Assignment #8 or 9...

one or the other...

1. We have used the definition for the $Zf\bar{f}$ coupling:

$$\frac{-ig}{\cos\theta_W}\gamma^\mu \frac{1}{2}(c_V^f - c_A^f\gamma^5)$$

where
$$c_V^f = T_f^3 - 2Q_f \sin^2\theta_W$$
$$c_A^f = T_f^3$$

The partial width $\Gamma_{f\bar{f}}$ is given by

$$\Gamma_{f\bar{f}} = \frac{N_c G_F M_Z^3}{6\pi\sqrt{2}}\left((c_A^f)^2 + (c_V^f)^2\right)$$

The $Zf\bar{f}$ coupling can be expressed in terms of left- and right-handed couplings using,

$$\frac{1}{2}(c_V^f - c_A^f\gamma^5) = c_L^f \frac{1}{2}(1-\gamma^5) + c_R^f\frac{1}{2}(1+\gamma^5)$$

Where
$$c_R^f + c_L^f = c_V^f$$
$$c_L^f - c_R^f = c_A^f$$

and hence,
$$c_L^f = \frac{1}{2}(c_V^f + c_A^f)$$
$$c_R^f = \frac{1}{2}(c_V^f - c_A^f)$$

In this case, the cross sections for polarized $e^+e^- \to f\bar{f}$ are, when $\sqrt{s} \sim M_Z$,

$$\frac{d\sigma_{LL}}{d\Omega} = \frac{N_c\alpha^2}{4s}(1+\cos\theta)^2 |4rc_L^e c_L^f|^2$$
$$\frac{d\sigma_{LR}}{d\Omega} = \frac{N_c\alpha^2}{4s}(1-\cos\theta)^2 |4rc_L^e c_R^f|^2$$
$$\frac{d\sigma_{RL}}{d\Omega} = \frac{N_c\alpha^2}{4s}(1-\cos\theta)^2 |4rc_R^e c_L^f|^2$$
$$\frac{d\sigma_{RR}}{d\Omega} = \frac{N_c\alpha^2}{4s}(1+\cos\theta)^2 |4rc_R^e c_R^f|^2$$

Notice that these differ from the expressions in the lecture notes by a factor of 4.

This can be traced to the definition of $c_L^f = \frac{1}{2}(c_V^f + c_A^f)$ and $c_R^f = \frac{1}{2}(c_V^f - c_A^f)$ which is different from the one used in Halzen & Martin (p. 306, eqn. 13.59) which is $c_R^f = c_V^f - c_A^f$, $c_L^f = c_V^f + c_A^f$ which differs by a factor of 2 from the intended definition for this class.

Thus, we will write,

$$\frac{d\sigma_{LL}}{d\Omega} = 4N_c\frac{\alpha^2}{s}(1+\cos\theta)^2|r|^2(c_L^e)^2(c_L^f)^2$$

$$\frac{d\sigma_{LR}}{d\Omega} = 4N_c\frac{\alpha^2}{s}(1-\cos\theta)^2|r|^2(c_L^e)^2(c_R^f)^2$$

$$\frac{d\sigma_{RL}}{d\Omega} = 4N_c\frac{\alpha^2}{s}(1-\cos\theta)^2|r|^2(c_R^e)^2(c_L^f)^2$$

$$\frac{d\sigma_{RR}}{d\Omega} = 4N_c\frac{\alpha^2}{s}(1+\cos\theta)^2|r|^2(c_R^e)^2(c_R^f)^2$$

The total spin-averaged cross section is

$$\frac{d\sigma}{d\Omega} = \frac{N_c\alpha^2}{s}(1+\cos^2\theta)|r|^2\left[(c_L^e)^2+(c_R^e)^2\right]\left[(c_L^f)^2+(c_R^f)^2\right] + 2\frac{N_c\alpha^2}{s}\cos\theta\,|r|^2\left[(c_L^e)^2-(c_R^e)^2\right]\left[(c_L^f)^2-(c_R^f)^2\right]$$

