PURDUE UNIVERSITY DEPARTMENT OF PHYSICS

Physics 56400

Introduction to Elementary Particle Physics I

Lecture 23

Fall 2019 Semester

Prof. Matthew Jones

Massless Standard Model

- The standard model, when constructed with massless particles, is gauge invariant and renormalizable
- The gauge symmetry appears to be
$$SU(3) \times SU(2) \times U(1)$$
- QCD is a pure massless gauge theory and can be considered separately
- The electromagnetic and weak interactions come from gauge invariance under $SU(2) \times U(1)$

Massless Standard Model

- Massless gauge bosons only have two degrees of freedom corresponding to transverse polarization states (like the photon)
- Massive gauge bosons need an additional degree of freedom for longitudinal polarization
- Also, massive fermions are described by terms in the Lagrangian that are of the form $\bar{u}_R u_L$ but this can't be constructed from left-handed doublets and right-handed signlets.

Spontaneous Symmetry Breaking

- The strategy proposed by Higgs (and others) is the following:

1. Introduce a new set of fields that transform under $SU(2) \times U(1)$
2. Allow the fields to have a potential energy function that has a minimum that is NOT located at the origin
3. Expand as a power series about the minimum
4. Terms that are of the form of mass couplings will emerge…

Higgs Fields

- Step 1 – introduce a doublet of fields that transform under SU(2)xU(1):

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$$

- These transform under SU(2) as follows:

$$\phi \to \phi' = e^{-i\vec{\sigma}\cdot\vec{\theta}(x)/2}\phi \qquad SU(2)$$

$$\phi \to \phi' = e^{-i\,I\,\theta'(x)/2}\phi \qquad U(1)$$

- Covariant derivative:

$$D_\mu = \partial_\mu + \frac{ig}{2}\,\vec{\sigma}\cdot\overrightarrow{W_\mu} + \frac{ig'}{2}\,I\,B_\mu$$

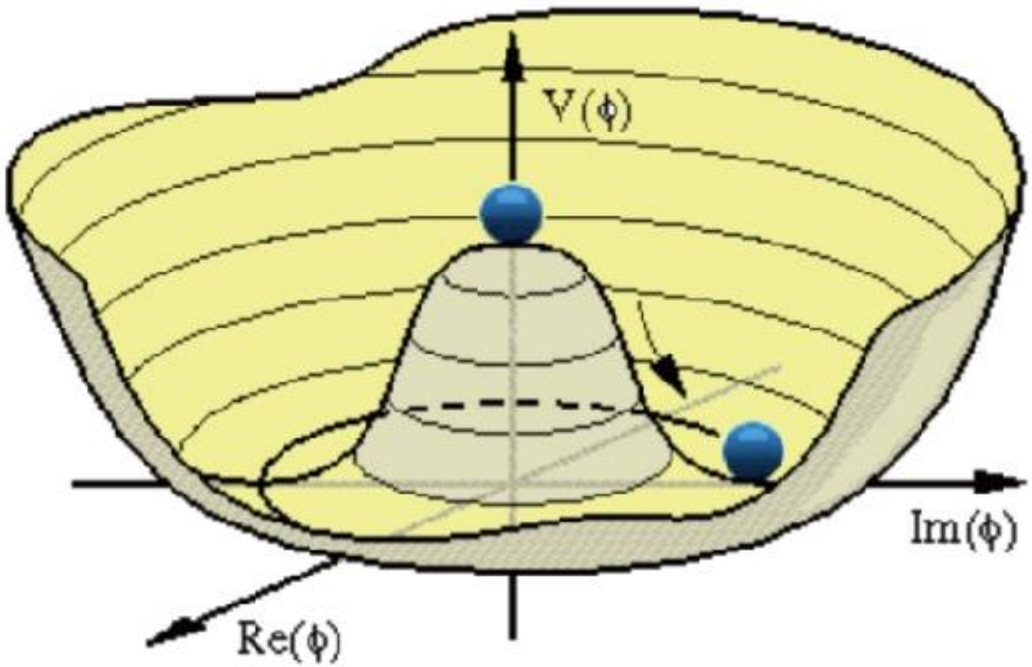
Higgs Field

- The theory so far is gauge invariant and is symmetric under SU(2)xU(1)
- Step 2: construct an appropriate potential energy function

