PURDUE UNIVERSITY DEPARTMENT OF PHYSICS

Physics 56400

Introduction to Elementary Particle Physics I

Lecture 3

Fall 2019 Semester

Prof. Matthew Jones

Sources of Particle Beams

- We have seen how studying the macroscopic processes of particle scattering can tell us about their microscopic interactions.
- What types of particles can we use as targets?
  - Generally stable particles like electrons, nuclei
- What types of particles can we use as beams?
  - Naturally occurring radioactive emissions
  - Cosmic rays
  - Charged, stable particles, accelerated by electric fields
  - Secondary beams

Nuclear Decays

- Alpha decay ($\alpha = {}^4_2He$):

$$^{241}_{95}Am \rightarrow {}^{237}_{93}Np + {}^4_2He$$

- Beta decay ($\beta = e^{\pm}$):

$$^{90}Sr \rightarrow {}^{90}Y + e^- + \bar{\nu}_e$$

(this is a 3-body decay)

- Gamma decay (γ is a high energy photon):

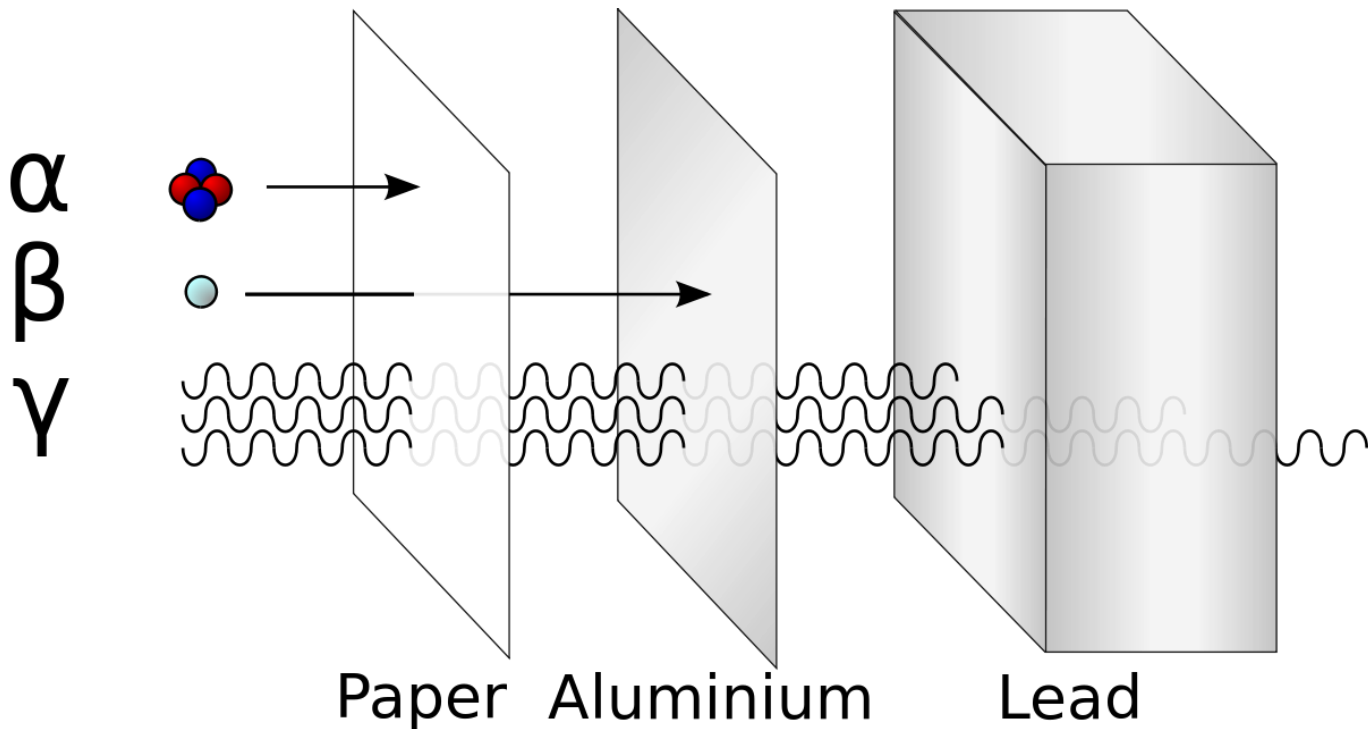
$$^{137}Cs \rightarrow {}^{137}Ba^* + e^- + \bar{\nu}_e$$

$$\quad\quad\quad\quad \hookrightarrow \; {}^{137}Ba + \gamma$$

- These are classified as "ionizing radiation".

