Purdue University Department of Physics

Physics 56400

Introduction to Elementary Particle Physics I

Lecture 10

Fall 2019 Semester

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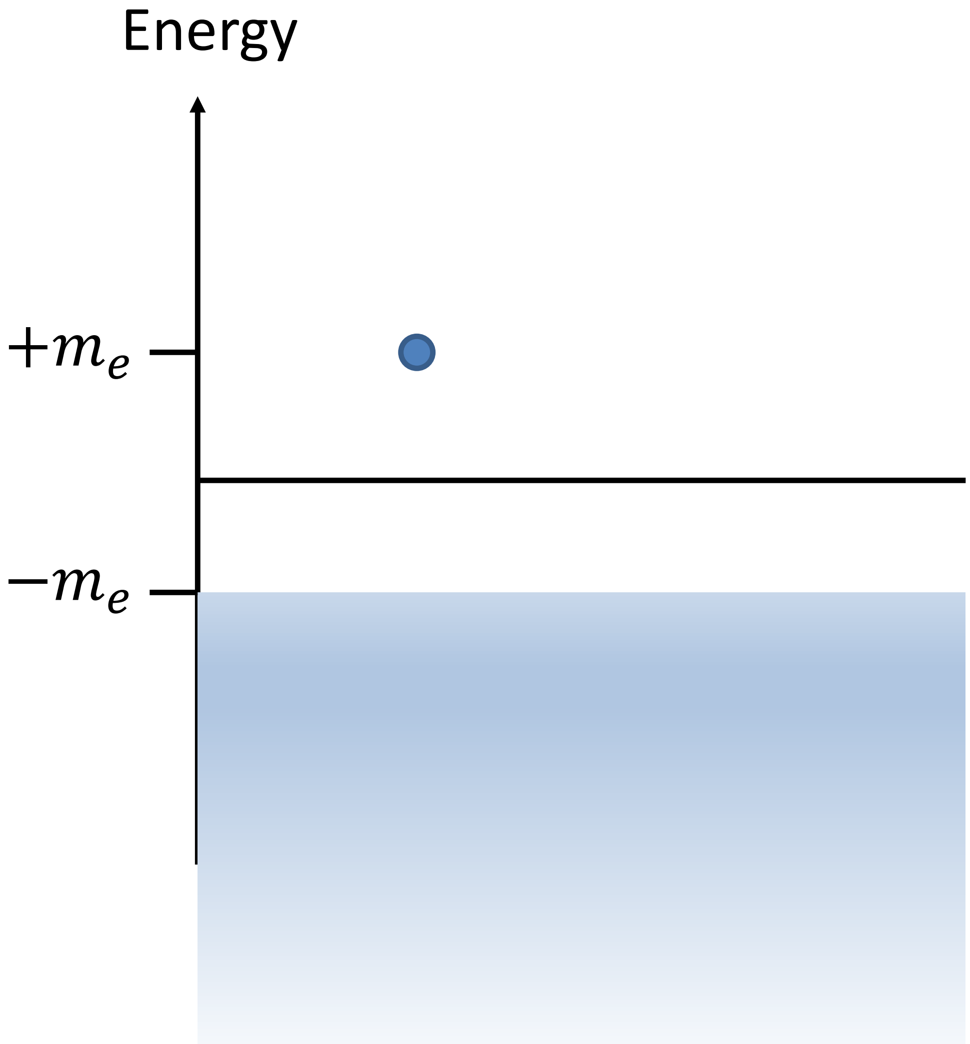
Elementary Particles

- Atomic physics:
  - Proton, neutron, electron, photon
- Nuclear physics:
  - Alpha, beta, gamma rays
- Cosmic rays:
  - Something charged (but what?)
- Relativistic quantum mechanics (Dirac, 1928)
  - Some solutions described electrons (with positive energy)
  - Other solutions described electrons with *negative energy*
  - Dirac came up with an elegant explanation for to think about this…

