PURDUE UNIVERSITY DEPARTMENT OF PHYSICS

Physics 56400
Introduction to Elementary Particle Physics I

Lecture 19

Fall 2019 Semester

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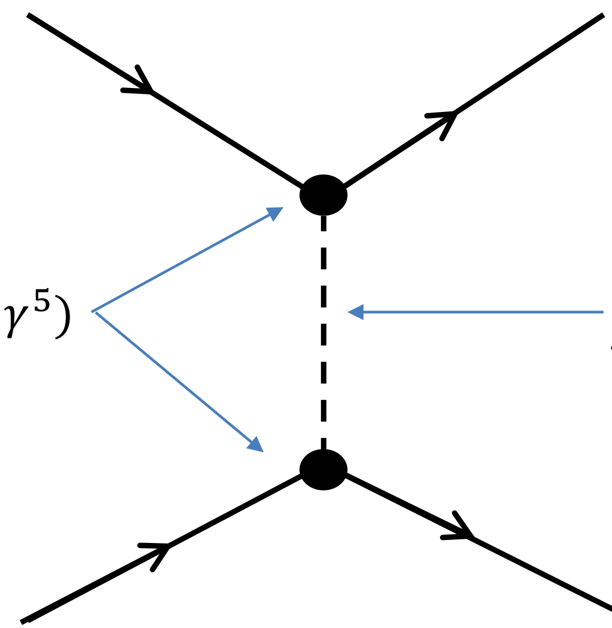
Summary of Tuesday's Lecture

- Weak interactions couple to fermion states with left-handed chirality
- This is a purely empirical observation, but consistent with all measurements
- Left-handed charged currents follow the form

$$J_e^\mu = \overline{u}_e \gamma^\mu \frac{1}{2}(1-\gamma^5)\, v_{\nu_e}$$

- The Fermi 4-point interaction parameterized all the dynamics as a universal weak coupling with strength G_F
- Some cross sections calculated using the 4-point interaction violated unitarity at high energies
- This deficiency was mitigated by introducing a massive, charged, vector boson.

Charged Weak Current



$$-\frac{ig}{\sqrt{2}}\gamma^{\mu}\frac{1}{2}(1-\gamma^{5})$$

$$\frac{-i\left(g^{\mu\nu}-\frac{q^{\mu}q^{\nu}}{M_W^2}\right)}{q^2-M_W^2}$$

$$\frac{-i\left(g^{\mu\nu}-\frac{q^{\mu}q^{\nu}}{M_W^2}\right)}{q^2-M_W^2}\approx\frac{ig^{\mu\nu}}{M_W^2}$$

$$G_F=\frac{\sqrt{2}g^2}{8\,M_W^2}$$

Family Structure of Matter

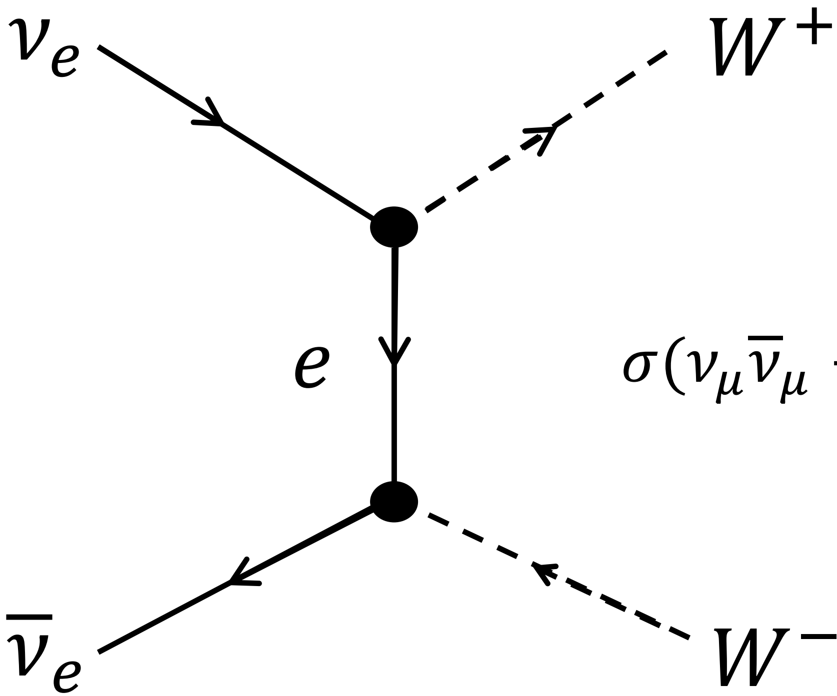
$$\begin{pmatrix} u \\ d \end{pmatrix} \quad \begin{pmatrix} c \\ s \end{pmatrix} \quad \begin{pmatrix} t \\ b \end{pmatrix}$$

