PURDUE UNIVERSITY DEPARTMENT OF PHYSICS

Physics 56400

Introduction to Elementary Particle Physics I

Lecture 11

Fall 2019 Semester

Prof. Matthew Jones

Hadronic Isospin Multiplets

- Nucleons: $\begin{pmatrix} p \\ n \end{pmatrix}$
- Pions: $\begin{pmatrix} \pi^+ \\ \pi^0 \\ \pi^- \end{pmatrix}$
- Delta resonances: $\begin{pmatrix} \Delta^{++} \\ \Delta^+ \\ \Delta^0 \\ \Delta^- \end{pmatrix}$

Baryon Antiparticles

- Baryons have baryon number B=+1
- Anti-baryons will have baryon number B=-1
- Baryon number is conserved in strong interactions
- Production of anti-protons must proceed as follow:

$$p + p \rightarrow p + p + p + \bar{p}$$

or

$$p + n \rightarrow p + n + p + \bar{p}$$

- What is the minimum beam energy needed to make anti-protons in a fixed target experiment?

Baryon Antiparticles

Beam proton: m $p_b = (E_b, \vec{p}_b)$

Target proton: $p_t = (m_p, \vec{0})$

Final state: $p_f = (4E_p, 4\vec{p}_p)$

$$|\vec{p}_p| = \frac{|\vec{p}_b|}{4} = \frac{\sqrt{E_b^2 - m_p^2}}{4}$$

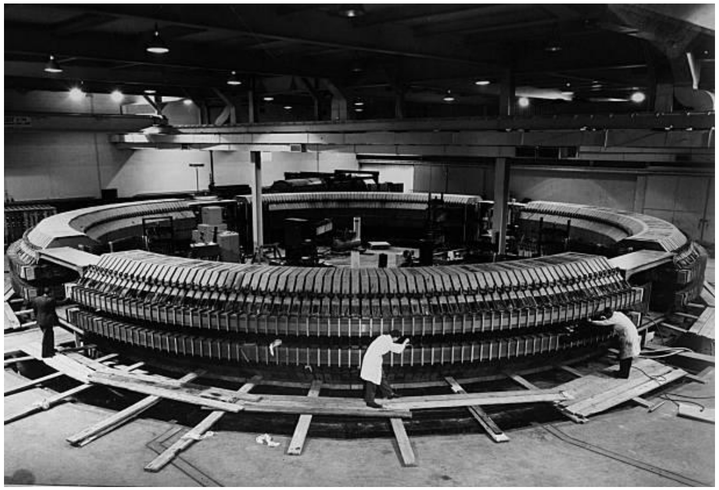
$$E_p = \frac{E_b + m_p}{4} = \sqrt{|\vec{p}_p|^2 + m_p^2} = \sqrt{\frac{E_b^2 - m_p^2}{16} + m_p^2}$$

$$\frac{E_b^2 + 2E_b m_p + m_p^2}{16} = \frac{E_b^2 + 15m_p^2}{16}$$

$$E_b = 7m_p = 6.57\ \text{GeV}$$

$T_b = 6m_p = 5.63\ \text{GeV}$ (kinetic energy of beam proton)

Anti-protons



Brookhaven Cosmotron (1953)

$E = 3\ \mathrm{GeV}$

Berkeley Bevatron (1954)

$E = 6\ \mathrm{GeV}$

Anti-proton discovered in 1955 by Chamberlain and Segré. Nobel prize in 1959.

Baryon Anti-particles

- Using the Dirac sea idea, an anti-proton is equivalent to the absence of a negative-energy proton.
- If a negative energy proton has isospin $I_3 = +1/2$ then its absence looks like $I_3 = -1/2$.
- Anti-protons: $\begin{pmatrix} \bar{n} \\ \bar{p} \end{pmatrix} = \begin{pmatrix} 1/2 \\ -1/2 \end{pmatrix}$
- Anti-deltas: $\begin{pmatrix} \bar{\Delta}^{+} \\ \bar{\Delta}^{0} \\ \bar{\Delta}^{-} \\ \bar{\Delta}^{--} \end{pmatrix} = \begin{pmatrix} 3/2 \\ 1/2 \\ -1/2 \\ -3/2 \end{pmatrix}$
- Anti-baryons also have odd-parity.
- Pions:
  - π^+ and π^- are anti-particles,
  - π^0 is its own anti-particle.

Vector Mesons

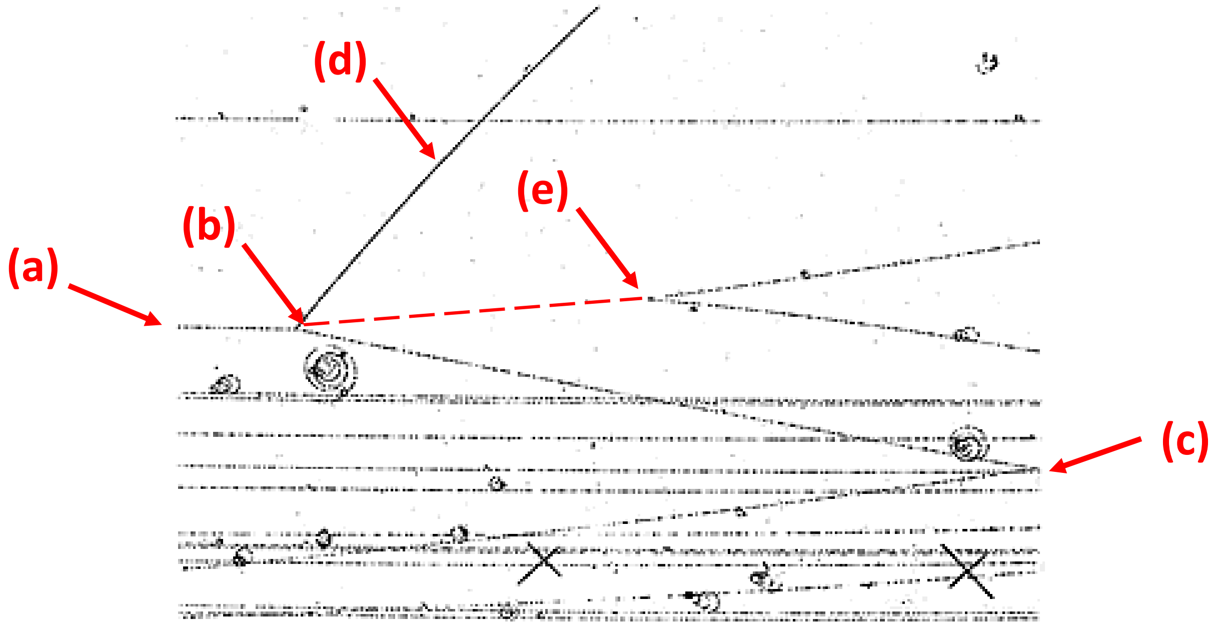
- e^+e^- collisions can produce states of spin-1:
- The $\rho(770)$ multiplet contains three charge states:

