spin 1/2 baryons, positive parity

S = 0	n	p	$m \sim 940$ MeV
S = -1	Σ^-	Σ^0 Σ^+	$m \sim 1190$ MeV
S = -2	Ξ^-	Ξ^0	$m \sim 1315$ MeV

spin 3/2 baryons, positive parity

S = 0	Δ^- Δ^0 Δ^+ Δ^{++}	$m \sim 1230$ MeV
S = -1	Σ^{*-} Σ^{*0} Σ^{*+}	$m \sim 1385$ MeV
S = -2	Ξ^{*-} Ξ^{*0}	$m \sim 1530$ MeV
S = -3	Ω^-	$m \sim 1670$ MeV

All particles with the same strangeness have approximately the same mass.

If $H|\psi\rangle = E|\psi\rangle$
$= M|\psi\rangle$ then the strong interaction, which gives the particles their mass, doesn't distinguish between p, n or $\Sigma^+, \Sigma^0, \Sigma^-$.

This implies the existance of another symmet.
Hence, we can assign these states quantum numbers that are conserved by the strong interaction.

$\Longrightarrow$ Isospin.

p and n form a doublet, just like a spin ½ particle has two spin states.

The π^-, π^0, π^+ form a triplet, just like a spin 1 particle.

By convention
$$|p\rangle = |\tfrac{1}{2}\ {+\tfrac{1}{2}}\rangle_I$$
$$|n\rangle = |\tfrac{1}{2}\ {-\tfrac{1}{2}}\rangle_I$$
$$|\pi^+\rangle = |1\ {+1}\rangle_I$$
$$|\pi^0\rangle = |1\ 0\rangle_I$$
$$|\pi^-\rangle = |1\ {-1}\rangle_I$$

Similarly,

$$|\Delta^{++}\rangle = |\tfrac{3}{2}\ {+\tfrac{3}{2}}\rangle_I$$
$$|\Delta^{+}\rangle = |\tfrac{3}{2}\ {+\tfrac{1}{2}}\rangle_I$$
$$|\Delta^{0}\rangle = |\tfrac{3}{2}\ {-\tfrac{1}{2}}\rangle_I$$
$$|\Delta^{-}\rangle = |\tfrac{3}{2}\ {-\tfrac{3}{2}}\rangle_I \ .$$

This is how we classified the mesons and baryons:

$I_z =$	-1	$-1/2$	0	$+1/2$	$+1$
$S=+1$		K^0		K^+	
$S=0$	π^-		π^0, η, η'		π^+
$S=-1$		K^-		$\bar{K}^0$	

	-1	$-1/2$	0	$+1/2$	$+1$
		K^{*0}		K^{*+}	
	ρ^-		ρ^0, ω, ϕ		ρ^+
		K^{*-}		$\bar{K}^{*0}$	

$I_z =$	-1	$-1/2$	0	$+1/2$	$+1$
$S=0$		n		p	
	Σ^-		Σ^0, Λ		Σ^+
		Ξ^-		Ξ^0	

$I_z =$	$-3/2$	-1	$-1/2$	0	$+1/2$	$+1$	$+3/2$
	Δ^-		Δ^0		Δ^+		Δ
		Σ^{*-}		Σ^{*0}		Σ^{*+}	
			Ξ^{*-}		Ξ^{*0}		
				Ω^-			

Although the η, η' have $I_z = 0$ their masses are very different from the pion mass. $m_\eta = 548$ MeV, $m_{\eta'} = 958$ MeV. These are isospin singlet states:

$$|\eta\rangle = |0\ 0\rangle_I$$
$$|\eta'\rangle = |0\ 0\rangle_I$$

Similarly, $|\Lambda^0\rangle = |0\ 0\rangle$, $|\omega\rangle = |0\ 0\rangle_I$ and $|\phi\rangle = |0\ 0\rangle_I$.

But $|\rho^0\rangle = |1\ 0\rangle_I$ is part of the ρ triplet.

Using the table of Clebsch-Gordon coefficients.

Problem: two particles have angular momentum states $|j_1 m_1\rangle |j_2 m_2\rangle$. What is the state that describes the total angular momentum $|J M\rangle$?

We must conserve $M = m_1 + m_2$ but the total angular momentum could add up in various ways:

same as $|j_1 m_1\rangle|j_2 m_2\rangle$

$$|j_1 j_2 J M\rangle = \sum |j_1 j_2 m_1 m_2\rangle \underbrace{\langle j_1 j_2 m_1 m_2 | j_1 j_2 J M\rangle}$$

these are complex numbers - the C-G coefficients.