The Dirac Sea

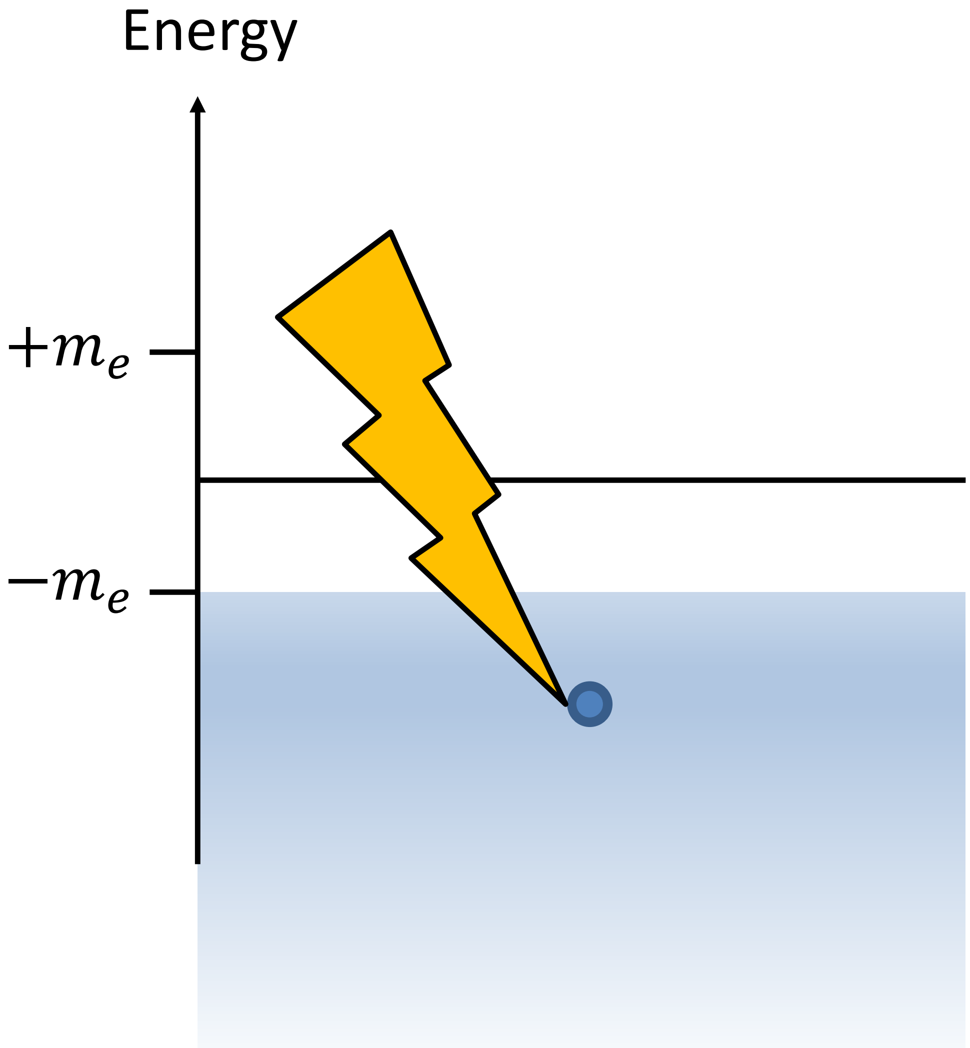
- Dirac proposed that all electrons have negative charge.
- Normal electrons have positive energy.
- All negative-energy states are populated and form the Dirac sea.
- The Pauli exclusion principle explains why positive-energy electrons can’t reach a lower energy state
  - those states are already populated
- The only way to observe an electron in the sea would be to give it a positive energy.

The Dirac Sea



Lowest positive-energy state corresponds to an electron at rest. It can't fall into the sea because those states are already filled.

