PURDUE UNIVERSITY DEPARTMENT OF PHYSICS

Physics 56400

Introduction to Elementary Particle Physics I

Lecture 18

Fall 2019 Semester

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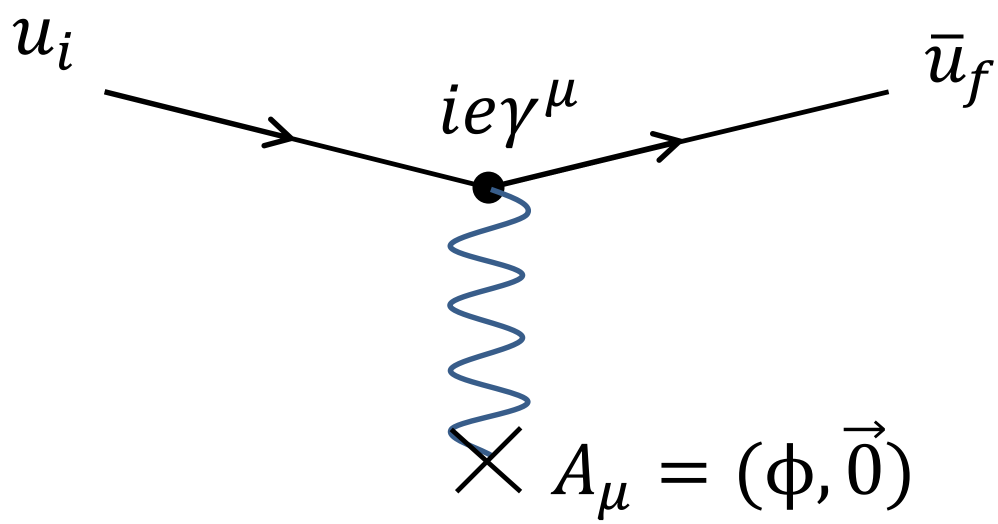
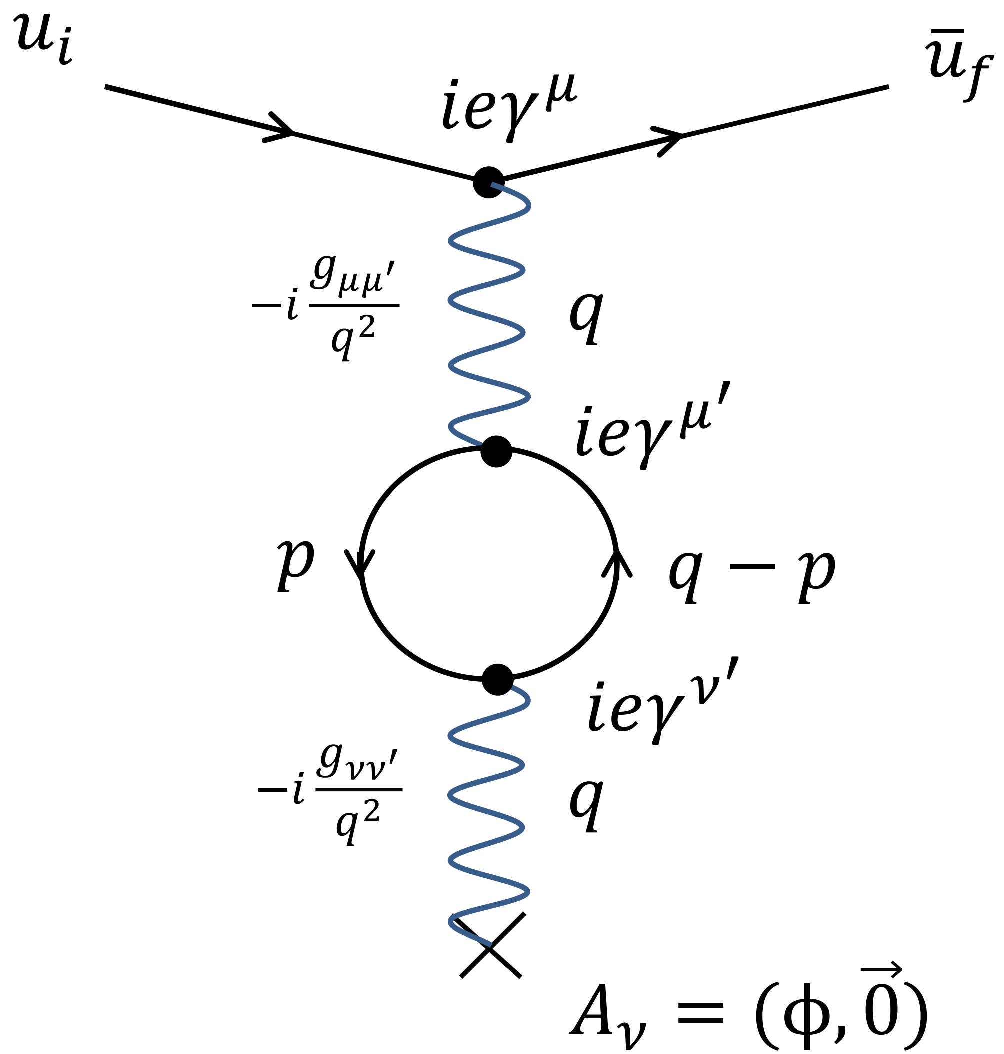
Higher Order Feynman Diagrams

