PURDUE UNIVERSITY DEPARTMENT OF PHYSICS

Physics 56400
Introduction to Elementary Particle Physics I

Lecture 13

Fall 2019 Semester

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Family Structure of the Standard Model

$$\begin{pmatrix} u \\ d \end{pmatrix} \qquad \begin{pmatrix} c \\ s \end{pmatrix}$$

$$\begin{pmatrix} \nu_e \\ e \end{pmatrix} \qquad \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}$$

- The quarks listed here are strong eigenstates which are not the same as the eigenstates that participate in the weak interaction:

$$d' = d\ \cos\theta_C + s\ \sin\theta_C$$
$$s' = -d\ \sin\theta_C + s\ \cos\theta_C$$

- The Cabibbo angle, $\theta_C = 13.02°$ is a parameter in the unitary transformation from the strong to the weak basis.

Family Structure of the Standard Model

$$\begin{pmatrix} u \\ d' \end{pmatrix} \qquad \begin{pmatrix} c \\ s' \end{pmatrix}$$

$$\begin{pmatrix} \nu_e \\ e \end{pmatrix} \qquad \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}$$

- The weak interaction *only* allows transitions between members of the same doublet:

$$u \leftrightarrow d'$$

$$c \leftrightarrow s'$$

$$\nu_e \leftrightarrow e$$

$$\nu_\mu \leftrightarrow \mu$$

Family Structure of the Standard Model

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- The weak interaction *only* allows transitions between members of the same doublet:

$$u \leftrightarrow d' = d\ \cos\theta_C + s\ \sin\theta_C$$

$$c \leftrightarrow s' = -d\ \sin\theta_C + s\ \cos\theta_C$$

$$\nu_e \leftrightarrow e$$

$$\nu_\mu \leftrightarrow \mu$$

- Transitions between families only occurs because the weak basis and strong basis are not the same.
- Flavor changing neutral currents are suppressed by the GIM mechanism.

Third Family?

- The τ lepton has a mass of $1776.82\ \mathrm{MeV}$ and a lifetime of $0.3\ \mathrm{ps}$, coincidentally similar to the properties of D mesons.
- The τ lepton was uncovered in e^+e^- collisions at SLAC in 1975 via

$$e^+e^- \to \mu^\pm e^\mp + \not{p}_T$$

- Interpreted as evidence for production of τ pairs and subsequent decay via

$$\tau^- \to e^-\nu_\tau\bar{\nu}_e$$
$$\tau^- \to \mu^-\nu_\tau\bar{\nu}_\mu$$

Three Families of Matter

$$\begin{pmatrix} u \\ d \end{pmatrix} \quad \begin{pmatrix} c \\ s \end{pmatrix} \quad \begin{pmatrix} ? \\ ? \end{pmatrix}$$

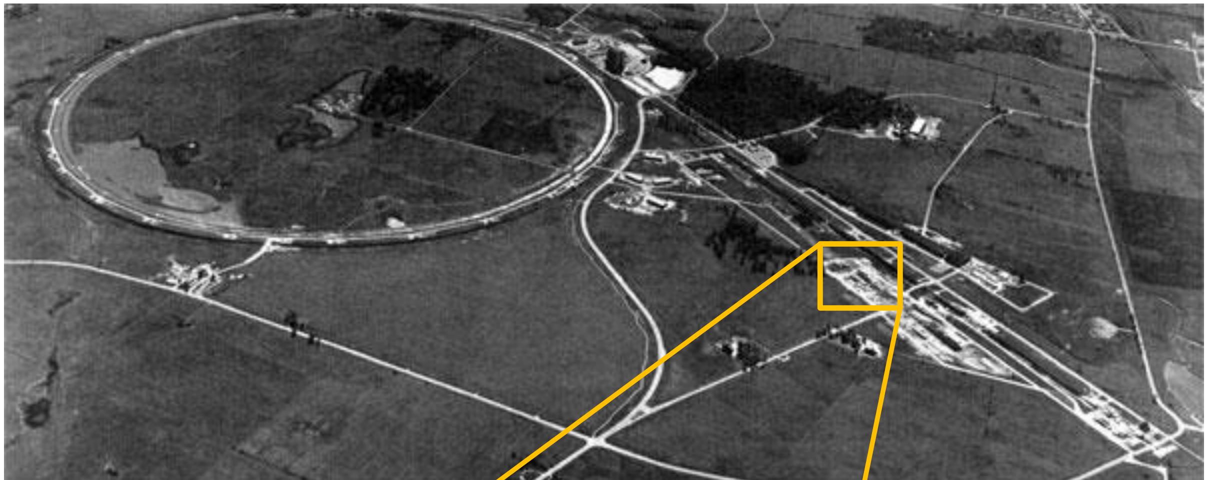
$$\begin{pmatrix} \nu_e \\ e \end{pmatrix} \quad \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix} \quad \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}$$

