PURDUE UNIVERSITY DEPARTMENT OF PHYSICS

Physics 56400
Introduction to Elementary Particle Physics I

Now in PowerPoint!

Lecture 25
Fall 2018 Semester

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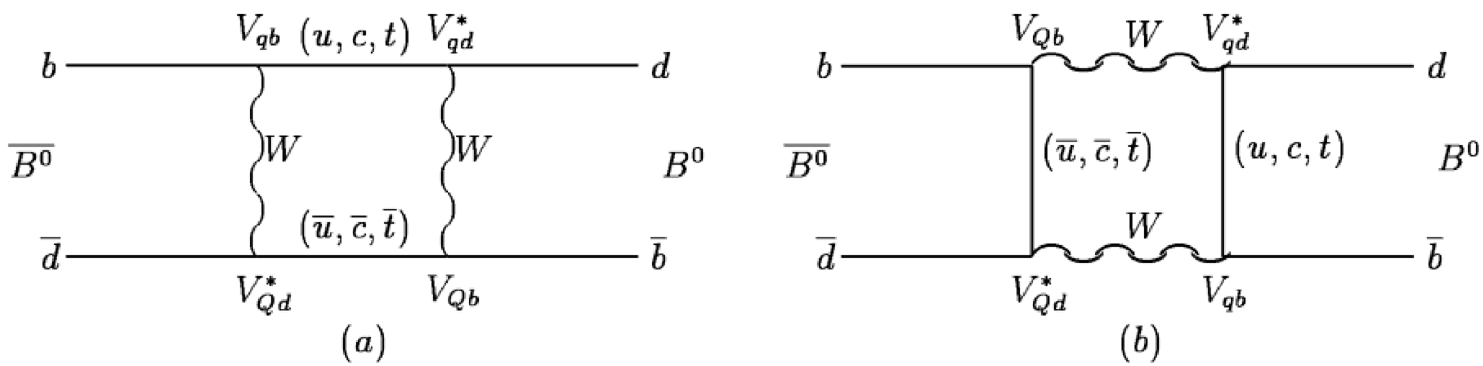
Neutrino Oscillations

- Kaon, B-meson, and neutrino oscillations are a natural consequence of a coupled oscillator system.
- Most features have good analogies in classical mechanics: https://youtu.be/kqOARsCJC-8
- General features:
  - Two oscillators that are essentially identical
  - Weak coupling
  - Motion of one mass excites motion in the other
  - Slow oscillation of motion between the two masses

B-Meson Oscillations

- Strong eigenstates have definite flavor (b or $\bar{b}$)
- Weak coupling via box diagrams:



$$H\begin{pmatrix} B^0 \\ \bar{B}^0 \end{pmatrix} = \begin{pmatrix} M & M_{12} \\ M_{12} & M \end{pmatrix}\begin{pmatrix} B^0 \\ \bar{B}^0 \end{pmatrix}$$

B-Meson Oscillations

- Physical masses are eigenvalues of the Hamiltonian:

$$m_H = M + M_{12}$$
$$m_L = M - M_{12}$$

- The CP-eigenstates are

$$B_L = \frac{1}{\sqrt{2}}(B^0 + \bar{B}^0)$$
$$B_H = \frac{1}{\sqrt{2}}(B^0 - \bar{B}^0)$$

- How does this relate to the mechanical example?

B-Meson Oscillations

- There are two modes of oscillation:
  - Even (both pendula swing the same direction)
  - Odd (pendula swing in opposite directions)
- These have different energies:
  - Even modes have less energy
  - Odd modes store energy in the coupling
- Probability of mixed/unmixed oscillates:

$$P(B^0, t|B^0) = |\langle B^0|\Psi(t)\rangle|^2 = \frac{e^{-\Gamma t}}{2}(1 + \cos \Delta m t)$$

$$P(\bar{B}^0, t|B^0) = |\langle \bar{B}^0|\Psi(t)\rangle|^2 = \frac{e^{-\Gamma t}}{2}(1 - \cos \Delta m t)$$

