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The weak interaction maximally violates parity but (almost) conserves CP.

So although there are no right-handed neutrinos, there are right-handed anti-neutrinos.

Decays that occur through a weak interaction should conserve CP.

Consider the neutral kaons produced in a strong interaction. $s\bar{s}$ quarks produced in pairs so we know what the quark content is when a K^0 is formed.

$$\left.\begin{aligned} K^0 &= (\bar{s}d) \\ \bar{K}^0 &= (s\bar{d}) \end{aligned}\right\} \text{ Strong eigenstates.}$$

$$CP|K^0\rangle = \eta|\bar{K}^0\rangle$$
$$CP|\bar{K}^0\rangle = \eta'|K^0\rangle$$
$$(CP)^2|K^0\rangle = \eta\eta'|K^0\rangle$$

By convention we take $\eta = \eta' = 1$.

$|K^0\rangle$ and $|\bar{K}^0\rangle$ are not CP eigenstates.

But we can define

$$|K_S^0\rangle = \frac{1}{\sqrt{2}}\left(|K^0\rangle + |\bar{K}^0\rangle\right) \qquad CP \text{ even}$$

$$|K_L^0\rangle = \frac{1}{\sqrt{2}}\left(|K^0\rangle - |\bar{K}^0\rangle\right) \qquad CP \text{ odd}$$

How can these decay?

$$K_L^0 \to 3\pi \qquad CP \text{ odd}$$

$$K_S^0 \to 2\pi \qquad CP \text{ even.}$$

Example: $\pi^+ \, \pi^- \xrightarrow{C} \pi^- \, \pi^+ \xrightarrow{P} \pi^+ \, \pi^-$

Although the invariant amplitude for the weak decay is similar, there is very little phase space available for the 3π final state.

$$\Gamma = \int \frac{|\mathcal{M}|^2}{2M} \, dQ$$

$$\int dQ_{3\pi} \ll \int dQ_{2\pi}$$

$$\tau = \frac{\hbar}{\Gamma} \qquad \tau_{K_S^0} = .9 \times 10^{-10}\,s$$

$$\tau_{K_L^0} = .5 \times 10^{-7}\,s$$

If you start with a pure K_S^0 state then at a later time t, the amplitude of the system is

$$A_S(t) = A_S(0)\, e^{-(\Gamma_S/2 + i m_S)t}$$

so that $|A_S(t)|^2 = |A_S(0)|^2 e^{-\Gamma_S t}$

Similarly, $A_L(t) = A_L(0)\, e^{-(\Gamma_L/2 + i m_L)t}$.

In a strong interaction, K^0 and $\bar{K}^0$ are created. What is the probability of observing a K_S^0 decay or a K_L^0 decay at time t?

$$|K_S\rangle = \frac{1}{\sqrt{2}}\left(|K^0\rangle + |\bar{K}^0\rangle\right)$$

$$|K_L\rangle = \frac{1}{\sqrt{2}}\left(|K^0\rangle - |\bar{K}^0\rangle\right)$$

or $|K^0\rangle = \frac{1}{\sqrt{2}}\left(|K_S^0\rangle + |K_L^0\rangle\right)$

$$|\bar{K}^0\rangle = \frac{1}{\sqrt{2}}\left(|K_S^0\rangle - |K_L^0\rangle\right)$$

So $|K_S^0(t)\rangle = e^{-(\Gamma_S/2 + i m_S)t}\,|K_S^0\rangle$

$$= \frac{1}{\sqrt{2}}\, e^{-(\Gamma_S/2 + i m_S)t}\left(|K^0\rangle + \cancel{|\bar{K}^0\rangle}\right)$$

So $P_{K_S}(t) = \frac{1}{2}\, e^{-\Gamma_S t}$

$$P_{K_L}(t) = \frac{1}{2}\, e^{-\Gamma_L t}$$

If the beam has momentum p, then

$$d = \gamma\beta c t = \frac{p}{m} c t$$

In practice you can create a pure K_L^0 beam by letting all the K_S^0 component decay away. (we'll come back to this).