Now, $(c_L^f)^2 + (c_R^f)^2 = \frac{1}{2}\left((c_V^f)^2 + (c_A^f)^2\right)$

and $(c_L^f)^2 - (c_R^f)^2 = c_V^f c_A^f$

So $$\frac{d\sigma}{d\Omega} = \frac{N_c \alpha^2}{4s}(1+\cos^2\theta)|r|^2\left[(c_V^e)^2+(c_A^e)^2\right]\left[(c_V^f)^2+(c_A^f)^2\right] + \frac{2N_c\alpha^2}{s}\cos\theta\,|r|^2\left[c_V^e c_A^e\right]\left[c_V^f c_A^f\right]$$

Recall that $$r \equiv \frac{\sqrt{2}\,G_F M_Z^2}{s - M_Z^2 + i\Gamma_Z M_Z}\left(\frac{s}{e^2}\right)$$
$$= \frac{\sqrt{2}\,G_F M_Z^2}{s - M_Z^2 + i\Gamma_Z M_Z}\left(\frac{s}{4\pi\alpha}\right)$$

When $s = M_Z^2$, this is $r = \dfrac{-i\sqrt{2}\,G_F M_Z^3}{4\pi\alpha\,\Gamma_Z}$

and thus, $$\frac{d\sigma}{d\Omega} = \frac{N_c G_F^2 M_Z^4}{32\pi^2\Gamma_Z^2}(1+\cos^2\theta)\left[(c_V^e)^2+(c_A^e)^2\right]\left[(c_V^f)^2+(c_A^f)^2\right] + \frac{N_c G_F^2 M_Z^4}{4\pi^2\Gamma_Z^2}\cos\theta\left[c_V^e c_A^e\right]\left[c_V^f c_A^f\right]$$

Next, usind $$\int(1+\cos^2\theta)\,d\Omega = \frac{16\pi}{3}$$
and $$\int\cos\theta\,d\Omega = 0$$

we have $$\sigma = \frac{N_c G_F^2 M_Z^4}{6\pi\,\Gamma_Z^2}\left[(c_V^e)^2+(c_A^e)^2\right]\left[(c_V^f)^2+(c_A^f)^2\right]$$

But if we write $$\Gamma_f = \frac{N_c G_F M_Z^3}{6\pi\sqrt{2}}\left[(c_V^f)^2+(c_A^f)^2\right]$$

then $$\Gamma_e\Gamma_f = \frac{N_c G_F^2 M_Z^6}{72\pi^2}\left[(c_V^e)^2+(c_A^e)^2\right]\left[(c_V^f)^2+(c_A^f)^2\right]$$

Therefore, $$\sigma = \frac{12\pi\,\Gamma_e\Gamma_f}{M_Z^2\,\Gamma_Z^2}$$

2. The differential cross section for $e^+e^- \to \mu^+\mu^-$ can be written

$$\frac{d\sigma}{d\Omega}(e_L^- e_R^- \to \mu_L^- \mu_R^+) = \frac{\alpha^2}{4s}(1+\cos\theta)^2 \left|1 + 4rc_L^e c_L^\mu\right|^2$$

with similar expressions for $LR \to RL$, $RL \to LR$ and $RL \to RL$.

When the final state particle is a fermion with charge Q_f (such that $Q_\mu = -1$) then we can write

$$\frac{d\sigma}{d\Omega}(e_L^- e_R^+ \to f_L \bar{f}_R) = \frac{N_c\alpha^2}{4s}(1+\cos\theta)^2 \left|-Q_f + 4rc_L^e c_L^f\right|^2$$

$$\frac{d\sigma}{d\Omega}(e_L^- e_R^+ \to f_R \bar{f}_L) = \frac{N_c\alpha^2}{4s}(1-\cos\theta)^2 \left|-Q_f + 4rc_L^e c_R^f\right|^2$$