Neutrino Oscillations

- We know that neutrinos seem to carry flavor quantum numbers:

$$\binom{\nu_e}{e} \qquad \binom{\nu_\mu}{\mu} \qquad \binom{\nu_\tau}{\tau}$$

- Evidence for neutrino oscillations:
  - Solar neutrino problem: $\nu_e \rightarrow \nu_\mu$
  - Atmospheric neutrinos: $\nu_\mu \rightarrow \nu_\tau$
- Flavor eigenstates are not eigenstates of the Hamiltonian.
  - Flavor eigenstates: $\nu_\alpha = \nu_e, \nu_\mu, \nu_\tau$
  - Mass eigenstates: $\nu_i = \nu_1, \nu_2, \nu_3$
- Unitary transformation:

$$|\nu_\alpha\rangle = \sum_i U^*_{\alpha i} |\nu_i\rangle$$

$$|\nu_i\rangle = \sum_\alpha U_{\alpha i} |\nu_\alpha\rangle$$

Neutrino Mixing

- CKM matrix (quarks) → PMNS matrix (neutrinos)
- First consider only two families:

$$U = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$$

$$|\nu_\alpha\rangle = \sum_{i=1,2} U^*_{\alpha i} \, |\nu_i\rangle$$

$$|\nu(t)\rangle = \sum_{i=1,2} U^*_{\alpha i} \, e^{-iE_i t} |\nu_i\rangle = \sum_{i=1,2} U^*_{\alpha i} \, e^{-iE_i t} \sum_{\beta=e,\mu} U_{\beta i} |\nu_\beta\rangle$$

$$\langle \nu_\beta | \nu(t) \rangle = \sum_i U_{\beta i} U^*_{\alpha i} \, e^{-iE_i t}$$

Two Family Neutrino Mixing

$$P\left(\nu_e \to \nu_\mu\right) = \left|\left\langle \nu_\mu \middle| \nu(t) \right\rangle\right|^2 = \left|\sum_{i=1,2} U_{\mu i} U_{ei}^{*}\, e^{-iE_i t}\right|^2$$

$$= \left|\cos\theta \sin\theta\, e^{-iE_1 t} - \cos\theta \sin\theta\, e^{-iE_2 t}\right|^2$$

$$= \sin^2\theta \cos^2\theta \left|e^{-iE_1 t} - e^{-iE_2 t}\right|^2$$

$$= \frac{\sin^2 2\theta}{4}\left(2 - 2\, Re\left(e^{-i(E_1 - E_2)t}\right)\right)$$

$$= \sin^2 2\theta\ \sin^2\left(\frac{E_1 - E_2}{2}\right) t$$

Two Family Neutrino Mixing

- The neutrinos are ultrarelativistic, so

$$E_i = \sqrt{p^2 + m_i^2} \approx p + \frac{m_i^2}{2E}$$

$$P(\nu_e \to \nu_\mu) = \sin^2 2\theta \ \sin^2 \frac{\Delta m^2 t}{4E}$$

- But in natural units, $t = L$ so

$$P(\nu_e \to \nu_\mu) = \sin^2 2\theta \ \sin^2 \frac{\Delta m^2 L}{4E}$$

- The amplitude of the oscillation probability tells us about the mixing angle.
- The oscillation frequency tells us about the difference of the squared masses.