$$V(\phi) = \mu\phi^\dagger\phi + \lambda\left(\phi^\dagger\phi\right)^2$$

- If the parameter $\mu < 0$ then the minimum is not located at the origin. We must have $\lambda > 0$ or else the vacuum is unstable

Higgs Potential



The theory is still symmetric, but the minimum is not located at the origin. The symmetry is broken when the Higgs field is expanded about an arbitrarily chosen point at the minimum.

Higgs Field

- We chose to expand the Higgs field about a particular point at the minimum of the potential energy function:

$$\phi_0 = \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix}$$

$$\phi_0^\dagger \phi_0 = v^2/2$$

- The potential energy is minimized when

$$\frac{\partial V}{\partial v^2} = \frac{\mu}{2} + \frac{\lambda v^2}{2} = 0$$

$$v = \sqrt{-\mu/\lambda}$$

- Recall that $\mu < 0$ for the potential to have the right shape.

Weak Mixing

- Next, we write the gauge fields in terms of the physical fields:

$$D_\mu = \partial_\mu + \frac{ig}{2}\begin{pmatrix} 0 & W_\mu^1 - iW_\mu^2 \\ W_\mu^1 + iW_\mu^2 & 0 \end{pmatrix} + \frac{ig}{2}\begin{pmatrix} W_\mu^3 & 0 \\ 0 & -W_\mu^3 \end{pmatrix} + \frac{ig'}{2}\begin{pmatrix} B_\mu & 0 \\ 0 & B_\mu \end{pmatrix}$$

- The charged components of the W fields are fine but the neutral components can mix:

$$B_\mu = \cos\theta_W\, A_\mu + \sin\theta_W\, Z_\mu$$
$$W_\mu^3 = \sin\theta_W\, A_\mu - \cos\theta_W\, Z_\mu$$

Weak Mixing

- The ratio of the weak couplings is also expressed in terms of the weak mixing angle:

$$\frac{g'}{g} = \tan\theta_W$$

- Now the covariant derivative can be written:

$$D_\mu = \partial_\mu + \frac{ig}{\sqrt{2}}\begin{pmatrix} 0 & W_\mu^+ \\ W_\mu^- & 0 \end{pmatrix} + \frac{g\sin\theta_W}{2}\begin{pmatrix} (\tan\theta_W - \cot\theta_W)Z_\mu & 0 \\ 0 & (\tan\theta_W + \cot\theta_W)Z_\mu \end{pmatrix}$$

- We have written $W_\mu^\pm = W_\mu^1 \mp iW_\mu^2$
- Notice that the A_μ terms have cancelled!

Weak Mixing

- Even at the minimum, the covariant derivatives are still affected by the vacuum expectation value of the Higgs field:

$$\phi_0 = \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix}$$

- $\big(D_\mu \phi_0\big)(D^\mu \phi_0)^\dagger$ will have terms that are of the form:

$$\frac{g^2}{2}\left[\begin{pmatrix} 0 & W_\mu^+ \\ W_\mu^- & 0 \end{pmatrix}\begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix}\right]\left[\begin{pmatrix} 0 & W_\mu^+ \\ W_\mu^- & 0 \end{pmatrix}\begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix}\right]^\dagger$$

$$= \frac{g^2 v^2}{8}\big(W_\mu^1 W^{1\mu} + W_\mu^2 W^{2\mu}\big)$$

- This is exactly the same form as a mass term for the charged weak vector bosons.

