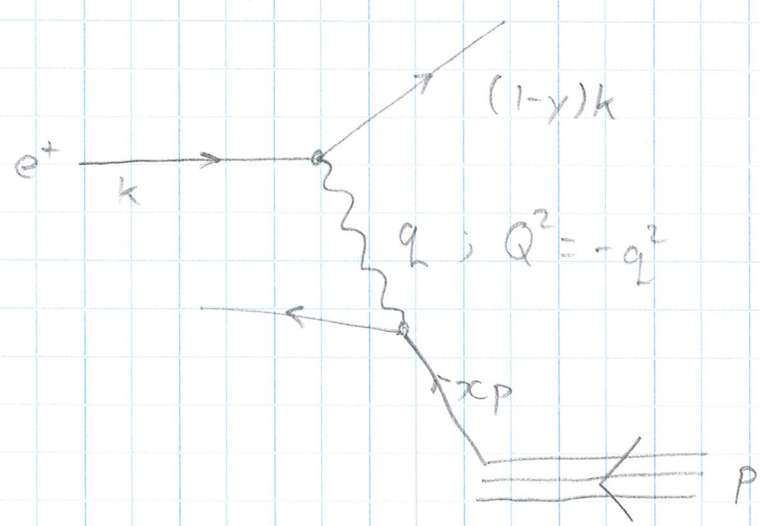
$$\hat{s} = x_1 x_2 s$$



The cross section factors into a hard part $\hat{\sigma}$ which is calculated perturbatively (eg, Feynman diagrams) and is convoluted with parton distribution functions

$f_{q/p}(x)$ is the probability density for finding q in the proton with momentum fraction x.

Simpler process



$$x = \frac{Q^2}{2p\cdot q} \qquad y = \frac{p\cdot q}{p\cdot k}$$

$$Q^2 = xys$$

Q^2 is the photon virtuality. Large Q^2 has a smaller Compton wavelength and probes smaller structure within the proton.

$$\frac{d^2\sigma}{dx\,dQ^2} = \frac{4\pi\alpha^2}{x\,Q^4}\left(\frac{1+(1-y)^2}{2} F_2^p(x) \right)$$

$$F_2^p(x) = x\left(e_u^2\, u(x) + e_d^2\, d(x) \right) = x\left(\frac{4}{9} u(x) + \frac{1}{9} d(x) \right)$$

$u(x) = f_{u/p}(x) \qquad d(x) = f_{d/p}(x)$

Isospin symmetry: $f_{u/p}(x) = f_{d/n}(x)$

$f_{d/p}(x) = f_{u/n}(x)$

$$F_2^n(x) = x\left(\frac{4}{9} d(x) + \frac{1}{9} u(x) \right)$$

We observe that

$$\int x\,u(x)\,dx \approx 2\int x\,d(x)\,dx$$

So u-quarks carry about twice as much momentum as d quarks

But $\int u(x)\,dx \longrightarrow \infty$

$\int d(x)\,dx \longrightarrow \infty$

Why? We actually are sensitive to

$$F_2^p(x) = x\left(e_u^2\left(u(x) + \bar{u}(x)\right) + e_d^2\left(d(x) + \bar{d}(x)\right) \right)$$

There are an infinite number of $q\bar{q}$ pairs, but most carry almost no momentum (ie, $x \to 0$).

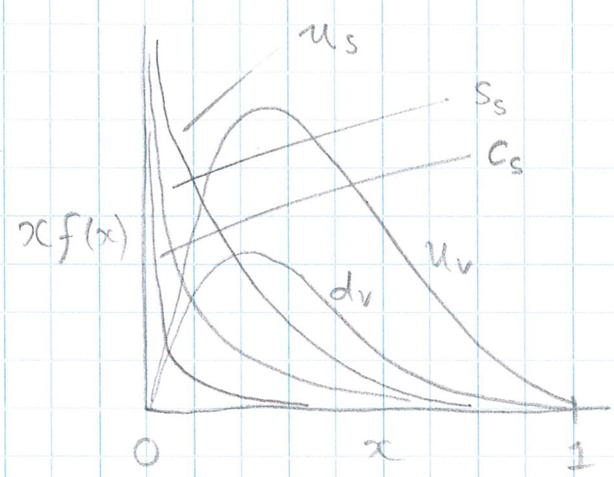
Valence quarks:

$$u_v(x) = u(x) - \bar{u}(x)$$

$$d_v(x) = d(x) - \bar{d}(x)$$

$$\int dx \left(u(x) - \bar{u}(x) \right) = 2$$

$$\int dx \left(d(x) - \bar{d}(x) \right) = 1 .$$



Virtual quarks are the "sea" quarks.

Momentum sum rule: $\sum_i \int dx\, x\, q_i(x) = 1$

Quarks only account for about half the momentum of the proton.

The rest is due to gluons.