- Rutherford scattering:



- This is the first term in the perturbative expansion of the amplitude.
- The next order in perturbation theory introduces more diagrams.

Higher-Order Corrections



Higher-Order Corrections

- As usual, we must sum over all possible unobserved intermediate states.
- In this case it means we integrate over the unobserved momentum p but this diverges logarithmically:

$$-i\,\frac{g_{\mu\nu}}{q^2} \to \left(-i\,\frac{g_{\mu\mu'}}{q^2}\right)\, I^{\mu'\nu'}(q)\left(-i\,\frac{g_{\nu'\nu}}{q^2}\right)$$

$$= \left(-\frac{i}{q^2}\right) I_{\mu\nu}(q)\left(-\frac{i}{q^2}\right)$$

- The calculation is lengthy, but

$$I_{\mu\nu}(q) \sim -i g_{\mu\nu} \int_{m^2}^{\infty} \frac{dp^2}{p^2}$$

Regularization

- For now, we keep everything finite by taking an (arbitrary) upper limit of integration.

$$I(q^2) = \frac{\alpha}{3\pi}\log\left(\frac{M^2}{-q^2}\right)$$

- This diverges as $M \to \infty$ but we can absorb this divergence into the physical parameters of the theory, namely the electric charge which must be measured anyway.

Renormalization

- Suppose we want to measure e in a low-energy scattering experiment with small momentum transfers

$$-i\mathcal{M} = (ie\overline{u}\gamma_0 u)\left(-\frac{i}{q^2}\right)\left(1 - \frac{\alpha}{3\pi}\log\left(\frac{M^2}{m^2}\right) - \frac{\alpha}{15\pi}\frac{q^2}{m^2} + \mathcal{O}(e^4)\right)(-iZe)$$

- We don't know what e is – that's what we want to measure.
- We can *define* e to be the observed coupling strength at low momentum transfer.

$$-i\mathcal{M} = (ie_R\overline{u}\gamma_0 u)\left(-\frac{i}{q^2}\right)\left(1 - \frac{e_R^2}{60\pi^2}\frac{q^2}{m^2}\right)(-iZe_R)$$

$$e_R \equiv e\left(1 - \frac{e^2}{12\pi^2}\log\frac{M^2}{m^2}\right)^{1/2}$$

Renormalization

- What have we assumed?
- Physical measurements include all possible Feynman amplitudes.
- Perturbation theory accounts for just the lowest order terms in a power series.
- We need to define the parameters in the theory in terms of physical measurements (that are influenced by all orders).
- We have calculated the dependence (to lowest order) of the process on q^2.

Vacuum Polarization

- We can also describe this qualitatively.
- A static point charge is surrounded by an electric field which has an energy density.
- This energy can create virtual electron-positron pairs, but they must annihilate shortly after they form
  - Energy must be conserved
  - Consistent with Heisenberg's uncertainty principle
- The point charge is screened by the virtual e^+e^- pairs which polarize the vacuum
- The further we penetrate the cloud of e^+e^- pairs, the more we are influenced by the point charge at the origin.