Weak Mixing

- The mass of the charged weak vector bosons is just the coefficient:

$$\frac{g^2 v^2}{8}\left(W_\mu^1 W^{1\mu} + W_\mu^2 W^{2\mu}\right) = \frac{M_W^2}{2}\left(W_\mu^1 W^{1\mu} + W_\mu^2 W^{2\mu}\right)$$

$$M_W = \frac{gv}{2}$$

- Mass terms for the Z^0 emerge in the same way:

$$\frac{g^2 v^2}{8\cos^2\theta_W} Z_\mu Z^\mu = \frac{M_Z^2}{2} Z_\mu Z^\mu$$

$$M_Z = \frac{gv}{2\cos\theta_W}$$

Weak Mixing

$$M_W = \frac{gv}{2} \qquad\qquad M_Z = \frac{gv}{2\cos\theta_W}$$

$$G_F = 1.166 \times 10^{-5}\ \mathrm{GeV}^{-2} = \frac{g^2}{4\sqrt{2}\, M_Z^2\, cos^2\,\theta_W}$$

$$\alpha = \frac{e^2}{4\pi} = \frac{1}{137.036}$$

$$M_W = 80.398\ \mathrm{GeV}$$

$$M_Z = 91.1876\ \mathrm{GeV}$$

- Now we have a prediction from the theory:

$$\cos\theta_W = 0.921$$

$$\frac{M_W}{M_Z} = 0.882$$

- This is not bad agreement but the relations are expected to be modified by electromagnetic radiative corrections.

Higgs Vacuum Expectation Value

$$v = \frac{2M_W}{g}$$

$$g^2 = \frac{8M_W^2 G_F}{\sqrt{2}}$$

$$v = \sqrt{\frac{1}{\sqrt{2}\,G_F}} = 246\ \text{GeV}$$

- The vacuum potential energy of the Higgs field is:

$$V(\phi_0) = -\frac{\mu^2}{2\lambda} = \frac{\mu v^2}{2} < 0$$

Fermion Masses

- We can't make gauge invariant mass terms for the fermions by putting in masses explicitly
- Instead, we start with massless Fermions and let them couple to the Higgs field
  - Left-handed doublet: $e_L = \begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L$
  - Higgs doublet: $\phi_0 = \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix}$
  - Right-handed singlet: $e_R = (e^-)_R$
- Consider the following expression:

$$\lambda_e\left(\bar{e}_L\phi_0 e_R + \ \bar{e}_R\phi_0^\dagger e_L\right) = \frac{\lambda_e v}{\sqrt{2}}\ (\bar{e}_L^- e_R^- + \bar{e}_R^- e_L^-)$$

- Then, the electron mass is expressed $m_e = (\lambda_e v)/\sqrt{2}$.

Higgs Dynamics

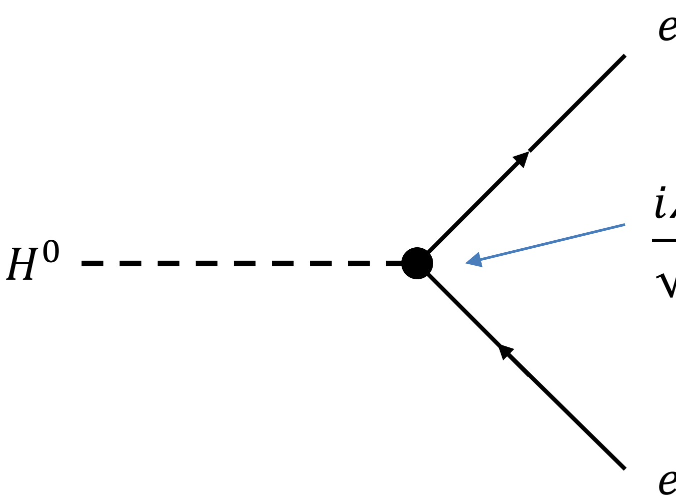
- So far, we have just created a theory of massless particles with a funky gauge symmetry and expanded it about the minimum of a potential energy function.
- The Higgs is a complex doublet of fields and three of its degrees of freedom gave mass terms to the W and Z
- The remaining (real) part of the Higgs field can be written as an expansion about the potential energy minimum:

$$\phi(x) = \begin{pmatrix} 0 \\ \dfrac{v}{\sqrt{2}} + \dfrac{H(x)}{\sqrt{2}} \end{pmatrix}$$

- The field, $H(x)$ is a real (hence electrically neutral), scalar field.
- Anything that couples to v will also couple to $H(x)$

Higgs Couplings

- This is a fundamentally unique property of the Higgs boson – the strength of its coupling depends directly on the mass.