$$\begin{pmatrix} \nu_e \\ e \end{pmatrix} \quad \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix} \quad \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}$$

- These are grouped into “weak isospin doublets”.
- The charged weak interaction couples within each doublet.
- This is like the idea of strong isospin (the strong interaction doesn’t distinguish based on electric charge)

Charged Weak Current

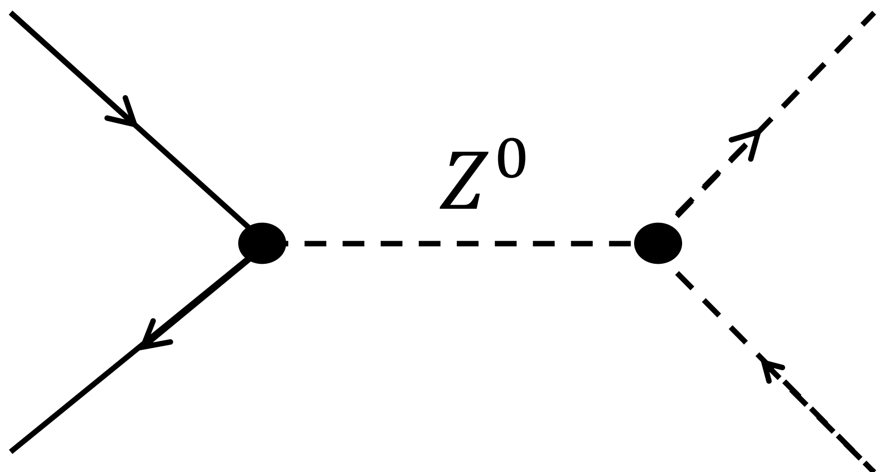
- Introducing the W boson avoided unitarity violation in $\nu_e \overline{\nu}_e \to \mu^+ \mu^-$
- But it persists in the process $\nu_\mu \overline{\nu}_\mu \to W^+ W^-$



$$\sigma(\nu_\mu \overline{\nu}_\mu \to W^+ W^-) \sim \left(\frac{g}{M_W}\right)^4 s$$

Weak Neutral Currents

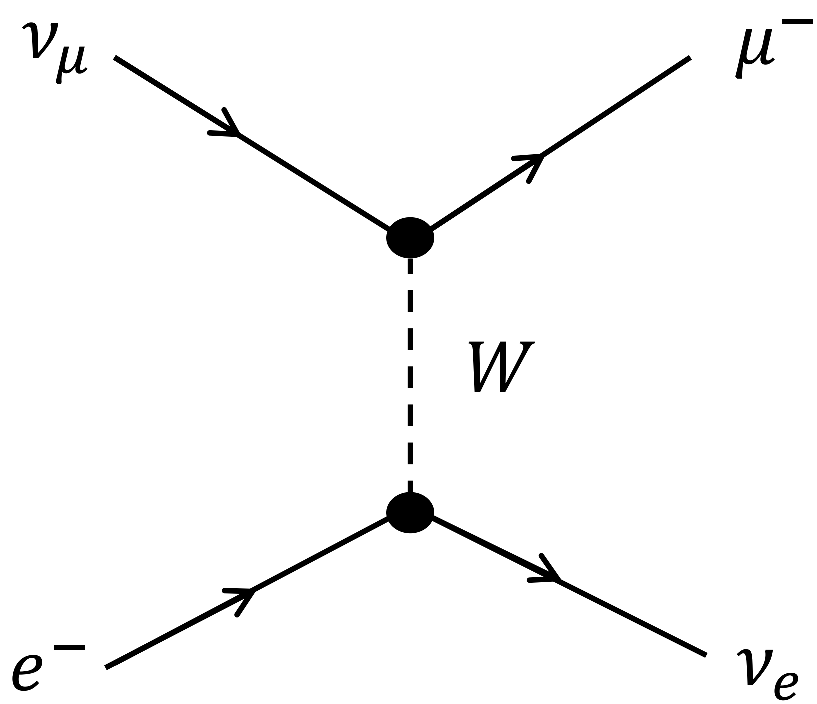
- To fix the problem with unitarity, another vector boson was introduced that would cancel the bad behavior in the amplitude:



- Its mass would be about the same as the W and its couplings to fermions and W's would be highly constrained.

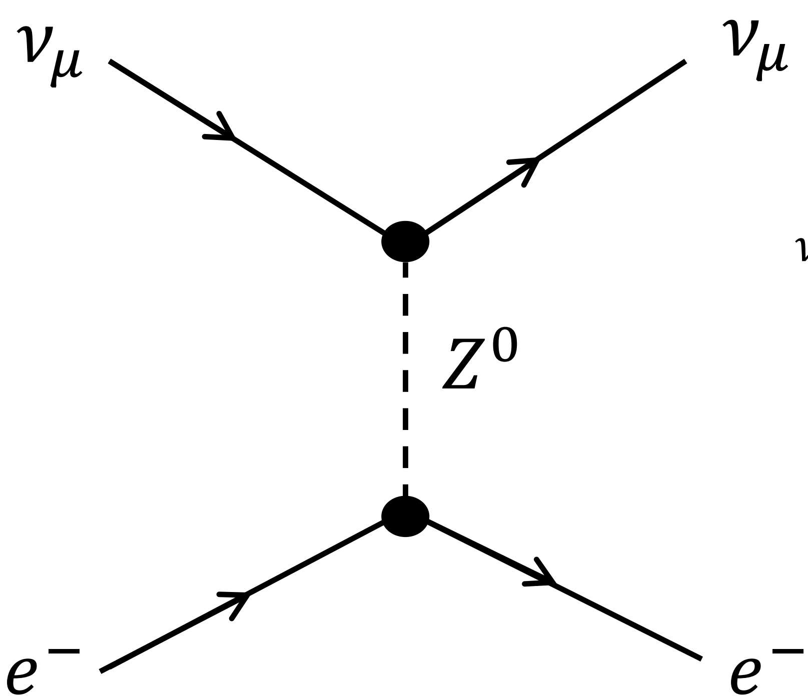
Evidence for Weak Neutral Currents

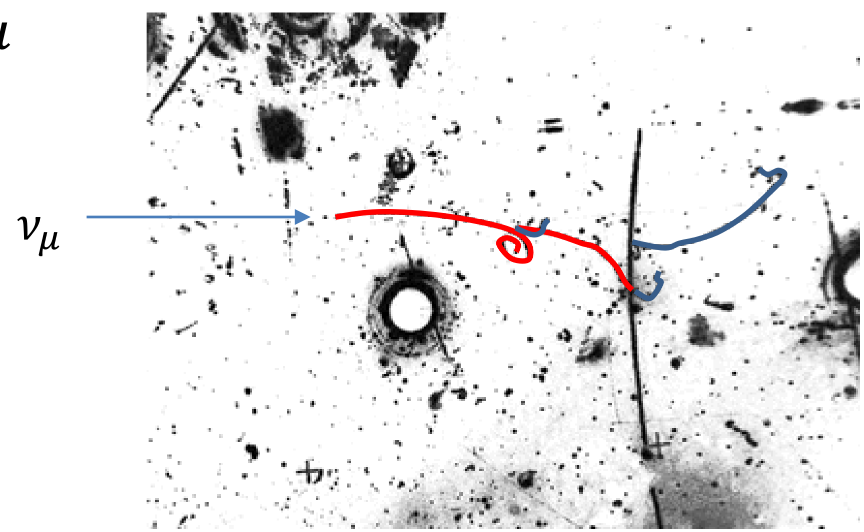
- If the weak interaction only involved charged currents, then neutrino scattering would have to change the lepton charge:



Evidence for Weak Neutral Currents

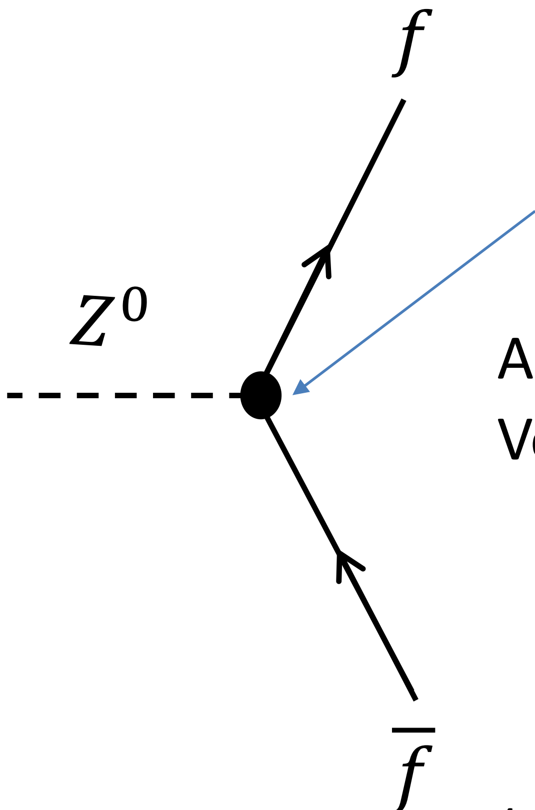
- But, weak neutral currents would allow elastic neutrino-electron scattering:





CERN, 1973

Weak Neutral Currents



$$-\frac{ig'}{2}\,\gamma^\mu(c_V - c_A\gamma^5)$$

Axial coupling: $\boldsymbol{c_A = I_3}$

Vector coupling: $\boldsymbol{c_V = I_3 - 2Q_f \sin^2\theta_W}$

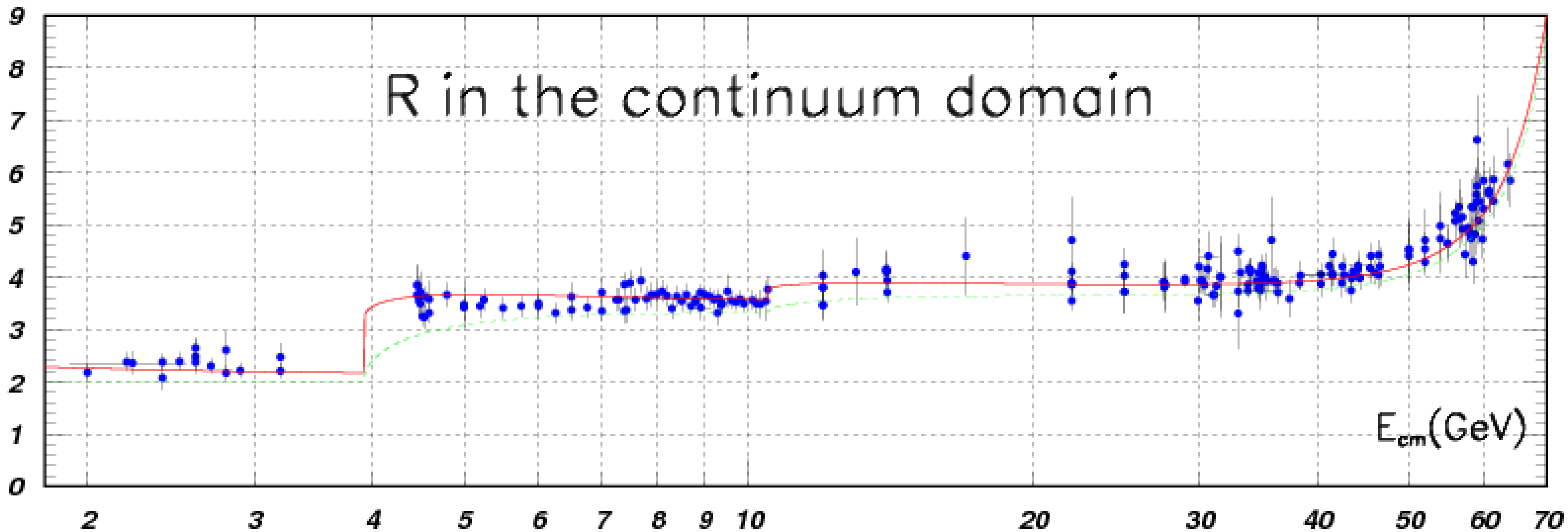
$$\boldsymbol{g\sin\theta_W = g'\cos\theta_W = e}$$