$$\begin{pmatrix} \rho^+ \\ \rho^0 \\ \rho^- \end{pmatrix}$$

- They form an isospin triplet (I=1).
- They have odd parity.
- They decay to pairs of pions:

$$\rho^+ \to \pi^+\pi^0$$
$$\rho^0 \to \pi^+\pi^-$$

Strange Mesons



a) Incident proton
b) Interaction with target nucleus
c) Outgoing proton
d) Outgoing charged particle
e) Decay of a long-lived neutral particle, $V^0 \rightarrow \pi^+\pi^-$

Strange Mesons

- First observed in high-altitude cloud chamber experiments
- Strange mesons were easily produced in strong interactions, but only in pairs.
- Strange mesons had long lifetimes so they must decay weakly even though there were no leptons in the final state.
- Proposal:
  - These particles carry a new quantum number (strangeness)
  - Strangeness is conserved in strong interactions
  - Strange particles are produced in particle/anti-particle pairs to conserve strangeness
  - Strange particles cannot decay strongly
  - The lightest positively charged strange meson assigned $S=+1$

Strange Hadrons

- Types of strange particles:
  - Neutral mesons: $V^0 \to \pi^+\pi^-$
  - Charged mesons: $\tau^+ \to \pi^+\pi^-\pi^+$ Parity -1

    $\theta^+ \to \pi^+\pi^0$ Parity +1
  - Neutral baryons: $\Lambda \to p\pi^-$
- The τ-θ puzzle:
  - The τ^+ and θ^+ had exactly the same charge, mass, lifetime (they seemed to be the same particle)
  - They decayed to final states with different parity (they seemed to have opposite quantum numbers)
  - Parity must not necessarily be conserved in weak decays
- Both are now know as the charged kaon, K^+

Strange Hadrons

- Charged kaons: K^+ and K^-, $m_{K^+} = 493.7\ \text{MeV}$
- Neutral kaons: K^0 and $\overline{K}^0$, $m_{K^0} = 497.6\ \text{MeV}$
- These seemed to form two isospin doublets:

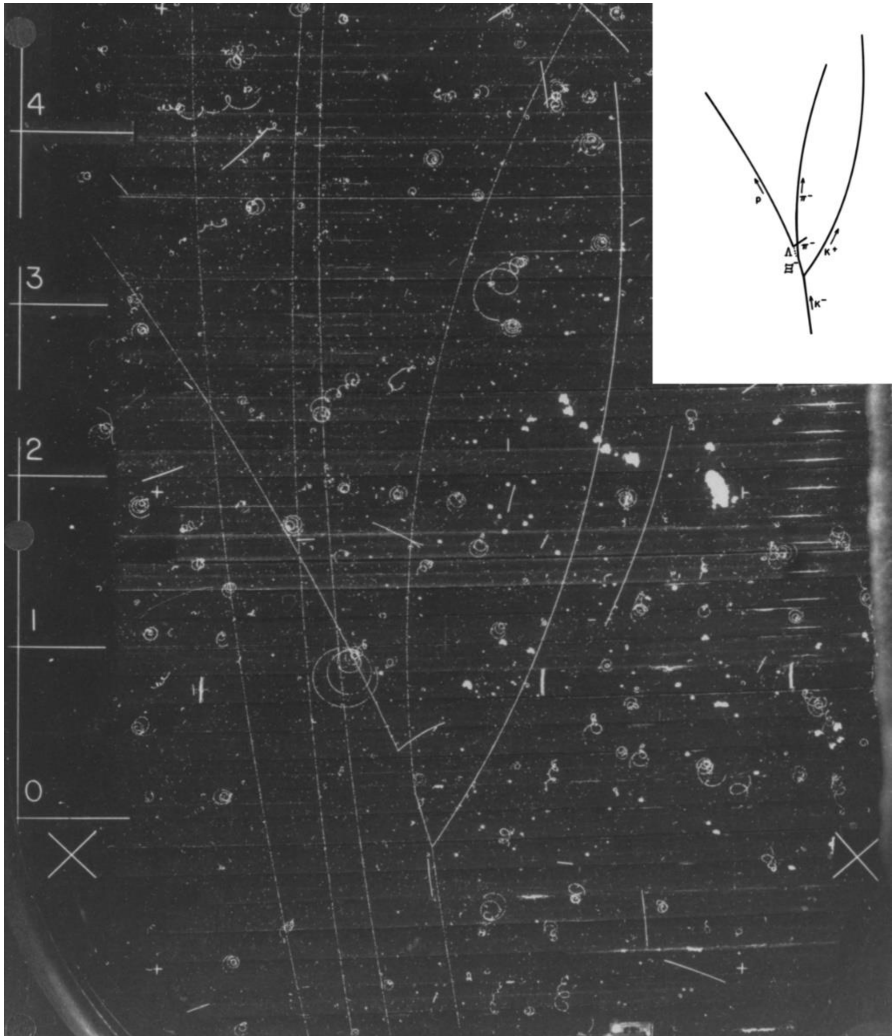
$$\begin{pmatrix} K^+ \\ K^0 \end{pmatrix} \qquad \begin{pmatrix} \overline{K}^0 \\ K^- \end{pmatrix}$$