Example: Spin ½ + spin ½ can add up to a total spin 1 or total spin 0 state

$$|1\ 1\rangle = |\tfrac{1}{2}\ {+\tfrac{1}{2}}\rangle|\tfrac{1}{2}\ {+\tfrac{1}{2}}\rangle$$

$$|1\ 0\rangle = \tfrac{1}{\sqrt{2}}|\tfrac{1}{2}\ {+\tfrac{1}{2}}\rangle|\tfrac{1}{2}\ {-\tfrac{1}{2}}\rangle + \tfrac{1}{\sqrt{2}}|\tfrac{1}{2}\ {-\tfrac{1}{2}}\rangle|\tfrac{1}{2}\ {+\tfrac{1}{2}}\rangle$$

$$|1\ {-1}\rangle = |\tfrac{1}{2}\ {-\tfrac{1}{2}}\rangle|\tfrac{1}{2}\ {-\tfrac{1}{2}}\rangle$$

$$|0\ 0\rangle = \tfrac{1}{\sqrt{2}}|\tfrac{1}{2}\ {+\tfrac{1}{2}}\rangle|\tfrac{1}{2}\ {-\tfrac{1}{2}}\rangle - \tfrac{1}{\sqrt{2}}|\tfrac{1}{2}\ {-\tfrac{1}{2}}\rangle|\tfrac{1}{2}\ {+\tfrac{1}{2}}\rangle$$

Another more elaborate example:

Spin $\frac{3}{2}$ plus spin 1.

row of table

$$\left|\tfrac{3}{2}\ \tfrac{1}{2}\right\rangle\left|1\ 0\right\rangle = \sqrt{\tfrac{3}{5}}\left|\tfrac{5}{2}\ \tfrac{1}{2}\right\rangle + \sqrt{\tfrac{1}{15}}\left|\tfrac{3}{2}\ \tfrac{1}{2}\right\rangle - \sqrt{\tfrac{1}{3}}\left|\tfrac{1}{2}\ \tfrac{1}{2}\right\rangle$$

Alternatively

column of table.

$$\left|\tfrac{3}{2}\ \tfrac{1}{2}\right\rangle = \sqrt{\tfrac{2}{5}}\left|\tfrac{3}{2}\ \tfrac{3}{2}\right\rangle\left|1\ -1\right\rangle + \sqrt{\tfrac{1}{15}}\left|\tfrac{3}{2}\ \tfrac{1}{2}\right\rangle\left|1\ 0\right\rangle - \sqrt{\tfrac{8}{15}}\left|\tfrac{3}{2}\ -\tfrac{1}{2}\right\rangle\left|1\ +1\right\rangle$$

3/2×1

		5/2
		+5/2
+3/2	+1	1

		5/2	3/2
		+3/2	+3/2
+3/2	0	2/5	3/5
+1/2	+1	3/5	−2/5

		5/2	3/2	1/2
		+1/2	+1/2	+1/2
+3/2	−1	1/10	2/5	1/2
+1/2	0	3/5	1/15	−1/3
−1/2	+1	3/10	−8/15	1/6

		5/2	3/2	1/2
		−1/2	−1/2	−1/2
+1/2	−1	3/10	8/15	1/6
−1/2	0	3/5	−1/15	−1/3
−3/2	+1	1/10	−2/5	1/2

		5/2	3/2
		−3/2	−3/2
−1/2	−1	3/5	2/5
−3/2	0	2/5	−3/5

		5/2
		−5/2
−3/2	−1	1

35. CLEBSCH-GORDAN COEFFICIENTS, SPHERICAL HARMONICS, AND d FUNCTIONS

Note: A square-root sign is to be understood over *every* coefficient, *e.g.*, for −8/15 read $-\sqrt{8/15}$.

Notation:

		J	J	...
		M	M	...
m_1	m_2			
m_1	m_2	Coefficients		
.	.			
.	.			