$$\frac{d\sigma}{d\Omega}(e_R^- e_L^+ \to f_L \bar{f}_R) = \frac{N_c\alpha^2}{4s}(1-\cos\theta)^2 \left|-Q_f + 4rc_R^e c_L^f\right|^2$$

$$\frac{d\sigma}{d\Omega}(e_R^- e_L^+ \to f_R \bar{f}_L) = \frac{N_c\alpha^2}{4s}(1+\cos\theta)^2 \left|-Q_f + 4rc_R^e c_R^f\right|^2$$

where $$r = \frac{\sqrt{2}\, G_F M_Z^2}{s - M_Z^2 + i\Gamma_Z M_Z}\left(\frac{s}{4\pi\alpha}\right)$$

Then, $$\frac{d\sigma}{d\Omega} = \frac{N_c\alpha^2}{16s}\left[A_0(1+\cos^2\theta) + 2A_1\cos\theta\right]$$

where $$A_0 = \left[\left|-Q_f + 4c_L^e c_L^f\right|^2 + \left|-Q_f + 4c_L^e c_R^f\right|^2 + \left|-Q_f + 4c_R^e c_L^f\right|^2 + \left|-Q_f + 4c_R^e c_R^f\right|^2\right]$$

$$A_1 = \left[\left|-Q_f + 4c_L^e c_L^f\right|^2 - \left|-Q_f + 4c_L^e c_R^f\right|^2 - \left|-Q_f + 4c_R^e c_L^f\right|^2 + \left|-Q_f + 4c_R^e c_R^f\right|^2\right]$$

Since
$$\int_0^1 (1+\cos^2\theta)\, d(\cos\theta) = \left(y + \tfrac{1}{3}y^3\right)\Big|_0^1 = \frac{4}{3}$$
$$\int_{-1}^0 (1+\cos^2\theta)\, d(\cos\theta) = \left(y + \tfrac{1}{3}y^3\right)\Big|_{-1}^0 = \frac{4}{3}$$
$$\int_0^1 \cos\theta\, d(\cos\theta) = \tfrac{1}{2}y^2\Big|_0^1 = \frac{1}{2}$$
$$\int_{-1}^0 \cos\theta\, d(\cos\theta) = \tfrac{1}{2}y^2\Big|_{-1}^0 = -\frac{1}{2}$$

we have

$$A_{FB} = \frac{\displaystyle\int_0^1 \frac{d\sigma}{d(\cos\theta)}\, d(\cos\theta) - \int_{-1}^0 \frac{d\sigma}{d(\cos\theta)}\, d(\cos\theta)}{\displaystyle\int_0^1 \frac{d\sigma}{d(\cos\theta)}\, d(\cos\theta) + \int_{-1}^0 \frac{d\sigma}{d(\cos\theta)}\, d(\cos\theta)}$$

$$= \frac{3\left(|-Q_f + 4rc_L^e c_L^f|^2 - |-Q_f + 4rc_L^e c_R^f|^2 - |-Q_f + 4rc_R^e c_L^f|^2 + |-Q_f + 4rc_R^e c_R^f|^2\right)}{4\left(|-Q_f + 4rc_L^e c_L^f|^2 + |-Q_f + 4rc_L^e c_R^f|^2 + |-Q_f + 4rc_R^e c_L^f|^2 + |-Q_f + 4rc_R^e c_R^f|^2\right)}$$

In order to graph this as a function of $\sqrt{s}$, first calculate r, then calculate

$$W_{LL} = Q_f^2 - 2Q_f \mathrm{Re}(r)\, c_L^e c_L^f + |r|^2 (c_L^e)^2 (c_L^f)^2$$
$$W_{LR} =$$
$$W_{RL} = \quad \text{etc.}$$
$$W_{RR} =$$