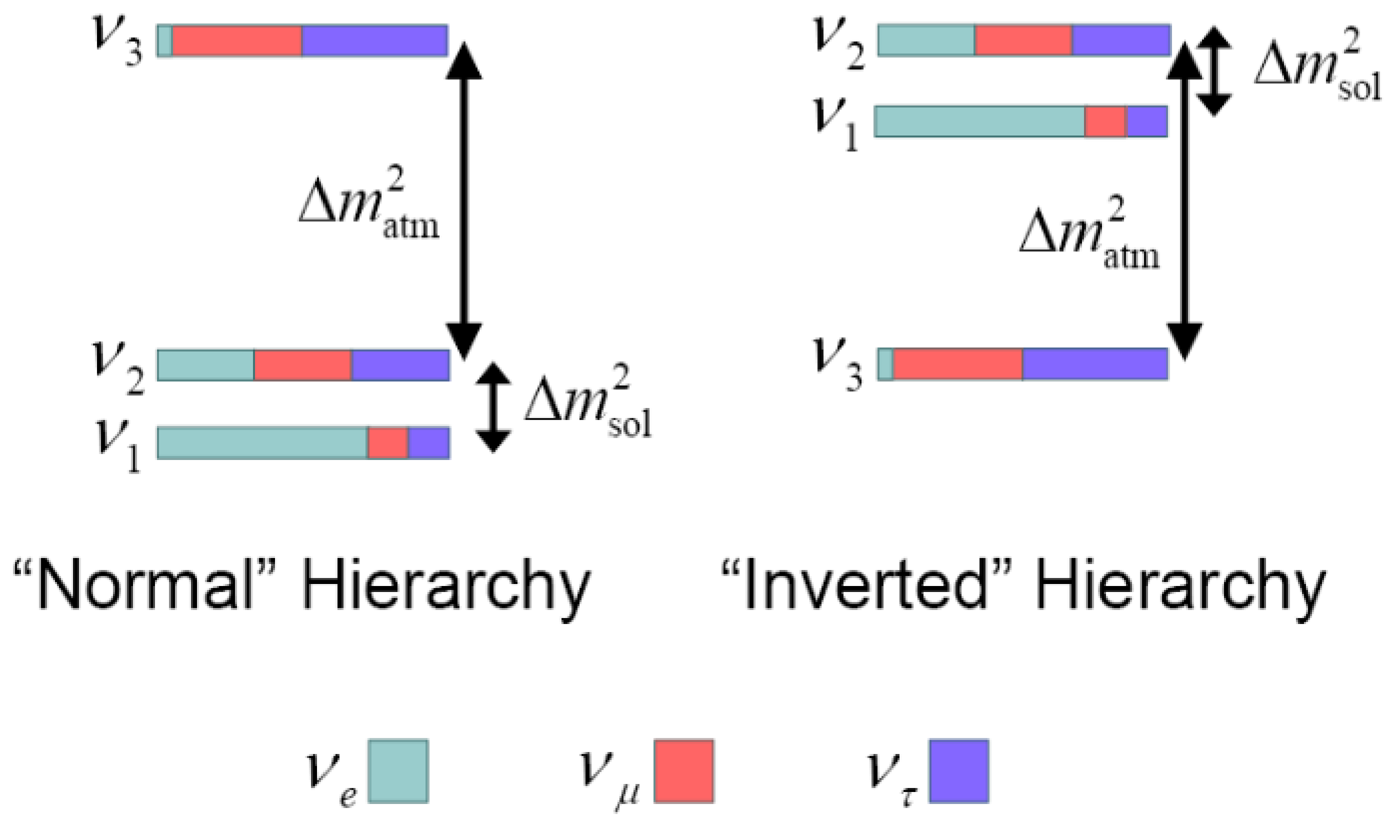
Three-Family Neutrino Mixing

- CKM matrix (quarks) → PMNS matrix (neutrinos)
- Parameterized by three angles and one phase
  - θ_{12} (solar/reactor neutrinos),
  - θ_{23} (atmospheric neutrinos)
  - θ_{13}, δ_{CP}
  - Three Δm^2 parameters
- What kind of experiments can we do?
  - Neutrino disappearance
  - Neutrino appearance
  - Neutrino vs anti-neutrino
  - Amplitude as a function of L

Oscillation Measurements

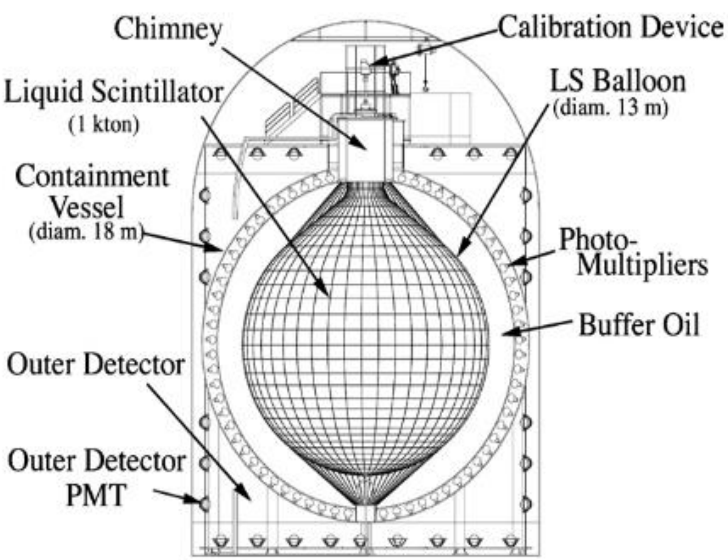
1. What are the values of Δm_{12}^2, Δm_{23}^2 and Δm_{13}^2?
2. Is the mass hierarchy normal or inverted?
   - Normal hierarchy: $\Delta m_{21}^2 \ll \quad (\Delta m_{32}^2 \simeq \Delta m_{31}^2 > 0)$
   - Inverted hierarchy: $\Delta m_{21}^2 \ll -(\Delta m_{31}^2 \simeq \Delta m_{32}^2 < 0)$
3. What are the mixing angles?
4. What is the CP-violating phase?
5. Direct measurements of neutrino masses
6. Is the neutrino its own anti-particle?