- The τ neutrino was inferred, but not yet directly observed.
- Within each family we must have

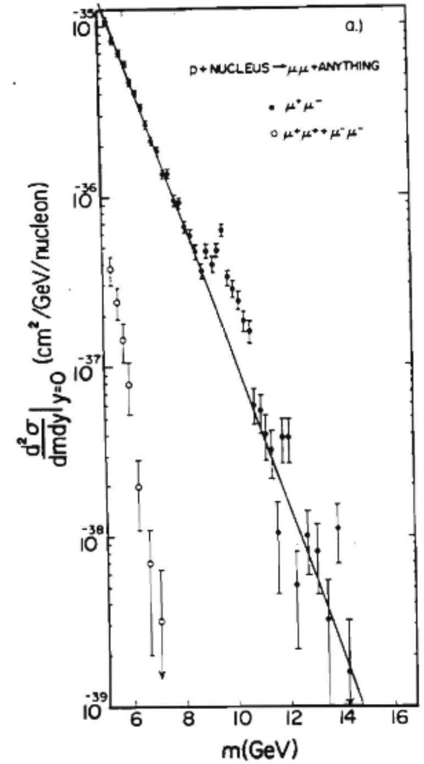
$$3\,Q_q + 3\,Q_{q'} + Q_\nu + Q_\ell = 0$$

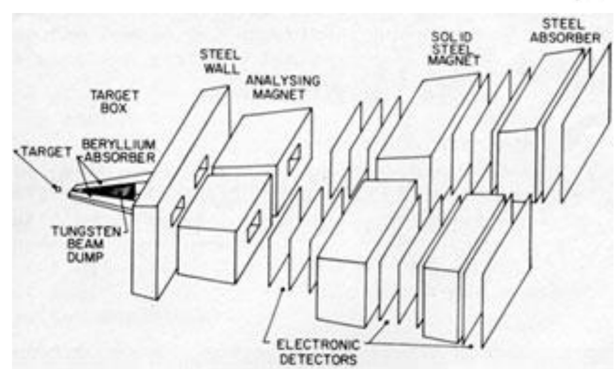
- One might expect that there should also be quarks in the 3rd family.