The Dirac Sea

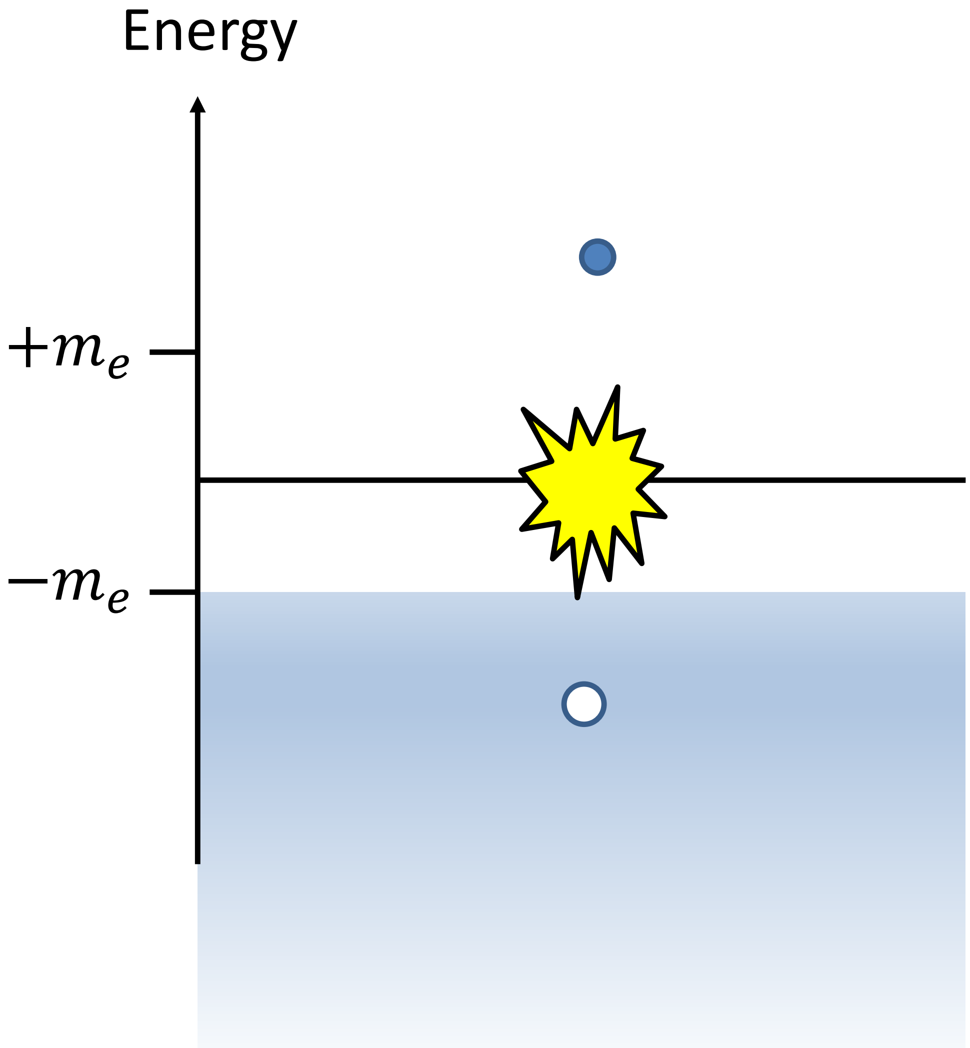


Now we have a positive energy electron, and a hole in the sea.

The absence of a negative charge looks like a positive charge.

This describes pair production.

The Dirac Sea

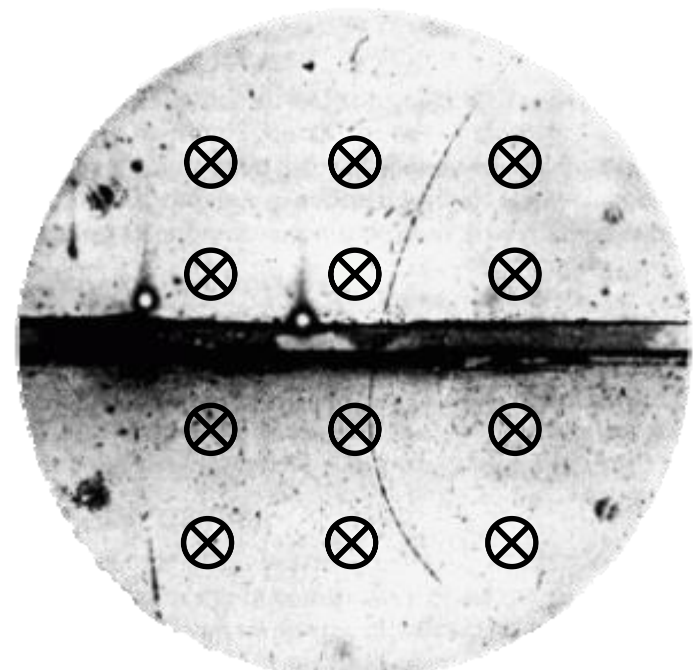


Now that there is a hole in the sea, a positive energy electron can fall into it, releasing energy.

This corresponds to electron-positron annihilation.

Positrons

- This explanation was not immediately interpreted as a prediction for a new particle
- However, Carl Anderson observed "positive electrons" in cosmic rays in 1933.
- Nobel prize in 1934.
- Anti-particles are a natural consequence of special relativity and quantum mechanics.



Nuclear Forces

- If all the positive charge of an atom is contained in the tiny nucleus, why doesn't electrostatic repulsion blow it apart?
- If there is another force that binds the nucleus together, why don't we observe it in macroscopic experiments?
- Yukawa proposed that it must be a short-range force.

$$V(r) = \frac{e^{-r/r_0}}{r} = \frac{e^{-mr/\hbar c}}{r}$$

- If $r_0 \sim 1\ \text{fm}$, then $m \sim 197\ \text{MeV}$
- This is consistent with E&M:
  - The photon is massless, making $V(r)$ observable over macroscopic distances

Searching for Yukawa's π-meson

- In 1936, Carl Anderson and Seth Neddermeyer observed a charged particle with intermediate mass in cosmic rays.
- Its mass was consistent with Yukawa's meson but it did not interact with nuclear material.
- In 1947, the pi meson was observed in cosmic rays

Properties of Elementary Particles

- What distinguishes elementary particles?
  - Mass
  - Charge
  - Spin (intrinsic angular momentum)
  - Lifetime and decays
  - Interactions with other particles
  - Other quantum numbers to be discovered…

Nucleons

- We already know about protons and neutrons

	p	n
Mass	938.27 MeV	939.57 MeV
Charge	$+1$	0
Spin	½	½
Lifetime	(stable)	$882\ s$
Decays	$-$	$n \to p + e^- + \bar{\nu}$

- When we don't distinguish between them, we just call them "nucleons", N.