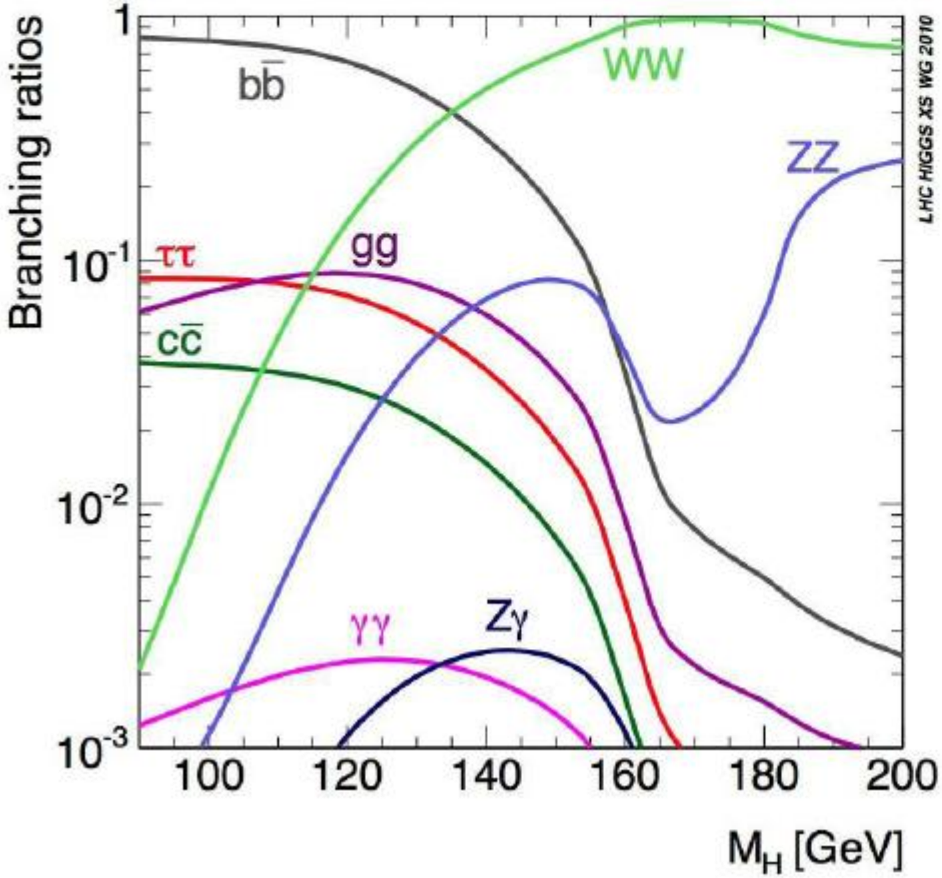
$$\frac{i\lambda_e}{\sqrt{2}} = \frac{im_e}{v} = \frac{ig}{2}\frac{m_e}{M_W} = i\frac{e}{2\sin\theta_W}\frac{m_e}{M_W}$$