1/2×1/2

M	m_1	m_2	J=1	J=0
+1	+1/2	+1/2	1	
0	+1/2	−1/2	1/2	1/2
0	−1/2	+1/2	1/2	−1/2
−1	−1/2	−1/2	1	

1×1/2

M	m_1	m_2	J=3/2	J=1/2
+3/2	+1	+1/2	1	
+1/2	+1	−1/2	1/3	2/3
+1/2	0	+1/2	2/3	−1/3
−1/2	0	−1/2	2/3	1/3
−1/2	−1	+1/2	1/3	−2/3
−3/2	−1	−1/2	1	

2×1/2

M	m_1	m_2	J=5/2	J=3/2
+5/2	+2	+1/2	1	
+3/2	+2	−1/2	1/5	4/5
+3/2	+1	+1/2	4/5	−1/5
+1/2	+1	−1/2	2/5	3/5
+1/2	0	+1/2	3/5	−2/5
−1/2	0	−1/2	3/5	2/5
−1/2	−1	+1/2	2/5	−3/5
−3/2	−1	−1/2	4/5	1/5
−3/2	−2	+1/2	1/5	−4/5
−5/2	−2	−1/2	1	

3/2×1/2

M	m_1	m_2	J=2	J=1
+2	+3/2	+1/2	1	
+1	+3/2	−1/2	1/4	3/4
+1	+1/2	+1/2	3/4	−1/4
0	+1/2	−1/2	1/2	1/2
0	−1/2	+1/2	1/2	−1/2
−1	−1/2	−1/2	3/4	1/4
−1	−3/2	+1/2	1/4	−3/4
−2	−3/2	−1/2	1	

2×1

M	m_1	m_2	J=3	J=2	J=1
+3	+2	+1	1		
+2	+2	0	1/3	2/3	
+2	+1	+1	2/3	−1/3	
+1	+2	−1	1/15	1/3	3/5
+1	+1	0	8/15	1/6	−3/10
+1	0	+1	2/5	−1/2	1/10
0	+1	−1	1/5	1/2	3/10
0	0	0	3/5	0	−2/5
0	−1	+1	1/5	−1/2	3/10
−1	0	−1	2/5	1/2	1/10
−1	−1	0	8/15	−1/6	−3/10
−1	−2	+1	1/15	−1/3	3/5
−2	−1	−1	2/3	1/3	
−2	−2	0	1/3	−2/3	
−3	−2	−1	1		

1×1

M	m_1	m_2	J=2	J=1	J=0
+2	+1	+1	1		
+1	+1	0	1/2	1/2	
+1	0	+1	1/2	−1/2	
0	+1	−1	1/6	1/2	1/3
0	0	0	2/3	0	−1/3
0	−1	+1	1/6	−1/2	1/3
−1	0	−1	1/2	1/2	
−1	−1	0	1/2	−1/2	
−2	−1	−1	1		

3/2×1

M	m_1	m_2	J=5/2	J=3/2	J=1/2
+5/2	+3/2	+1	1		
+3/2	+3/2	0	2/5	3/5	
+3/2	+1/2	+1	3/5	−2/5	
+1/2	+3/2	−1	1/10	2/5	1/2
+1/2	+1/2	0	3/5	1/15	−1/3
+1/2	−1/2	+1	3/10	−8/15	1/6
−1/2	+1/2	−1	3/10	8/15	1/6
−1/2	−1/2	0	3/5	−1/15	−1/3
−1/2	−3/2	+1	1/10	−2/5	1/2
−3/2	−1/2	−1	3/5	2/5	
−3/2	−3/2	0	2/5	−3/5	
−5/2	−3/2	−1	1		

$$Y_1^0 = \sqrt{\frac{3}{4\pi}}\cos\theta$$

$$Y_1^1 = -\sqrt{\frac{3}{8\pi}}\sin\theta\, e^{i\phi}$$

$$Y_2^0 = \sqrt{\frac{5}{4\pi}}\left(\frac{3}{2}\cos^2\theta - \frac{1}{2}\right)$$

$$Y_2^1 = -\sqrt{\frac{15}{8\pi}}\sin\theta\cos\theta\, e^{i\phi}$$

$$Y_2^2 = \frac{1}{4}\sqrt{\frac{15}{2\pi}}\sin^2\theta\, e^{2i\phi}$$

$$Y_\ell^{-m} = (-1)^m Y_\ell^{m*}$$

$$d^{\ell}_{m,0} = \sqrt{\frac{4\pi}{2\ell+1}}\, Y_\ell^m\, e^{-im\phi}$$

$$\langle j_1 j_2 m_1 m_2 | j_1 j_2 J M\rangle = (-1)^{J-j_1-j_2}\langle j_2 j_1 m_2 m_1 | j_2 j_1 J M\rangle$$