Pi Mesons

- Pions come in three varieties:

	$\pi^{\pm}$	π^{0}
Mass	139 MeV	135 MeV
Charge	± 1	0
Spin	0	0
Lifetime	26 ns	$8.4 \times 10^{-17}\ s$
Decays	$\pi^{\pm} \rightarrow \mu^{\pm}\nu$	$\pi^{0} \rightarrow \gamma\gamma$

- Produced in nuclear collisions
- Interact strongly with nuclei

Hadrons

- Particles that interact strongly are hadrons.
- There are two types:
  - Baryons (like the proton and neutron)
  - Mesons (like pions)
- Baryon number seems to be a conserved quantity.

$$p + p \rightarrow p + n + \pi^{+}$$

$$\underbrace{p + n}_{B=+2} \rightarrow \underbrace{p + n + \pi^{0}}_{B=+2}$$

Leptons

- Electrons and muons are somewhat different.

	$e^{\pm}$	$\mu^{\pm}$
Mass	0.511 MeV	106 MeV
Charge	± 1	± 1
Spin	½	½
Lifetime	(stable)	2.2 μs
Decays	$-$	$\mu^{\pm} \rightarrow e^{\pm} \nu \bar{\nu}$

- Lepton number and flavor seem to be conserved quantities.
- Neither interact strongly with nuclei
- Both are associated with beta decay

Beta Decay

- Nuclear beta decay:

$$n \rightarrow p + e^{-} + \bar{\nu}_e$$

  - The electron anti-neutrino cancels the electron's lepton-number
- Muon decay:

$$\mu^{-} \rightarrow e^{-} + \bar{\nu}_e + \nu_\mu$$

  - The electron anti-neutrino cancels the electron's lepton number
  - The muon lepton number is carried by the muon neutrino
- Both are classified as weak decays because the lifetimes are so long

Neutrinos

- Neutrinos do not interact strongly or electromagnetically
- Their weak interactions are so rare that we almost never observe them directly.
- If nuclear beta decay had a 2-body final state, then the electron would be mono-energetic
  - Momentum/energy conservation

$$E_e = \frac{m_p^2 + m_e^2 - m_n^2}{2m_p}$$

- In 1930, Pauli postulated the neutrino

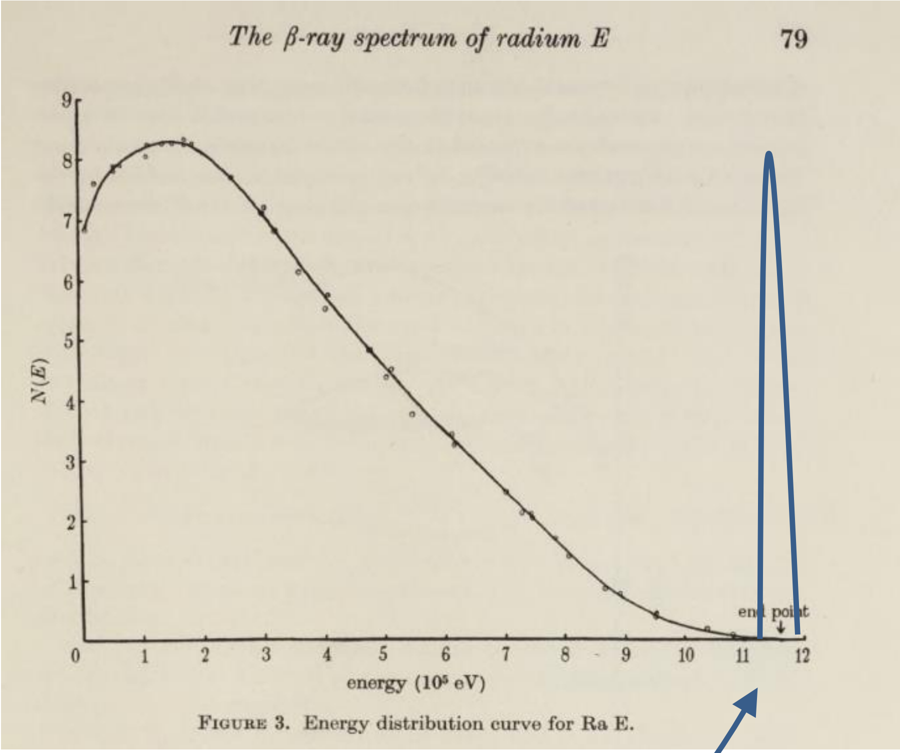


FIGURE 3. Energy distribution curve for Ra E.