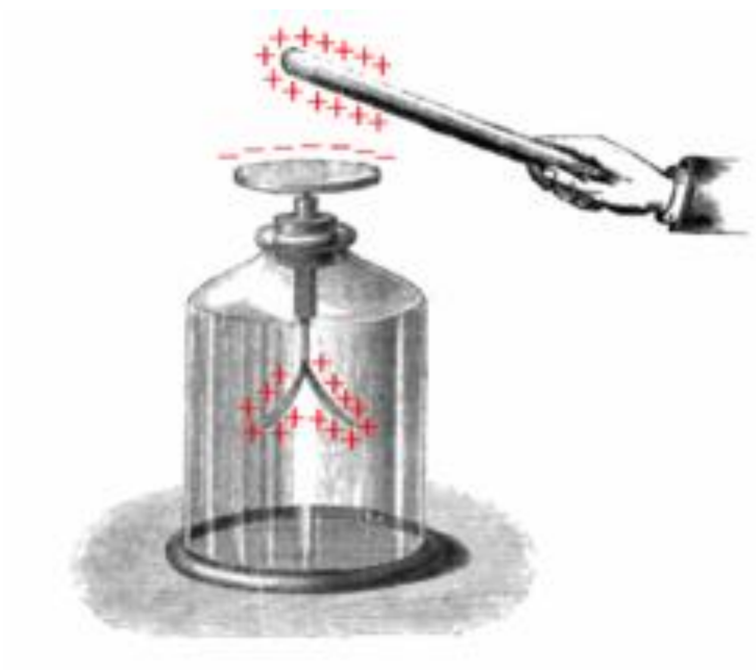
Nuclear Decays

- Types of radiation were classified by their different interactions with matter.



Detecting Ionizing Radiation

- Rate of ionizing radiation was measured using electroscopes.



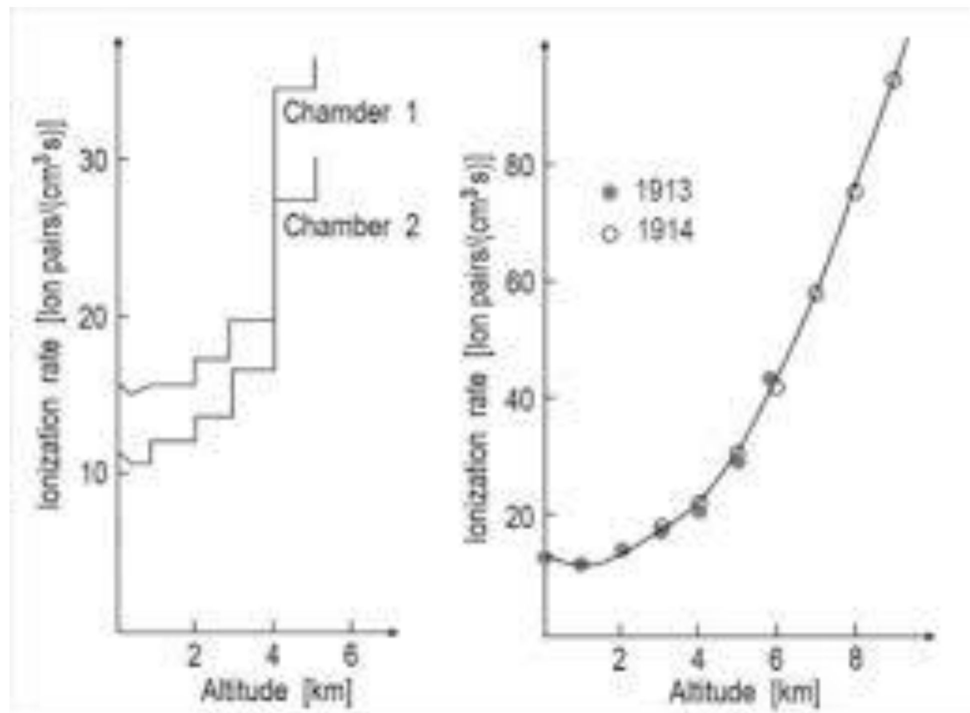
Ionizing radiation produces positive and negative ions in the air they move through.

Negative ions are attracted to the positively charged leaves of the electroscope.

Discharge rate is related to the amount of ionizing radiation present.