Running Coupling Constant

- With this definition, we see that the electromagnetic coupling strength depends on momentum transfer.
- The potential due to a point charge has the form:

$$V(r) = -\frac{Ze_R^2}{4\pi r} - \frac{Ze_R^4}{60\pi^2 m^2}\delta(\vec{r})$$

- The closer an electron gets to the nucleus, the more its wavefunction overlaps with the delta-function at the origin.
- Lamb shift: modification to energy levels of Hydrogen for electrons that are in S-wave states, for which the wave-function at the origin is finite.

Running Coupling Constant

- Now we can predict how the coupling constant measured at one scale, will evolve to a different value for other scales.
- This is equivalent to summing, to all orders, certain classes of diagrams (but not all diagrams).

$$\alpha(Q^2) = \frac{\alpha(\mu^2)}{1 - \frac{\alpha(\mu^2)}{3\pi}\log\left(\frac{Q^2}{\mu^2}\right)}$$

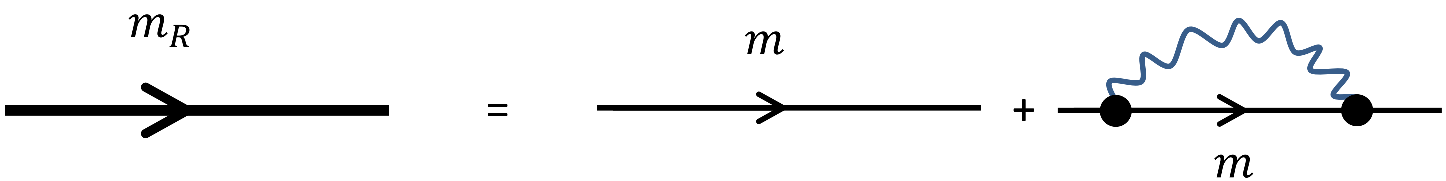
- This tells us what the value of α will be when the momentum transfer is Q^2 in terms of the value measured when $Q^2 = \mu^2$.
- Usually, we define α to be measured at low momentum transfers, for which $\mu^2 = m^2$.

Running Coupling Constant

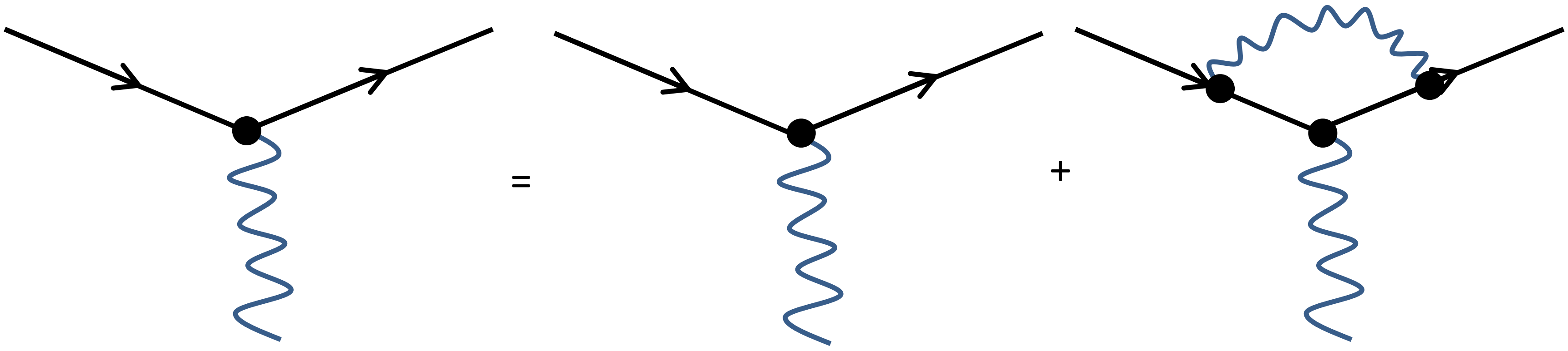
- At low momentum transfers we determine, *from experiment*, that
$$\alpha(m^2) \approx \frac{1}{137}$$
- In high-energy electron scattering experiments, when $Q^2 \sim (100\ \mathrm{GeV})^2$,
$$\alpha(Q^2) \rightarrow \frac{1}{134}$$
- But at these energies there can be many other virtual fermions (muons, quarks) in the loops which also need to be counted.
- In practice,
$$\alpha\left((100\ \mathrm{GeV})^2\right) \sim \frac{1}{128}$$