Hadronic Resonances

- Particles that decay strongly have short lifetimes
  - They decay instantaneously
- Strong decays are forbidden when they violate a conservation law

$$p \nrightarrow \pi^+ + \pi^0$$

(violates baryon number conservation)

- Elastic scattering cross sections tell us about microscopic structure
  - Hard sphere scattering
  - Coulomb scattering
- Strongly decaying hadrons are observed as resonances in the elastic and inelastic cross sections

Hadronic Resonances

- Elastic pion-proton scattering cross section:

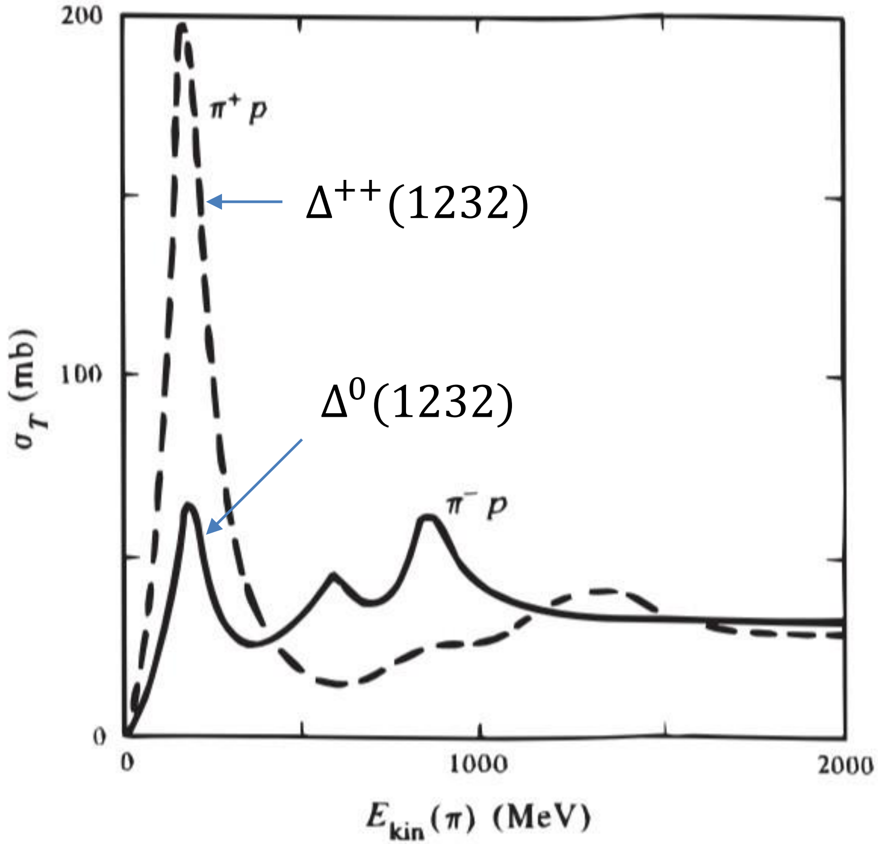


Figure 5.35: Total cross section as a function of pion kinetic energy for the scattering of positive and negative pions from protons. (1 mb = 1 millibarn = 10^{-27} cm^2.)

The first peak occurs at

$$E_{kin}(\pi) \sim 200 \text{ MeV}$$

$$p_p = (m_p, \vec{0})$$
$$p_\pi = (E_\pi, \vec{p}_\pi)$$

$$\begin{aligned} E_{cm}^2 &= (p_p + p_\pi)^2 \\ &= (m_p + E_\pi)^2 - |\vec{p}_\pi|^2 \\ &= (m_p + E_\pi)^2 - (E_\pi^2 - m_\pi^2) \\ &= m_p^2 + m_\pi^2 + 2m_p E_\pi \\ &= m_p^2 + m_\pi^2 + 2m_p(m_\pi + E_{kin}) \end{aligned}$$

$$E_{cm} = 1239 \text{ MeV}$$

Hadronic Resonances

- Resonance are often labeled with their mass in MeV
- Except for charge, their properties are similar
- In fact, there are four $\Delta(1232)$ resonances

	$\boldsymbol{\Delta^-}$	$\boldsymbol{\Delta^0}$	$\boldsymbol{\Delta^+}$	$\boldsymbol{\Delta^{++}}$
Mass	1232 MeV	1231 MeV	1235 MeV	1231 MeV
Charge	-1	0	$+1$	$+2$
Spin	3/2	3/2	3/2	3/2
Width	117 MeV	117 MeV	117 MeV	117 MeV
Decays	$n + \pi^-$	$n+\pi^0, p+\pi^+$	$n+\pi^+, p+\pi^0$	$p+\pi^+$

- When we don't distinguish between them, we just call them Δ… The decays are all just $\Delta \rightarrow N\pi$.

Isospin

- Electrons have spin ½ but we don't think of $|e \uparrow\rangle$ and $|e \downarrow\rangle$ as distinctly different particles
  - They are just two states of the same particle
  - They are symmetric unless we put them in a magnetic field
- The properties of the hadron multiplets are almost the same (except for charge):
  - Nucleon doublet
  - Pion triplet
  - Delta quadruplet
- Maybe these are just different states of the same strongly interacting particle
- We can only distinguish between them because of the electromagnetic interaction

Isospin

$I = \frac{1}{2}$ doublet: $\begin{pmatrix} p \\ n \end{pmatrix} = \begin{pmatrix} +\frac{1}{2} \\ -\frac{1}{2} \end{pmatrix}$