Cosmic Rays

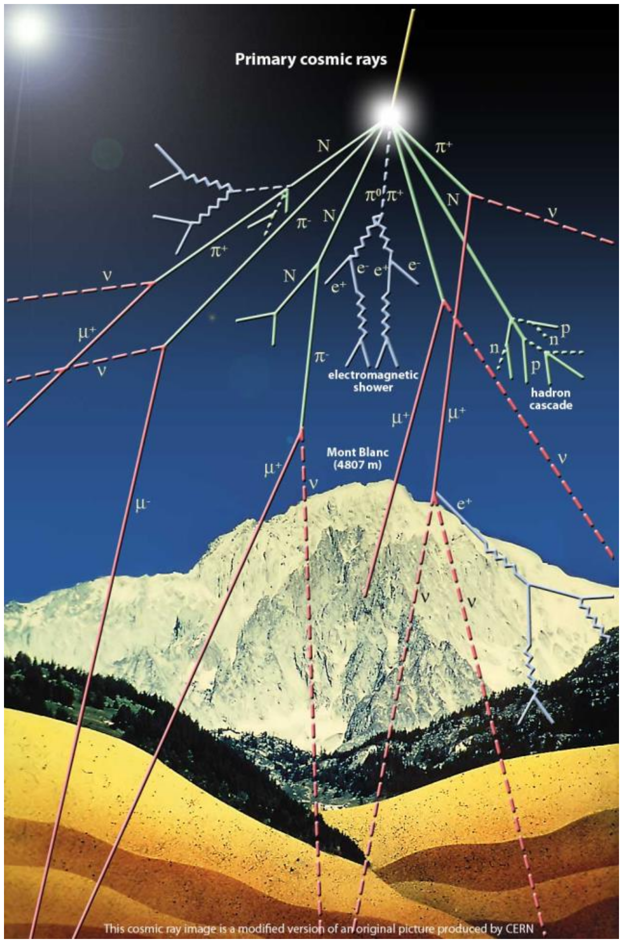
- Viktor Hess observed that electroscopes discharged even in the absence of known sources of ionizing radiation.
- He hypothesized that if the source was terrestrial, then the ionization rate would decrease with increasing altitude.



Cosmic Rays

- Hess speculated that these were rays of great penetrating power entering the earth's atmosphere from above.
- Questions:
  - Did they really enter from above?
  - Were they neutral (like γ-rays) or charged (like β-rays)?
- Subsequent measurements demonstrated that they were charged (interacted with earth's magnetic field) and that they lost energy when moving down through material.

Cosmic Rays



A primary cosmic ray (mostly protons) hits an atomic nucleus at high altitude.

Secondary particles include charged pions.

Pions interact strongly with atomic nuclei and are quickly attenuated.

But charged pions decay to muons which do not interact with nuclear matter.

Almost all of the cosmic ray flux at sea level is due to muons.

Energy Units

- We will use electron-volts as a unit of energy

$$E = mc^2$$

- The dimensions of mass are eV/c^2
- Typical mass scales:

$$m_e = 0.511\,MeV/c^2$$

$$m_p = 938.3\,MeV/c^2$$

$$m_n = 939.6\,MeV/c^2$$

1 amu = 931.5 MeV/c^2
from ^{12}C

$$m_\gamma = 0 \text{ (exactly massless)}$$

$$m_\nu \approx 0 \text{ (almost, but not quite massless)}$$

Energy Conservation

- Energy is always conserved. Consider the decay:

$$^{241}_{95}Am \rightarrow {}^{237}_{93}Np + \alpha$$

- The Am and Np nuclei are very heavy and remain essentially at rest.

$$E_\alpha = m_{Am}c^2 - m_{Np}c^2$$

$$m_{Am} = 241.056829\,u$$

$$m_{Np} = 237.048173\,u$$

$$E_\alpha = (4.008656\,u) \cdot (931.5\,MeV/c^2) \cdot c^2$$

$$m_\alpha = 4.002603\,u$$

- Kinetic energy:

$$T = E_\alpha - m_\alpha c^2 = 5.64\,MeV$$

Energy Conservation

- The kinetic energy of the α-particle is much less than its rest mass.
- It is reasonable to use a non-relativistic approximation:

$$T = \frac{1}{2}mv^2$$

- Velocity of the α-particle is

$$v = 1.65\ \times 10^8\, m/s$$
$$c = 2.998\ \times 10^8\, m/s$$
$$\frac{v}{c} = 0.055$$

