PURDUE UNIVERSITY DEPARTMENT OF PHYSICS

Physics 56400

Introduction to Elementary Particle Physics I

Lecture 2

Fall 2019 Semester

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Cross Sections

- Reaction rate:

$$R = \mathcal{L} \cdot \sigma$$

- The cross section is proportional to the probability of observing a particular outcome in a collision
- Example:
  - High energy protons might interact with a large cross section but only rarely do they produce a Higgs particle
  - The rates of both processes are proportional to the luminosity

Cross Sections

- In quantum mechanics, one of the only questions we are allowed to ask is,

"what is the probability of observing a system in a final state $|f\rangle$ given that it started in an initial state $|i\rangle$?"

- Quantum mechanics states that this probability is proportional to

$$P \sim |\langle f|i\rangle|^2$$

- If the initial and final states describe "free particles", then

$$P \sim |\langle f|U|i\rangle|^2$$

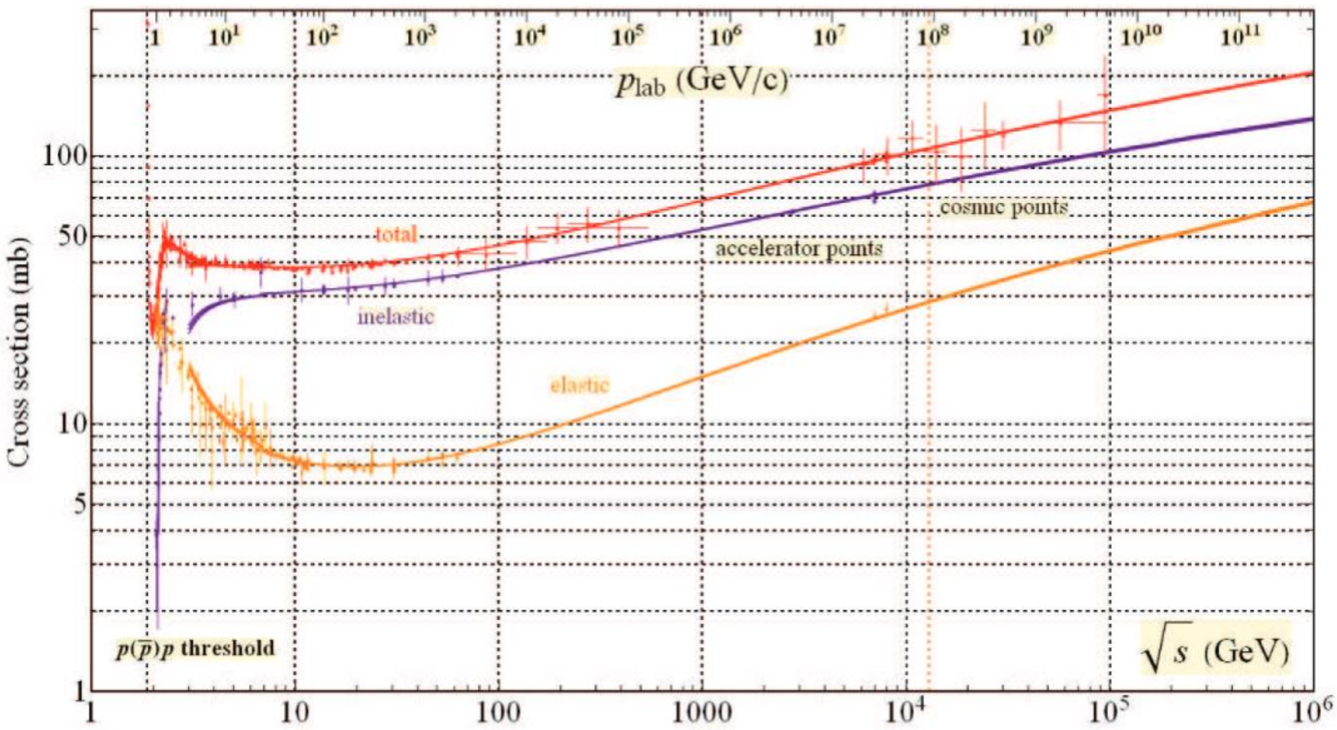
where the operator U evolves the initial state, from time $t \to -\infty$ to the asymptotic time $t \to +\infty$.