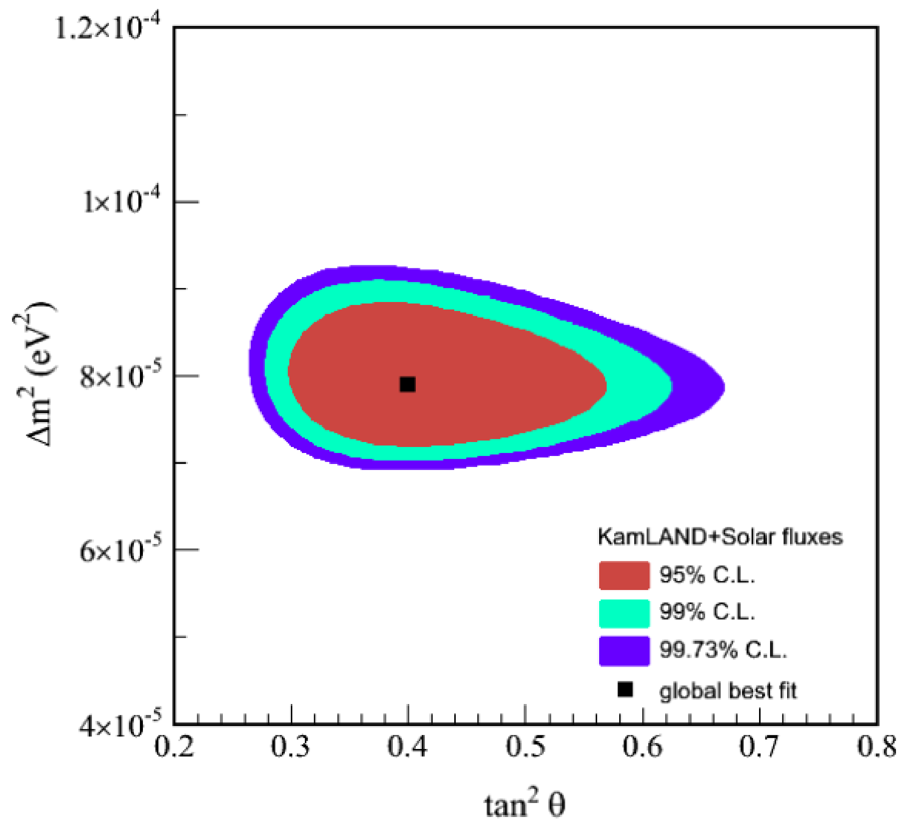
Neutrino Mass Hierarchy



Solar and Reactor Neutrinos

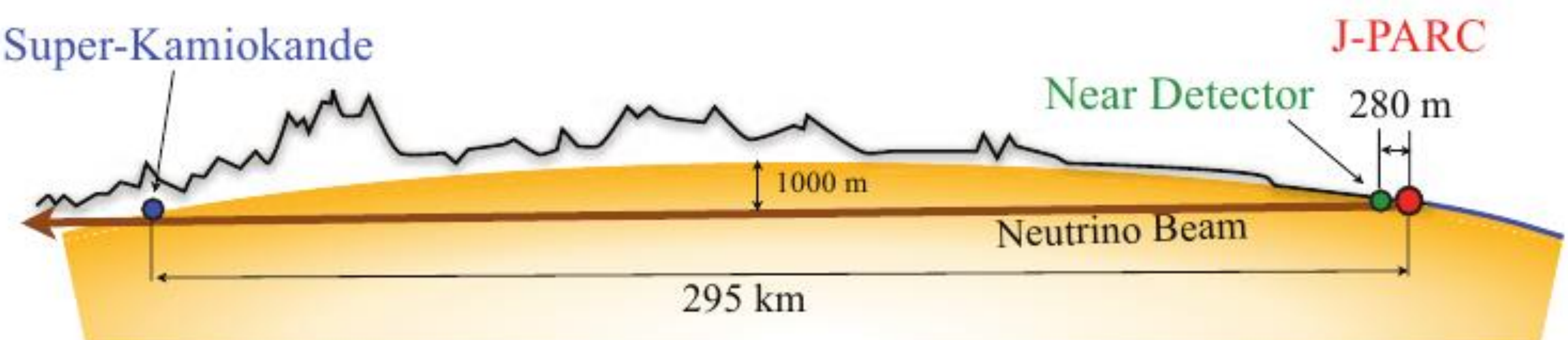
- KamLAND detects neutrinos from several Japanese nuclear reactors: $L \sim 175\ km$