Energy Conservation

- In β-decay of ^{90}Sr, the energy released is 0.546 MeV.
- This is large compared to the rest mass of the electron so we probably can't use classical mechanics to find its velocity.
- In special relativity, the relation between energy and momentum is

$$E^2 = m^2c^4 + p^2c^2$$

- The total energy is the rest mass-energy plus the kinetic energy:

$$E = mc^2 + T$$

Energy Conservation

- In the case of ^{90}Sr decay, the electron can have up to 0.546 MeV of kinetic energy.

$$E = (0.511\, MeV/c^2) \cdot c^2 + 0.546\, MeV$$
$$= 1.057\, MeV$$

- Momentum is

$$pc = \sqrt{E^2 - m^2c^4} = 0.925\, MeV$$

- What is the velocity?

Lorentz Transformations

- In any reference frame, the mass remains invariant but p and E transform according to

$$E' = \gamma E + \gamma\beta pc$$
$$p'c = \gamma pc + \gamma\beta E$$

where $\beta = v/c$ and $\gamma = 1/\sqrt{1 - \beta^2}$

- At rest, $p = 0$ and $E = mc^2$
- When boosted with velocity βc the momentum and energy are

$$E' = \gamma mc^2$$
$$p'c = \gamma\beta mc^2$$

- The boost velocity is

$$\beta = p'c/E'$$

Lorentz Transformations

- In the case of ^{90}Sr decay,

$$\beta = \frac{(0.925\,MeV/c)\cdot c}{1.057\,MeV} = 0.875$$

- This is a significant fraction of the speed of light, so relativistic kinematics is needed for the analysis.

4-vectors

- Momentum is a vector:

$$\vec{p} = m\gamma\vec{v}$$
$$|\vec{p}|^2 = m\gamma\beta c$$

- Energy/momentum and position/time transform in the same way and it is convenient to treat them as different components of a single vector:

$$p = (E/c, \vec{p})$$
$$x = (ct, \vec{x})$$

- Lorentz transformation for a boost along the x-axis:

$$L = \begin{pmatrix} \gamma & \gamma\beta & 0 & 0 \\ \gamma\beta & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

4-vectors

- Components after a boost along the x-axis:

$$E' = \gamma E + \gamma\beta p_x c$$
$$p'_x c = \gamma\beta E + \gamma p_x c$$
$$p'_y c = p_y c$$
$$p'_z c = p_z c$$

- The mass remains invariant:

$$m'^2 c^4 = E'^2 - |\vec{p}'|^2 \mathrm{c}^2 = \cdots = m^2 c^4$$

- This is why we call it the “invariant mass”… it is the same in all reference frames.

4-vectors

- It will be convenient to refer to the components of 4-element vectors using a Greek-letter index:

$$x^\mu \qquad \mu = 0,1,2,3$$

- When we refer only to the space components (ie, $\vec{x}$ or $\vec{p}$), we use a Latin-letter index:

$$x^i \qquad i = 1,2,3$$

- We write the elements of the Lorentz transformation matrix as

$$L^{\mu}{}_{\nu}$$

Einstein summation convention

- A repeated index in an expression implies summation:

$$L^{\mu}{}_{\nu} x^{\nu} \equiv \sum_{\nu=0}^{3} L^{\mu}{}_{\nu} x^{\nu}$$

- Summation is performed only between upper and lower indices.
- Upper index 4-vectors (contravariant) and lower index 4-vectors (covariant) have a relative minus sign in their space components:

$$x_{\mu} = (x^0, -x^1, -x^2, -x^3)$$

Metric tensor

- We can lower or raise an index by multiplying by the metric tensor:

$$x_\mu = g_{\mu\nu} x^\nu$$

$$g_{\mu\nu} = g^{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

- Any quantity that is of the form

$$a^\mu b_\mu = g_{\mu\nu} a^\mu b^\nu$$

is Lorentz invariant.

- In particular,

$$p^\mu p_\mu c^2 = E^2 - |\vec{p}|^2 c^2 = m^2 c^4$$

Other conventions

- The (+,-,-,-) convention is called the Minkowski metric.
- This choice of metric tensor is not unique.
- We could also write:

$$-s^2 = x_\mu x^\mu = \vec{x} \cdot \vec{x} - (x^0)^2$$

which corresponds to

$$g^{\mu\nu} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

- Another convention:

$$x = (x^1, x^2, x^3, x^4)$$
$$x^4 = ix^0$$

- Then, $x \cdot x = |\vec{x}|^2 - (x^0)^2$ which corresponds to $g = I_{4\times4}$