$$d^{j}_{m',m} = (-1)^{m-m'} d^{j}_{m,m'} = d^{j}_{-m,-m'}$$

3/2×3/2

M	m_1	m_2	J=3	J=2	J=1	J=0
+3	+3/2	+3/2	1			
+2	+3/2	+1/2	1/2	1/2		
+2	+1/2	+3/2	1/2	−1/2		
+1	+3/2	−1/2	1/5	1/2	3/10	
+1	+1/2	+1/2	3/5	0	−2/5	
+1	−1/2	+3/2	1/5	−1/2	3/10	
0	+3/2	−3/2	1/20	1/4	9/20	1/4
0	+1/2	−1/2	9/20	1/4	−1/20	−1/4
0	−1/2	+1/2	9/20	−1/4	−1/20	1/4
0	−3/2	+3/2	1/20	−1/4	9/20	−1/4
−1	+1/2	−3/2	1/5	1/2	3/10	
−1	−1/2	−1/2	3/5	0	−2/5	
−1	−3/2	+1/2	1/5	−1/2	3/10	
−2	−1/2	−3/2	1/2	1/2		
−2	−3/2	−1/2	1/2	−1/2		
−3	−3/2	−3/2	1			

$$d^{1}_{0,0} = \cos\theta \qquad d^{1/2}_{1/2,1/2} = \cos\frac{\theta}{2} \qquad d^{1/2}_{1/2,-1/2} = -\sin\frac{\theta}{2}$$

$$d^{1}_{1,1} = \frac{1+\cos\theta}{2} \qquad d^{1}_{1,0} = -\frac{\sin\theta}{\sqrt{2}} \qquad d^{1}_{1,-1} = \frac{1-\cos\theta}{2}$$

2×3/2

M	m_1	m_2	J=7/2	J=5/2	J=3/2	J=1/2
+7/2	+2	+3/2	1			
+5/2	+2	+1/2	3/7	4/7		
+5/2	+1	+3/2	4/7	−3/7		
+3/2	+2	−1/2	1/7	16/35	2/5	
+3/2	+1	+1/2	4/7	1/35	−2/5	
+3/2	0	+3/2	2/7	−18/35	1/5	
+1/2	+2	−3/2	1/35	6/35	2/5	2/5
+1/2	+1	−1/2	12/35	5/14	0	−3/10
+1/2	0	+1/2	18/35	−3/35	−1/5	1/5
+1/2	−1	+3/2	4/35	−27/70	2/5	−1/10
−1/2	+1	−3/2	4/35	27/70	2/5	1/10
−1/2	0	−1/2	18/35	3/35	−1/5	−1/5
−1/2	−1	+1/2	12/35	−5/14	0	3/10
−1/2	−2	+3/2	1/35	−6/35	2/5	−2/5
−3/2	0	−3/2	2/7	18/35	1/5	
−3/2	−1	−1/2	4/7	−1/35	−2/5	
−3/2	−2	+1/2	1/7	−16/35	2/5	
−5/2	−1	−3/2	4/7	3/7		
−5/2	−2	−1/2	3/7	−4/7		
−7/2	−2	−3/2	1			

2×2

M	m_1	m_2	J=4	J=3	J=2	J=1	J=0
+4	+2	+2	1				
+3	+2	+1	1/2	1/2			
+3	+1	+2	1/2	−1/2			
+2	+2	0	3/14	1/2	2/7		
+2	+1	+1	4/7	0	−3/7		
+2	0	+2	3/14	−1/2	2/7		
+1	+2	−1	1/14	3/10	3/7	1/5	
+1	+1	0	3/7	1/5	−1/14	−3/10	
+1	0	+1	3/7	−1/5	−1/14	3/10	
+1	−1	+2	1/14	−3/10	3/7	−1/5	
0	+2	−2	1/70	1/10	2/7	2/5	1/5
0	+1	−1	8/35	2/5	1/14	−1/10	−1/5
0	0	0	18/35	0	−2/7	0	1/5
0	−1	+1	8/35	−2/5	1/14	1/10	−1/5
0	−2	+2	1/70	−1/10	2/7	−2/5	1/5
−1	+1	−2	1/14	3/10	3/7	1/5	
−1	0	−1	3/7	1/5	−1/14	−3/10	
−1	−1	0	3/7	−1/5	−1/14	3/10	
−1	−2	+1	1/14	−3/10	3/7	−1/5	
−2	0	−2	3/14	1/2	2/7		
−2	−1	−1	4/7	0	−3/7		
−2	−2	0	3/14	−1/2	2/7		
−3	−1	−2	1/2	1/2			
−3	−2	−1	1/2	−1/2			
−4	−2	−2	1				