- The neutral Λ is an isospin singlet, $m_\Lambda = 1116\ \text{MeV}$
- A triplet of strange baryons was observed:

  $\Sigma^+, \Sigma^0, \Sigma^-$ with similar mass, $m_\Sigma \sim 1193\ \text{MeV}$
- Decays:
  - $\Sigma^+ \to p\pi^0, n\pi^+$ with lifetime 80 ns
  - $\Sigma^- \to n\pi^-$ with lifetime 148 ns

  (the two decays above:) Not particle/anti-particle!
  - $\Sigma^0 \to \Lambda\gamma$ with lifetime 10^{-19} seconds (electromagnetic decay conserves strangeness)

Doubly Strange Baryons



Production and decay of a xi minus

$$K^- p \rightarrow K^+ \boldsymbol{\Xi}^-$$
$$\boldsymbol{\Xi}^- \rightarrow \Lambda \pi^-$$
$$\Lambda \rightarrow p \pi^-$$

The Ξ^- has S=-2 and decays weakly.

The Ξ is sometimes called a “cascade”.

Strange Resonances

- Strange vector mesons: $K^*(892)$

$$K^{*+} \to K^+\pi^0, K^0\pi^+$$
$$K^{*0} \to K^+\pi^-, K^0\pi^0$$

- They also (rarely) decay electromagnetically:

$$K^* \to K\gamma \;\; (BF \sim 10^{-3})$$

- Vector mesons with no strangeness:

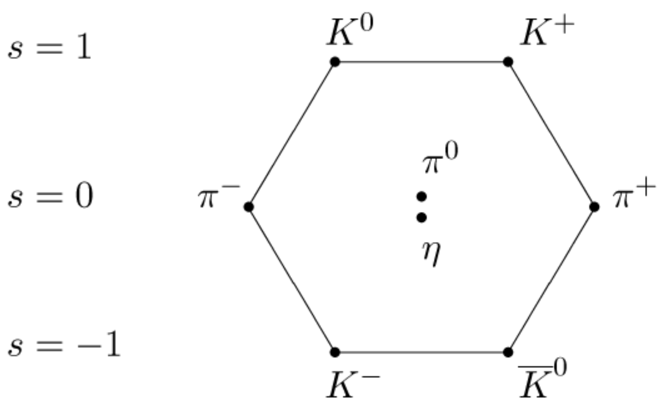
$$\phi(1020) \to K^+K^-, K^0\bar{K}^0$$
$$\omega(782) \to \pi^+\pi^-\pi^0, \pi^0\gamma$$

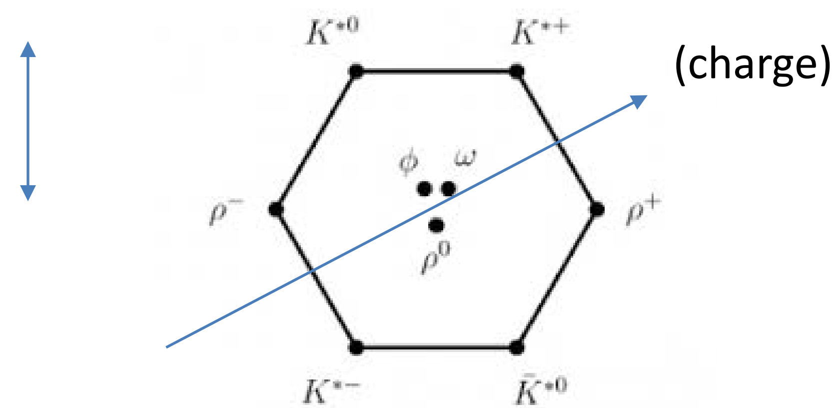
- Strange baryon resonances:

$$\Sigma^{*-}, \Sigma^{*0}, \Sigma^{*+}$$

Meson Multiplets

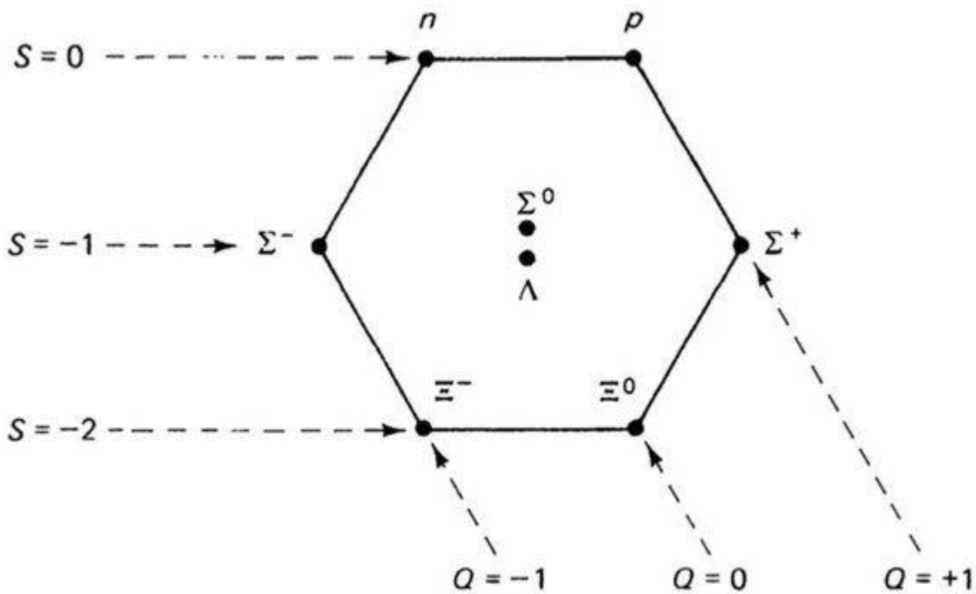
- Plot I_3 on the x-axis and strangeness on the y-axis:

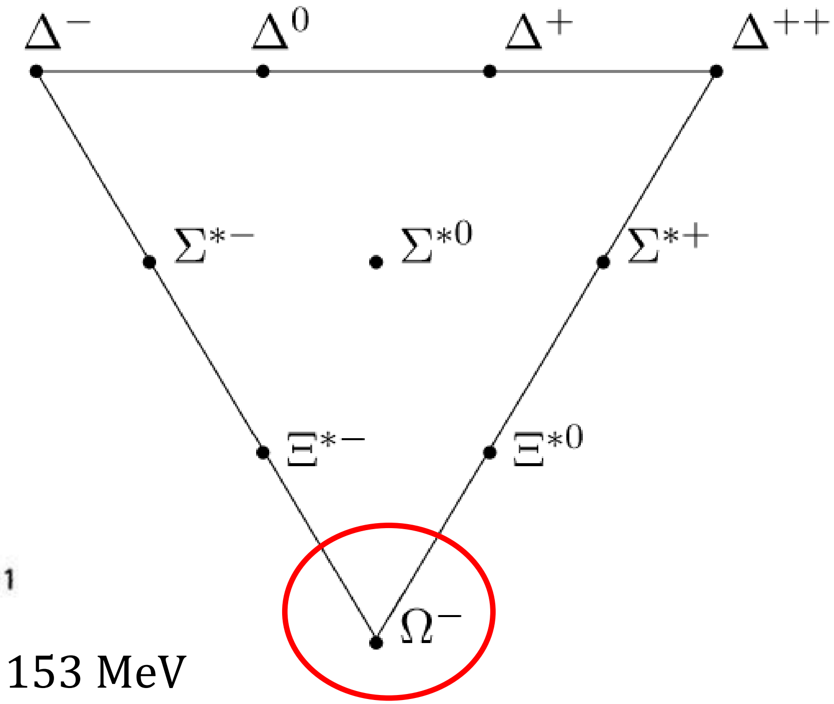




Baryon Multiplets

- Do the same for the baryons:





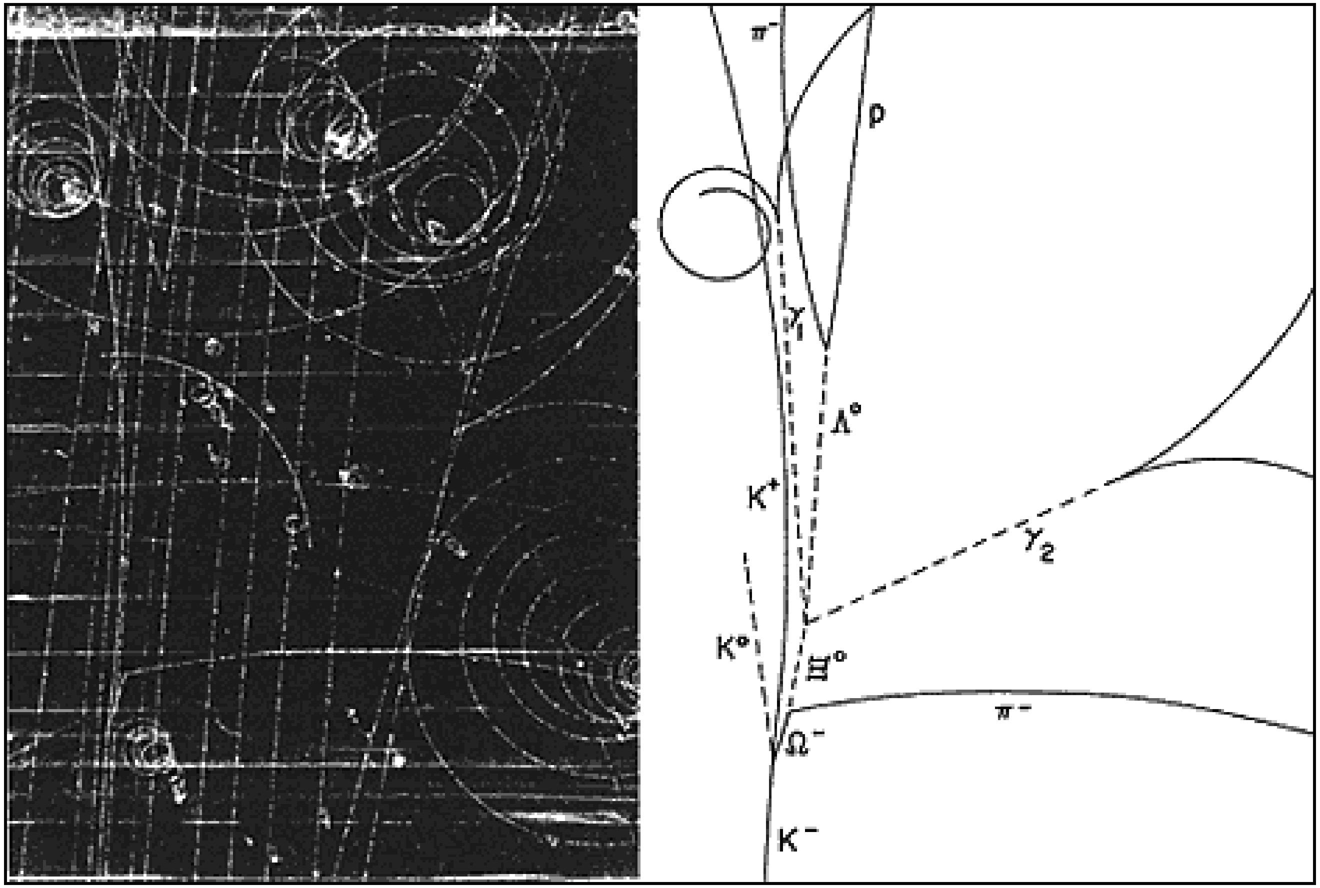
$$m(\Sigma^*) - m(\Delta) = 153 \text{ MeV}$$
$$m(\Xi^*) - m(\Sigma^*) = 145 \text{ MeV}$$

Therefore, the Ω^- mass is estimated to be about

$$m(\Omega^-) = m(\Sigma^*) + 150 \text{ MeV} = 1530 \text{ MeV} + 150 \text{ MeV} = \mathbf{1680\ MeV}$$

Predicted independently by Gell-Mann and Ne'eman in 1961.