Natural Units

- It becomes cumbersome to keep track of all the factors of c and $\hbar$.
- We can define a new system of units in which c and $\hbar$ have unit magnitude (natural units).
- Then energy, momentum and mass all have the same units (energy).
- We can convert distance units to energy using the factor

$$\hbar c = 197.3269788\ \mathrm{MeV} \cdot \mathrm{fm}$$
$$(\hbar c)^2 = 0.3893793656\ \mathrm{GeV}^2 \cdot \mathrm{mbarn}$$

Natural Units

- In natural units, time and distance have dimensions of 1/energy (eg, MeV^{-1})
- Cross sections have units 1/energy^2 (eg, GeV^{-2})
- The lifetime of an unstable particle is the reciprocal of the "width":

$$\tau = \frac{1}{\Gamma}$$

- The "width" has units of energy and is the same as the decay rate.

Natural Units

- To convert back into CGS units, multiply by the appropriate factors of $\hbar c$ and c:
- To get time in seconds,

$$\tau = \frac{\hbar c}{c}\frac{1}{\Gamma} = \frac{(197.327\ \mathrm{MeV} \cdot \mathrm{fm})(10^{-13}\ \mathrm{cm} \cdot \mathrm{fm}^{-1})}{(29.98\ \mathrm{cm} \cdot \mathrm{ns}^{-1})\ \Gamma}$$

$$= \frac{6.582\ \times 10^{-13}\ \mathrm{MeV} \cdot \mathrm{ns}}{\Gamma}$$

- Another convenient number is

$$\hbar = 6.582\ \times 10^{-13}\ \mathrm{MeV} \cdot \mathrm{ns}$$

Derivative Operators

- In quantum mechanics, E and $\vec{p}$ are associated with operators:

$$E \to i\hbar \frac{\partial}{dt}$$
$$\vec{p} = -i\hbar\nabla$$

- In natural units we write this as

$$p^\mu \to i\partial^\mu$$

- For this to work, we need to define the derivative operator as follows:

$$\partial^\mu = \frac{\partial}{\partial x^\mu} = \left(\frac{\partial}{\partial t}, -\nabla\right)$$

- The covariant derivative is

$$\partial_\mu = \left(\frac{\partial}{\partial t}, \nabla\right)$$

- Note the minus sign! Be very careful not to mix these up!

Derivative Operators

- We also write the D'Alembertian operator as:

$$\Box = \partial^{\mu}\partial_{\mu} = \frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial y^2} - \frac{\partial^2}{\partial z^2}$$

- Gauss' law and Ampere's law can be written:

$$\partial_{\mu}F^{\mu\nu} = \mu_0 J^{\nu}$$

- The electromagnetic field tensor is

$$F^{\mu\nu} = \partial^{\mu}A^{\nu} - \partial^{\nu}A^{\mu}$$

$$A^{\mu} = \left(\phi, \vec{A}\right)$$

$$J^{\mu} = (\rho, \vec{J})$$

Cross Products

- In 3-vector notation, the expression

$$\vec{u} = \vec{a} \times \vec{b}$$

is well understood to mean

$$u_1 = a_2 \cdot b_3 - a_3 \cdot b_2$$
$$u_2 = a_3 \cdot b_1 - a_1 \cdot b_3$$
$$u_3 = a_1 \cdot b_2 - a_2 \cdot b_1$$

- How can we write this using the abstract index notation?
- Completely antisymmetric tensor:

$$\varepsilon^{ijk} = \varepsilon_{ijk} = \begin{cases} +1 \text{ for even permuations of } ijk \\ -1 \text{ for odd permutations } of\ ijk \end{cases}$$

- For example, $\varepsilon^{123} = +1$ whereas $\varepsilon^{132} = -1$
- $\varepsilon^{iij} = \varepsilon^{ijj} = 0$

Cross Products

- Now we can write

$$u^i = \varepsilon^{ijk} a^j b^k$$

- For example,

$$u^1 = \sum_{j,k} \varepsilon^{1jk} a^j b^k$$
$$= \varepsilon^{123} a^2 b^3 + \varepsilon^{132} a^3 b^2$$
$$= a^2 b^3 - a^3 b^2$$

Pauli Matrices

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

- Basic relations:

$$\sigma_i \sigma_j = \delta_{ij} + i\varepsilon_{ijk}\sigma_k$$

$$[\sigma_i, \sigma_j] = \sigma_i \sigma_j - \sigma_j \sigma_i = 2i\varepsilon_{ijk}\sigma_k$$