Observation of $\Upsilon \rightarrow \mu^+\mu^-$

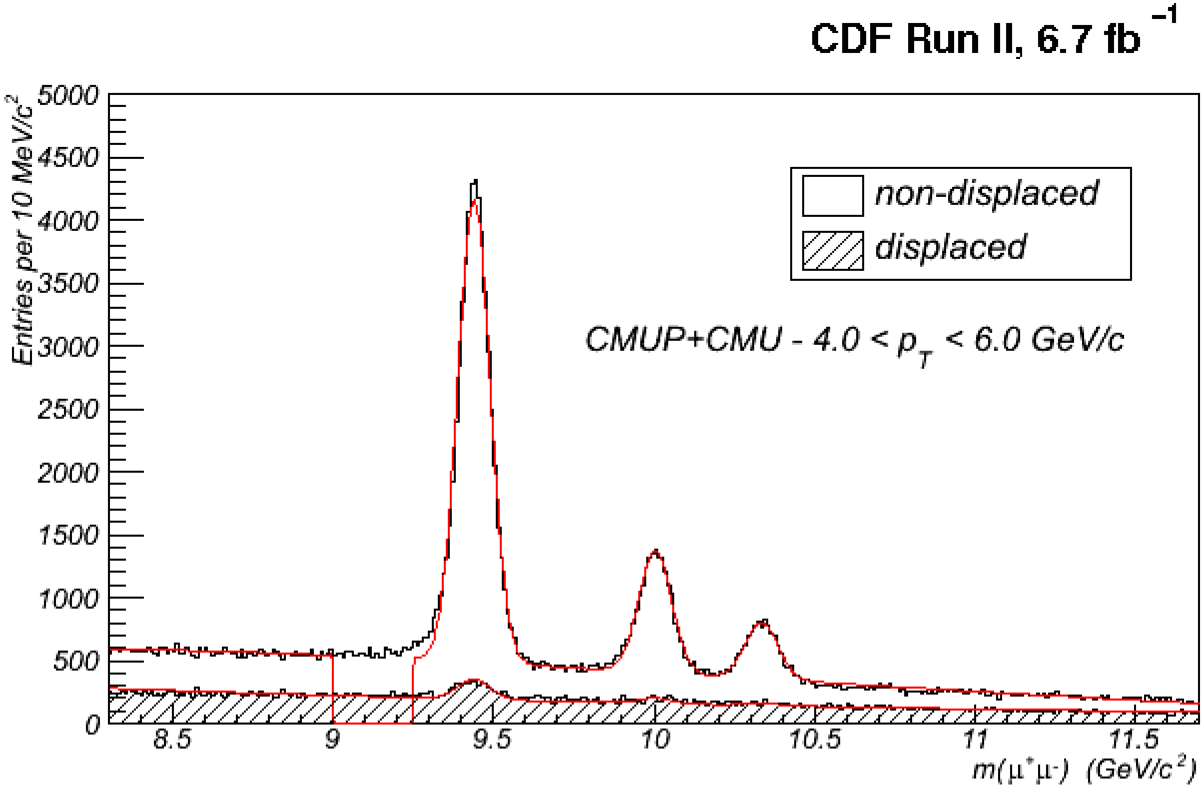


Fermilab, 1977





$\Upsilon(nS) \to \mu^+\mu^-$



- There are three narrow Υ resonances corresponding to $n = 1,2,3$ at masses below the $B\bar{B}$ threshold, $\Gamma = 50, 30, 20 \text{ keV}$
- The $\Upsilon(4S)$ has just enough energy to decay to B^+B^- or $B^0\bar{B}^0$, $\Gamma = 20 \text{ MeV}$

Charm Mesons

- Pseudoscalar (↑↓) D mesons:

$$D^+ = (c\bar{d})$$
$$D^0 = (c\bar{u})$$
$$D_s^+ = (c\bar{s})$$

- Vector (↑↑) D mesons: D^{*+}, D^{*0}, D_s^{*+}

$D^*(2007)^0$ DECAY MODES	Fraction (Γ_i/Γ)	p (MeV/c)
$D^0\pi^0$	(64.7 ± 0.9) %	43
$D^0\gamma$	(35.3 ± 0.9) %	137

$D^*(2010)^\pm$ DECAY MODES	Fraction (Γ_i/Γ)	p (MeV/c)
$D^0\pi^+$	(67.7 ± 0.5) %	39
$D^+\pi^0$	(30.7 ± 0.5) %	38
$D^+\gamma$	$(\ 1.6\pm0.4)$ %	136

D_s^{*+} DECAY MODES	Fraction (Γ_i/Γ)	p (MeV/c)
$D_s^+\gamma$	(93.5 ± 0.7) %	139
$D_s^+\pi^0$	$(\ 5.8\pm0.7)$ %	48

Bottom Mesons

- Pseudoscalar (↑↓) B mesons:

$$B^+ = (u\bar{b})$$
$$B^0 = (d\bar{b})$$
$$B_s^0 = (s\bar{b})$$

- Vector (↑↑) B mesons: B^{*+}, B^{*0}, B_s^{*0}

B^* $I(J^P) = \frac{1}{2}(1^-)$

I, J, P need confirmation. Quantum numbers shown are quark-model predictions.

Mass $m_{B^*} = 5324.65 \pm 0.25$ MeV
$m_{B^*} - m_B = 45.18 \pm 0.23$ MeV
$m_{B^{*+}} - m_{B^+} = 45.34 \pm 0.23$ MeV

B^* DECAY MODES	Fraction (Γ_i/Γ)	p (MeV/c)
$B\gamma$	dominant	45

Heavy Mesons

- Heavy mesons are (almost) non-relativistic systems and their spectra can be interpreted in terms of Hydrogen-atom quantum numbers
- Spin-spin interaction:

$$\Delta E \sim \frac{\vec{s}_1 \cdot \vec{s}_2}{m_1 m_2}$$

- This explains why the mass splitting in the $B^* - B$ system is smaller than in the $D^* - D$ system
- We can estimate that

$$m_b/m_c \sim 140 \text{ MeV}/45 \text{ MeV} \sim 3$$

Heavy Flavored Baryons

- Charm baryons:

$$\Lambda_c^+ = (udc)$$

$$\Sigma_c^{++} = (uuc) \qquad \Sigma_c^+ = (udc) \qquad \Sigma_c^0 = (ddc)$$

$$\Xi_c^+ = (usc) \qquad \Xi_c^0 = (dsc)$$

$$\Omega_c^0 = (ssc)$$

- Bottom baryons:

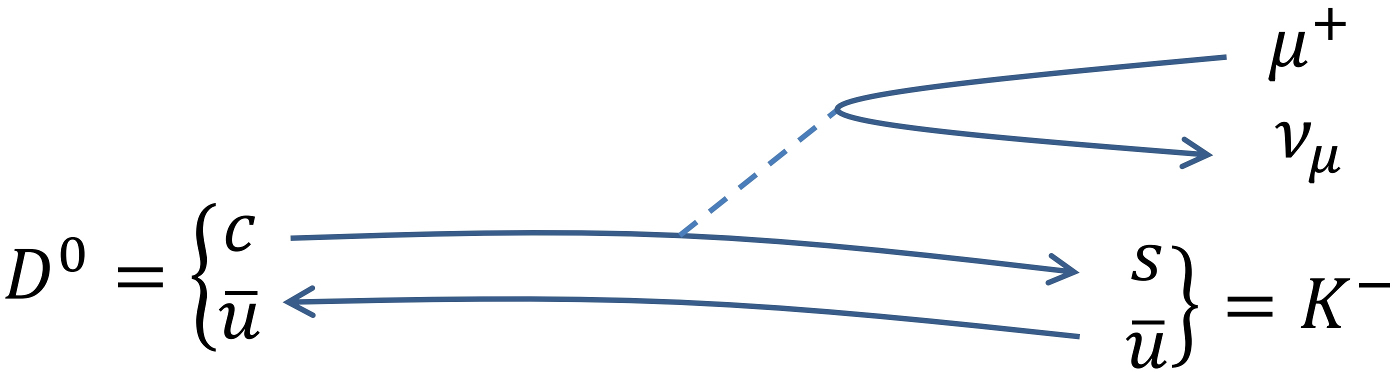
$$\Lambda_b^0 = (udb)$$

$$\Sigma_b^+ = (uub) \qquad \Sigma_b^0 = (udb) \qquad \Sigma_b^- = (ddb)$$

$$\Xi_b^0 = (usb) \qquad \Xi_b^- = (dsb)$$

$$\Omega_b^- = (ssb)$$

Semileptonic Decays



- The $\bar{u}$ quark is called the "spectator" quark because it doesn't directly participate in the decay.

D^0 DECAY MODES	Fraction (Γ_i/Γ)
	Semileptonic modes
$K^- e^+ \nu_e$	(3.530 ± 0.028) %
$K^- \mu^+ \nu_\mu$	(3.31 ± 0.13) %
$K^*(892)^- e^+ \nu_e$	(2.15 ± 0.16) %
$K^*(892)^- \mu^+ \nu_\mu$	(1.86 ± 0.24) %

Semileptonic Decays

B^+ DECAY MODES		Fraction (Γ_i/Γ)
		Semileptonic and leptonic modes
$\ell^+ \nu_\ell$ anything	[sss]	(10.99 $\pm$ 0.28) %
$\overline{D}^0 \ell^+ \nu_\ell$	[sss]	(2.20 $\pm$ 0.10) %
$\overline{D}^*(2007)^0 \ell^+ \nu_\ell$	[sss]	(4.88 $\pm$ 0.10) %

B^0 DECAY MODES		Fraction (Γ_i/Γ)
$\ell^+ \nu_\ell$ anything	[sss]	(10.33 $\pm$ 0.28) %
$D^- \ell^+ \nu_\ell$	[sss]	(2.20 $\pm$ 0.10) %
$D^*(2010)^- \ell^+ \nu_\ell$	[sss]	(4.88 $\pm$ 0.10) %

- The weak interaction couples with equal strength to final states containing μ or e.
- This effect is called “lepton universality”.