Cross Sections

- We will see that cross sections work like probabilities:
$$\sigma \sim |\langle f|U|i\rangle|^2$$
- The time evolution operator is expressed in terms of the Hamiltonian, H.
- We are *not* allowed to ask *how* the initial state turns into the final state, we can only calculate the probability.
- In practice, we need to account for all possible intermediate states, and add their amplitudes (complex numbers).
- The Standard Model of particle physics tells us what the possible intermediate states could be and how the initial and final states couple to them.

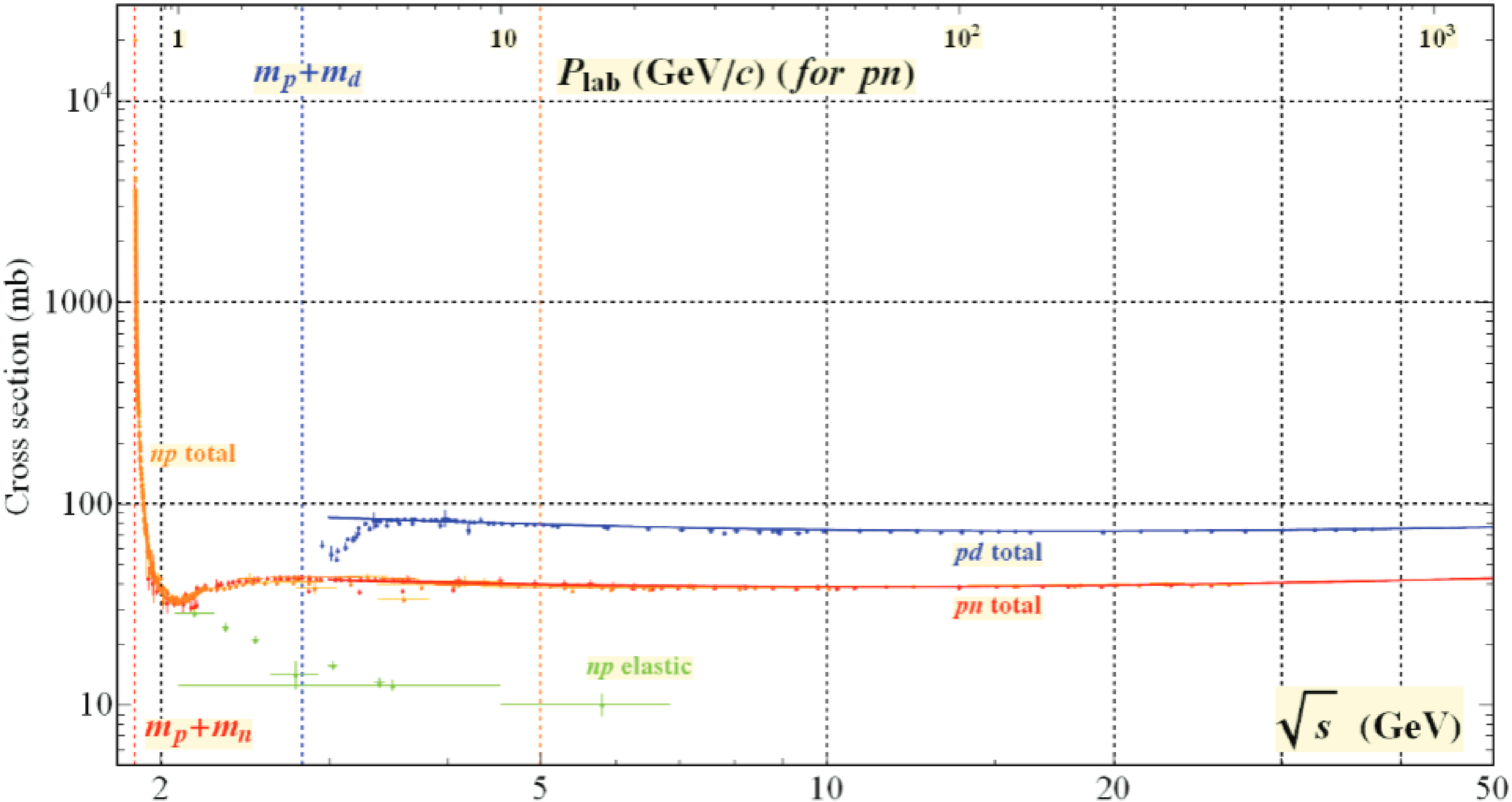
Cross Sections

- The cross section could be a function of several independent variables
  – For example, the beam energy:



Cross Sections

The deuteron is a bound state of a proton and a neutron.
It is a stable isotope of hydrogen (heavy hydrogen).