$$d^{3/2}_{3/2,3/2} = \frac{1+\cos\theta}{2}\cos\frac{\theta}{2}$$

$$d^{3/2}_{3/2,1/2} = -\sqrt{3}\,\frac{1+\cos\theta}{2}\sin\frac{\theta}{2}$$

$$d^{3/2}_{3/2,-1/2} = \sqrt{3}\,\frac{1-\cos\theta}{2}\cos\frac{\theta}{2}$$

$$d^{3/2}_{3/2,-3/2} = -\frac{1-\cos\theta}{2}\sin\frac{\theta}{2}$$

$$d^{3/2}_{1/2,1/2} = \frac{3\cos\theta-1}{2}\cos\frac{\theta}{2}$$

$$d^{3/2}_{1/2,-1/2} = -\frac{3\cos\theta+1}{2}\sin\frac{\theta}{2}$$

$$d^{2}_{2,2} = \left(\frac{1+\cos\theta}{2}\right)^2$$

$$d^{2}_{2,1} = -\frac{1+\cos\theta}{2}\sin\theta$$

$$d^{2}_{2,0} = \frac{\sqrt{6}}{4}\sin^2\theta$$

$$d^{2}_{2,-1} = -\frac{1-\cos\theta}{2}\sin\theta$$

$$d^{2}_{2,-2} = \left(\frac{1-\cos\theta}{2}\right)^2$$

$$d^{2}_{1,1} = \frac{1+\cos\theta}{2}(2\cos\theta-1)$$

$$d^{2}_{1,0} = -\sqrt{\frac{3}{2}}\sin\theta\cos\theta$$

$$d^{2}_{1,-1} = \frac{1-\cos\theta}{2}(2\cos\theta+1)$$

$$d^{2}_{0,0} = \left(\frac{3}{2}\cos^2\theta - \frac{1}{2}\right)$$

Figure 35.1: The sign convention is that of Wigner (*Group Theory*, Academic Press, New York, 1959), also used by Condon and Shortley (*The Theory of Atomic Spectra*, Cambridge Univ. Press, New York, 1953), Rose (*Elementary Theory of Angular Momentum*, Wiley, New York, 1957), and Cohen (*Tables of the Clebsch-Gordan Coefficients*, North American Rockwell Science Center, Thousand Oaks, Calif., 1974). The coefficients here have been calculated using computer programs written independently by Cohen and at LBNL.

It was an observed fact that the strong interaction conserved isospin.

Example :

$$|\Sigma^{*-}\rangle = |1\,-1\rangle_I = \frac{1}{\sqrt{2}}|1\,-1\rangle|1\,0\rangle + \frac{1}{\sqrt{2}}|1\,0\rangle|1\,-1\rangle$$

$$|\Lambda\rangle|\pi^-\rangle = |0\,0\rangle|{+1}\,-1\rangle = |1\,-1\rangle$$

$$|\Sigma^-\rangle|\pi^0\rangle = |1\,-1\rangle|1\,0\rangle = \frac{1}{\sqrt{2}}|2\,-1\rangle + \frac{1}{\sqrt{2}}|1\,-1\rangle$$

So $$\frac{\Gamma(\Sigma^{*-}\to\Lambda\pi^-)}{\Gamma(\Sigma^{*-}\to\Sigma^-\pi^0)} = \frac{|p_\Lambda|^2}{|p_\Sigma|^2\cdot\frac{1}{2}}$$

$$E_\Lambda = \frac{M_{\Sigma^*}^2 + m_\Lambda^2 - m_\pi^2}{2M_{\Sigma^*}} = \frac{(1385\text{ MeV})^2 + (1116\text{ MeV})^2 - (1}{2(1385\text{ MeV})} = 1130\text{ M}$$

$$p_\Lambda = \sqrt{E_\Lambda^2 - m_\Lambda^2} = 212\text{ MeV}$$

Similarly, $p_\Sigma = 126$ MeV

$$\frac{\Gamma(\Sigma^{*-}\to\Lambda\pi^-)}{\Gamma(\Sigma^{*-}\to\Sigma^-\pi^0)} = \frac{2(212)^2}{(126)^2} = 5.7$$

Particle data book reports the measured ratio to be 7.4 ± 0.6 .

Not too far off...