Semileptonic Decays

- Lifetimes of charm and bottom hadrons:

$$\tau(D^0) = 0.410\ \text{ps}$$

$$\tau(B^0) = 1.520\ \text{ps}$$

- What suppresses the b-quark decay rate?
- Weak interactions seem to prefer to couple within the same family.

$$c \leftrightarrow s$$

- But the b-quark can't decay to the more massive top quark, so it must switch families.

Weak Decays of Bottom Quarks

B^0 DECAY MODES		Fraction (Γ_i/Γ)
$D^- \ell^+ \nu_\ell$	[sss]	(2.20 ± 0.10) %
$D^*(2010)^- \ell^+ \nu_\ell$	[sss]	(4.88 ± 0.10) %
$\pi^- \ell^+ \nu_\ell$	[sss]	$(1.50 \pm 0.06) \times 10^{-4}$
$\rho^- \ell^+ \nu_\ell$	[sss]	$(2.94 \pm 0.21) \times 10^{-4}$

- Transitions from the 3rd to the 1st family are suppressed by a factor of about 150.
- Again, the weak eigenstates are not the same as the strong eigenstates.
- However, they are related by a unitary transformation.

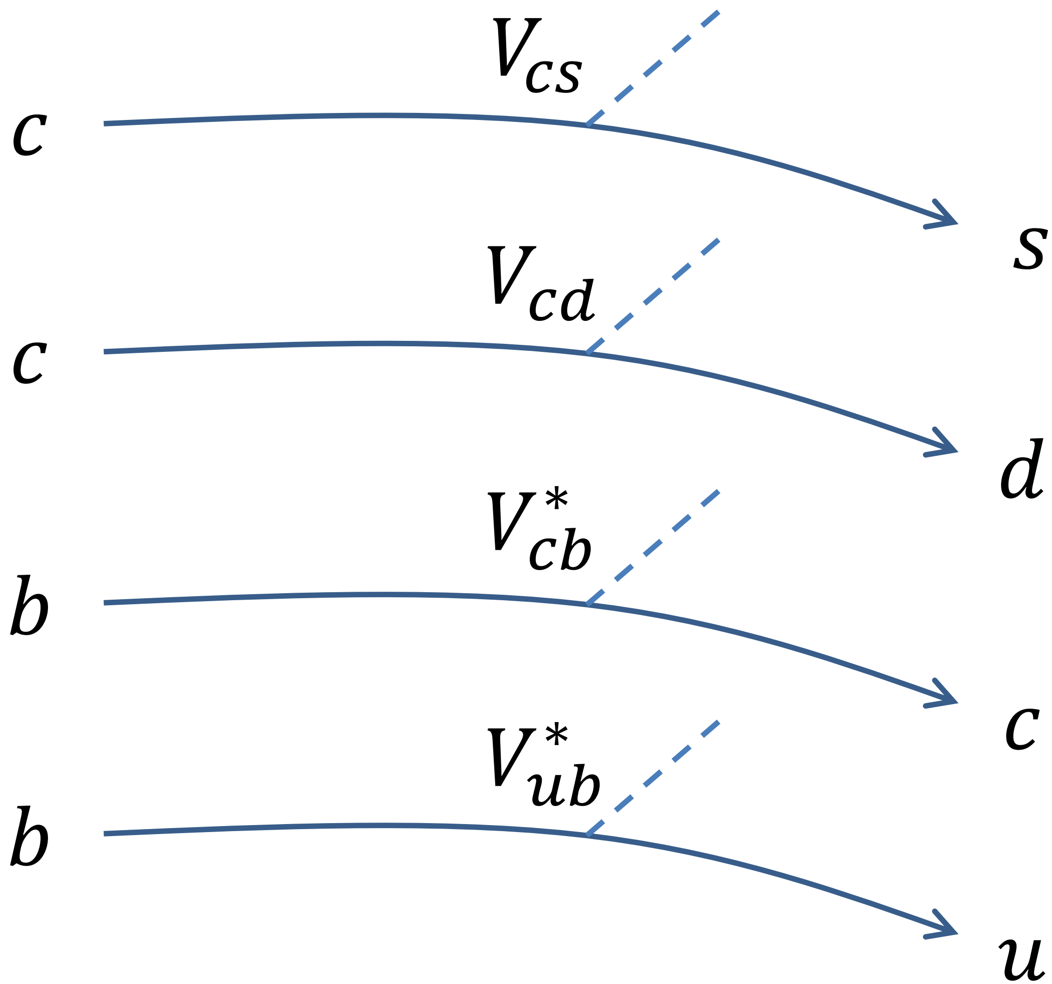
The CKM Matrix

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

- The matrix is unitary and can be described by four real numbers (three angles and one phase).
- One convenient parameterization is to expand it in powers of $\lambda = \sin\theta_C$ (Wolfenstein parameterization):

$$V_{CKM} = \begin{pmatrix} 1 - \lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 - \lambda^2/2 & A\lambda^2 \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1 \end{pmatrix}$$