The Omega Baryon



Discovered in 1964 using an 80-inch bubble chamber at the 33 GeV Brookhaven AGS.

$$m(\Omega^-) = 1672 \text{ MeV}$$

S=-3

Interpreting Hadron Multiplets

- Group theoretic approach (Gell-Mann):
$$\mathbf{3} \otimes \overline{\mathbf{3}} = \mathbf{8} \oplus \mathbf{1}$$
$$\mathbf{3} \otimes \mathbf{3} \otimes \mathbf{3} = \mathbf{10} \oplus \mathbf{8} \oplus \mathbf{8} \oplus \mathbf{1}$$
- Constituent particle approach (Zweig):
  - Three types of quarks: u, d, s
  - Mesons are made of quark/anti-quark pairs
  - Baryons are made of three quarks
  - Quarks have charge $q_u = +2/3, q_d = q_s = -1/3$.
  - Quarks have spin 1/2

Quark Composition of the Mesons

- Pseudoscalar mesons are $|q \uparrow \overline{q'} \downarrow\rangle$ states
- Vector mesons are $|q \uparrow \overline{q'} \uparrow\rangle$ states

Meson	Quarks
π^+	$(u\bar{d})$
π^0	$(u\bar{u})$ or $(d\bar{d})$
π^-	$(d\bar{u})$
K^+	$(u\bar{s})$
K^0	$(d\bar{s})$
$\bar{K}^0$	$(s\bar{d})$
K^-	$(s\bar{u})$
η	?
η'	?

Meson	Quarks
ρ^+	$(u\bar{d})$
ρ^0	$(u\bar{u})$ or $(d\bar{d})$
ρ^-	$(d\bar{u})$
K^{*+}	$(u\bar{s})$
K^{*0}	$(d\bar{s})$
$\bar{K}^{*0}$	$(s\bar{d})$
K^{*-}	$(s\bar{u})$
ω	?
ϕ	?

Quark Composition of the Baryons

- Spin $^{1}/_{2}$ baryons are combinations with $|\uparrow\uparrow\downarrow\rangle$
- Spin $^{3}/_{2}$ baryons have $|\uparrow\uparrow\uparrow\rangle$

J=1/2 Baryon	Quarks
p	(uud)
n	(udd)
Σ^{+}	(uus)
Σ^{0}	(uds)
Σ^{-}	(dds)
Ξ^{0}	(uss)
Ξ^{-}	(dss)
Λ	?

J=3/2 Baryon	Quarks
Δ^{++}	(uuu)
Δ^{+}	(uud)
Δ^{0}	(udd)
Δ^{-}	(ddd)
Σ^{*+}	(uus)
Σ^{*0}	(uds)
Σ^{*-}	(dds)
Ξ^{*0}	(uss)
Ξ^{*-}	(dss)
Ω^{-}	(sss)

The Quark Model

- We can start to think of mesons as bound states of quarks (like the Hydrogen atom)
- Spins that are $\uparrow\uparrow$ are excited states of $\uparrow\downarrow$
- The potential energy function must be of the form

$$V = V(r) + \kappa \frac{\vec{s}_1 \cdot \vec{s}_2}{m_1 m_2}$$

- If the strong interaction doesn't care about quark flavor, then all mesons have the same ground-state energy.
- Empirical mass formula:

$$m = m_1 + m_2 + \kappa \frac{\vec{s}_1 \cdot \vec{s}_2}{m_1 m_2}$$

- What are the expectation values of the $\vec{s}_1 \cdot \vec{s}_2$ operator?

The Quark Model

Recall that $\vec{S} = \vec{s}_1 + \vec{s}_2$ and that $\left|\vec{S}\right|^2 = S(S+1)$

$$\left|\vec{S}\right|^2 = (\vec{s}_1 + \vec{s}_2) \cdot (\vec{s}_1 + \vec{s}_2)$$

$$= |\vec{s}_1|^2 + |\vec{s}_2|^2 + 2\,\vec{s}_1 \cdot \vec{s}_2$$

$$\vec{s}_1 \cdot \vec{s}_2 = \frac{1}{2}\left(\left|\vec{S}\right|^2 - |\vec{s}_1|^2 - |\vec{s}_2|^2\right)$$

$$= \frac{1}{2}\left(S(S+1) - \frac{3}{4} - \frac{3}{4}\right)$$

$$\vec{s}_1 \cdot \vec{s}_2 = \begin{cases} -3/4 & \text{when } S = 0 \\ +1/4 & \text{when } S = 1 \end{cases}$$

(pseudoscalar)
(vector)

Quark Model Wave Functions

- The neutral mesons are orthogonal linear combinations of $(u\bar{u})$, $(d\bar{d})$ and $(s\bar{s})$ states.