Another example:

$$K^{*0} \to K^+\pi^-$$
$$\to K^0\pi^0$$

$$|K^{*0}\rangle = |\tfrac{1}{2}\ -\tfrac{1}{2}\rangle$$

$$|K^+\rangle|\pi^-\rangle = |\tfrac{1}{2}\ +\tfrac{1}{2}\rangle|1\ -1\rangle = \tfrac{1}{\sqrt{3}}|\tfrac{3}{2}\ -\tfrac{1}{2}\rangle - \sqrt{\tfrac{2}{3}}|\tfrac{1}{2}\ -\tfrac{1}{2}\rangle$$
$$|K^0\rangle|\pi^0\rangle = |\tfrac{1}{2}\ -\tfrac{1}{2}\rangle|1\ 0\rangle = \sqrt{\tfrac{2}{3}}|\tfrac{3}{2}\ -\tfrac{1}{2}\rangle + \sqrt{\tfrac{1}{3}}|\tfrac{1}{2}\ -\tfrac{1}{2}\rangle$$

So $$\frac{\Gamma(K^{*0} \to K^+\pi^-)}{\Gamma(K^{*0} \to K^0\pi^0)} = \frac{\frac{2}{3}|p_{K^+}|^2}{\frac{1}{3}|p_{K^0}|^2} \approx 2$$

So if $\Gamma_{K^+\pi^-} + \Gamma_{K^0\pi^0} = \Gamma_{total}$ then

$$Br(K^{*0} \to K^+\pi^-) = \frac{2}{3}$$
$$Br(K^{*0} \to K^0\pi^0) = \frac{1}{3}$$

This is what is observed.

Gell-Mann - Nishijima formula:

$$Q = \frac{1}{2}(B+S) + I_3$$

Eg, proton has $I_3 = +\frac{1}{2}$

$B = +1$

$S = 0$

so $Q = +1$.

Λ has $B = 1$

$S = -1$

$I_3 = 0$

so $Q = 0$.

K^+ has $S = +1$

$B = 0$

$I_3 = +\frac{1}{2}$

so $Q = +1$.

Strangeness - conserved in strong interactions

Isospin - two components, strong interaction independent of their mixture.

Rigerous group theory : Gell-Mann $\rightarrow$ "Quarks"

Substructure : Zweig $\rightarrow$ "Aces"

1964.

Sakata had previously proposed that particles are bound states of fundamental baryons:

p, n, Λ

eg, $K^+ = \bar{\Lambda} p$ $\quad \pi^+ = p\bar{n}$, $\Sigma^- = \Lambda n \bar{p}$

This explained the quantum numbers but had other problems. (Why are p, n, Λ special? why are Σ's not regarded as being special?)

The quark model proposed the fundamental constituents as u, d, s.

$I_3(u) = +\frac{1}{2}$

$I_3(d) = -\frac{1}{2}$

$I_3(s) = 0$

u, d have strangeness 0,

s has strangeness -1.

Baryons are made out of 3 quarks.

Mesons are quark - antiquark pairs.

So quarks have baryon number $\frac{1}{3}$.

Gell-Mann Nishijima formula:

$$Q = \frac{1}{2}(B+S) + I_3$$

$$Q_u = \frac{1}{2}\left(\frac{1}{3} + 0\right) + \frac{1}{2}$$

$$= \frac{1}{2} + \frac{1}{6} = +\frac{2}{3}$$

$$Q_d = \frac{1}{2}\left(\frac{1}{3} + 0\right) - \frac{1}{2}$$

$$= -\frac{1}{3}$$

$$Q_s = \frac{1}{2}\left(\frac{1}{3} - 1\right) + 0$$

$$= -\frac{1}{3}$$

So this is strange ... fractional electric charges.

In this scheme we can write the quark content of the hadrons:

$p = (uud)$ $\quad$ $n = (udd)$

$\Lambda = (uds)$ $\quad$ $\Sigma^0 = (uds)$

$\Sigma^+ = (uus)$

$\Sigma^- = (dds)$

$\Xi^0 = (uss)$

$\Xi^- = (dss)$ $\quad$ $\Omega^- = (sss)$

$K^+ = (u\bar{s})$ $\quad$ $K^- = (\bar{u}s)$

$K^0 = (d\bar{s})$ $\quad$ $\bar{K}^0 = (\bar{d}s)$

$\pi^+ = (u\bar{d})$ $\quad$ $\pi^0 = \frac{1}{\sqrt{2}}(u\bar{u} + d\bar{d})$ $\quad$ $\pi^- = (\bar{u}d)$

$\eta = \frac{1}{\sqrt{2}}(u\bar{u} - d\bar{d})$