Weak Decays



Cabibbo favored
$|V_{cs}| = 0.973$

Cabibbo suppressed
$|V_{cd}| = 0.225$

Cabibbo suppressed
$|V_{cb}| = 0.041$

Doubly Cabibbo suppressed
$|V_{cb}| = 0.0035$

B Mixing

- The neutral B mesons are similar to the neutral K mesons: their mass eigenstates have definite CP
- B mesons have many possible final states, so unlike the neutral kaon system, their lifetimes are almost identical.
- The B^0 and $\bar{B}^0$ strong eigenstates will oscillate.
- CP eigenstates:

$$B_L = \frac{1}{\sqrt{2}}(B^0 + \bar{B}^0)$$
$$B_H = \frac{1}{\sqrt{2}}(B^0 - \bar{B}^0)$$

B Mixing

- Time dependence of an initially pure B^0 state:

$$|\Psi(t)\rangle = \frac{1}{2}\left(e^{-im_H t-\Gamma t/2} + e^{-im_L t-\Gamma t/2}\right)B^0 + \frac{1}{2}\left(e^{-im_H t-\Gamma t/2} - e^{-im_L t-\Gamma t/2}\right)\bar{B}^0$$

- What is the probability of finding the B-meson in a B^0 state at time t?

$$P(B^0, t|B^0) = |\langle B^0|\Psi(t)\rangle|^2 = \frac{e^{-\Gamma t}}{2}(1+\cos\Delta m t)$$

- What is the probability of finding the B-meson in a $\bar{B}^0$ state at time t?

$$P(\bar{B}^0, t|B^0) = |\langle \bar{B}^0|\Psi(t)\rangle|^2 = \frac{e^{-\Gamma t}}{2}(1-\cos\Delta m t)$$

Observing B Mixing

- Bottom quarks are produced in $b\bar{b}$ pairs by the strong interaction.
- When the $\bar{b}$-quark forms a B^0 meson, it will oscillate $B^0 \leftrightarrow \bar{B}^0$ with frequency Δm.
- The b-quark can form B^-, $\bar{B}^0$, and also $\bar{B}_s^0$ or Λ_b if there is sufficient energy.
- If it forms a $\bar{B}^0$ or $\bar{B}_s^0$ then they will also oscillate with frequency Δm or Δm_s.
- If it forms a B^- or Λ_b then it will not oscillate and will always decay in a b-quark state.

Observing B Mixing

- First, produce $b\bar{b}$, for example via $e^+e^- \to b\bar{b}$ at high energy (eg, LEP).
- Leptons from semi-leptonic B-decays can be distinguished from other backgrounds
  - They have higher momentum because the b-quarks are highly boosted
  - They can have large p_T^{rel} (momentum transverse to the b-quark direction) because $m_b \gg m_c$
  - They are emitted close to the production point (because b-hadrons have relatively short lifetimes)
- Count events with same/opposite sign leptons: N_{++}/N_{+-}.
- If there were no mixing then $N_{++} = 0$.
- The observed ratio depends on Δm
  - There are several other ways to measure Δm

Dynamics of Oscillations

$$\begin{pmatrix} B^0(t) \\ \bar{B}^0(t) \end{pmatrix} = e^{-iHt} \begin{pmatrix} B^0 \\ \bar{B}^0 \end{pmatrix}$$

$$H \begin{pmatrix} B^0 \\ \bar{B}^0 \end{pmatrix} = \begin{pmatrix} M & M_{12} \\ M_{12} & M \end{pmatrix} \begin{pmatrix} B^0 \\ \bar{B}^0 \end{pmatrix}$$