$I = 1$ triplet: $\begin{pmatrix} \pi^+ \\ \pi^0 \\ \pi^- \end{pmatrix} = \begin{pmatrix} +1 \\ 0 \\ -1 \end{pmatrix}$

$I = \frac{3}{2}$ multiplet: $\begin{pmatrix} \Delta^{++} \\ \Delta^+ \\ \Delta^0 \\ \Delta^- \end{pmatrix} = \begin{pmatrix} +\frac{3}{2} \\ +\frac{1}{2} \\ -\frac{1}{2} \\ -\frac{3}{2} \end{pmatrix}$

Isospin

- The different charge states have different isospin components along the I_z (or I_3) axis.
- This is completely made up and has no geometric meaning (it has nothing to do with the z-axis).
- Algebraically, it is the same as angular momentum.
- Fundamentally it is a representation of the group SU(2), which is the same as the group of rotations.
- If the strong interaction conserves isospin, then we can predict branching ratios and relative cross sections.
- You should review Clebsch-Gordon coefficients…

Clebsch-Gordon Coefficients

34. CLEBSCH-GORDAN COEFFICIENTS, SPHERICAL HARMONICS, AND d FUNCTIONS

Note: A square-root sign is to be understood over *every* coefficient, *e.g.*, for $-8/15$ read $-\sqrt{8/15}$.

Notation:

		J	J	...
		M	M	...
m_1	m_2	Coefficients		
m_1	m_2			
⋮	⋮			

$$Y_1^0 = \sqrt{\frac{3}{4\pi}}\cos\theta$$

$$Y_1^1 = -\sqrt{\frac{3}{8\pi}}\sin\theta\, e^{i\phi}$$

$$Y_2^0 = \sqrt{\frac{5}{4\pi}}\left(\frac{3}{2}\cos^2\theta - \frac{1}{2}\right)$$

$$Y_2^1 = -\sqrt{\frac{15}{8\pi}}\sin\theta\cos\theta\, e^{i\phi}$$

$$Y_2^2 = \frac{1}{4}\sqrt{\frac{15}{2\pi}}\sin^2\theta\, e^{2i\phi}$$

$$Y_\ell^{-m} = (-1)^m Y_\ell^{m*}$$

$$d^{\ell}_{m,0} = \sqrt{\frac{4\pi}{2\ell+1}}\, Y_\ell^m\, e^{-im\phi}$$

$$\langle j_1 j_2 m_1 m_2 | j_1 j_2 J M\rangle = (-1)^{J-j_1-j_2}\langle j_2 j_1 m_2 m_1 | j_2 j_1 J M\rangle$$

1/2×1/2

m_1	m_2	1
		+1
+1/2	+1/2	1

m_1	m_2	1	0
		0	0
+1/2	−1/2	1/2	1/2
−1/2	+1/2	1/2	−1/2

m_1	m_2	1
		−1
−1/2	−1/2	1

1×1/2

m_1	m_2	3/2
		+3/2
+1	+1/2	1

m_1	m_2	3/2	1/2
		+1/2	+1/2
+1	−1/2	1/3	2/3
0	+1/2	2/3	−1/3

m_1	m_2	3/2	1/2
		−1/2	−1/2
0	−1/2	2/3	1/3
−1	+1/2	1/3	−2/3

m_1	m_2	3/2
		−3/2
−1	−1/2	1

2×1

m_1	m_2	3
		+3
+2	+1	1

m_1	m_2	3	2
		+2	+2
+2	0	1/3	2/3
+1	+1	2/3	−1/3

m_1	m_2	3	2	1
		+1	+1	+1
+2	−1	1/15	1/3	3/5
+1	0	8/15	1/6	−3/10
0	+1	2/5	−1/2	1/10

m_1	m_2	3	2	1
		0	0	0
+1	−1	1/5	1/2	3/10
0	0	3/5	0	−2/5
−1	+1	1/5	−1/2	3/10

m_1	m_2	3	2	1
		−1	−1	−1
0	−1	2/5	1/2	1/10
−1	0	8/15	−1/6	−3/10
−2	+1	1/15	−1/3	3/5

m_1	m_2	3	2
		−2	−2
−1	−1	2/3	1/3
−2	0	1/3	−2/3

m_1	m_2	3
		−3
−2	−1	1

1×1

m_1	m_2	2
		+2
+1	+1	1

m_1	m_2	2	1
		+1	+1
+1	0	1/2	1/2
0	+1	1/2	−1/2

m_1	m_2	2	1	0
		0	0	0
+1	−1	1/6	1/2	1/3
0	0	2/3	0	−1/3
−1	+1	1/6	−1/2	1/3

m_1	m_2	2	1
		−1	−1
0	−1	1/2	1/2
−1	0	1/2	−1/2

m_1	m_2	2
		−2
−1	−1	1

3/2×1

m_1	m_2	5/2
		+5/2
+3/2	+1	1

m_1	m_2	5/2	3/2
		+3/2	+3/2
+3/2	0	2/5	3/5
+1/2	+1	3/5	−2/5

m_1	m_2	5/2	3/2	1/2
		+1/2	+1/2	+1/2
+3/2	−1	1/10	2/5	1/2
+1/2	0	3/5	1/15	−1/3
−1/2	+1	3/10	−8/15	1/6