$$\boldsymbol{M_Z = \frac{M_W}{\cos\theta_W}}$$

(this follows from the unification of electromagnetic and weak interactions via the Higgs mechanism.)

Evidence for the Z Boson

- Elastic neutrino scattering
- "R" ratio at e^+e^- colliders:



- Forward-backward asymmetries in e^+e^- collisions

Forward-Backward Asymmetries

- Pure quantum electrodynamics:

$$\frac{d\sigma}{d\Omega} = n_c Q_f^2 \frac{\alpha^2}{4s}(1 + \cos^2\theta)$$

- Forward/backward cross-section:

$$\sigma_F = 2\pi \int_0^1 \frac{d\sigma}{d\Omega} d(\cos\theta) = n_c Q_f^2 \frac{2\pi\alpha^2}{3s}$$

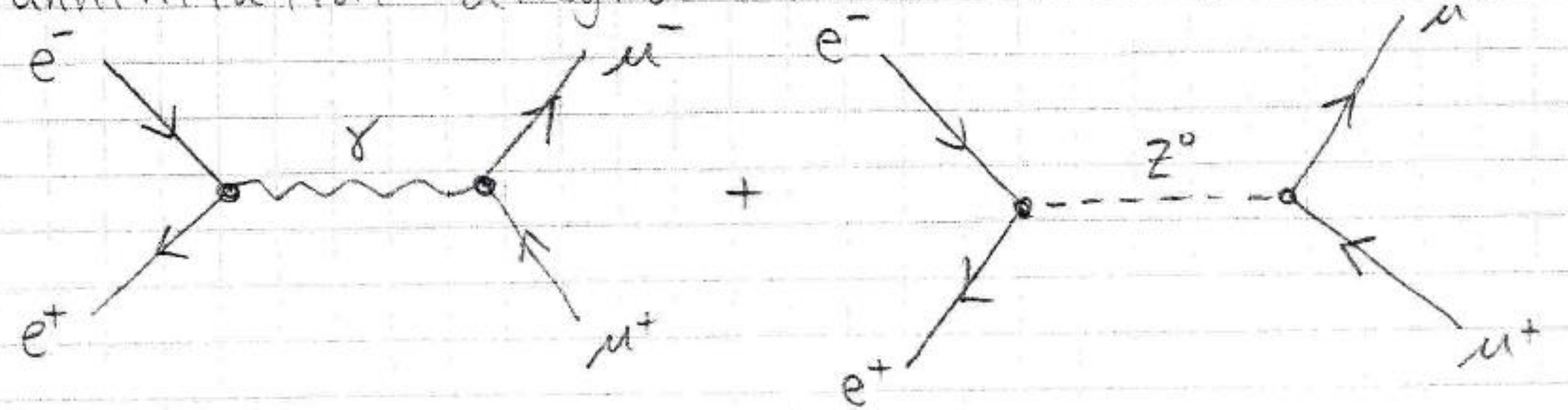
$$\sigma_B = 2\pi \int_{-1}^0 \frac{d\sigma}{d\Omega} d(\cos\theta) = n_c Q_f^2 \frac{2\pi\alpha^2}{3s}$$