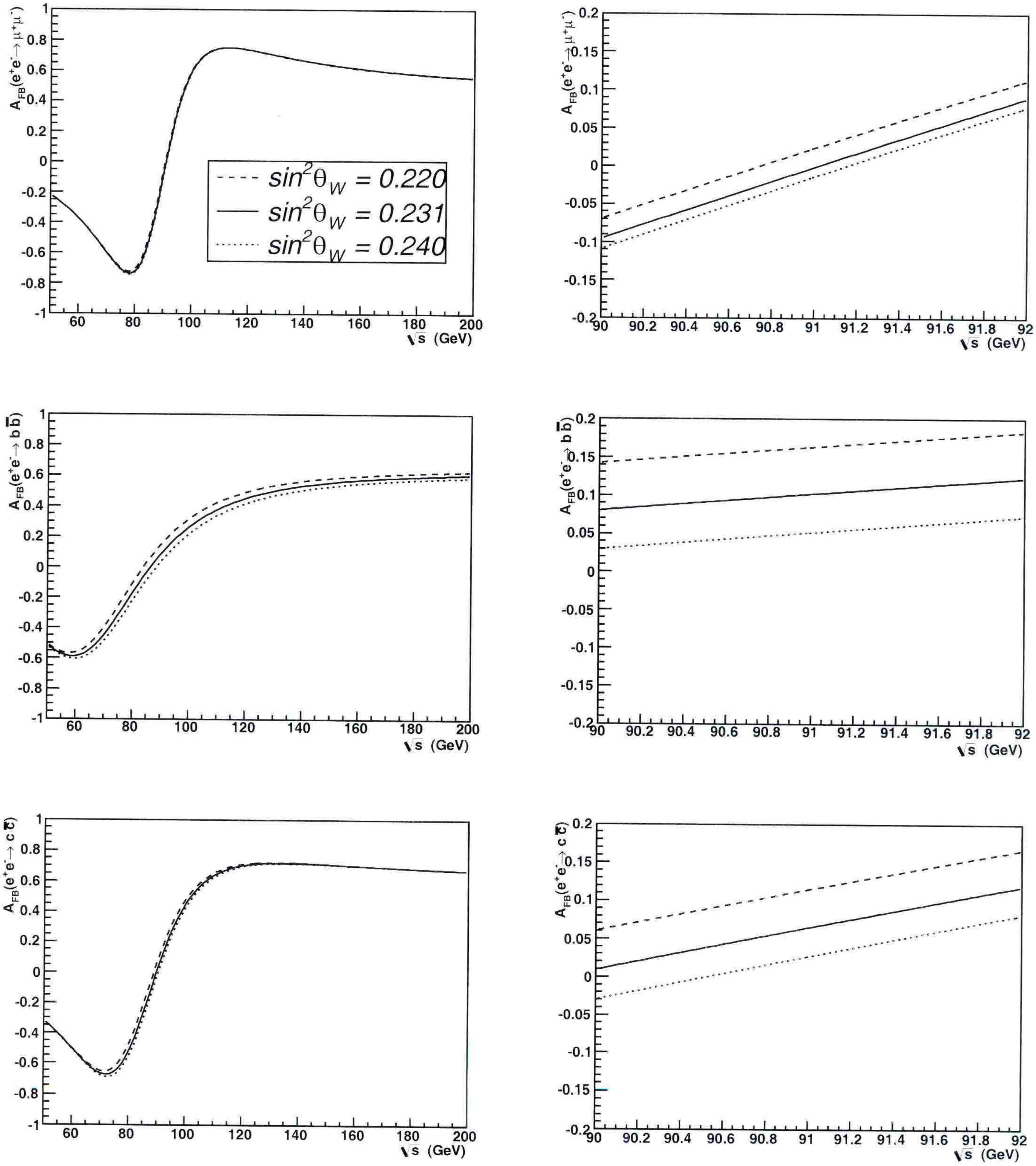
where c_L^f and c_R^f are calculated in terms of T_f^3 and $\sin^2\theta_W$.