m_1	m_2	5/2	3/2	1/2
		−1/2	−1/2	−1/2
+1/2	−1	3/10	8/15	1/6
−1/2	0	3/5	−1/15	−1/3
−3/2	+1	1/10	−2/5	1/2

m_1	m_2	5/2	3/2
		−3/2	−3/2
−1/2	−1	3/5	2/5
−3/2	0	2/5	−3/5

m_1	m_2	5/2
		−5/2
−3/2	−1	1

2×1/2

m_1	m_2	5/2
		+5/2
+2	+1/2	1

m_1	m_2	5/2	3/2
		+3/2	+3/2
+2	−1/2	1/5	4/5
+1	+1/2	4/5	−1/5

m_1	m_2	5/2	3/2
		+1/2	+1/2
+1	−1/2	2/5	3/5
0	+1/2	3/5	−2/5

m_1	m_2	5/2	3/2
		−1/2	−1/2
0	−1/2	3/5	2/5
−1	+1/2	2/5	−3/5

m_1	m_2	5/2	3/2
		−3/2	−3/2
−1	−1/2	4/5	1/5
−2	+1/2	1/5	−4/5

m_1	m_2	5/2
		−5/2
−2	−1/2	1

3/2×1/2

m_1	m_2	2
		+2
+3/2	+1/2	1

m_1	m_2	2	1
		+1	+1
+3/2	−1/2	1/4	3/4
+1/2	+1/2	3/4	−1/4

m_1	m_2	2	1
		0	0
+1/2	−1/2	1/2	1/2
−1/2	+1/2	1/2	−1/2

m_1	m_2	2	1
		−1	−1
−1/2	−1/2	3/4	1/4
−3/2	+1/2	1/4	−3/4

m_1	m_2	2
		−2
−3/2	−1/2	1

Isospin

- Consider $\pi^+ p$ scattering…
- We have to add spin-1 to spin-1/2:

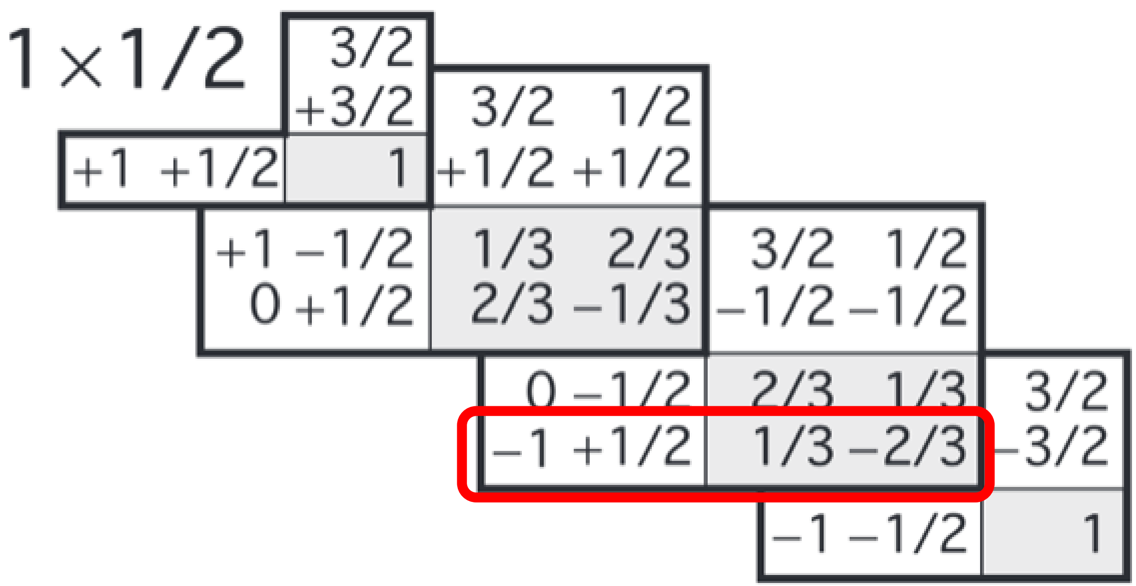
$$|1,+1\rangle \left|\frac{1}{2},+\frac{1}{2}\right\rangle = \left|\frac{3}{2},+\frac{3}{2}\right\rangle$$

- This can proceed only via the Δ^{++} resonance
- That was easy.