What if you put a piece of material in the beam so that it interacts via the strong interaction at time t $\left(t = \frac{m}{pc} d\right)$ would you expect to get K^+ or K^-?

$$\underset{S=+1}{K^0 p} \rightarrow \underset{S=+1}{K^+ n} \qquad \underset{S=+1}{K^0 n} \not\rightarrow \underset{S=-1}{K^- p}$$

$$\underset{S=-1}{\bar{K}^0 n} \rightarrow \underset{S=-1}{K^- p} \qquad \underset{S=-1}{\bar{K}^0 p} \not\rightarrow \underset{S=+1}{K^+ n}$$

Seeing a K^+ means the state was K^0
seeing a K^- means it was $\bar{K}^0$.

$$|K^0(t)\rangle = \frac{1}{\sqrt{2}}\left(|K_S^0(t)\rangle + |K_L^0(t)\rangle\right)$$

$$= \frac{1}{\sqrt{2}}\left(e^{-(\Gamma_S/2 + i m_S)t}|K_S^0\rangle + e^{-(\Gamma_L/2 + i m_L)t}|K_L^0\rangle\right)$$

Initially if it was a $K^0 = \frac{1}{\sqrt{2}}\left(|K_S^0\rangle + |K_L^0\rangle\right)$

then $P_{K^0}(t) = |\langle K^0 | K^0(t)\rangle|^2 = \frac{1}{4}\left|e^{-(\Gamma_S/2 + i m_S)t} + e^{-(\Gamma_L/2 + i m_L)t}\right|^2$

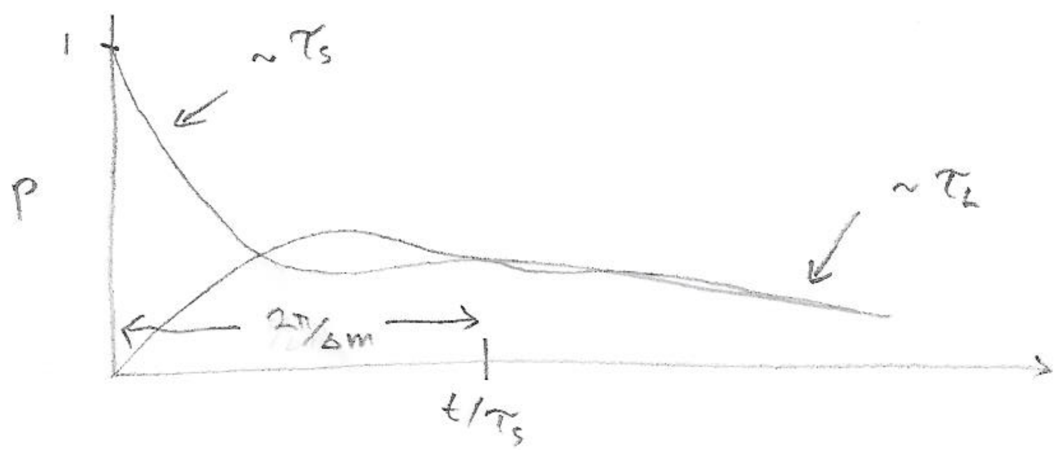
Recall: $|a+b|^2 = |a|^2 + |b|^2 + 2\mathrm{Re}(a^* b)$

$$P_{K^0}(t) = \frac{1}{4}\left(e^{-\Gamma_s t} + e^{-\Gamma_L t} + 2\mathrm{Re}\, e^{-(\Gamma_s + \Gamma_L)t/2}\, e^{i(m_s - m_L)t}\right)$$

$$= \frac{1}{4}\left(e^{-\Gamma_s t} + e^{-\Gamma_L t} + 2e^{-\bar{\Gamma} t}\cos \Delta m t\right)$$

where $\bar{\Gamma} = \frac{1}{2}(\Gamma_s + \Gamma_L)$ and $\Delta m = m_s - m_L$.

If $m_s \neq m_L$ then we expect to see oscillations in the probability of observing a K^0 at time t.



Measured $m_L - m_s = 3.5 \times 10^{-12}$ MeV/c^2

Another effect: What is the probability of observing a K_S^0 decay in a K_L^0 beam?