$$\pi^0 = \frac{1}{\sqrt{2}}\left(u\bar{u} - d\bar{d}\right)$$

$$\eta \approx \frac{1}{\sqrt{6}}\left(u\bar{u} + d\bar{d} - 2s\bar{s}\right)$$

$$\eta' \approx \frac{1}{\sqrt{3}}\left(u\bar{u} + d\bar{d} + s\bar{s}\right)$$

- Vector mesons:

$$\rho^0 = \frac{1}{\sqrt{2}}\left(u\bar{u} - d\bar{d}\right)$$

$$\omega \approx \frac{1}{\sqrt{2}}\left(u\bar{u} + d\bar{d}\right)$$

$$\phi \approx s\bar{s}$$

Color Quantum Numbers

- Pauli's exclusion principle states that identical Fermions can't occupy the same state.
- There is apparently a problem with the spin 3/2 baryons:

$$\Delta^{++} = u\uparrow u\uparrow u\uparrow$$
$$\Delta^{-} = d\uparrow d\uparrow d\uparrow$$
$$\Omega^{-} = s\uparrow s\uparrow s\uparrow$$

- Also, the exchange of identical fermions must be anti-symmetric.
- It was proposed that quarks must carry an additional quantum number that we now call "color".
- The "color" part of the wave function is completely anti-symmetric.
- Physical hadrons are color singlets – they have no net color charge.

Color Wave Functions

- Mesons always have the color symmetric wave function:

$$\psi_c = \frac{1}{\sqrt{3}}(R\bar{R} + G\bar{G} + B\bar{B})$$

- Baryons always have the color anti-symmetric wave function:

$$\psi_c = \frac{1}{\sqrt{6}}(RGB - RBG + GBR - GRB + BRG - BGR)$$

- These factor from the rest of the hadron spin-flavor wave function.
- The baryon spin-flavor wave function must be symmetric.
- Color eventually became a central concept in the description of the strong interaction.

Orbital Angular Momentum

- Bound states of quarks behave sort of like hydrogen atoms except
  - The masses of the constituents are similar
  - The quarks are light enough that they are relativistic
- Nevertheless, excited hadrons can be associated with the same quantum numbers used to describe the hydrogen atom
- Spectroscopic notation:

$$n^{2s+1}L_j$$

- Orbital quantum numbers are written S, P, D, F for L=0,1,2,3,…
- Many excited mesons have been assigned such quantum numbers…

Excited Meson States

Table 14.2: Suggested $q\bar{q}$ quark-model assignments for some of the observed light mesons. Mesons in bold face are included in the Meson Summary Table. The wave functions f and f' are given in the text. The singlet-octet mixing angles from the quadratic and linear mass formulae are also given for the well established nonets. The classification of the 0^{++} mesons is tentative and the mixing angle uncertain due to large uncertainties in some of the masses. Also, the $f_0(1710)$ and $f_0(1370)$ are expected to mix with the $f_0(1500)$. The latter is not in this table as it is hard to accommodate in the scalar nonet. The light scalars $a_0(980)$, $f_0(980)$, and $f_0(600)$ are often considered as meson-meson resonances or four-quark states, and are therefore not included in the table. See the "Note on Scalar Mesons" in the Meson Listings for details and alternative schemes.

$n\,^{2s+1}\ell_J$	J^{PC}	I = 1 $u\bar{d}$, $\bar{u}d$, $\frac{1}{\sqrt{2}}(d\bar{d}-u\bar{u})$	I = $\frac{1}{2}$ $u\bar{s}$, $d\bar{s}$; $\bar{d}s$, $-\bar{u}s$	I = 0 f'	I = 0 f	θ_{quad} [°]	θ_{lin} [°]
$1\,^1S_0$	0^{-+}	$\boldsymbol{\pi}$	$\boldsymbol{K}$	$\boldsymbol{\eta}$	$\boldsymbol{\eta'(958)}$	−11.5	−24.6
$1\,^3S_1$	1^{--}	$\boldsymbol{\rho(770)}$	$\boldsymbol{K^*(892)}$	$\boldsymbol{\phi(1020)}$	$\boldsymbol{\omega(782)}$	38.7	36.0
$1\,^1P_1$	1^{+-}	$\boldsymbol{b_1(1235)}$	$\boldsymbol{K_{1B}}^\dagger$	$h_1(1380)$	$\boldsymbol{h_1(1170)}$		
$1\,^3P_0$	0^{++}	$\boldsymbol{a_0(1450)}$	$\boldsymbol{K_0^*(1430)}$	$\boldsymbol{f_0(1710)}$	$\boldsymbol{f_0(1370)}$		
$1\,^3P_1$	1^{++}	$\boldsymbol{a_1(1260)}$	$\boldsymbol{K_{1A}}^\dagger$	$\boldsymbol{f_1(1420)}$	$\boldsymbol{f_1(1285)}$		
$1\,^3P_2$	2^{++}	$\boldsymbol{a_2(1320)}$	$\boldsymbol{K_2^*(1430)}$	$\boldsymbol{f_2'(1525)}$	$\boldsymbol{f_2(1270)}$	29.6	28.0
$1\,^1D_2$	2^{-+}	$\boldsymbol{\pi_2(1670)}$	$\boldsymbol{K_2(1770)}^\dagger$	$\eta_2(1870)$	$\boldsymbol{\eta_2(1645)}$		
$1\,^3D_1$	1^{--}	$\boldsymbol{\rho(1700)}$	$\boldsymbol{K^*(1680)}$		$\boldsymbol{\omega(1650)}$		
$1\,^3D_2$	2^{--}		$\boldsymbol{K_2(1820)}$				
$1\,^3D_3$	3^{--}	$\boldsymbol{\rho_3(1690)}$	$\boldsymbol{K_3^*(1780)}$	$\boldsymbol{\phi_3(1850)}$	$\boldsymbol{\omega_3(1670)}$	32.0	31.0
$1\,^3F_4$	4^{++}	$\boldsymbol{a_4(2040)}$	$\boldsymbol{K_4^*(2045)}$		$\boldsymbol{f_4(2050)}$		