Isospin

- Consider $\pi^- p$ scattering:
- We have to add spin-1 to spin-1/2:

$$|1,-1\rangle \left|\frac{1}{2},+\frac{1}{2}\right\rangle = \sqrt{\frac{1}{3}}\left|\frac{3}{2},-\frac{1}{2}\right\rangle - \sqrt{\frac{2}{3}}\left|\frac{1}{2},-\frac{1}{2}\right\rangle$$



Isospin

- Amplitude for observing the Δ^{++} state:
$$\langle \Delta^{++} | \pi^{+} p \rangle = 1$$
- Amplitude for observing the Δ^{0} state:
$$\langle \Delta^{0} | \pi^{-} p \rangle = \sqrt{1/3}$$
- Cross section $\propto$ probability $\propto |\langle f | i \rangle|^2$
- The Δ^{0} cross section in $\pi^{-} p$ scattering should be 1/3 the Δ^{++} cross section in $\pi^{+} p$ scattering.
- Remember, the strong interaction doesn't care. This is all just $\pi N \to \Delta$ and it conserves isospin.

Isospin

It's not exact, but what do you expect from a model with almost no content?

And besides, nobody had any better ideas at the time.

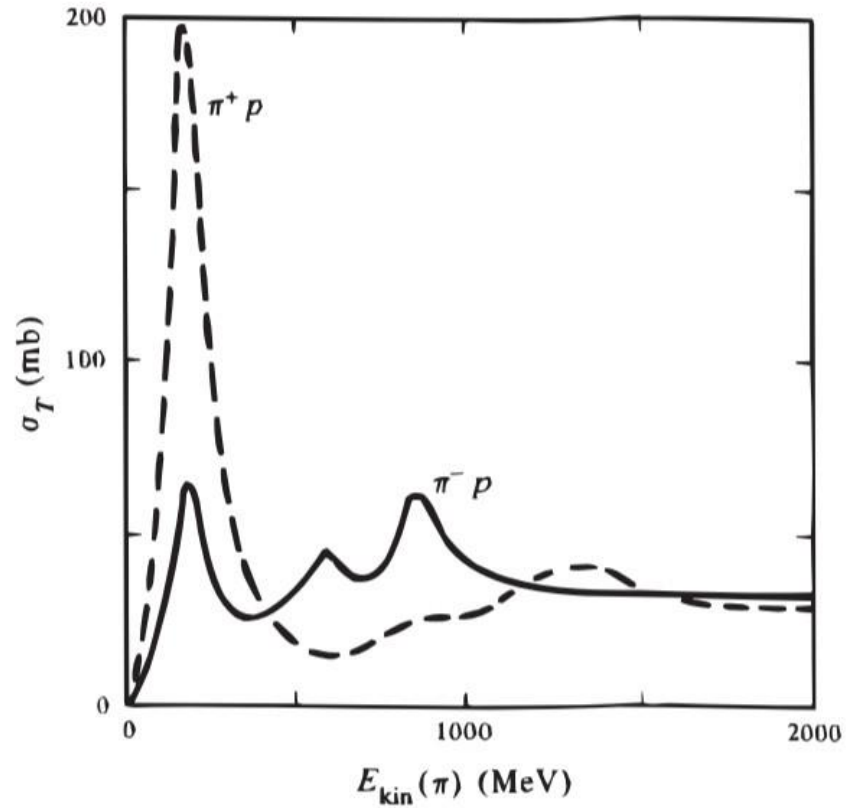


Figure 5.35: Total cross section as a function of pion kinetic energy for the scattering of positive and negative pions from protons. (1 mb = 1 millibarn = 10^{-27} cm^2.)

Weak Decays

- Weak decays do not conserve isospin!
- Examples:

$$n \rightarrow p + e^- + \bar{\nu}_e$$

$$\underbrace{\qquad}_{I_3 = -\frac{1}{2}} \quad \underbrace{\qquad}_{I_3 = +\frac{1}{2}}$$

$$\pi^+ \rightarrow \mu^+ + \nu_\mu$$

$$\underbrace{\qquad}_{I_3 = +1} \quad \underbrace{\qquad\qquad}_{I_3 = 0}$$

- This is *NOT* the same as "weak isospin" which we will use to describe the weak interaction.

Other Quantum Numbers

- How do these states change under parity transformations?
  - Even parity: $\Pi|\psi\rangle = +|\psi\rangle$
  - Odd parity: $\Pi|\psi\rangle = -|\psi\rangle$
- How can we tell?
- The proton is assigned a parity of $+1$
  - Therefore, the neutron also has a parity of $+1$
- The deuteron has spin-1 and parity of $(+1)^2$
- A π^- is captured on deuterium from an S-wave ground state $(L = 0)$ and emits two neutrons
  - Identical fermions must have odd parity and opposite spins
  - Pions have spin-0 so they the deuterons must have $L = 1$
- Parity of initial state: $(+1)^2(\pi)$
- Parity of final state: $(+1)^2(-1)^L = (-1)$

$$\pi = -1$$

Chinowsky and Steinberger, 1954

Parity

- Parity assignments:
  - N, Δ have parity +1
  - π have partiy -1
  - γ has parity +1
- Parity is conserved by electromagnetic and strong interactions
- Parity is violated in weak interactions