T2K and Daya Bay

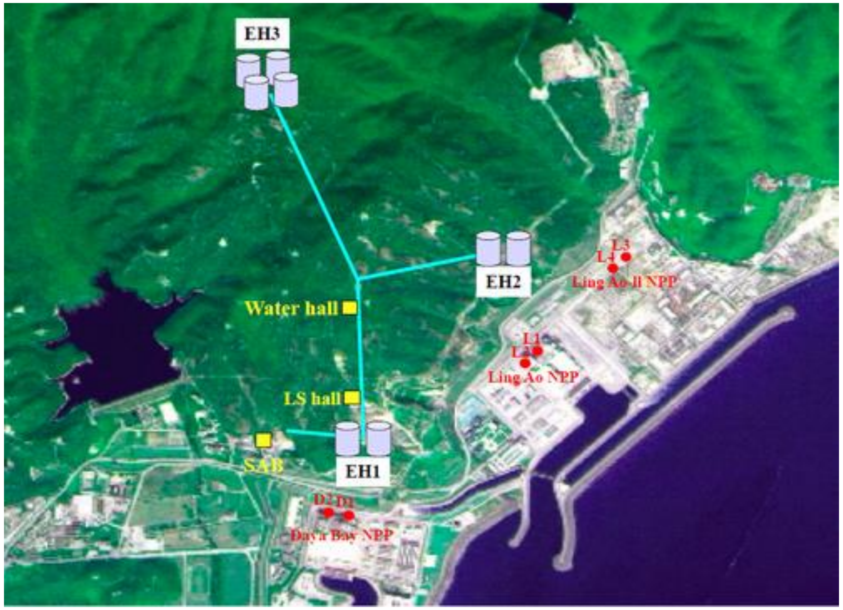
- These try to measure θ_{13} by looking for reactor anti-neutrino disappearance
- T2K is a long baseline $\nu_\mu \rightarrow \nu_e$ appearance experiment