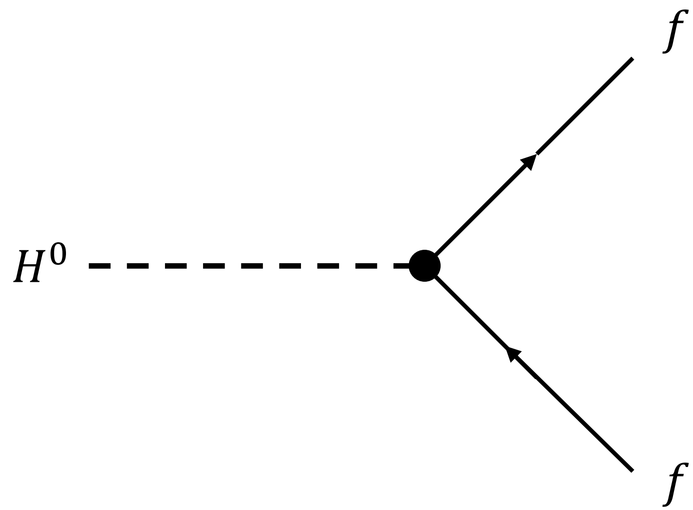
- This doesn't appear to introduce or remove any parameters from the model but it is significant.

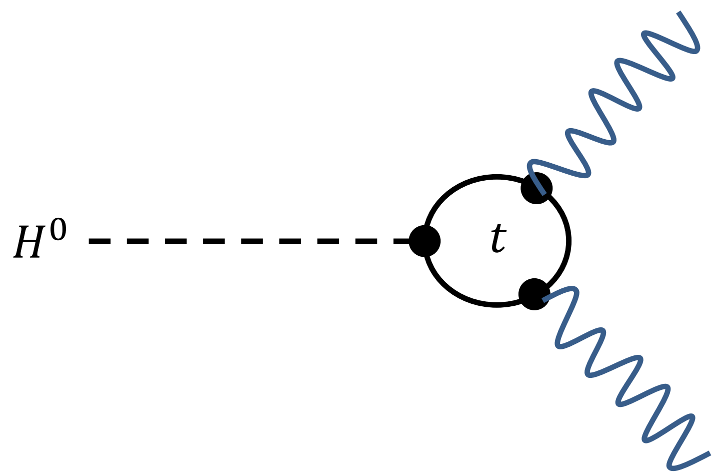
Higgs Decays

- If we managed to produce a Higgs, what would it decay into?
- A very massive Higgs would love to decay into top quarks but if that was forbidden by kinematics, then the next best thing is the bottom quark
- Then the charm quark, tau leptons, … and eventually the electron.
- It could also decay to ZZ^* or WW^* where one of the vector bosons has a mass that is far away from resonance.
- However, the Higgs could also decay via a virtual top quark in a loop and couple to gluons or photons

Higgs Decay

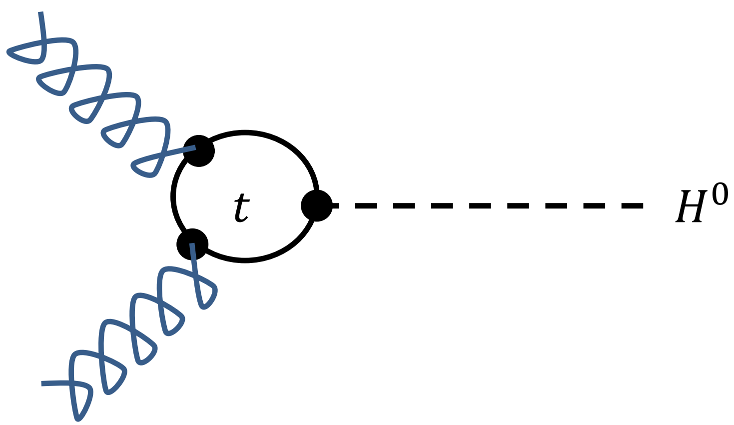

Branching ratios
1
10-1
10-2
10-3
100
120
140
160
180
200
MH [GeV]
bb̄
ττ
cc̄
gg
γγ
Zγ
WW
ZZ
LHC HIGGS XS WG 2010


H^0
f
f


H^0
t

Searching for Higgs Production

- Not found in $e^+e^- \to Z^* \to Z^0H^0$ at LEP
- Could be produced by gluon fusion at the Tevatron or at the LHC
- The $b\bar{b}$ final state is overwhelmed by QCD backgrounds: there are lots of 2-jet events and many of them are b-jets.