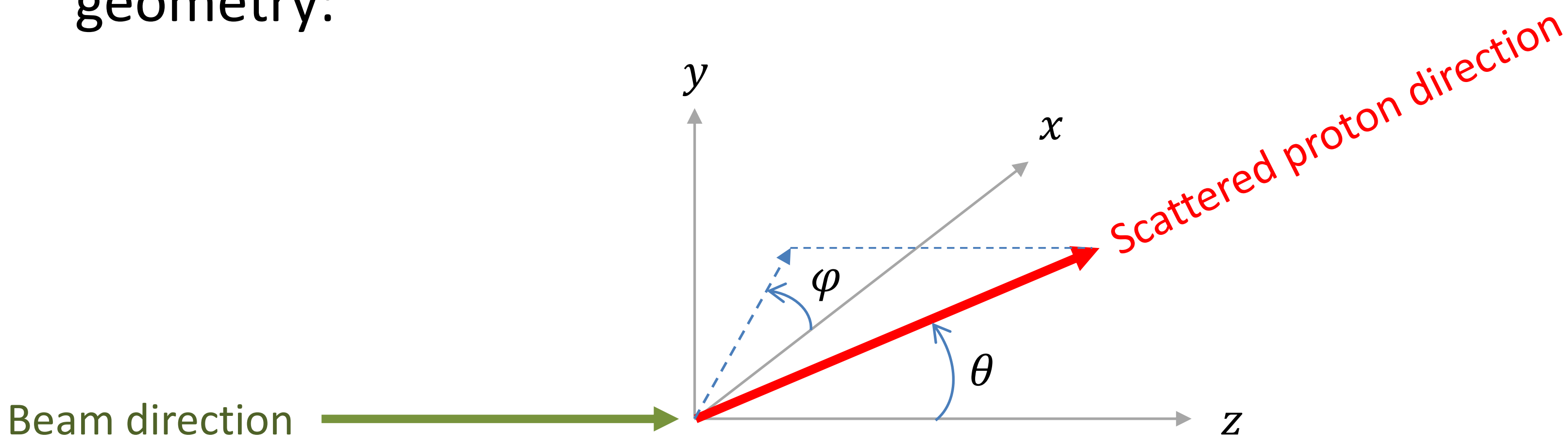


Differential Cross Sections

- There are lots of ways to describe the final state
- The total proton-proton cross section accounts for all possible interactions
- The elastic cross section describes the process where the protons retain their identity but just “bounce” off of each other
- We detect these by observing protons scattered at some angle with respect to the initial beam direction
- What can we learn by measuring the angular distribution of scattered protons?

Geometry

- We will use spherical coordinates to describe the scattering geometry:



- The angle θ is measured with respect to the z-axis
- The angle φ is measured with respect to the x-axis in the x-y plane.

(This is just the usual system of polar coordinates)

- In systems with azimuthal symmetry, we don't expect there to be any dependence on φ.

Geometry

- The differential element of solid angle is
$$d\Omega = d(\cos\theta)d\varphi = \sin\theta\, d\theta d\varphi$$
- This solid angle has the area dA on the surface of a sphere of radius R:
$$dA = R^2 d\Omega$$
- The total surface area of the sphere is
$$A = R^2 \int_{\pi}^{0} \sin\theta\, d\theta \int_{0}^{2\pi} d\varphi = 2\pi R^2 \int_{-1}^{1} d(\cos\theta)$$
$$= 4\pi R^2$$