- Forward-backward asymmetry:

$$A_{FB} = \frac{\sigma_F - \sigma_B}{\sigma_F + \sigma_B} = 0 \quad \text{(in pure QED)}$$

Forward-Backward Asymmetries

If the Z° couples to e^+e^- then there should be interference between Z° and γ annihilation diagrams:



$$\mathcal{M} = \mathcal{M}_\gamma + \mathcal{M}_Z$$

$$|\mathcal{M}|^2 = \left|\mathcal{M}_\gamma\right|^2 + |\mathcal{M}_Z|^2 + 2\mathrm{Re}(\mathcal{M}_\gamma^* \mathcal{M}_Z)$$

Left-Right Couplings

- The chiral decomposition of the Z coupling can be written in terms of separate coupling constants for right- and left-handed chiral fermions:

$$-\frac{ig}{\cos\theta_W}\gamma^\mu \frac{1}{2}(c_V + c_A\gamma^5) = -\frac{ig}{\cos\theta_W}\gamma^\mu \left(c_L \frac{1}{2}(1-\gamma^5) + c_R \frac{1}{2}(1+\gamma^5)\right)$$

$$c_L = \frac{1}{2}(c_V + c_A)$$

$$c_R = \frac{1}{2}(c_V - c_A)$$

- This is the same vertex factor as in QED when we set $c_A = 0$ in which case the photon couples equally to left- and right-handed fermions.

Left-Right Couplings

- Consider $e^+e^- \to \mu^+\mu^-$ in which both the e^- and μ^- have left-handed chirality:

$$-i\mathcal{M} = \left(\frac{ie^2}{q^2} + \frac{ig^2}{4\cos^2\theta_W}\frac{c_L^e c_L^\mu}{q^2 - M_Z^2}\right) \times \ \bar{v}_L(p_2)\gamma^\mu u_R(p_1)\bar{u}_L(k_1)\gamma_\mu v_R(k_2)$$

$$= \frac{ie^2}{s}(1 + rc_L^e c_L^\mu)\bar{v}_L(p_2)\gamma^\mu u_R(p_1)\bar{u}_L(k_1)\gamma_\mu v_R(k_2)$$

$$r = \frac{g^2}{4\cos^2\theta_W} \cdot \frac{1}{s - M_Z^2} \cdot \frac{s}{e^2} = \frac{\sqrt{2}M_Z^2 G_F}{s - M_Z^2}\left(\frac{s}{e^2}\right)$$

- Taking into account the finite width of the Z resonance,

$$r = \frac{\sqrt{2}M_Z^2 G_F}{s - M_Z^2 + i\Gamma_Z M_Z}\left(\frac{s}{e^2}\right)$$

Left-Right Couplings

- Differential cross section:

$$|\mathcal{M}_{LL}|^2 = \frac{e^4}{s^2}\left|1 + rc_L^e c_L^\mu\right|^2 (1 + \cos\theta)^2$$

- Likewise,

$$|\mathcal{M}_{RR}|^2 = \frac{e^4}{s^2}\left|1 + rc_R^e c_R^\mu\right|^2 (1 + \cos\theta)^2$$

- But,

$$|\mathcal{M}_{LR}|^2 = \frac{e^4}{s^2}\left|1 + rc_L^e c_R^\mu\right|^2 (1 - \cos\theta)^2$$

$$|\mathcal{M}_{RL}|^2 = \frac{e^4}{s^2}\left|1 + rc_R^e c_L^\mu\right|^2 (1 - \cos\theta)^2$$