Renormalization

- Other higher-order diagrams can be absorbed into the measured parameters of the theory



- Vertex corrections:



Anomalous Magnetic Moment

- To lowest order, Dirac's equation determines the magnetic moment of the electron to be
$$\vec{\mu} = -\frac{e}{2m}\vec{\sigma} = -g\,\frac{e}{2m}\;\vec{S}$$
- To lowest order, $g = 2$
- Including the higher order diagrams,
$$\vec{\mu} = -\frac{e}{2m}\left(1 + \frac{\alpha}{2\pi}\right)\vec{\sigma}$$
$$g = 2 + \frac{\alpha}{\pi} \approx 2.00232$$

Anomalous Magnetic Moment

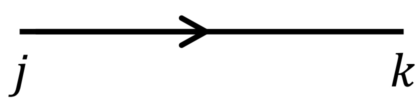
- One of the most precise calculations that has been performed in perturbation theory is the muon magnetic moment
- The g-2 experiment at Brookhaven (now at Fermilab) measures $g_\mu - 2$ from the spin-procession frequency of muons in a magnetic field.

Renormalizable Field Theories

- QED is an example of a renormalizable field theory
- At each order of perturbation theory, Feynman amplitudes can be split into a finite part and a divergent part
- The divergent parts can be absorbed into the definitions of the fundamental parameters (ie, m, e)
- This is not true for all field theories in which new types of divergences emerge at each higher order
- For example, Heavy Quark Effective Theory is an expansion in $1/m_Q$
  - It is not renormalizable but at each order, the relations between several experimental observables are calculable.

Quantum Chromodynamics

- The quantum field theory that describes the strong interaction is almost identical to QED except that quarks have three possible charges (color).
- Feynman rules for quarks:
  - Internal quark propagators:



$$\frac{i\left(\gamma^{\mu} p_{\mu} + m\right)}{p^2 - m^2} \ \delta_{jk}$$

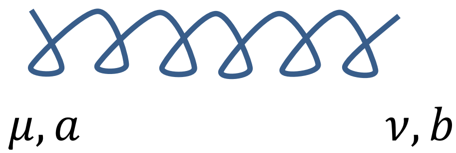
  - The delta function accounts for the conservation of quark color charge, $j, k = 1..3$

Quantum Chromodynamics

- We would like QCD to be gauge invariant
- We should have the freedom to redefine the labeling of the three colors without changing the observables
- Furthermore we would like QCD to be *locally* gauge invariant:
  - We should be able to redefine the colors independently at each point in space, provided the redefinition is a smoothly varying function over space.
  - This requirement dictates how the quarks must couple to the color fields
  - One consequence is that instead of one photon in QED, there are 8 gluons in QCD that couple to color charge.

Quantum Chromodynamics

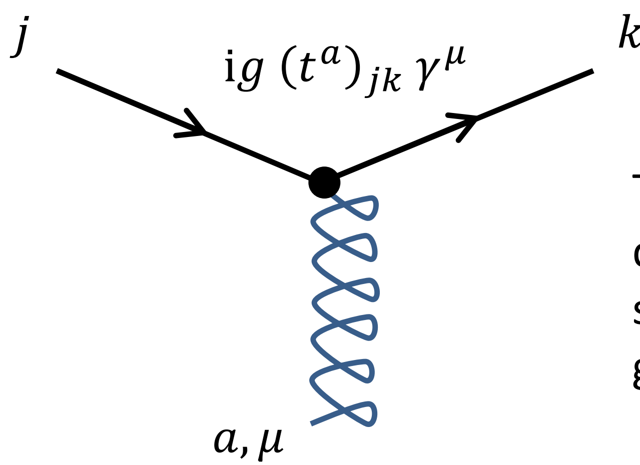
- Feynman rules for gluon propagators:



$$-i\,\frac{g_{\mu\nu}}{q^2}\,\delta_{ab}$$

  - Now, the indices a, b run over the 8 types of gluons.