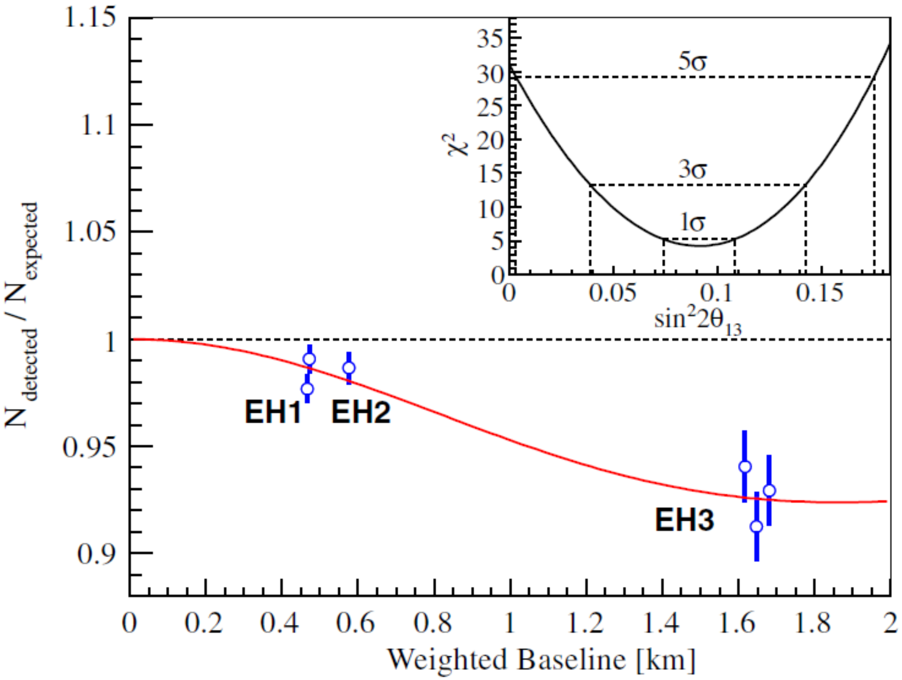


Daya Bay Experiment

- Measures survival probability of reactor anti-neutrinos:

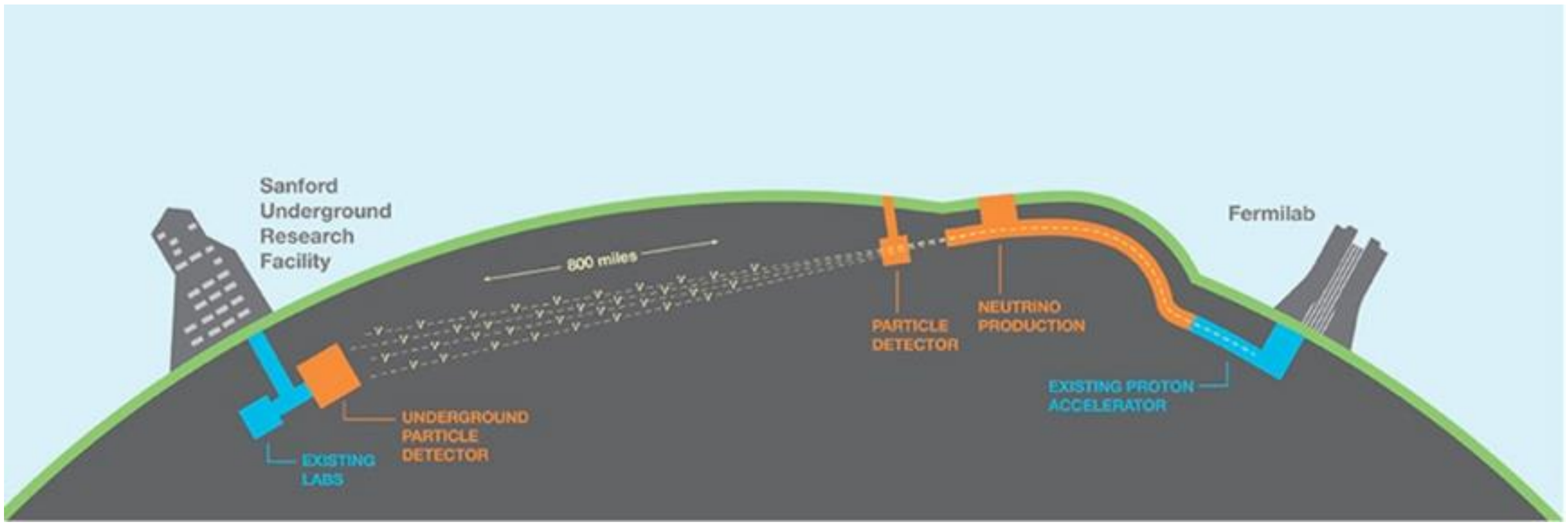
$$P_{\bar{\nu}_e \to \bar{\nu}_e} = 1 - \cos^4\theta_{13}\sin^2 2\theta_{12}\sin^2\Delta_{21} - \sin^2 2\theta_{13}(\cos^2\theta_{12}\sin^2\Delta_{31} + \sin^2\theta_{12}\sin^2\Delta_{32})$$





$$\sin^2 2\theta_{13} = 0.084 \pm 0.005$$

Deep Underground Neutrino Experiement



- Expects to measure mass hierarchy and δ_{CP} in about 5-10 years of running.
  - Sensitivity depends on the value of δ_{CP}