Differential Cross Sections

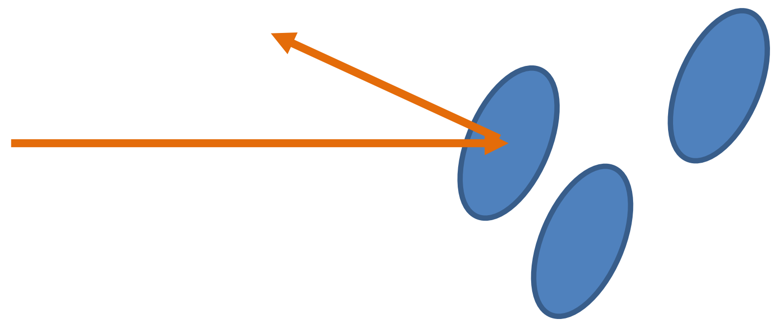
- We want to measure the rate at which protons are scattered with polar angles (θ, φ) into a small interval of solid angle:

$$dR = \mathcal{L}\frac{d\sigma}{d\Omega}d\Omega$$

- The function $d\sigma/d\Omega$ is the differential cross section.
  - In this case it is a function of the polar angles
- Let's calculate $d\sigma/d\Omega$ for a couple different models…

Brick Wall Scattering

- Suppose the target particles were flat disks:

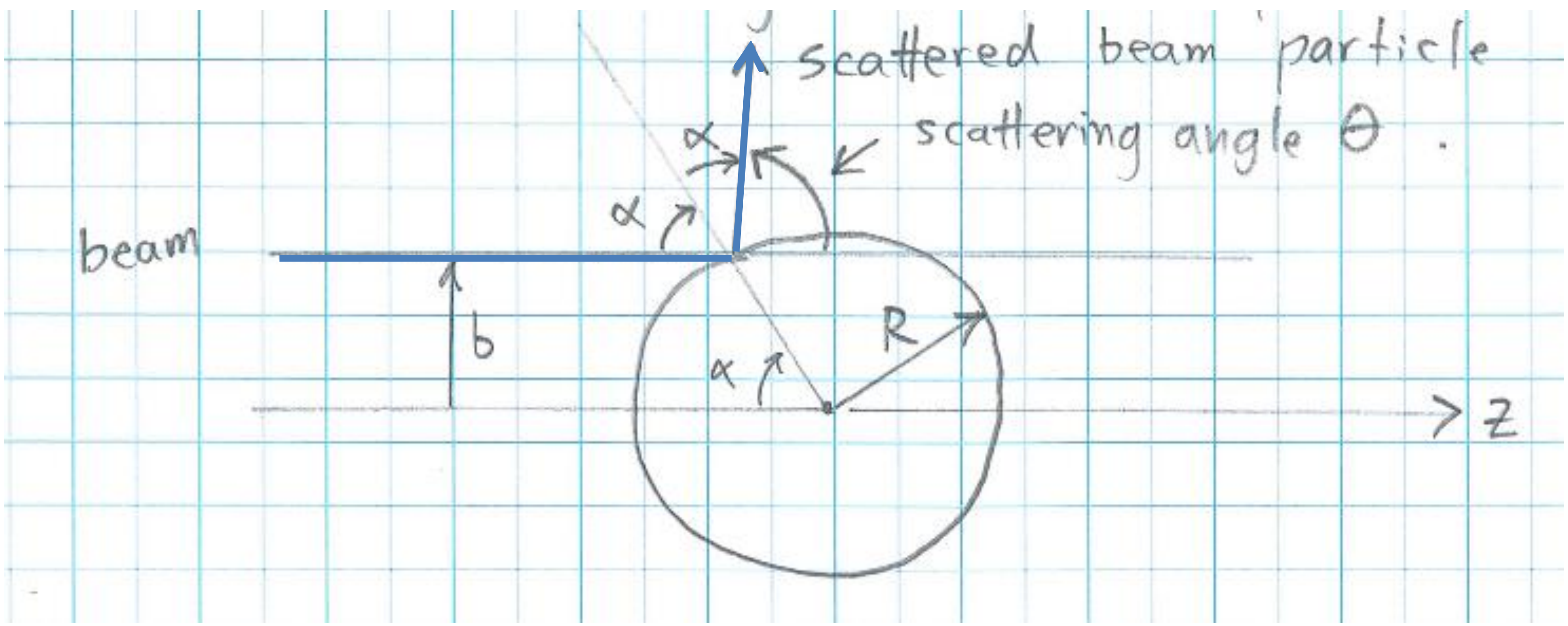


- Assuming they were all oriented the same way, the target would just reflect the beam particles
- All scattering angles would be the same
- The differential cross section would be a delta-function at that specific scattering angle

(not very interesting or realistic)