The muon forward-backward asymmetry would be the easiest to measure because muons can be easily identified and their charge is easily measured.

The b and c asymmetries require not only determining the parent quark in a jet of hadrons but also estimating its charge.

The b is easier to tag because of its larger mass and lifetime which can produce high p_T leptons with significant displacements from the e^+e^- collision point. The charge of the lepton is correlated with the original b-quark flavor, but the correlation is reduced by effects like $B^0 \bar{B}^0$ mixing.

The c is hardest because its shorter lifetime and lower mass cause c-jets to resemble light quark jets.



$A_{FB}(e^+e^- \to \mu^+\mu^-)$
$A_{FB}(e^+e^- \to b\bar{b})$
$A_{FB}(e^+e^- \to c\bar{c})$
$\sqrt{s}$ (GeV)
$\sin^2\theta_W = 0.220$
$\sin^2\theta_W = 0.231$
$\sin^2\theta_W = 0.240$

```
double afb(double *x,double *p) {
  double mz = 91.2;
  double gz = 2.4952;
  double alpha = 1/128.0;      //  Electroweak coupling at sqrt(s)=M_z
  double gf = 1.166e-5;
  double pi = 3.141592654;


  double sin2thw = p[0];
  double qf = p[1];
  double t3 = p[2];
  double cve = -0.5 + 2*sin2thw;
  double cae = -0.5;
  double cle = 0.5*(cve+cae);
  double cre = 0.5*(cve-cae);
  double cvf = t3 - 2*qf*sin2thw;
  double caf = t3;
  double clf = 0.5*(cvf+caf);
  double crf = 0.5*(cvf-caf);
  double roots = x[0];
  double s = roots*roots;
  TComplex den(s-mz*mz,gz*mz);
  TComplex num(sqrt(2)*gf*mz*mz*s/(4*pi*alpha),0);
  TComplex r = num/den;
  TComplex zll = TComplex(-qf + 4*r*cle*clf);
  TComplex zlr = TComplex(-qf + 4*r*cle*crf);
  TComplex zrl = TComplex(-qf + 4*r*cre*clf);
  TComplex zrr = TComplex(-qf + 4*r*cre*crf);

  double wll = pow(TComplex::Abs(zll),2);
  double wlr = pow(TComplex::Abs(zlr),2);
  double wrl = pow(TComplex::Abs(zrl),2);
  double wrr = pow(TComplex::Abs(zrr),2);

  double a0 = wll+wlr+wrl+wrr;
  double a1 = wll-wlr-wrl+wrr;
  return 3*a1/(4*a0);
}

void GraphAFB() {
  gROOT->SetStyle("Plain");
  TF1 *fafb0 = new TF1("afb",afb,50,200,3);
  fafb0->SetParameters(0.231,-1,-0.5);
  fafb0->SetLineWidth(1);
  TF1 *fafb1 = new TF1("afb1",afb,50,200,3);
  fafb1->SetParameters(0.22,-1,-0.5);
  fafb1->SetLineStyle(2);
  fafb1->SetLineWidth(1);
  TF1 *fafb2 = new TF1("afb2",afb,50,200,3);
  fafb2->SetParameters(0.24,-1,-0.5);
  fafb2->SetLineStyle(3);
  fafb2->SetLineWidth(1);

  TCanvas *c1 = new TCanvas("c1",NULL,0,0,700,800);
  c1->Divide(2,3);
  c1->cd(1);
  TH1F *h = gPad->DrawFrame(50,-1,200,1);