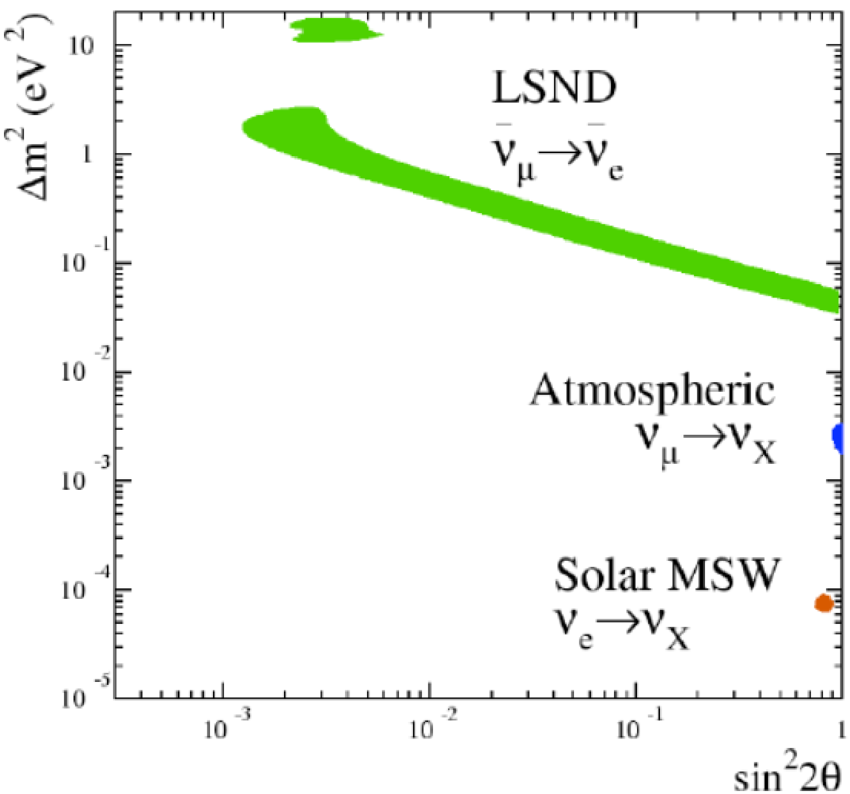
DUNE Timeline

Coincides with operation of Mu2e experiment.

Ideally, they expect to share the beam equitably…

The LSND Experiment

- This is a short baseline (30 m) experiment that looks for the appearance of $\bar{\nu}_e$ in a beam of $\bar{\nu}_\mu$