Strong Decays of Hadrons

- The "full width", Γ, is interpreted as a decay rate:

$$dN = -\Gamma N dt$$

$$N(t) = N(0)\, e^{-\Gamma t}$$

- The total decay rate is the sum of the decay rates to exclusive final states:

$$\Gamma_{total} = \sum_{i=1}^{n} \Gamma_i$$

- The branching fraction is the probability that a specific final state will be observed:

$$B_i = \frac{\Gamma_i}{\Gamma_{total}}$$

Strong Decays of Hadrons

- Not derived here:

statistical factor

$$d\Gamma = \left|\mathcal{M}_{fi}\right|^2 \frac{\mathrm{S}}{2\mathrm{M}} \left(\prod_{k=1}^{n} \frac{d^3\vec{p}_k}{(2\pi)^3 2E_k} \right) \times (2\pi)^4 \delta^4 \left(P - \sum_{k=1}^{n} p_k \right)$$

(reduced) matrix element squared

phase space of particles in the final state

4-momentum conserving delta-function

- The phase space is the density of allowed quantum mechanical states with a specific energy.

Strong Decays of Hadrons

- Two-body decays:

$$\Gamma = \frac{S|\vec{p}|}{8\pi M} \left|\mathcal{M}_{fi}\right|^2$$

- Strictly speaking though, because of Heisenberg's uncertainty principle, the energy (ie, mass) of the state might not be precisely known…

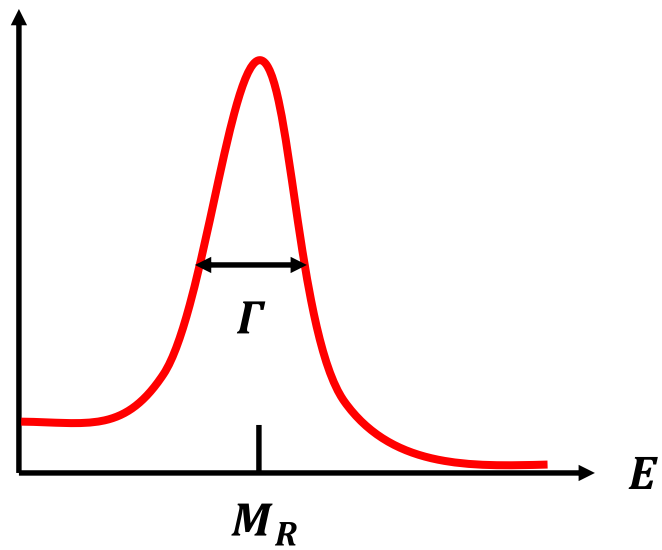
$$\Delta E \Delta t \sim \frac{\hbar}{2}$$

$$\Delta t = \frac{1}{\Gamma}$$

- In practice, we can replace the delta-function over energy with a Breit-Wigner function:

Strong Decays of Hadrons

$$\delta(E - M_R) \rightarrow \frac{1}{(E - M_R)^2 - \Gamma^2/4}$$



- The “mass” and “width” are parameters, determined from analyses of the resonance shape.

Strong Decays of Hadrons

- Strong decays will dominate unless they are forbidden by conservation laws
- Electromagnetic decays will also occur (unless they are forbidden) but with lower rates
- Weak decays generally dominate only when other decays are forbidden

Citation: M. Tanabashi *et al.* (Particle Data Group), Phys. Rev. D **98**, 030001 (2018)

$K^*(892)$ DECAY MODES

	Mode	Fraction (Γ_i/Γ)	Confidence level
Γ_1	$K\pi$	$\sim$ 100 %	
Γ_2	$(K\pi)^{\pm}$	(99.900 ± 0.009) %	
Γ_3	$(K\pi)^0$	(99.754 ± 0.021) %	
Γ_4	$K^0\gamma$	$(2.46 \pm 0.21) \times 10^{-3}$	
Γ_5	$K^{\pm}\gamma$	$(1.00 \pm 0.09) \times 10^{-3}$	
Γ_6	$K\pi\pi$	$< 7 \times 10^{-4}$	95%

$\Gamma \approx 46$ MeV

Quark Current Lines

- We can describe decay mechanisms by drawing the flow of quark currents
- These are not strictly speaking Feynman diagrams, but they borrow some of the concepts
  - Time flows from left to right
  - Anti-quark currents flow backwards

- Example: $K^{*0} \to K^+\pi^-$

$\pi^- = \begin{cases} d \\ \bar{u} \end{cases}$

$K^{*0} = \begin{cases} d \\ \bar{s} \end{cases}$

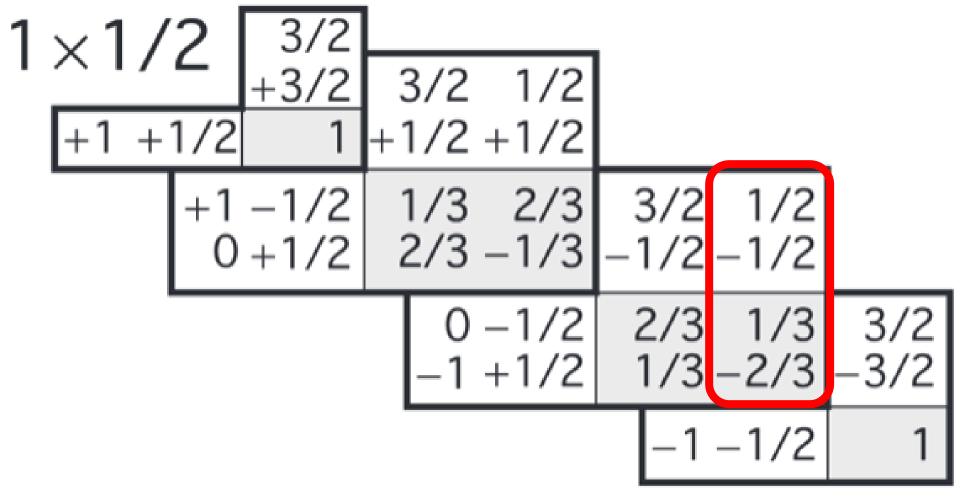
$p = 287 \text{ MeV}$

$K^+ = \begin{cases} u \\ \bar{s} \end{cases}$

  - Fractions of $K^+\pi^-$ and $K^0\pi^0$ are determined from isospin analysis…

Isospin Analysis of Strong Decays

$K^{*0} \rightarrow K^{+}\pi^{-}$ and $K^{*0} \rightarrow K^{0}\pi^{0}$



$$|K^{*0}\rangle = \sqrt{\frac{1}{3}}|K^{0}\pi^{0}\rangle - \sqrt{\frac{2}{3}}|K^{+}\pi^{-}\rangle$$