Hard Sphere Scattering

- Given that we are probably scattering from nuclei, perhaps they could be described as hard spheres…



"Impact parameter", b

$\sin\alpha = b/R$

Hard Sphere Scattering

- The scattering angle is $\theta = \pi - 2\alpha$

$$\begin{aligned}\cos\theta &= \cos(\pi - 2\alpha) \\ &= -\cos 2\alpha \\ &= \sin^2\alpha - \cos^2\alpha \\ &= 2\sin^2\alpha - 1 \\ &= \frac{2b^2}{R^2} - 1\end{aligned}$$

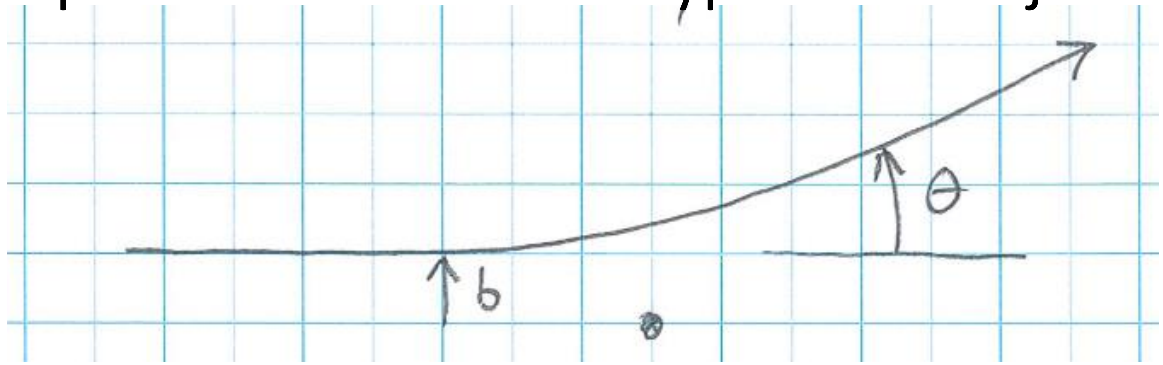
- Does this make sense?
  - As $b \to R, \cos\theta \to 1$
  - As $b \to 0, \cos\theta \to -1$

Hard Sphere Scattering

- The area of the hard sphere that will result in scattering angles larger than θ will be
$$A = \pi b^2$$
$$= \frac{\pi}{2} R^2 (1 + \cos\theta)$$
- The differential scattering cross section is
$$\frac{d\sigma}{d\Omega} = \frac{d\sigma}{d(\cos\theta)d\varphi} = \frac{R^2}{4}$$
  - This is isotropic
- Total cross section is $\sigma = \pi R^2$

Coulomb Scattering

- Assume that the beam has charge ze and the target nuclei has charge Ze
- The beam particle will follow a hyperbolic trajectory:



- Classical mechanics:

$$b = \frac{zZe^2}{mv_0} \cot\left({}^{\theta}\!/_{2}\right)$$

But we don’t know b on an event-by-event basis…

Coulomb Scattering

- Assume that the beam has intensity I.
- I is the number of incident beam particles per unit area per unit time.
- Total number of beam particles:

$$N = \int_0^T dt \int_0^\infty 2\pi\, b\, db\, I(b,t)$$

- We typically assume that the intensity is independent of time
- By definition,

$$d\sigma = \frac{dN(\theta,\varphi)}{T \cdot I_0} = b\, db\, d\varphi$$

Coulomb Scattering

- Differential cross section:

$$\frac{d\sigma}{d\Omega} = \frac{b\ db\ d\varphi}{d(\cos\theta)\ d\varphi} = \frac{b}{\sin\theta}\frac{db}{d\theta}$$

- But,

$$\frac{db}{d\theta} = \frac{d}{d\theta}\left(\frac{zZe^2}{mv_0}\cot\left({}^{\theta}/_{2}\right)\right)$$
$$= \frac{zZe^2}{mv_0}\csc^2\left({}^{\theta}/_{2}\right)$$