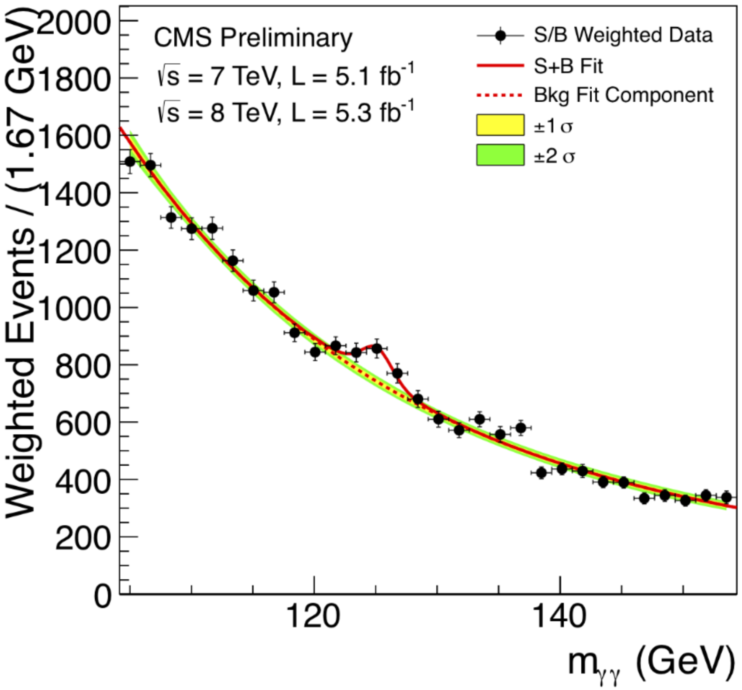


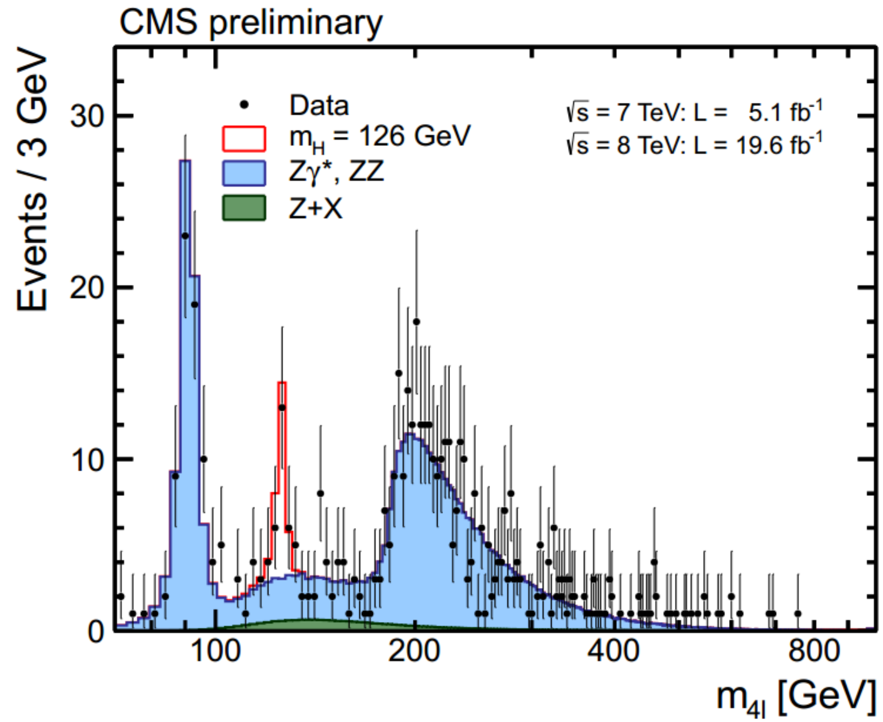
- Reconstructing jet 4-momentum is hard to do precisely.
- Searching for a small, broad bump on a large background is very difficult.