$$Br(K^{*0} \rightarrow K^{+}\pi^{-}) = 0.667$$

$$Br(K^{*0} \rightarrow K^{0}\pi^{0}) = 0.333$$

Electromagnetic Decays

- These are similar to transitions in hydrogen atoms
- Dipole transitions:

$$|\uparrow\uparrow\rangle \rightarrow \ |\uparrow\downarrow\rangle + \gamma$$

- The electromagnetic coupling constant (α) must be about 30 times smaller than the strong coupling constant (α_s).

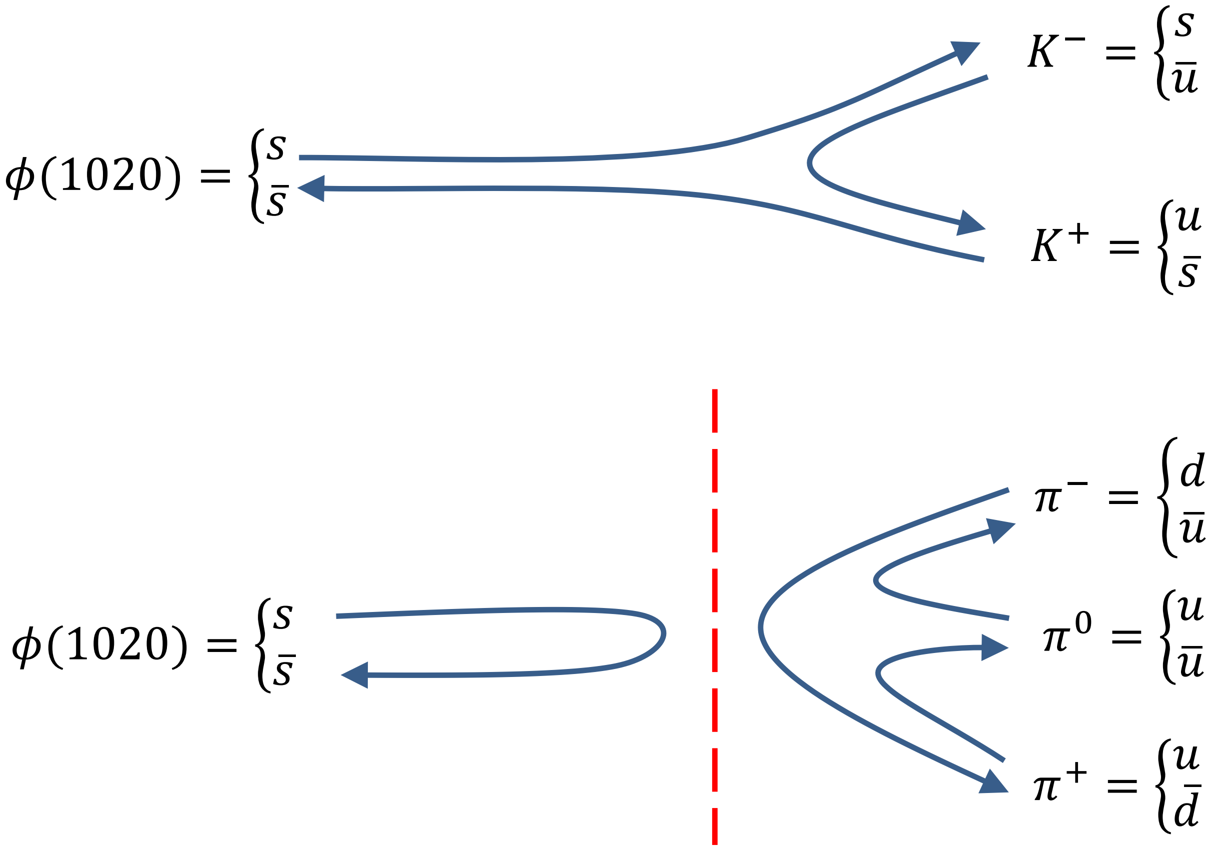
Zweig Suppression/OZI Rule

- Usually, decays that release a lot of kinetic energy (q^2) are enhanced because there are more quantum mechanical states with large energies than with small energies.
- But…

$\phi(1020)$ DECAY MODES

	Mode	Fraction (Γ_i/Γ)	Scale factor/ Confidence level
Γ_1	K^+K^-	(49.2 ± 0.5) %	S=1.3
Γ_2	$K^0_L K^0_S$	(34.0 ± 0.4) %	S=1.3
Γ_3	$\rho\pi + \pi^+\pi^-\pi^0$	(15.24 ± 0.33) %	S=1.2
Γ_4	$\rho\pi$		
Γ_5	$\pi^+\pi^-\pi^0$		
Γ_6	$\eta\gamma$	(1.303 ± 0.025) %	S=1.2
Γ_7	$\pi^0\gamma$	$(1.30 \pm 0.05) \times 10^{-3}$	
Γ_8	$\ell^+\ell^-$	—	

Zweig Suppression/OZI Rule



Allowed decay
$p = 127$ MeV

Suppressed decay
$p_{max} = 461$ MeV

$$\Gamma = 4.249 \text{ MeV}$$