  h->GetXaxis()->SetTitle("#sqrt{s}  (GeV)");
  h->GetYaxis()->SetTitle("A_{FB}(e^{+}e^{-}#rightarrow #mu^{+}#mu^{-})");
```

```
fafb0->Draw("CSAME");
fafb1->Draw("CSAME");
fafb2->Draw("CSAME");

TLegend *leg = new TLegend(0.4,0.15,0.85,0.50);
leg->SetFillStyle(0);
leg->SetTextFont(52);
leg->AddEntry(fafb1,"sin^{2}#theta_{W} = 0.220","L");
leg->AddEntry(fafb0,"sin^{2}#theta_{W} = 0.231","L");
leg->AddEntry(fafb2,"sin^{2}#theta_{W} = 0.240","L");
leg->Draw();
c1->cd(2);
TH1F *h = gPad->DrawFrame(90,-0.2,92,0.2);
h->GetXaxis()->SetTitle("#sqrt{s}  (GeV)");
h->GetYaxis()->SetTitle("A_{FB}(e^{+}e^{-}#rightarrow #mu^{+}#mu^{-})");
fafb0->Draw("CSAME");
fafb1->Draw("CSAME");
fafb2->Draw("CSAME");

TF1 *bafb0 = new TF1("bafb",afb,50,200,3);
bafb0->SetLineWidth(1);
TF1 *bafb1 = new TF1("bafb1",afb,50,200,3);
bafb1->SetLineStyle(2);
bafb1->SetLineWidth(1);
TF1 *bafb2 = new TF1("bafb2",afb,50,200,3);
bafb2->SetParameters(0.24,-1,-0.5);
bafb2->SetLineStyle(3);
bafb2->SetLineWidth(1);

c1->cd(3);
bafb0->SetParameters(0.231,-1/3.,-.5);
bafb1->SetParameters(0.220,-1/3.,-.5);
bafb2->SetParameters(0.240,-1/3.,-.5);
TH1F *h = gPad->DrawFrame(50,-1,200,1);
h->GetXaxis()->SetTitle("#sqrt{s}  (GeV)");
h->GetYaxis()->SetTitle("A_{FB}(e^{+}e^{-}#rightarrow b #bar{b})");
bafb0->Draw("CSAME");
bafb1->Draw("CSAME");
bafb2->Draw("CSAME");

c1->cd(4);
TH1F *h = gPad->DrawFrame(90,-0.2,92,0.2);
h->GetXaxis()->SetTitle("#sqrt{s}  (GeV)");
h->GetYaxis()->SetTitle("A_{FB}(e^{+}e^{-}#rightarrow b #bar{b})");
bafb0->Draw("CSAME");
bafb1->Draw("CSAME");
bafb2->Draw("CSAME");

TF1 *cafb0 = new TF1("cafb",afb,50,200,3);
cafb0->SetLineWidth(1);
TF1 *cafb1 = new TF1("cafb1",afb,50,200,3);
cafb1->SetLineStyle(2);
cafb1->SetLineWidth(1);
TF1 *cafb2 = new TF1("cafb2",afb,50,200,3);
cafb2->SetParameters(0.24,-1,-0.5);
cafb2->SetLineStyle(3);
cafb2->SetLineWidth(1);

c1->cd(5);
cafb0->SetParameters(0.231,2/3.,0.5);
```

```
  cafb1->SetParameters(0.220,2/3.,0.5);
  cafb2->SetParameters(0.240,2/3.,0.5);
  TH1F *h = gPad->DrawFrame(50,-1,200,1);
  h->GetXaxis()->SetTitle("#sqrt{s}   (GeV)");
  h->GetYaxis()->SetTitle("A_{FB}(e^{+}e^{-}#rightarrow c #bar{c})");
  cafb0->Draw("CSAME");
  cafb1->Draw("CSAME");
  cafb2->Draw("CSAME");

  c1->cd(6);
  TH1F *h = gPad->DrawFrame(90,-0.2,92,0.2);
  h->GetXaxis()->SetTitle("#sqrt{s}   (GeV)");
  h->GetYaxis()->SetTitle("A_{FB}(e^{+}e^{-}#rightarrow c #bar{c})");
  cafb0->Draw("CSAME");
  cafb1->Draw("CSAME");
  cafb2->Draw("CSAME");
}
```