Searching for Higgs Production

- Search for Higgs produced in association with something rare
  - eg, ZH or WH
- Search for a fully reconstructed final state
  - eg, $H^0 \rightarrow Z^* Z$
- Search for a rare decay with almost no background
  - eg, $H^0 \rightarrow \gamma\gamma$

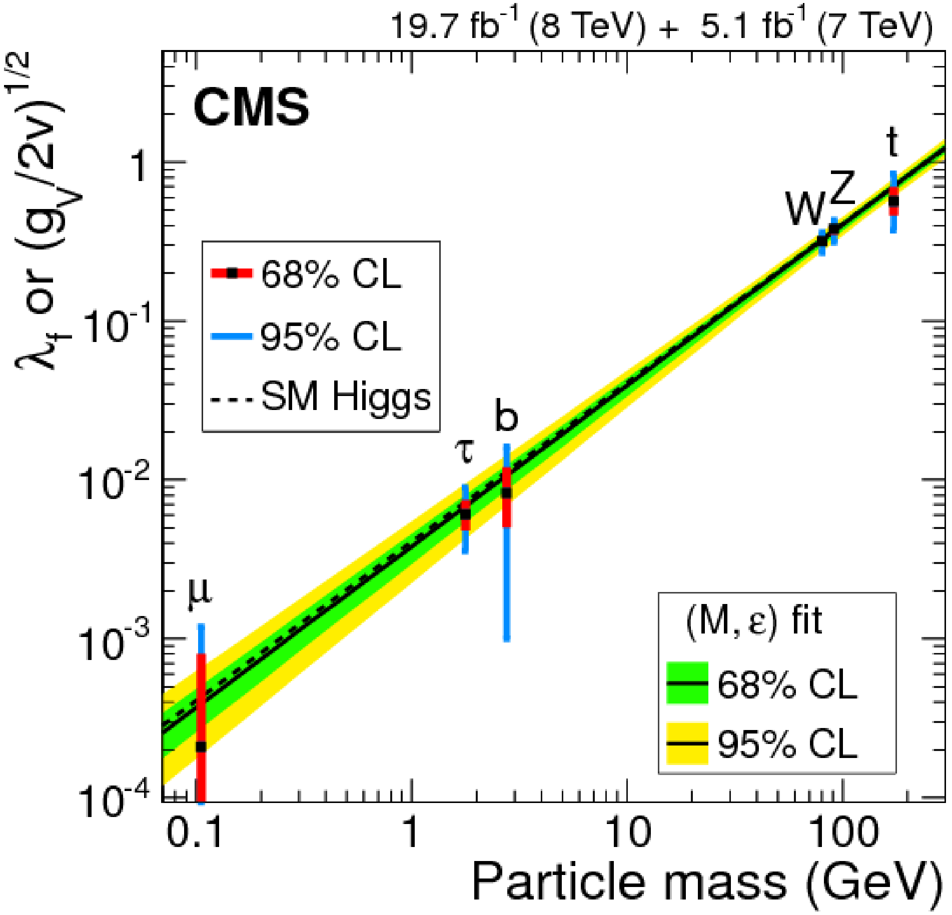
Higgs Observation at the LHC





Joint press release with ATLAS on July 4, 2012.

Higgs Coupling to Mass



The couplings do indeed appear to be proportional to the mass.

The Standard Model

- Start with massless fermions
- Impose an empirically determined gauge symmetry
  - This determines the possible types of interactions:
    - QCD comes from the $SU(3)$ color symmetry
    - Unification of electricity and magnetism with the weak interaction comes from $SU(2) \times U(1)$
- Introduce a Higgs field with a non-zero vacuum expectation value that couples to all the massless particles
- Expanding the Higgs about the potential energy minimum generates mass terms
- It also determines the couplings of real Higgs fields to other particles.
- Apparently, that seems to describe Nature quite well.