Far enough away from the K_S^0 decay region the beam can be pure $|K_L^0\rangle = \frac{1}{\sqrt{2}}(|K^0\rangle - |\bar{K}^0\rangle)$

But if there is some material in the beam then it could induce the reaction

$$\underset{S=-1}{\bar{K}^0} p \rightarrow \underset{S=-1}{\Lambda} \pi^+$$

But not $\underset{S=1}{K^0} p \rightarrow \bar{\Lambda} \pi^+$ since it violates baryon number. The material will attenuate the $\bar{K}^0$ beam more than the K^0 beam.

The beam that enters the material will be

$$|K_L\rangle = \frac{1}{\sqrt{2}}(|K^0\rangle - |\bar{K}^0\rangle)$$

the beam that emerges will be

$$|K'\rangle = \frac{1}{\sqrt{2}}(f|K^0\rangle - f'|\bar{K}^0\rangle)$$

where $f' < f$.

So $P_{K_S^0} = |\langle K_S^0 | K' \rangle|^2 = \left| \frac{1}{2}(\langle K^0| + \langle \bar{K}^0|)(f|K^0\rangle - f'|\bar{K}^0\rangle)\right|^2$

$$= \frac{1}{4}|f - f'|^2 > 0.$$

So we should see K_S^0 decays in the beam that emerges from the material.

"Regeneration".

What does $\Delta m \neq 0$ mean? Aren't the masses of quarks and antiquarks the same? Hamiltonian can couple K^0 to $\bar{K}^0$ states :

$$|\psi(t)\rangle = \begin{pmatrix} a(t) \\ b(t) \end{pmatrix} = a(t)|K^0\rangle + b(t)|\bar{K}^0\rangle$$

Schrodenger's equation :

$$i\frac{d}{dt}|\psi(t)\rangle = H|\psi(t)\rangle$$
$$= \left(M - \frac{i\Gamma}{2}\right)|\psi(t)\rangle$$

where $M - \frac{i\Gamma}{2} = \begin{pmatrix} M & p^2 \\ q^2 & M \end{pmatrix}$

in which p^2 and q^2 could be complex numbers. The mass eigenstates will diagonalize $M - \frac{i\Gamma}{2}$. Masses are the eigenvalues.

$$\det\left(M - \frac{i\Gamma}{2} - I\lambda\right) = \begin{vmatrix} M-\lambda & p^2 \\ q^2 & M-\lambda \end{vmatrix}$$
$$= (M-\lambda)^2 - p^2q^2 = 0$$

$$\text{So } (M-\lambda)^2 = p^2q^2$$
$$M-\lambda = \pm pq$$
$$\lambda = M \mp pq$$

Off-diagonal terms shift the masses of the physical states.

Eigenvectors:

$$\left(M - \frac{i\Gamma}{2}\right)\begin{pmatrix} a_1 \\ b_1 \end{pmatrix} = M - pq \begin{pmatrix} a_1 \\ b_1 \end{pmatrix}$$

$$\begin{pmatrix} M - M + pq & p^2 \\ q^2 & M - M + pq \end{pmatrix}\begin{pmatrix} a_1 \\ b_1 \end{pmatrix} = 0$$

$$\begin{pmatrix} pq & p^2 \\ q^2 & pq \end{pmatrix}\begin{pmatrix} a_1 \\ b_1 \end{pmatrix} = 0$$

$$a_1 q + b_1 p = 0$$

$$a_1 = \frac{p}{\sqrt{|p|^2+|q|^2}} \quad b_1 = \frac{-q}{\sqrt{|p|^2+|q|^2}}$$

In the case of the K^0 system

$$K_S = \frac{1}{\sqrt{|p|^2+|q|^2}}\left(p|K^0\rangle + q|\bar{K}^0\rangle\right)$$

$$K_L = \frac{1}{\sqrt{|p|^2+|q|^2}}\left(p|K^0\rangle - q|\bar{K}^0\rangle\right)$$

What is the probability of observing a K_S^0 decay in a beam of pure K_L^0 ?

$$|\langle K_S^0 | K_L^0 \rangle|^2 = \left(\frac{1}{|p|^2+|q|^2}\right)^2 \left(|p|^2 - |q|^2\right)^2$$

Should be non-zero if $|p| \neq |q|$ ...

In fact this was observed in 1964.

An example of CP violation.