Left-Right Couplings

- Differential cross sections:

$$\frac{d\sigma}{d\Omega}(e_L^- e_R^+ \to \mu_L^- \mu_R^+) = \frac{\alpha^2}{4s}(1+\cos\theta)^2\left|1 + r c_L^e c_L^\mu\right|^2$$
$$\frac{d\sigma}{d\Omega}(e_L^- e_R^+ \to \mu_R^- \mu_L^+) = \frac{\alpha^2}{4s}(1-\cos\theta)^2\left|1 + r c_L^e c_R^\mu\right|^2$$

(and likewise for RL and RR…)

…Average over incident helicity, sum over final state helicity:

$$\frac{d\sigma}{d\Omega}\left(e^- e^+ \to \mu^- \mu^+\right) = \frac{\alpha^2}{4s}\left(A_0(1+\cos^2\theta) + A_1\cos\theta\right)$$

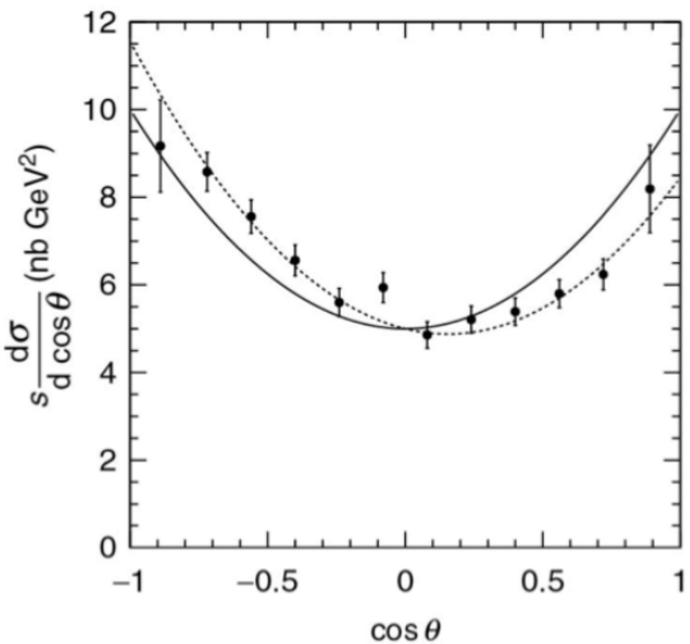
- Forward-backward asymmetry:

$$A_{FB} = \frac{3}{4}\frac{A_1}{A_0}$$

- Remember that this depends on r which is a function of s.

Forward-Backward Asymmetry

- The influence of the Z^0 was apparent even at energies much less than M_Z:



Results from the JADE experiment

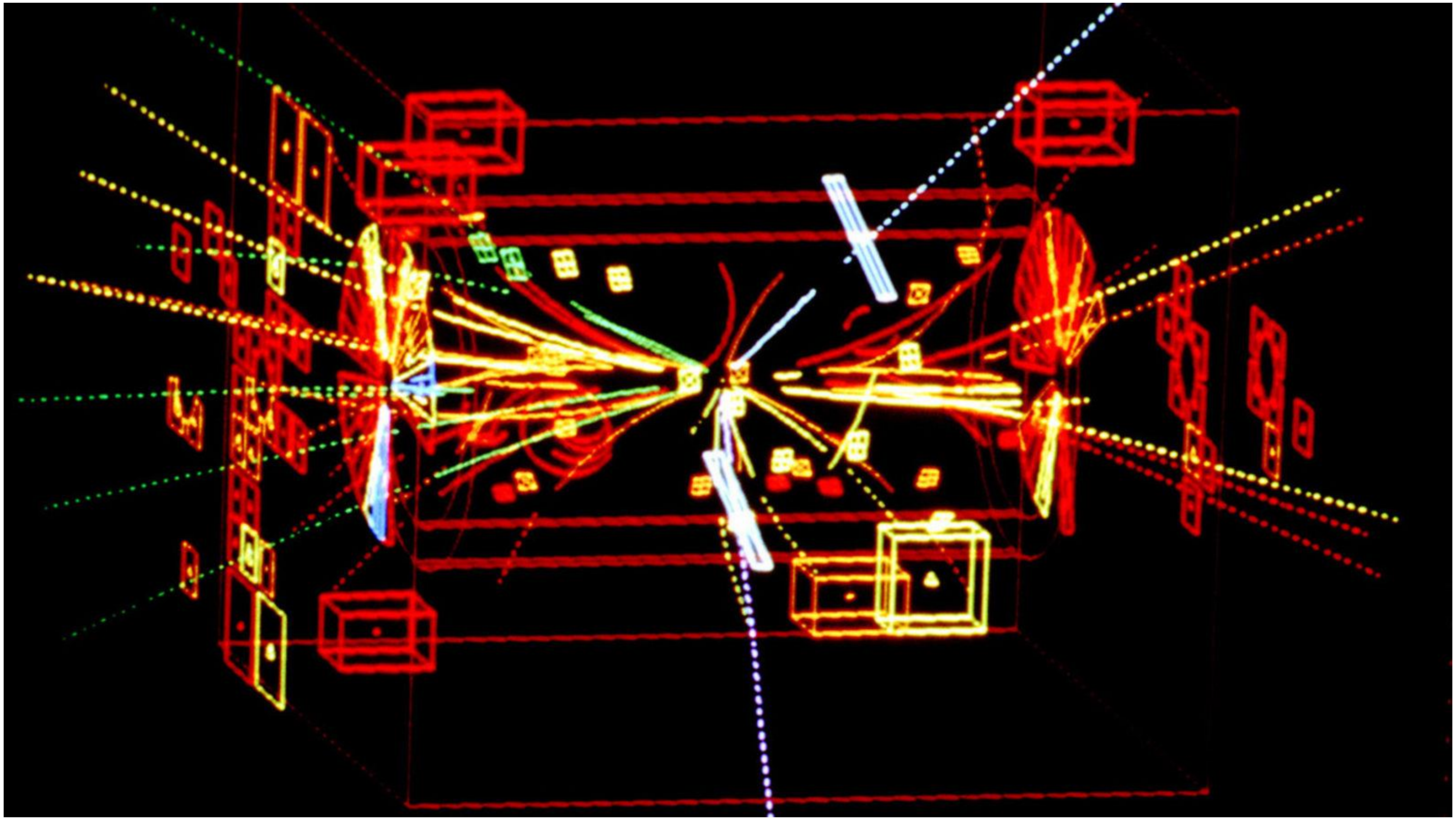
$$\sqrt{s} = 34.4\ \mathrm{GeV}$$

Analysis of the interference effects suggested that $M_Z \approx 90\ \mathrm{GeV}$.

Direct Production of W and Z

- In the early 1980's, e^+e^- colliders did not have sufficient energy to produce Z's directly.
- e^+e^- can only produce W^+W^- in pairs, so they certainly didn't have enough energy to produce them.
- It was proposed to turn the CERN SPS into a proton-anti-proton collider
  - Doubles center-of-mass energy for $u\bar{u}, d\bar{d} \to Z^0$
  - Can produce single W's via $u\bar{d} \to W^+$ and $\bar{u}d \to W^-$
  - Technical challenges associated with colliding anti-protons

Observation of W and Z



UA1 experiment at the $Sp\bar{p}S$
June 1, 1983

Construction of LEP (and SLC)

- Precision studies of the electroweak sector of the standard model motivated building high-energy e^+e^- colliders
- SLAC Linear Collider (SLC):
  - $\sqrt{s} = 91\ \mathrm{GeV}$ collisions (just sufficient to produce Z)
  - Highly polarized electron beam
- Large Electron Positron (LEP) Collider:
  - LEP1 : $\sqrt{s} \approx 91\ \mathrm{GeV}$
  - LEP1.5: $\sqrt{s} = 130/136\ \mathrm{GeV}$
  - LEP2: $\sqrt{s} = 161, 172, 183, 189\ \mathrm{GeV}$ (above W^+W^- threshold)