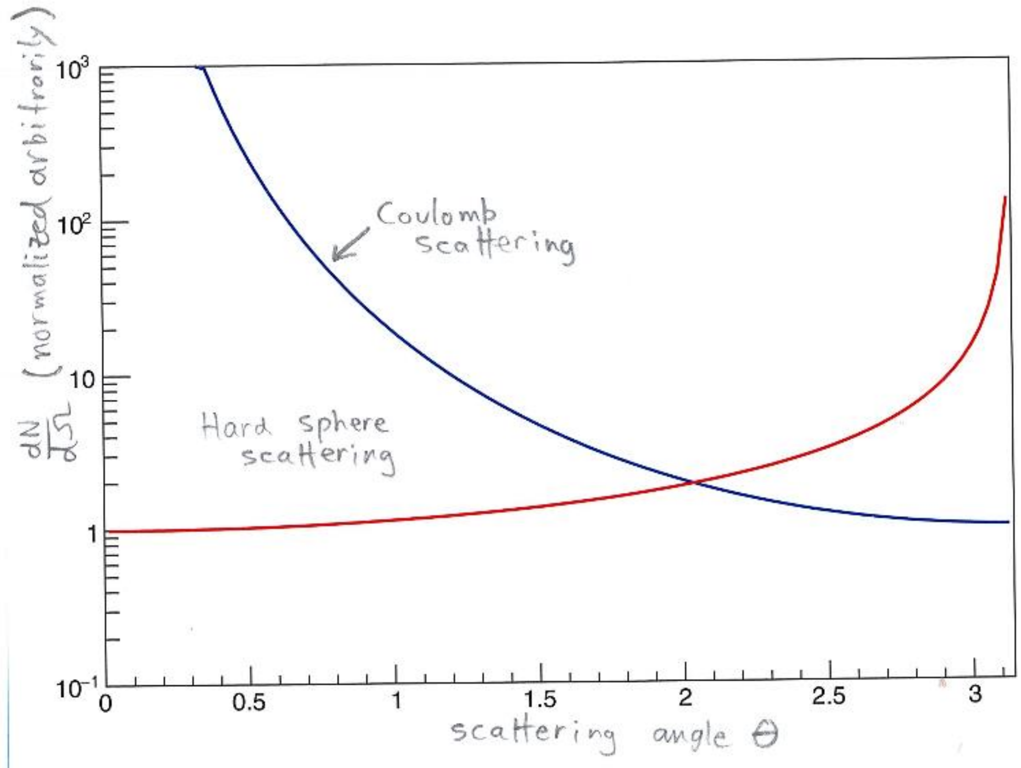
Coulomb Scattering

- Eventually, it turns out that

$$\frac{d\sigma}{d\Omega} = \frac{1}{4}\left(\frac{zZe^2}{mv_0}\right)^2 \frac{1}{\sin^4(\theta/_2)}$$

- This is the cross section for scattering from a single target nucleus

$$\frac{dR}{d\Omega} = \mathcal{L}\frac{d\sigma}{d\Omega}$$



Differential Cross Sections

- By studying the dependence of the differential cross section on various independent variables we can learn about the fundamental interactions on the microscopic scale.
- Exclusive cross section:
  - Final state is specified precisely (eg, exactly one proton in the case of elastic scattering)
- Inclusive cross section:
  - Final state includes the specified particle configuration
  - This is usually the sum of all possible exclusive cross sections that include the specified particle configuration

Decay Rates

- What else can we measure?
  - Lifetimes of unstable particles
  - Branching fractions to different final states
- Lifetime, decay rate and "partial width":

$$\Gamma = \frac{\hbar}{\tau}$$

- Branching fraction:

$$Br(i \to f) = \frac{\Gamma_f}{\Gamma_{total}}$$

$$\Gamma_{total} = \sum_j \Gamma_j$$

$$\Gamma_j \sim |\langle j|U|i\rangle|^2$$

Decay Rates

- Especially for nuclear transitions, we use the half-life rather than the decay rate.

$$N(t) = N_0 e^{-t/\tau} = N_0 2^{-t/t_{1/2}}$$

- Exercise: work out the relation between $t_{1/2}$ and τ
- The half-life of many radioactive elements is often expressed in years or days

$$t_{1/2}(^{22}\mathrm{Na}) = 2.602 \text{ years}$$
$$t_{1/2}(\,^{7}\mathrm{Be}) = 53.22 \text{ days}$$

- Decay rates are often expressed in Becquerel (decays per second) or in Curies.

$$1\ \mu\mathrm{Ci} = 37{,}000\ \mathrm{Bq}$$