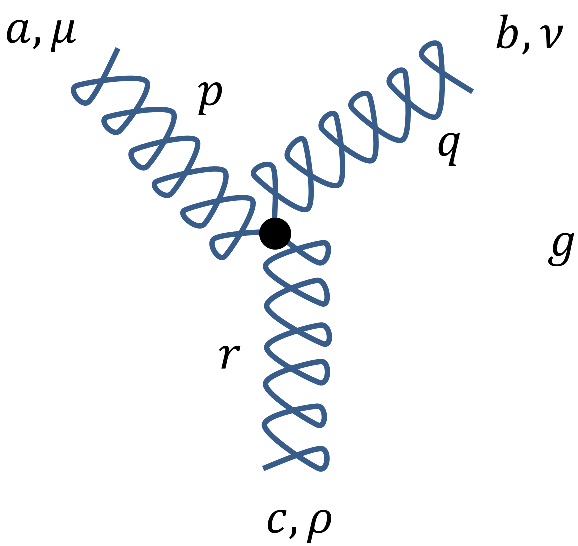
- Quark-gluon vertex:



The coefficients $(t^a)_{jk}$ are determined from the symmetry properties of the gauge group, SU(3).

Quantum Chromodynamics

- Another difference compared with QED is that the gluons themselves have color charge.
  - In QED, the photon does *not* have an electric charge.
- This means that gluons can couple to gluons



$$g\, f_{abc}[g^{\mu\nu}(p-q)^{\rho} + g^{\nu\rho}(q-r)^{\mu} + g^{\rho\mu}(r-p)^{\nu}]$$

$$p+q+r=0$$

- There is also a 4-gluon vertex…

Renormalization

- Gluons are massless which makes QCD a renormalizable field theory, just like QED.
- All the divergent parts of amplitudes can be absorbed into the definition of the strong coupling constant, α_s but now there are a bunch of extra coefficients from all the quark gluon vertices.

$$\alpha_s(Q^2) = \frac{\alpha_s(\mu^2)}{1 + \frac{\alpha_s(\mu^2)}{12\pi}(33 - n_f)\log\left(\frac{Q^2}{\mu^2}\right)}$$

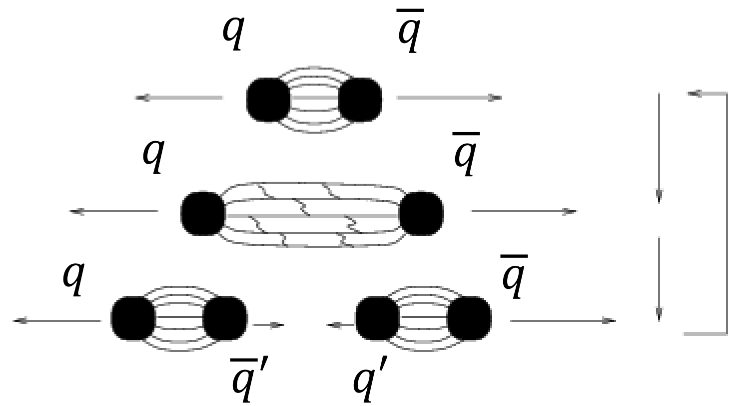
- This actually gets smaller as Q^2 increases!

Strong Coupling Constant

- High Q^2 corresponds to small distances
- Consequences of the running coupling constant:
  - At large momentum transfers, α_s is small enough that perhaps we can trust lowest order perturbation theory
  - At small distances, α_s is small, so the quarks in hadrons interact only weakly with each other (asymptotic freedom)
  - If you try to pull a quark out of a hadron, the strong force increases perhaps linearly with distance, just like stretching a spring.
  - The energy stored in the gluon field between two quarks can pop additional $q\overline{q}$ pairs out of the vacuum.

String Fragmentation

- A high energy quark will be connected to other sources of color by a “string” of gluons
- When sufficient energy is stored in the string, it will break into two strings, with a new q' and $\overline{q}'$ at each end.



- Eventually, the fragmentation stops and we are left with several hadrons in the final state which can then decay.

String Fragmentation

- A consequence of string fragmentation is that at high energies, an initial quark or gluon will turn into a collimated jet of hadrons
- The energy and momentum of the jet approximates that of the initial quark or gluon.
- There are only minor differences between quark and gluon jets.