- The physical masses are eigenvalues of the Hamiltonian

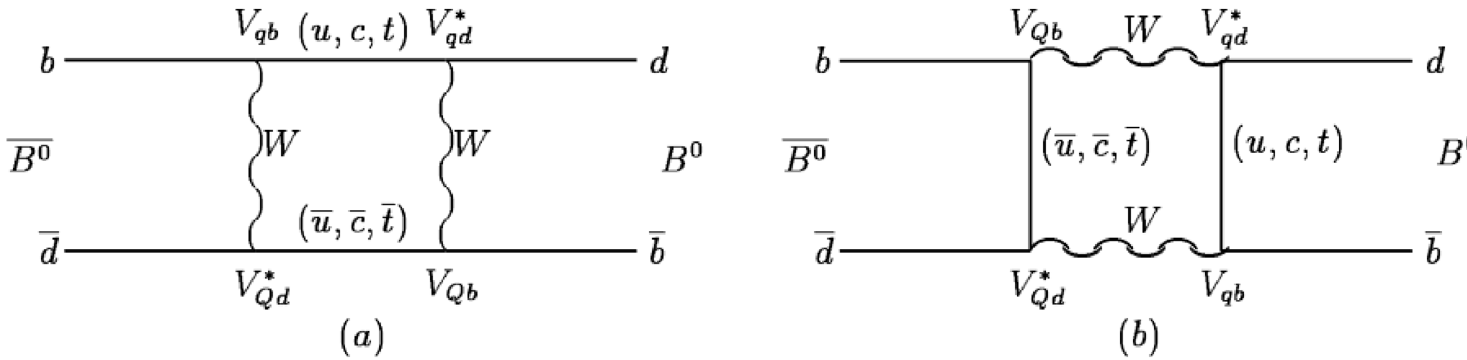
$$m_H = M + M_{12}$$
$$m_L = M - M_{12}$$

- Eigenvectors:

$$B_L = \frac{1}{\sqrt{2}}(B^0 + \bar{B}^0)$$
$$B_H = \frac{1}{\sqrt{2}}(B^0 - \bar{B}^0)$$

- What terms in the Hamiltonian lead to $B^0 \leftrightarrow \bar{B}^0$ couplings?

Dynamics of Oscillations



$$M_{12} \sim S_0\left(\frac{m_t^2}{M_W^2}\right)\left|V_{tq}^* V_{tb}\right|^2$$

$$S_0(x_t) \approx 0.784\, x_t^{0.76}$$

- The top quark dominates because it is much more massive than the c and u quarks
- Early measurement of Δm provided evidence that the top quark was *very* massive

Top Quark Production

- If the top quark was "light" then it could form a narrow $t\bar{t}$ bound state, just like the J/ψ and Υ.
- e^+e^- colliders that searched for top:
  - PETRA collider at DESY ($E_{cm} = 19\ \mathrm{GeV}$)
  - PEP collider at SLAC ($E_{cm} = 29\ \mathrm{GeV}$)
  - TRISTAN collider at KEK ($E_{cm} = 60\ \mathrm{GeV}$)
  - LEP and SLC colliders ($E_{cm} = 90\ \mathrm{GeV}$)
- If the top quark is more massive than the W-boson, then it would decay almost exclusively via

$$t \rightarrow W^+ b$$

followed by $W^+ \rightarrow \ell^+ \nu_\ell, q\bar{q}'$

Top Quark Decays

- The top quark decays almost exclusively to b because $V_{tb} \approx 1$.
- The signature for $t\bar{t}$ production at a high-energy hadron collider is then:
  - Evidence for two bottom quarks (semileptonic decays, displaced decay vertices)
  - Two leptons with opposite charge, possibly with opposite flavor
  - Large missing transverse energy (from the neutrinos)
- Observation of top quark production in $p\bar{p}$ collisions was announced in 1995

$$m_t = 173.0 \pm 0.4 \text{ GeV}$$

- The top quark decays before it can form a hadronic bound state.
  - No mesons or baryons containing top quarks

Three Families of Matter

$$\begin{pmatrix} u \\ d \end{pmatrix} \quad \begin{pmatrix} c \\ s \end{pmatrix} \quad \begin{pmatrix} t \\ b \end{pmatrix}$$

$$\begin{pmatrix} \nu_e \\ e \end{pmatrix} \quad \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix} \quad \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}$$

- We have seen that there are three types of interactions:
  - Electromagnetic (couples between photons and electric charges)
  - Strong interaction (couples only to quarks)
  - Weak interactions (prefers to couple within each family)
- Next, we will use relativistic quantum mechanics to calculate cross sections and decay rates
- Start with Quantum Electrodynamics
- The strong and weak interactions are similar