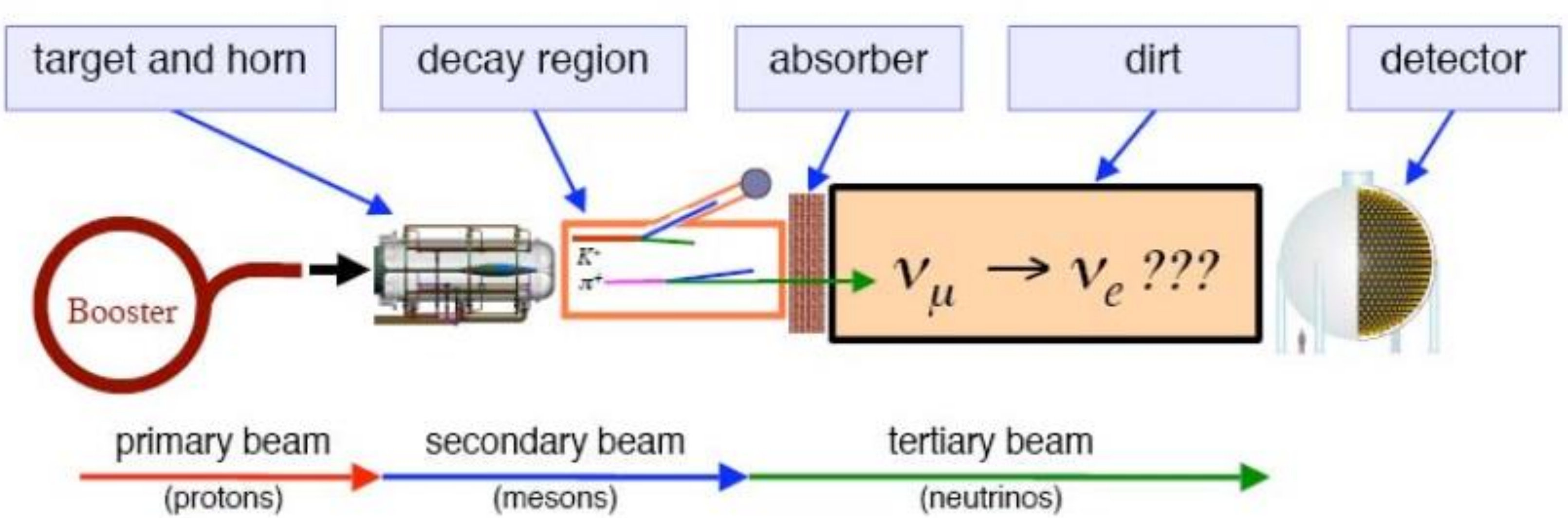


The problem is that $\Delta m^2_{nucl} + \Delta m^2_{atm} \neq \Delta m^2_{LSND}$

Is this evidence for a 4th generation of neutrino?

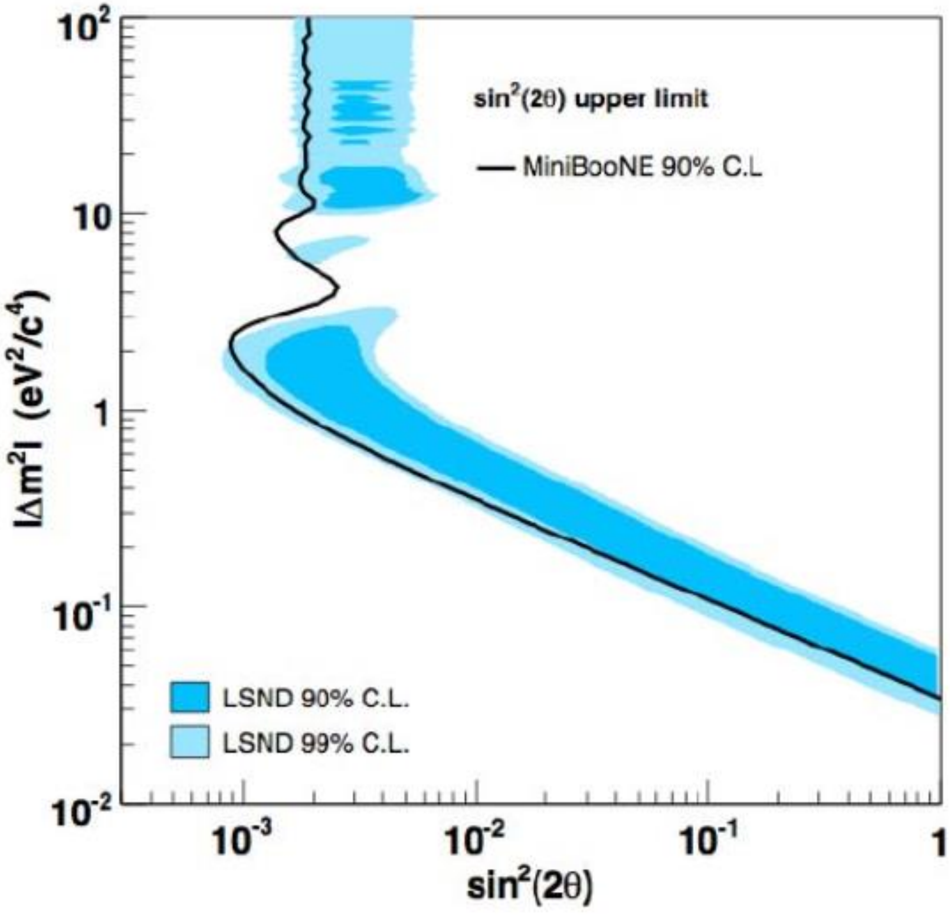
That would be weird…

MiniBoone Experiment



- This looks for ν_e appearing in a beam of ν_μ
- More challenging than initially expected due to some aspects of low energy nuclear physics

MiniBoone Experiment



This seems to rule out the LSND result...

Acknowledgements

- Some figures have been shamelessly stolen from Gary Cheng’s excellent summary:

http://www.astro.caltech.edu/~golwala/ph135c/20ChengNeutrinoOscillations.pdf